Assignment 5
for Discrete Structures II

5.1 Let $(A, \leq)$, $(C, \sqsubseteq)$ be posets. Prove that any continuous[1] function $f \colon A \to C$ is monotone. Hint: employ reasonable two element sets.

5.2 Find an example of a finite lattice $(A, \leq)$ and a monotone function $f \colon A \to A$ which is not continuous. Hint: a small number of elements is sufficient.

5.3 Give an example of a complete lattice $(A, \leq)$ and a monotone function $f \colon A \to A$ such that $f^\omega(\bot)$ is not a fixed point of f.

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[1] I.e., $f(\bigvee B) = \bigsqcup \{ f(x) \mid x \in B \}$ for any subset $B \subseteq A$.