Advancements of the OpenModelica Compiler toward a full implementation of event handling

Simulation hybrid systems

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2010-02-08

Outline

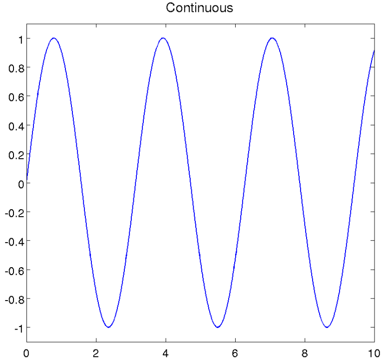
Hybrid models
Hybrid modeling

Hybrid

- Mixed systems with continuous and discrete components
- Modeling of discontinuous systems is a strength of Modelica
- Simulation needs handling with events and discontinuities

Applications

- Switched electric circuits
- Controlled systems
- PetriNets



Hybrid models
Hybrid modeling

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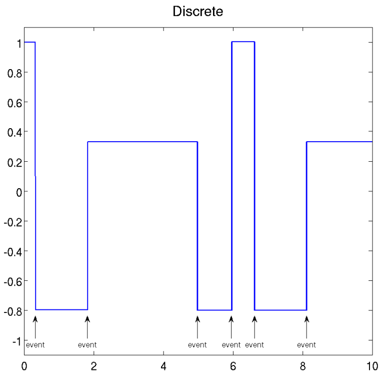

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Hybrid modeling

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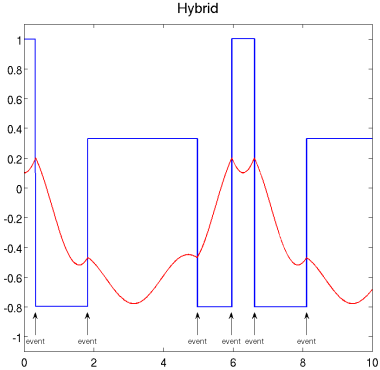
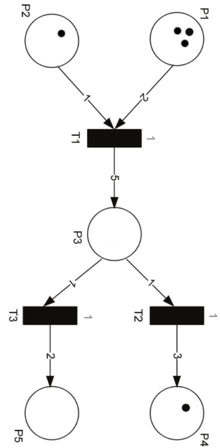


Hybrid models

Petri-Nets



Elements of Petri-Nets

- Fundamental items are places and transitions
- Directed edges connect items

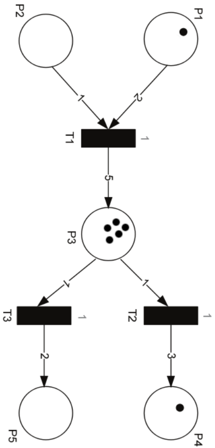
Modifications

- Differential equations describe firing speed for continuous behavior
- Edges may have weightings, threshold and inhibition
- Stochastic delay

$\Rightarrow$ Hybrid models

Hybrid models

Petri-Nets



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⇒ Hybrid models

Hybrid models
Petri-Nets in OpenModelica

Library in OpenModelica

- Continuous, discrete and stochastic places and transitions can be combined
- Combined PetriNets are versatile applicable
  For example: production or biological processes

Problems in OpenModelica

- "`when equations`" are not treated synchronously
- Some minor bugs in the use of arrays and functions

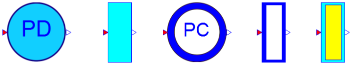


Figure: Elements of the PetriNet-Library

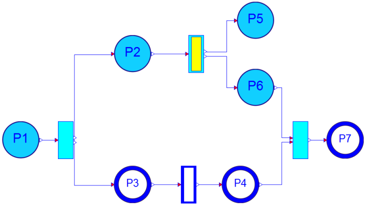
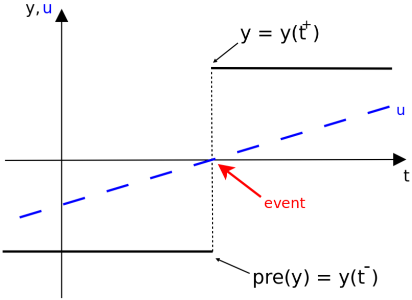


Figure: Example network

Hybrid models

Modelling events with Modelica

```
model example_if
  input Real u;
  Real y;
equation
  y = if u > 0 then 1 else -1;
end example_if;
```



- Conditional expressions like `u > 0` trigger events.
- If events occurs the value is stored twice.
- In this example `y` is the right limit and `pre(y)` is the left limit.

Hybrid models

Problems of simulate hybrid models

Why are there problems while integrating of discontinuous systems?

- Numerical integration calls for continuous differential equations
- Since all integrators approximate solutions with polynomials

Hybrid models

Problems of simulate hybrid models

Why are there problems while integrating of discontinuous systems?

- Numerical integration calls for continuous differential equations
- Since all integrators approximate solutions with polynomials

Solution

1. Numerical integration stops at an event
2. Make all discontinues changes
3. Numerical integration starts again

Implementation
Hybrid Modelica DAE-System

Flatten Modelica model:

$$0 = F(\underline{\dot{x}(t)}, \underline{x(t)}, \underline{y(t)}, \underline{u(t)}, \underline{p}, \underline{q(t_e)}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t)$$

↓ matching and sorting algorithm transform to

$$\underline{z} = \begin{pmatrix} \underline{\dot{x}(t)} \\ \underline{y(t)} \\ \underline{q(t_e)} \end{pmatrix} = \begin{pmatrix} \underline{f}(\underline{x(t)}, \underline{u(t)}, \underline{p}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t) \\ \underline{g}(\underline{x(t)}, \underline{u(t)}, \underline{p}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t) \\ \underline{h}(\underline{x(t)}, \underline{u(t)}, \underline{p}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t) \end{pmatrix}$$

Hybrid models

Synchronous equations

when example

```
model when_example
 Real x(start=0.1),y;
 discrete Real a(start=1);
equation
 y = sin(time*2);
 der(x) = a*y;
 when {y<-0.5,x>0.2} then
  a = x-pre(a);
 end when;
end when_example;
```

Hybrid models
Synchronous equations

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```

Incidence-Matrix

$$
\begin{array}{c} y = sin(time) \\ a = x - pre(a) \\ der(x) = a * y \end{array}
\qquad
\begin{array}{ccc} y & a & der(x) \end{array}
\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}
$$

Hybrid models
Synchronous equations

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Incidence-Matrix

$$\begin{array}{c} y = sin(time) \\ a = x - pre(a) \\ der(x) = a * y \end{array} \qquad \begin{matrix} y & a & der(x) \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Sorting equations

Sorting is based on all equations in due to the correct order of evaluation at all time points.

Implementation
Hybrid Modelica DAE-System

Flatten Modelica model:

$$0 = F(\underline{\dot{x}(t)}, \underline{x(t)}, \underline{y(t)}, \underline{u(t)}, \underline{p}, \underline{q(t_e)}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t)$$

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We get four blocks for code generation:

continuous $\underline{f}$ and $\underline{g}$ → derivative states and algebraic variables

discrete $\underline{h}$ → discrete algebraic variables

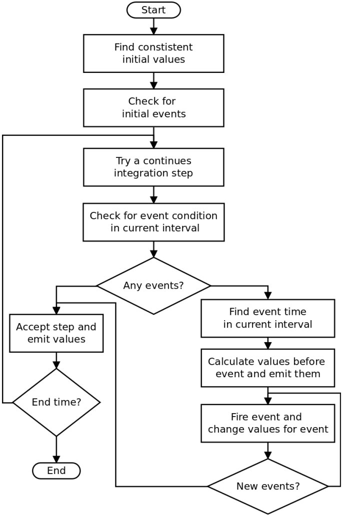
all $\underline{z}$ → all blocks together, in the right order

An additional block to manage the conditions for events:

$\underline{c(t)}$ → Zero Crossing functions

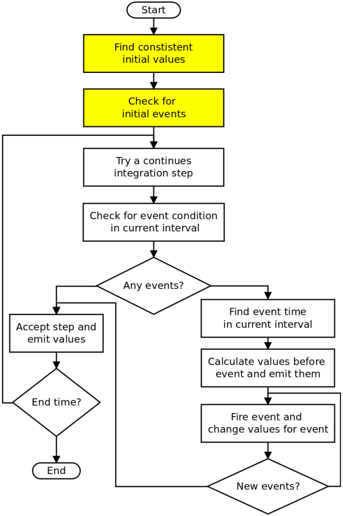
Implementation

Approach to simulate hybrid models



Implementation

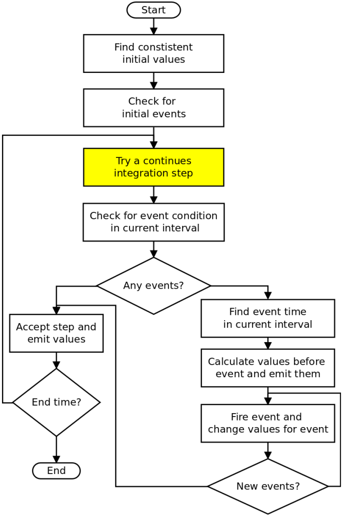
Approach to simulate hybrid models



Initial Step

- Initial-value problem is solved by a simplex-method
- Initial Zero-crossing functions and check for initial events

Implementation
Continuous integration



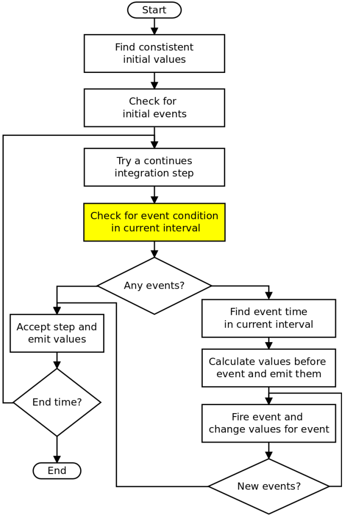
Integration step

- Integration method: $x_{i+1} = \Phi(x_i)$
- Calculate for the next step t_{i+1} the new state vector $\underline{x(t_{i+1})}$
- Evaluate continuous blocks $\underline{f}$ and $\underline{g}$

$$\underline{\dot{x}(t_{i+1})} = \underline{f}(\underline{x(t_{i+1})}, \underline{q(t_i)}, \underline{q_{pre}(t_i)}, \underline{c(t_{i+1})}, t)$$

$$\underline{y(t_{i+1})} = \underline{g}(\underline{x(t_{i+1})}, \underline{q(t_i)}, \underline{q_{pre}(t_i)}, \underline{c(t_{i+1})}, t)$$

Implementation
Check for event conditions

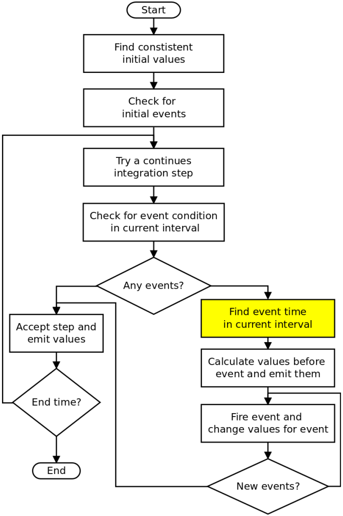


Check for zero-crossing

- Conditions are converted into zero-crossing functions
- $x < 2$ changes from `false` to `true` when $x - 2$ crosses zero
- If any zero-crossing becomes `true` an event is fired

Implementation

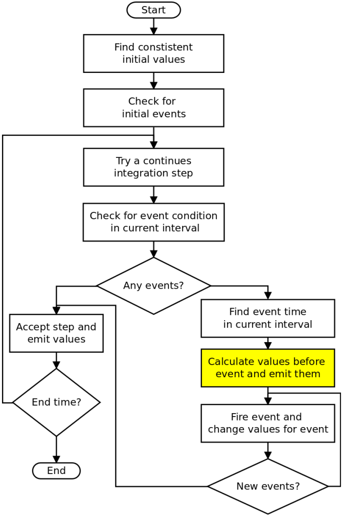
Find event time



Root-finding method

- Find root in interval $[t_i; t_{i+1}]$ as event time t_e
- Bisection is a very simple and robust method, but it is also relatively slow
- All methods approximate the root by setting limits on each side of t_e
- Additional we have $t_e - \epsilon$ and $t_e + \epsilon$

Implementation
Handle event



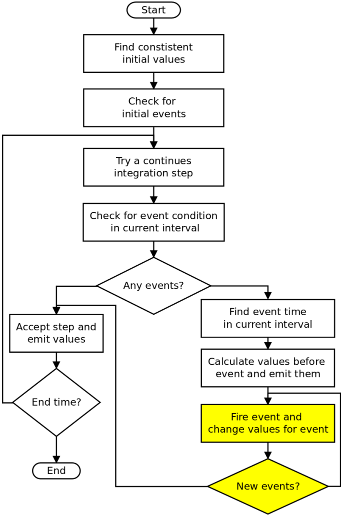
Handle event

1. Determine states at $t_e - \epsilon$ with interpolation
2. Determine continuous blocks by using functions $\underline{f}$ and $\underline{g}$
3. Save all variables as values for `pre()` and emit them to result file

$$\underline{\dot{x}(t_e - \epsilon)} = \underline{f}(\underline{x(t_e - \epsilon)}, \underline{q(t_i)}, \underline{q_{pre}(t_i)}, \underline{c(t_e - \epsilon)}, t)$$

$$\underline{y(t_e - \epsilon)} = \underline{g}(\underline{x(t_e - \epsilon)}, \underline{q(t_i)}, \underline{q_{pre}(t_i)}, \underline{c(t_e - \epsilon)}, t)$$

Implementation
Handle event



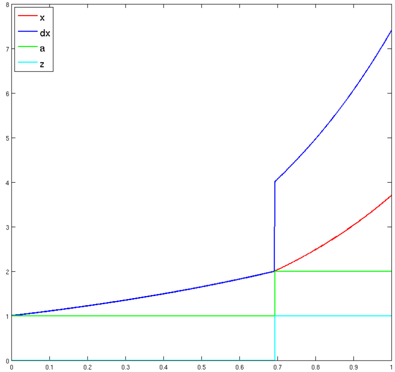
Handle event

1. Determine states at $t_e + \epsilon$ with interpolation
2. Evaluate all blocks by using function $\underline{z}$
3. Check for changes of discrete variables
4. Event Iteration

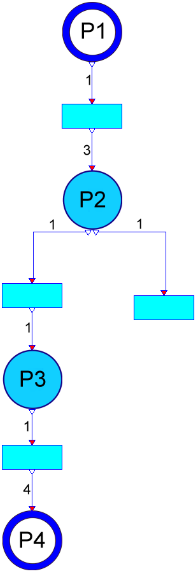
$$\underline{z} = \begin{matrix} \underline{\dot{x}(t_e+\epsilon)} \\ \underline{y(t_e+\epsilon)} \\ \underline{q(t_e+\epsilon)} \end{matrix} = \begin{matrix} \underline{f}(\underline{x(t_e+\epsilon)}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t) \\ \underline{g}(\underline{x(t_e+\epsilon)}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t) \\ \underline{h}(\underline{x(t_e+\epsilon)}, \underline{q_{pre}(t_e)}, \underline{c(t_e)}, t) \end{matrix}$$

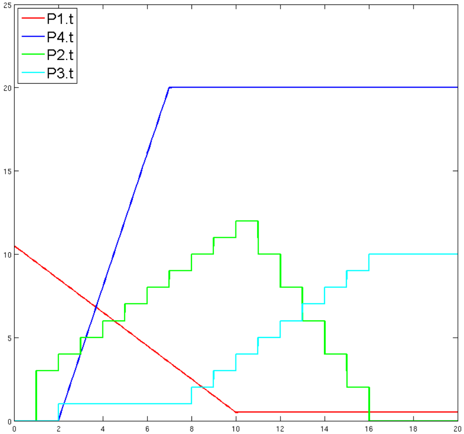
Examples
Eventiteration

```
model EventIteration
  Real x(start=1),dx;
  discrete Real a(start=1);
  Boolean y(start=false);
  Boolean z(start=false);
  Boolean h1,h2;
equation
  der(x) = dx;
  dx = a * x;
  h1 = x>=2; h2 = dx>=4;
  when h1 then
    y = true;
  end when;
  when y then
    a = 2.0;
  end when;
  when h2 then
    z = true;
  end when;
end EventIteration;
```



Examples
Petri-Net Example





Summary

- With this approach we can manage many synchronously appearing events.
- We are on the way to an optimal formulation of the PetriNet library in OpenModelica

- Work that remains to be done
  - Integrate further integrations methods and root-finding methods.
  - Adjust the code generation.