Revenue-maximizing and Truthful Online Auctions for Dynamic Spectrum Access

Ajay Gopinathan, Google
Niklas Carlsson, Linköping University
Zongpeng Li, University of Calgary
Chuan Wu, Hong Kong University

LINKÖPING UNIVERSITY

Motivation

- Spectrum scarcity has led to a vibrant secondary spectrum market
  - Primary users lease spectrum to secondary users
  - Lease on temporary basis

- Unique spatial and temporal characteristics
  - Co-located users may suffer from interference
  - Usage frequency and duration vary among users

- How can primary users maximize their revenue?
  - Auctions!

Why auctions?

- Auctions known as an efficient mechanism for maximizing economic welfare
  - Market determines the best price for leasing spectrum
  - Supply and demand variations are taken into account
- Welfare maximizing
  - Secondary users with higher valuations receive the spectrum ahead of users with lower valuations
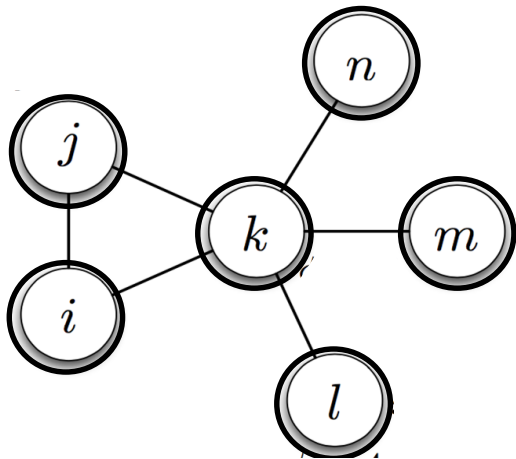
Challenges in secondary spectrum market

- Secondary user valuations for spectrum is private information
  - How do we truthfully elicit bids for spectrum?
- Timing at which spectrum is required is private information
  - Auction algorithm must work without knowledge of future bids!
- Spectrum allocation must be interference-free
  - Since allocation is NP-hard, approximation schemes must ensure secondary users cannot 'game' the auction.
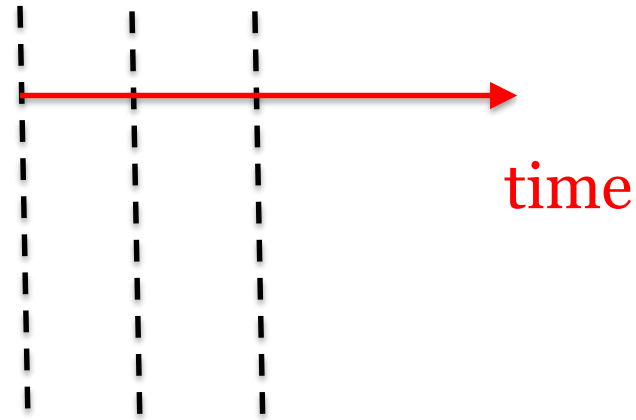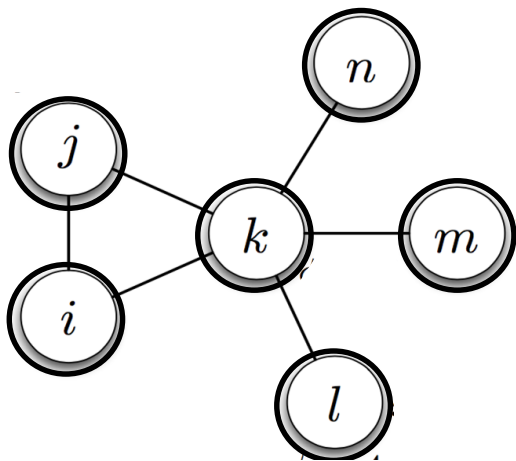
Our contributions

- Auction mechanism that is truthful in the online setting and interference-free

- Guarantees ⅕ fraction of the optimal revenue when spectrum assignment itself is optimal online algorithm

- An approximation algorithm that maintains truthful behavior that is also constant competitive with respect to the optimal online algorithm
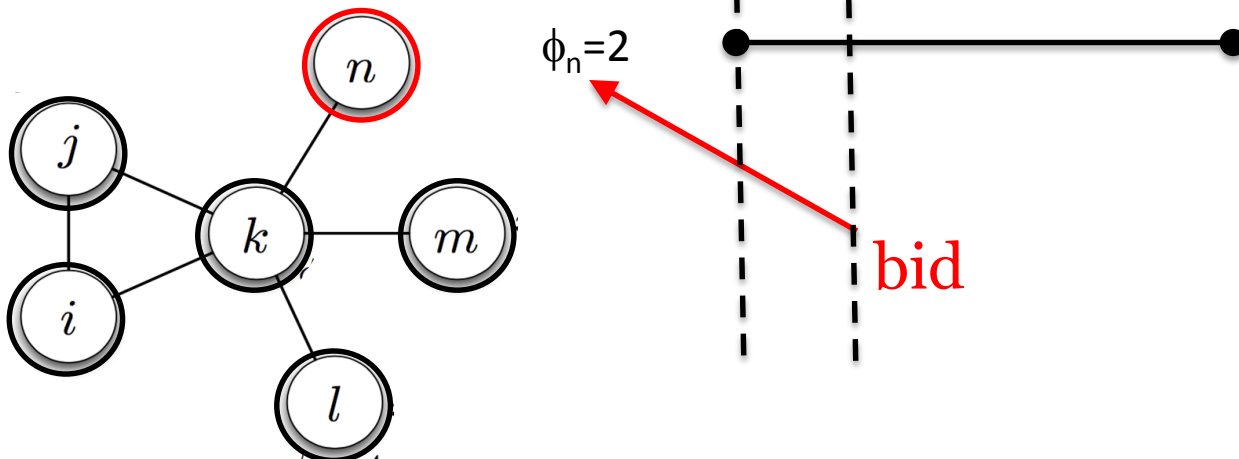
Online spectrum auction - model

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- Auctions run periodically
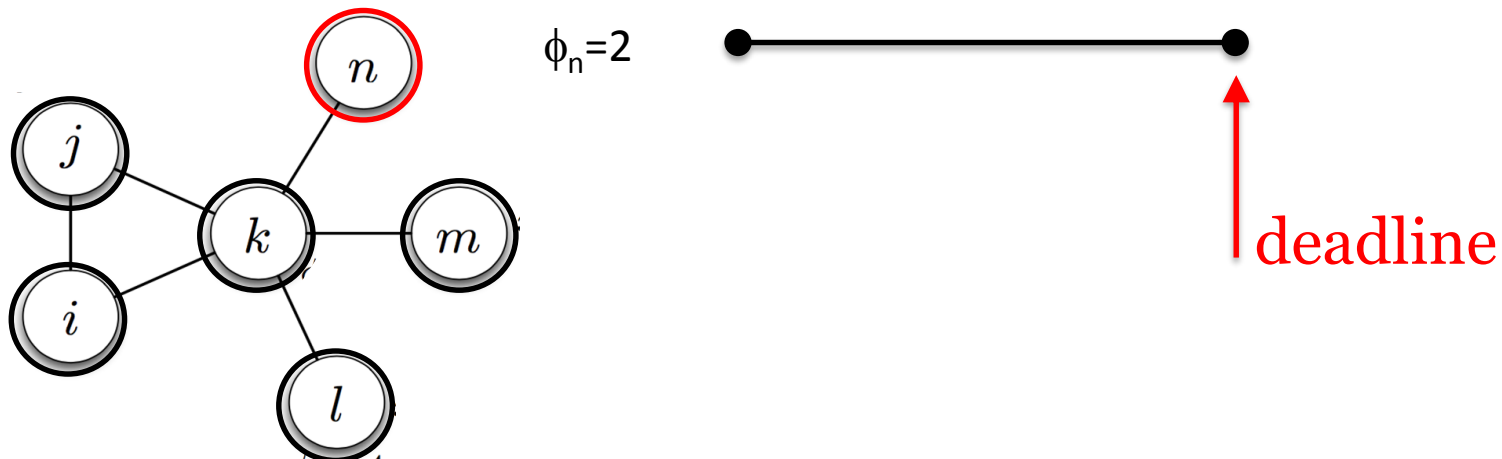

time

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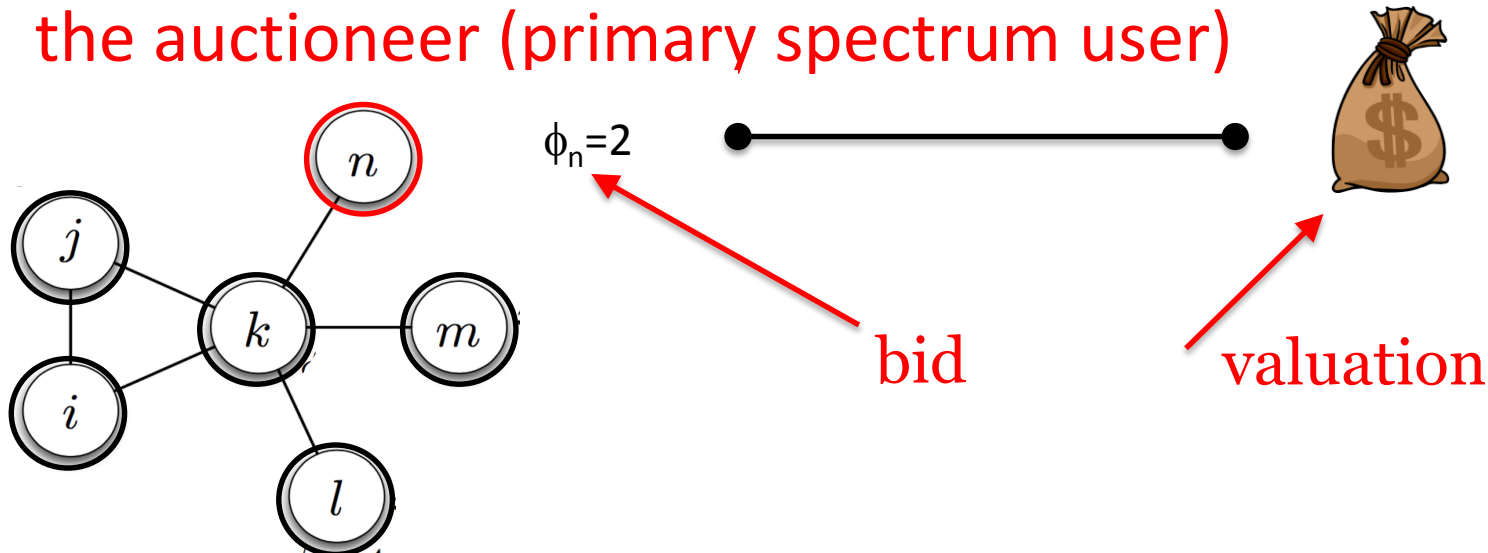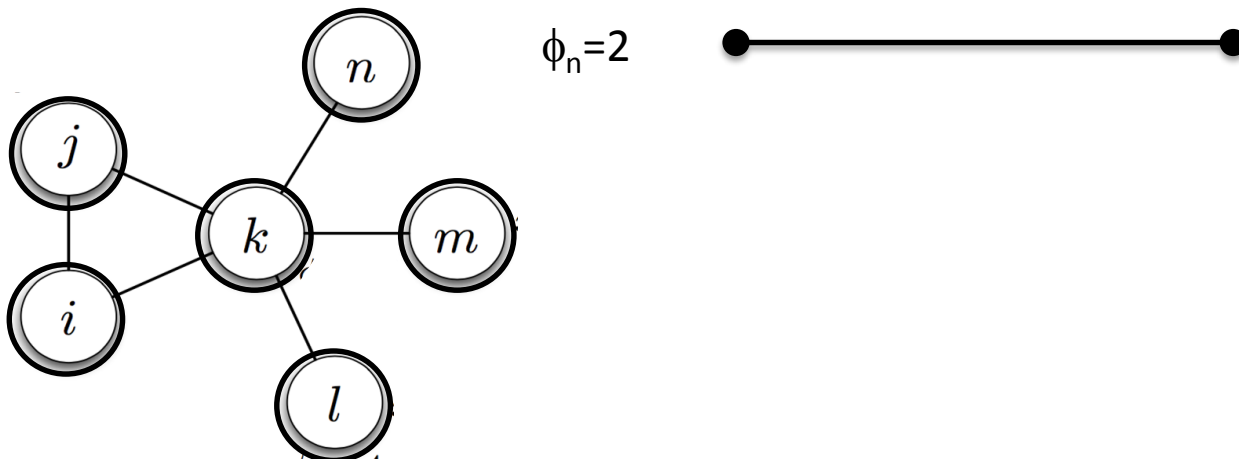- Users submit bids at the start of each timeslot

$\phi_n = 2$

bid

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- Users have deadlines by which they must be allocated the spectrum
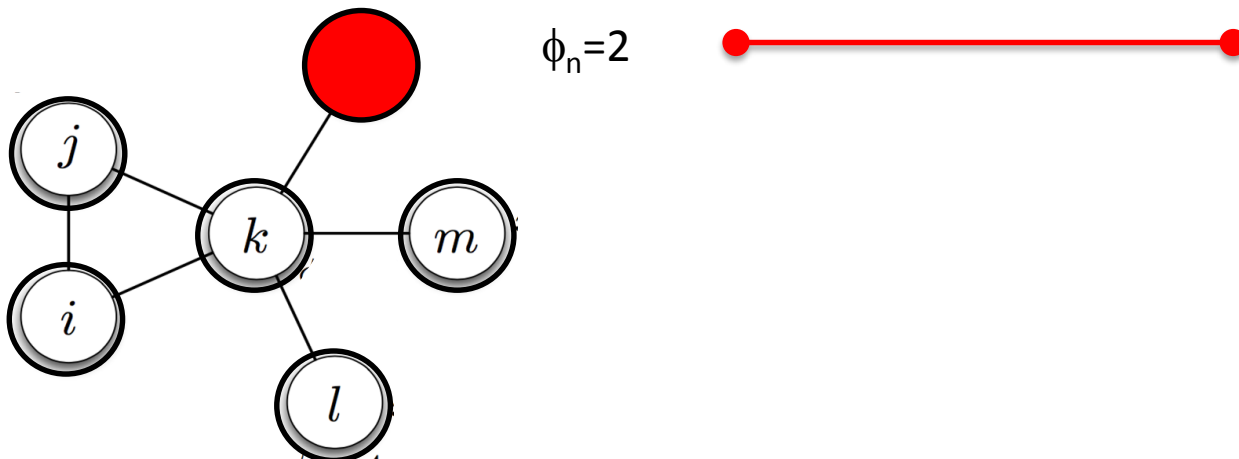
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deadline

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- The true valuation and deadline of users is unknown to the auctioneer (primary spectrum user)
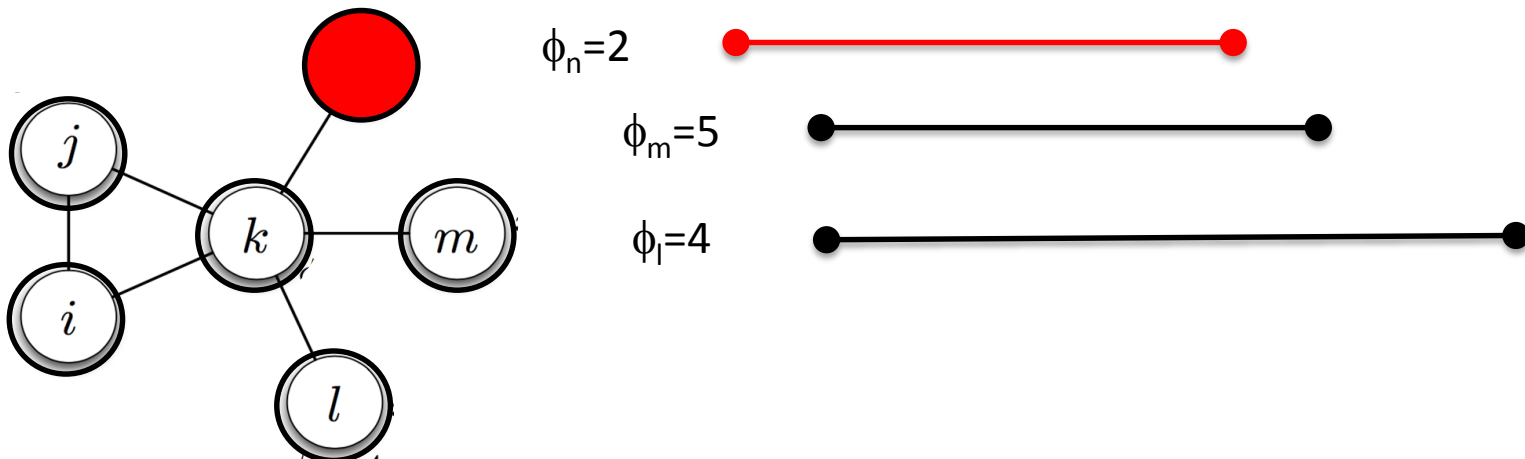
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valuation

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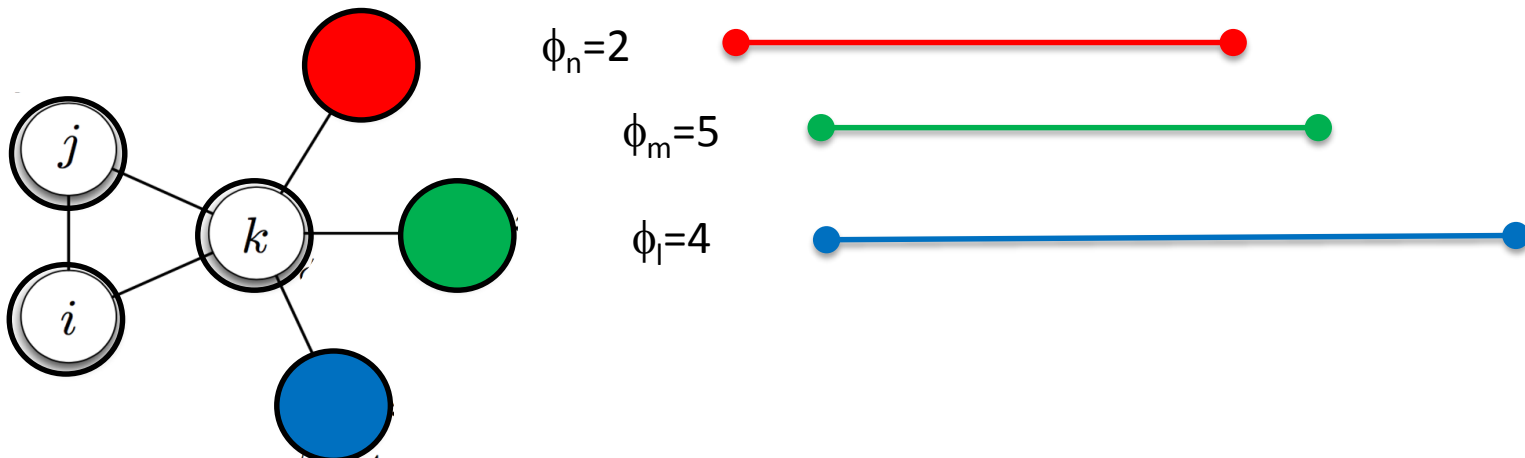
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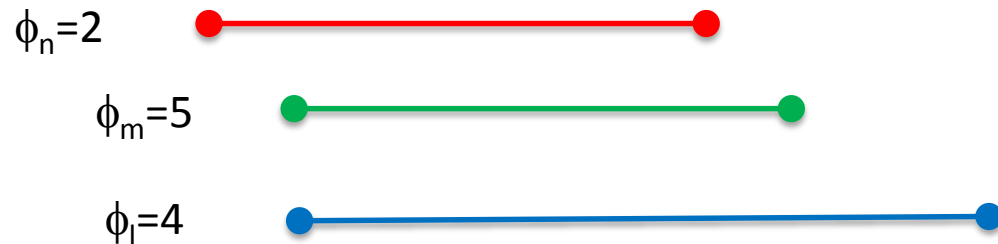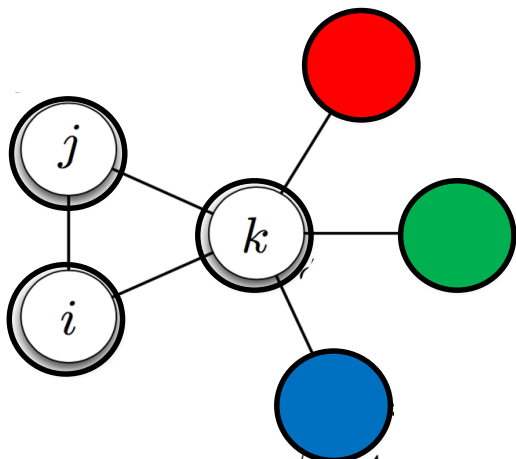

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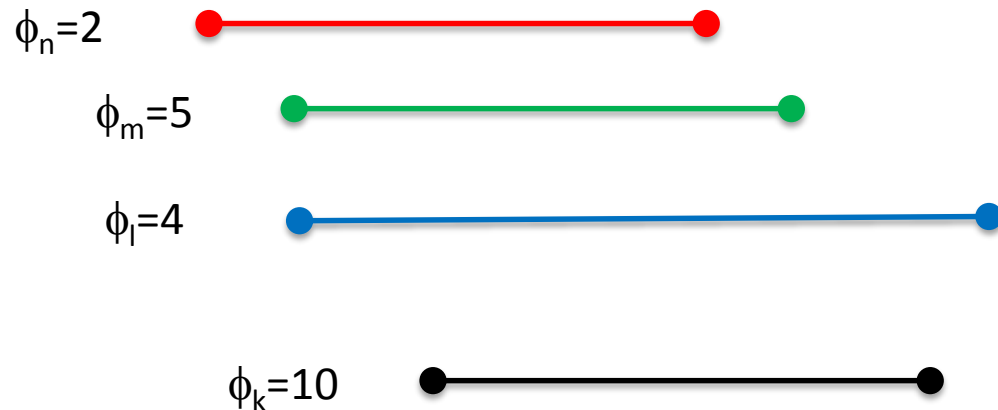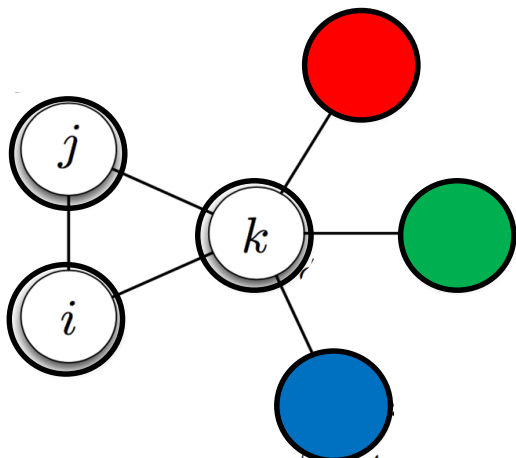
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Maximizing revenue

- Goal is to maximize revenue
- Problem: future, unknown bids may arrive with higher valuation
  - Should we assign spectrum now, or wait for higher bids?
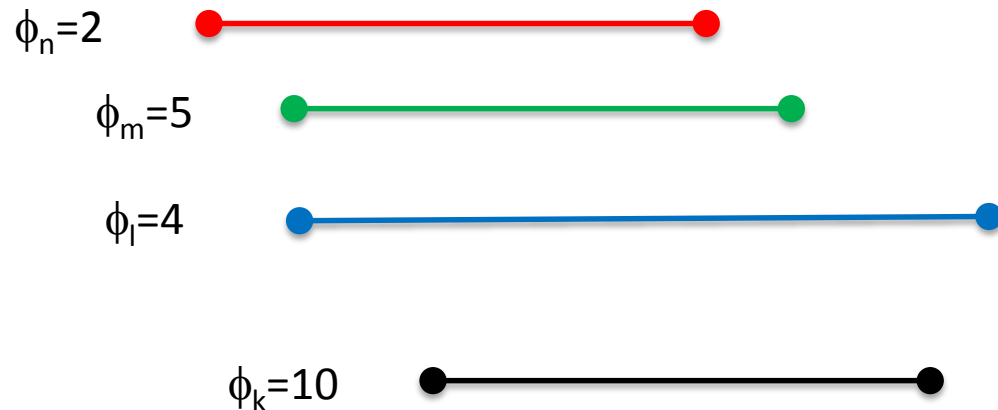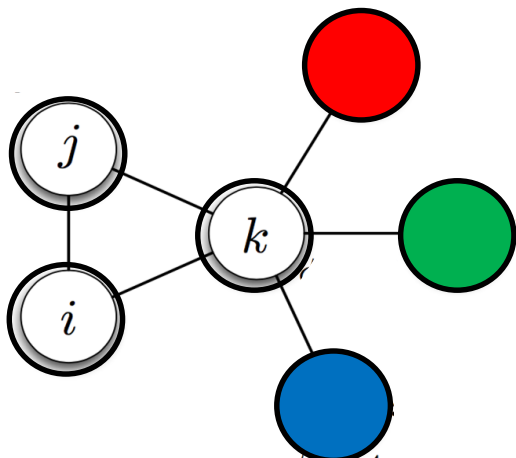
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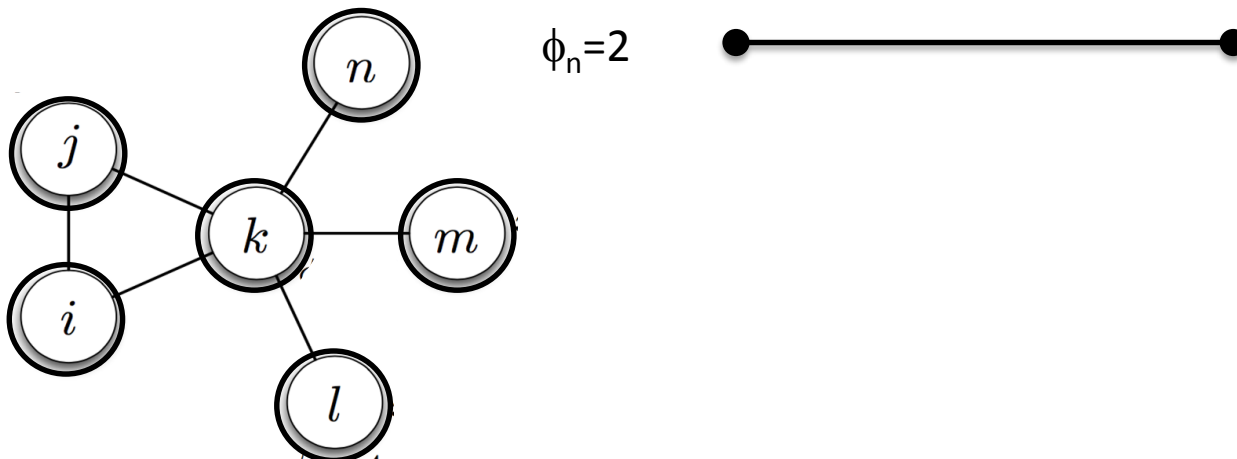

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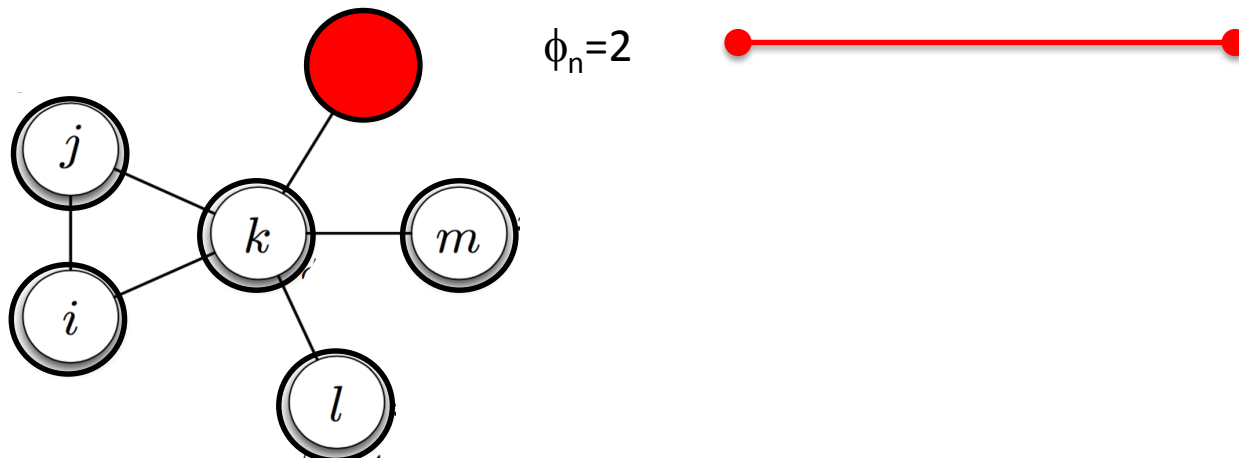
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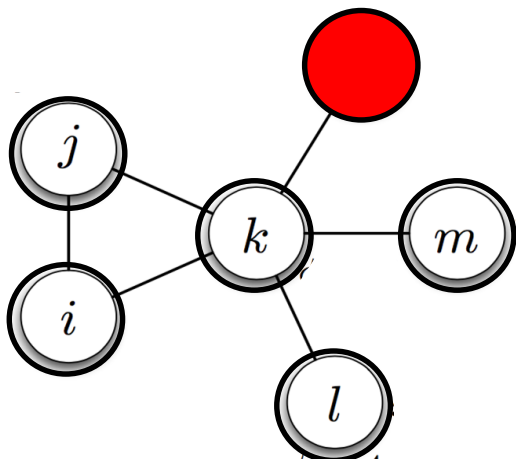
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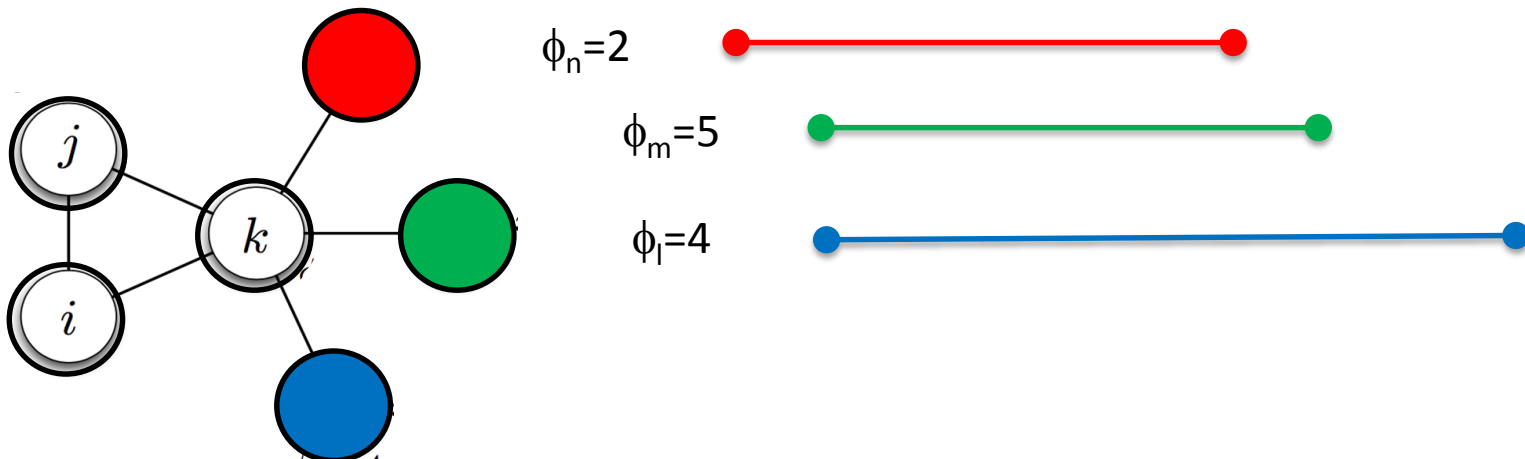 - Pre-empted users are not charged for their usage
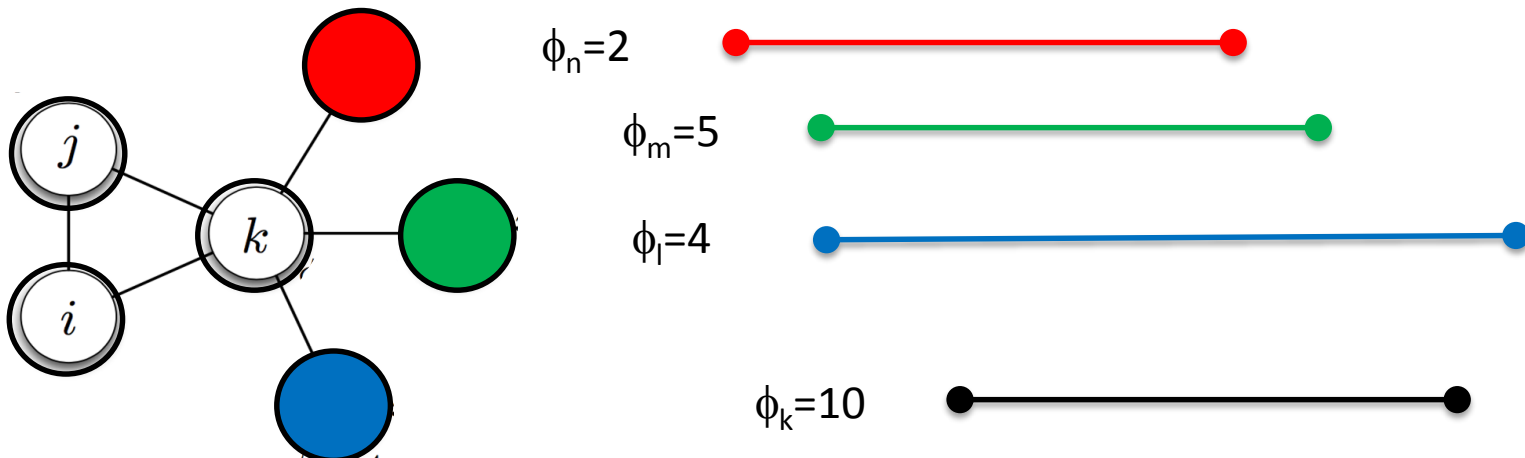
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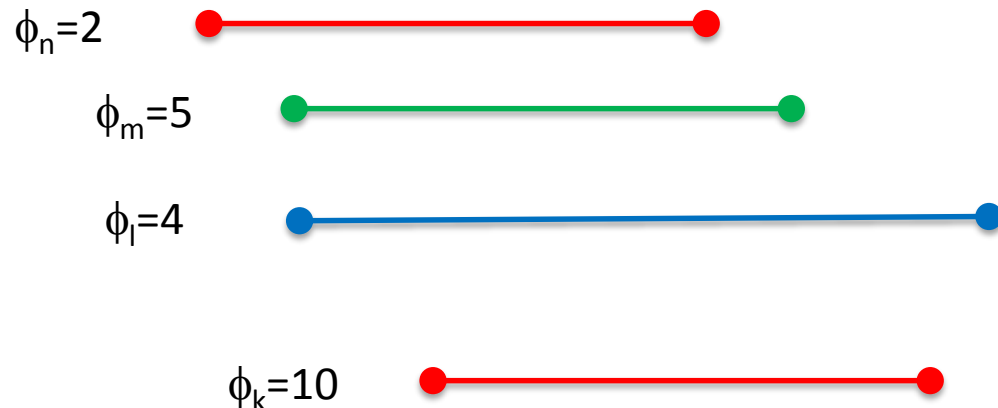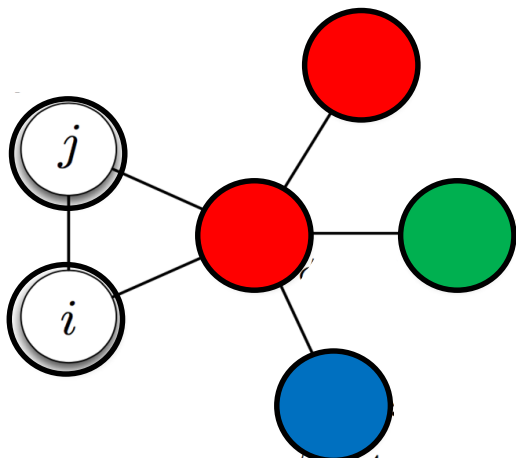
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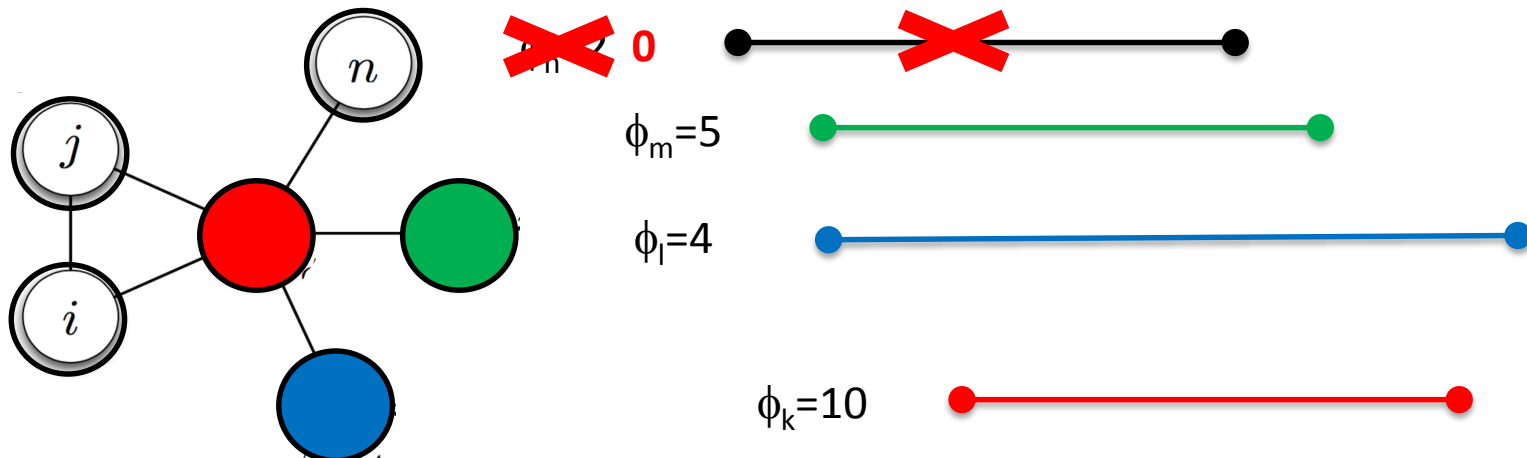$\phi_l = 4$

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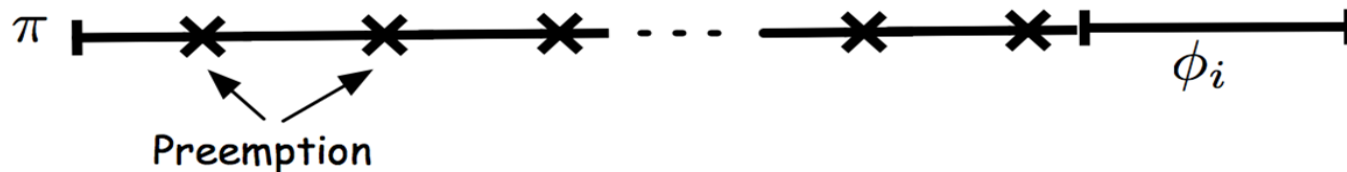$\phi_m = 5$

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Cost of preemption – worst case!

- Must avoid continuous pre-emption, as this can lead to zero revenue!
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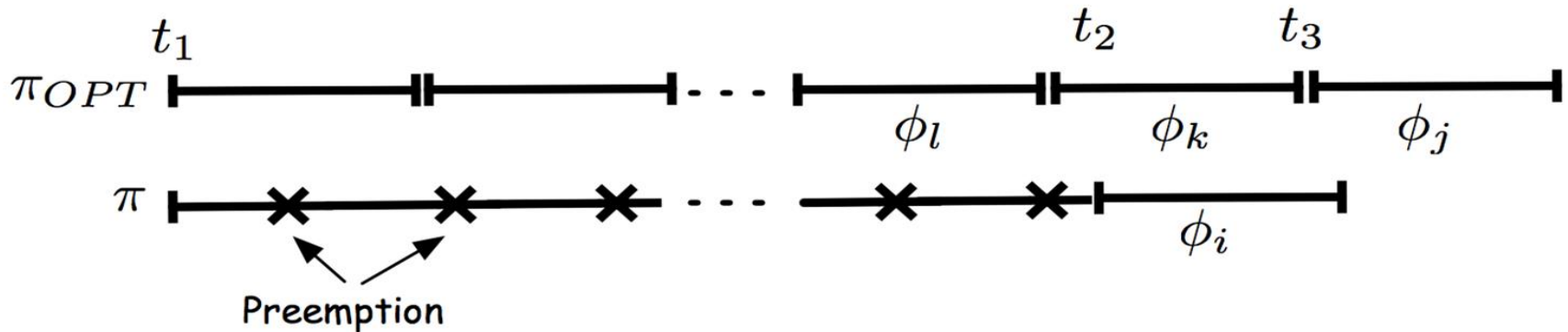
- Must avoid continuous pre-emption, as this can lead to zero revenue!
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- Inflate user bid as a function of time for which user has already used channel!
  - We show that this leads to revenue that is at least ⅕ fraction of the optimal (offline) solution

Auctions with optimal channel allocation

- Determine allocation using an integer linear program

$$\text{Maximize} \quad \sum_{i \in \mathcal{M}} \phi_i(v_i, \tau_i^k) x_i^k(t)$$

Subject To:

$$\sum_{k \in setK} x_i^k(t) \leq 1 \qquad \qquad \forall i \in \mathcal{M}$$
$$\sum_{j \in setN_i} x_j^k(t) + x_i^k \leq 1 \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{M}$$

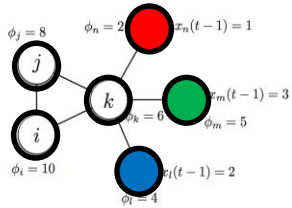$$x_i^k \in \{0, 1\} \quad \forall i \in \mathcal{M}, \forall k \in \mathcal{K}$$

- Determine payment using a combination of VCG mechanism, Myerson's virtual valuation, and artificial bid inflation
  - We prove that this leads to a truthful, 5-competitive auction with respect to optimal revenue

Greedy channel allocation

$\phi_j = 8$

$\phi_n = 2$ $x_n(t-1) = 1$

j

k $x_m(t-1) = 3$

$\phi_k = 6$ $\phi_m = 5$

i

$\phi_i = 10$

$x_l(t-1) = 2$

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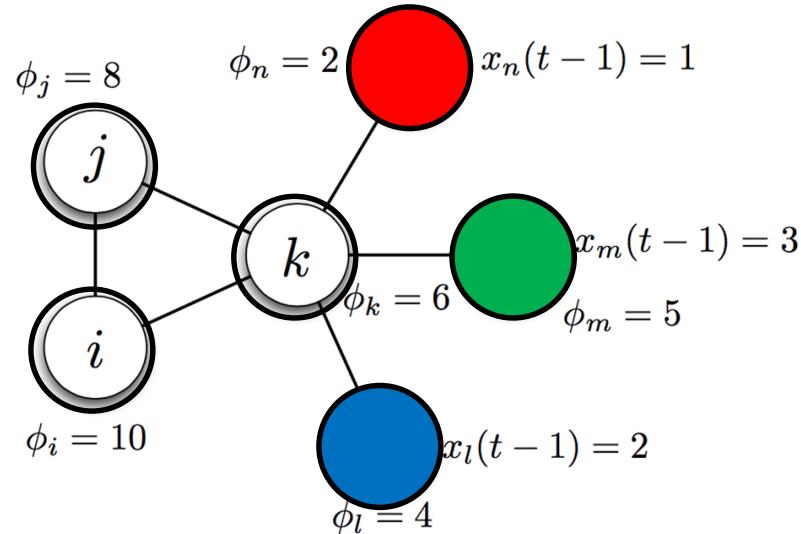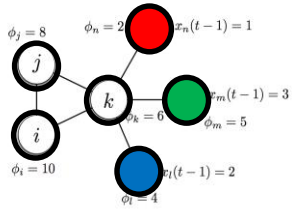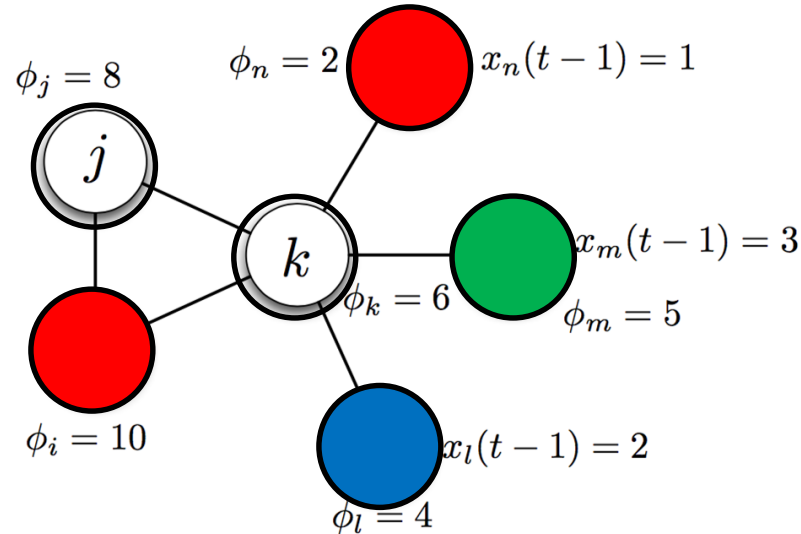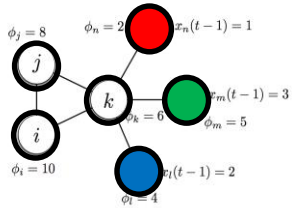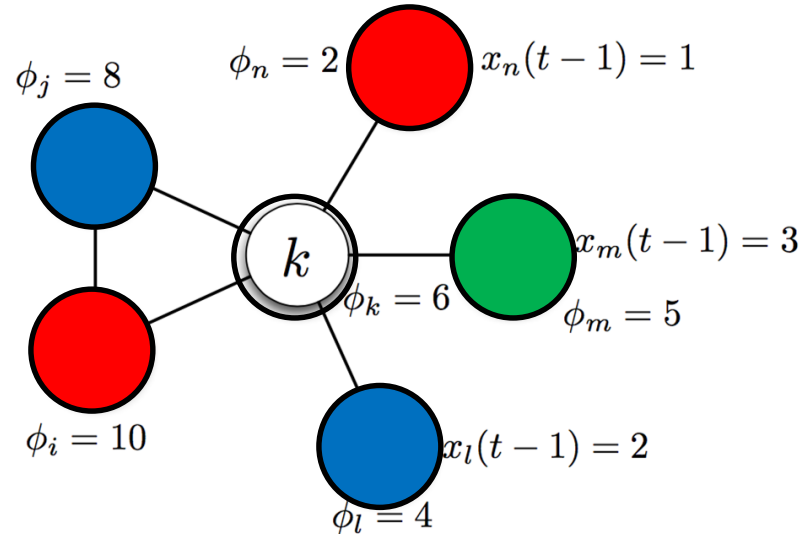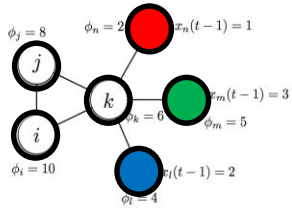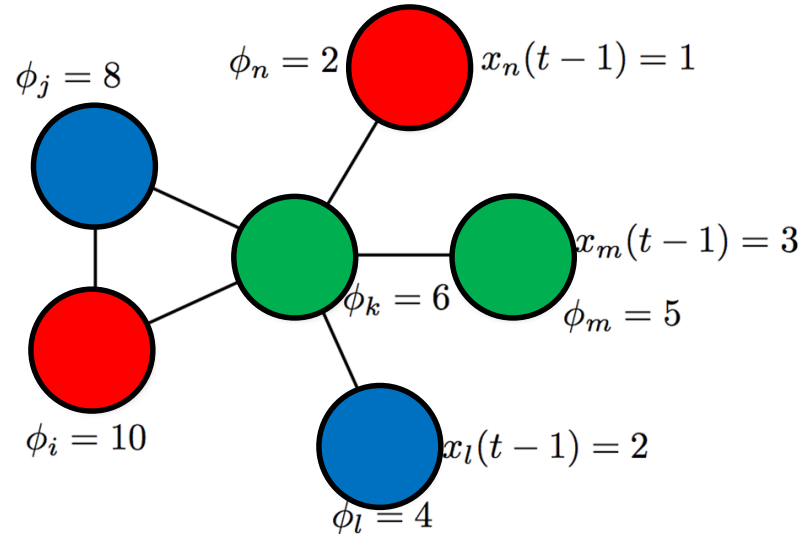- Naive greedy assignment leads to unnecessary preemption and lost revenue

Greedy channel allocation

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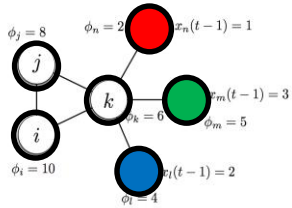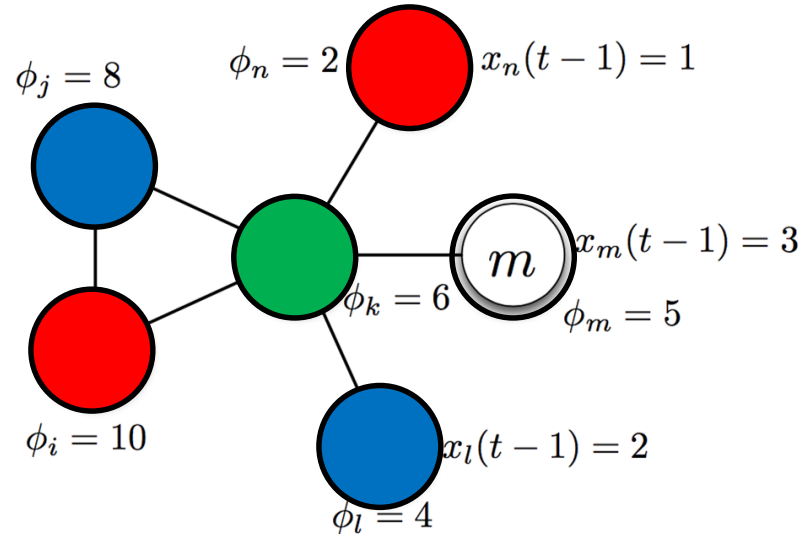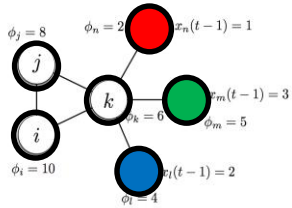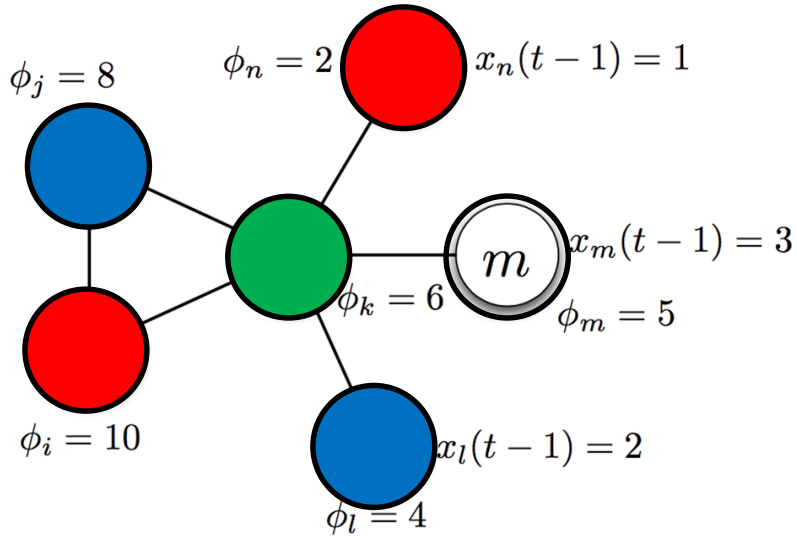- Naive greedy assignment leads to unnecessary preemption and lost revenue

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Greedy channel allocation

$$\sum \phi = 30$$

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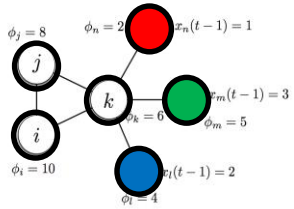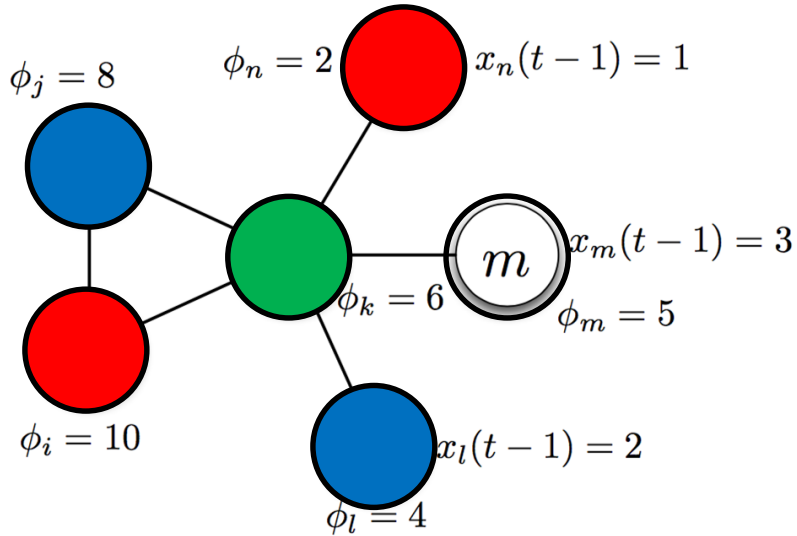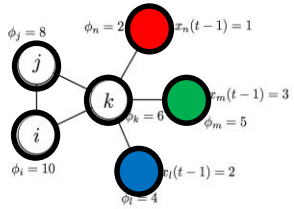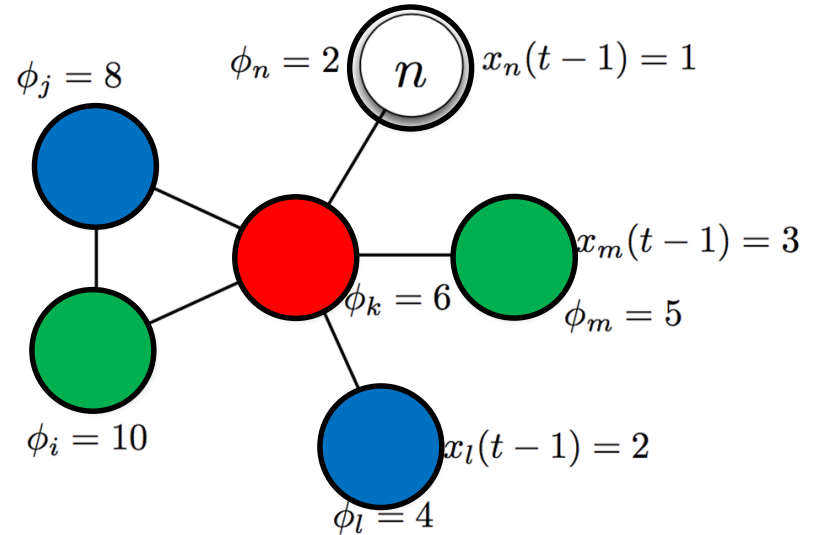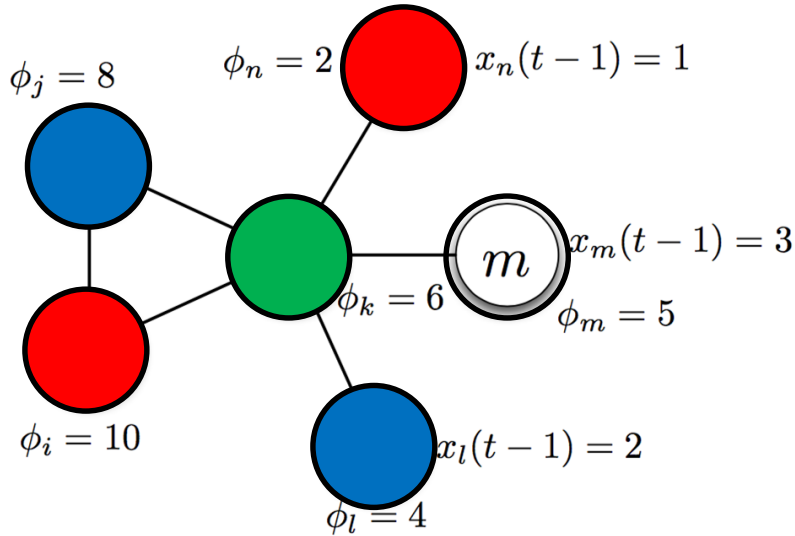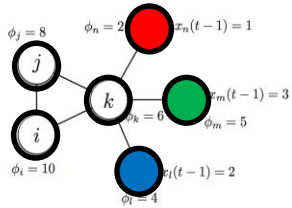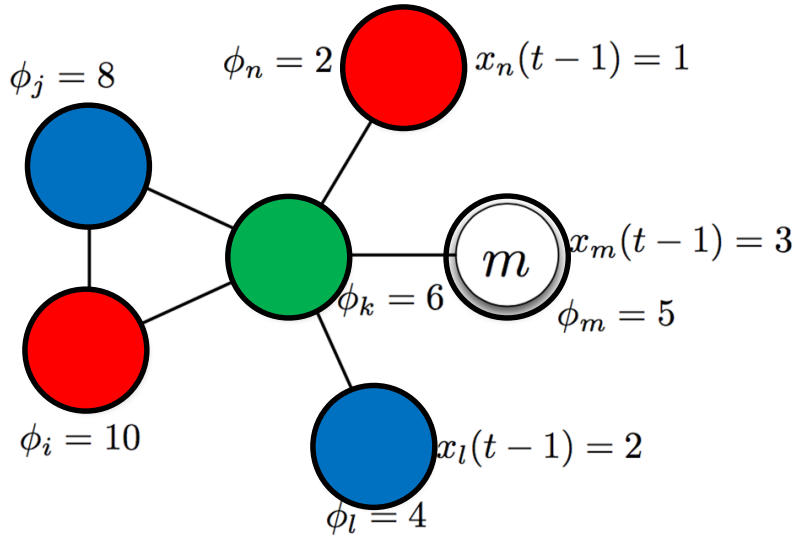
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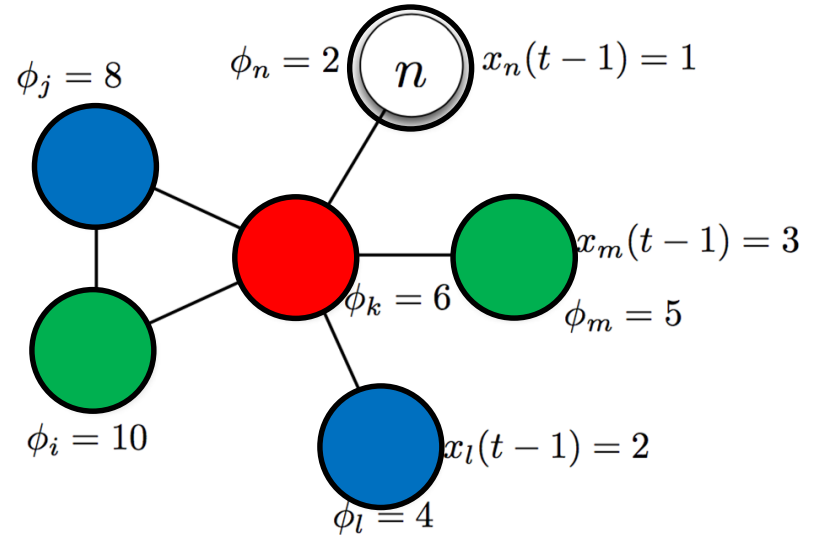
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Greedy channel allocation

$$\Sigma\,\phi = 30 \qquad\qquad \Sigma\,\phi = 33$$

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Greedy channel allocation

- Solution: we design a special channel ranking algorithm, that takes into account its previous allocation to avoid unnecessary preemption

- We prove that our allocation is monotone in bids, which is a crucial and necessary property to maintain a truthful auction

Conclusions

- Secondary spectrum auctions exhibit both temporal and spatial characteristics that are unique

- We designed an online auction that guarantees at least ⅕-fraction of the optimal (offline) revenue
  - Use bid inflation to ensure revenue is maximized

- We designed a greedy algorithm that maintains truthful behavior and is still constant competitive with respect to the optimal solution

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Questions?

Ajay Gopinathan, **Niklas Carlsson**, Zongpeng Li, Chuan Wu

Paper can be downloaded here:
http://www.ida.liu.se/~nikca/papers/wons16b.html

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