



Dynamic Content Allocation for Cloud-assisted Service of Periodic Workloads

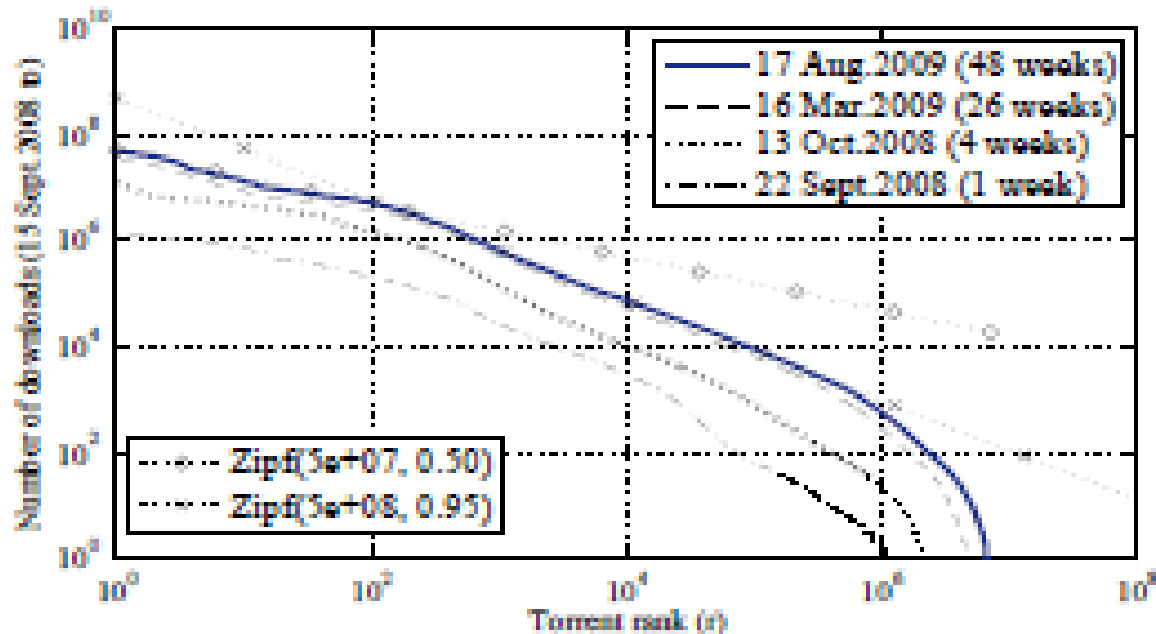
György Dán

Royal Institute of Technology (KTH)

Niklas Carlsson

Linköping University

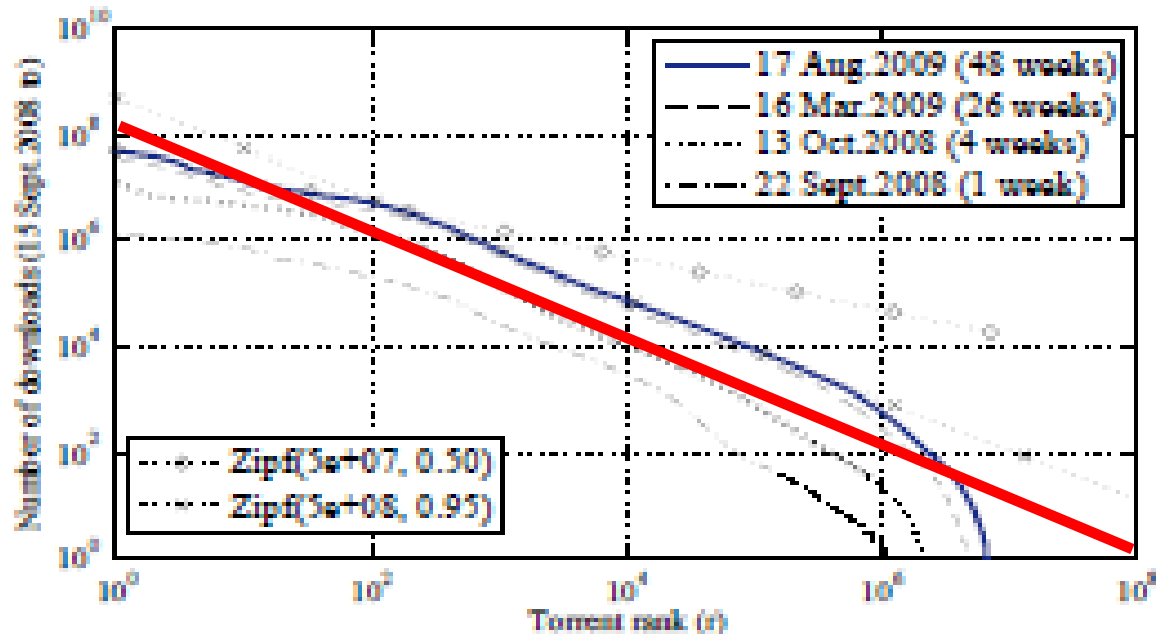
Internet Content Delivery



From: Dan and Carlsson, "Power-laws Revisited: A Large Scale Measurement Study of Peer-to-Peer Content Popularity", Proc. IPTPS 2010.

- Large amounts of data with varying popularity
- Multi-billion market (\$8B to \$20B, 2012-2015)
 - Goal: Minimize content delivery costs
- Migration to cloud data centers

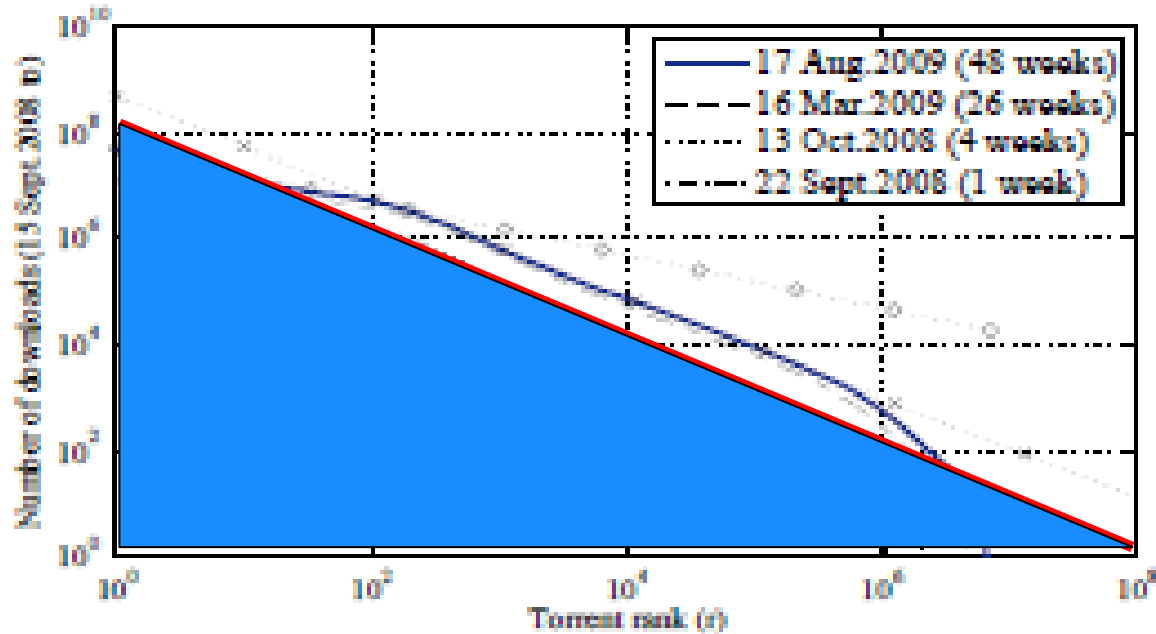
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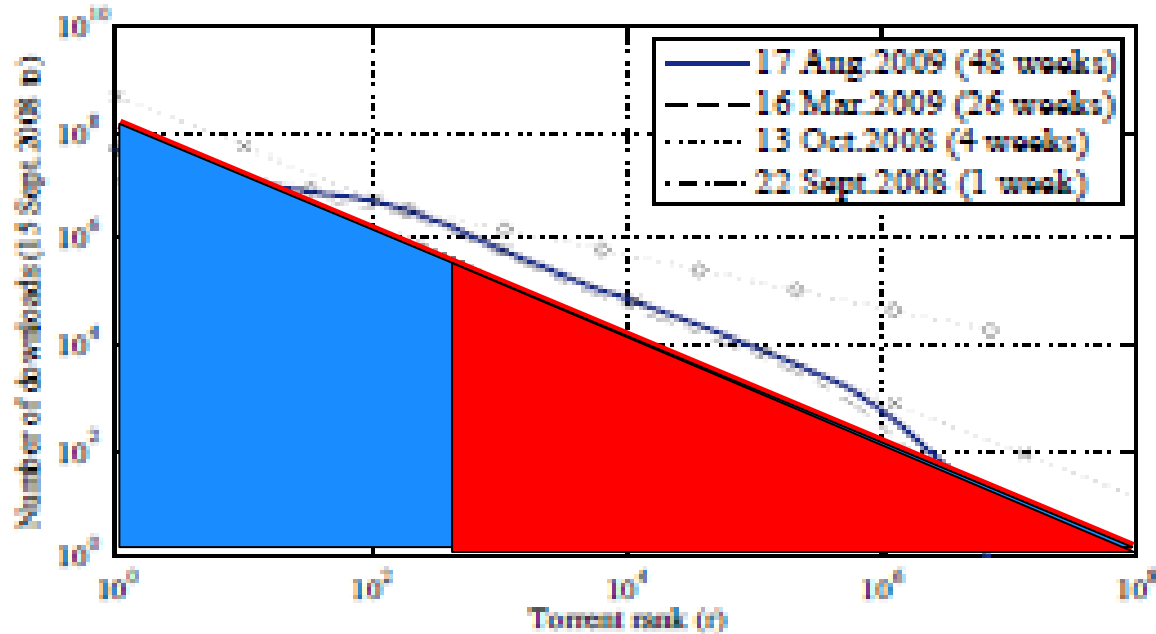
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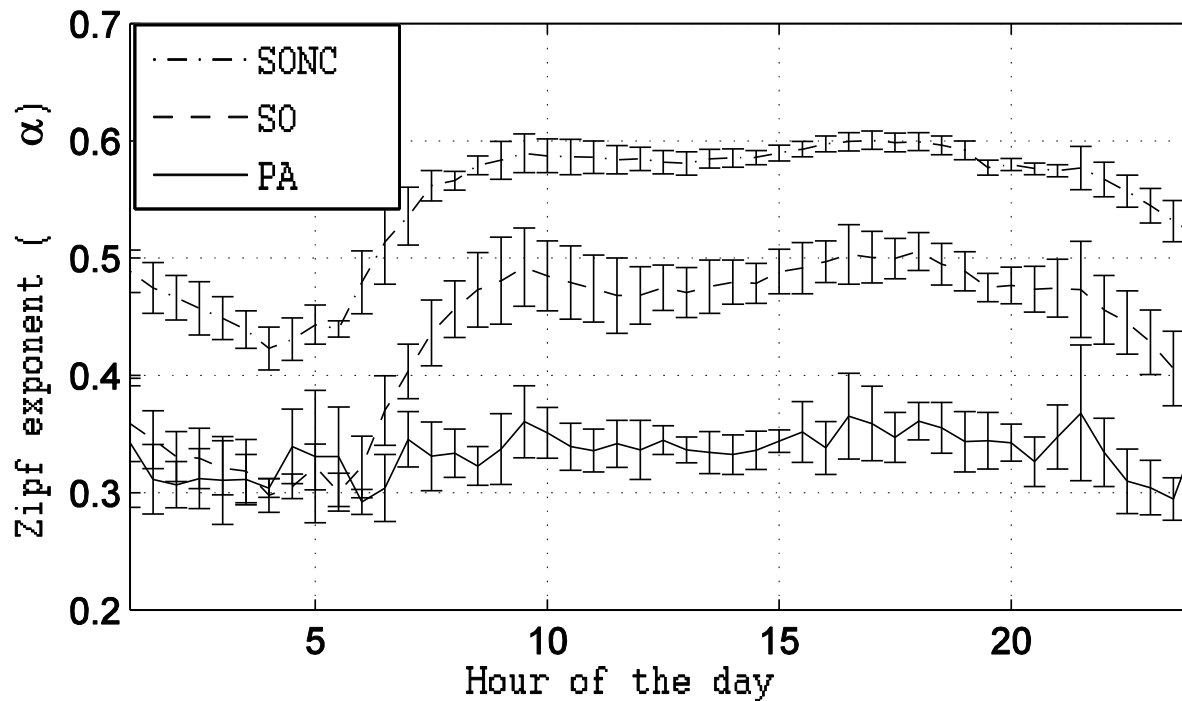


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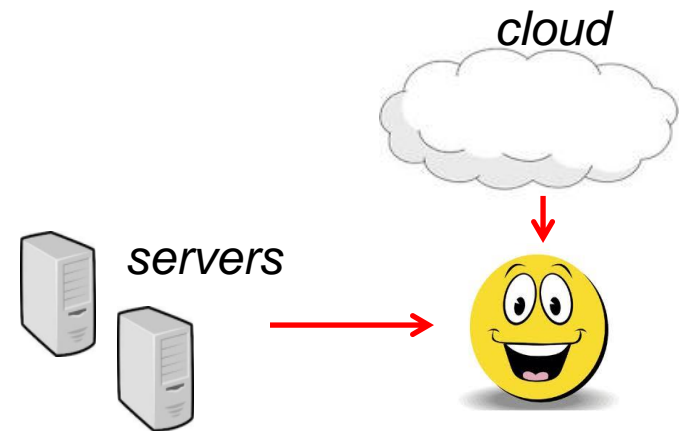
Periodic Workloads

- Characterization of Spotify traces
- In addition to diurnal traffic volumes ...
- ... we found that also the Zipf exponent vary with time-of-day



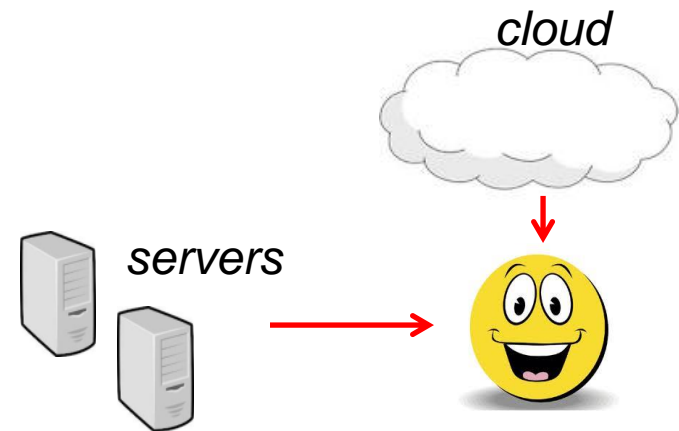
Content Delivery

- Cloud-based delivery
- Dedicated infrastructure



Content Delivery

- Cloud-based delivery
 - Flexible computation, storage, and bandwidth
 - Pay per volume and access
- Dedicated infrastructure
 - Limited storage
 - Capped unmetered bandwidth
 - Potentially closer to the user



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**Cloud bandwidth elastic;
however, flexible comes
at premium ...**



High-level problem

- Minimize content delivery costs

	Bandwidth	Cost
Cloud-based	Elastic/flexible	\$\$\$
Dedicated servers	Capped	\$

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How to get the best of two worlds?



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 - Improved workload models and prediction enables prefetching ...
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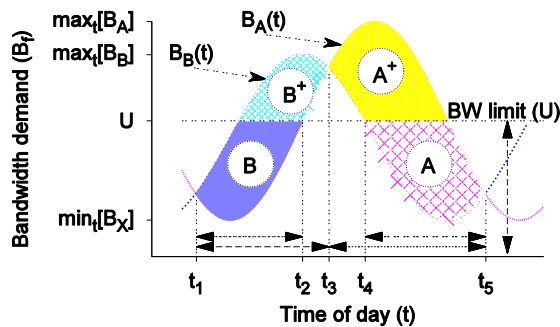
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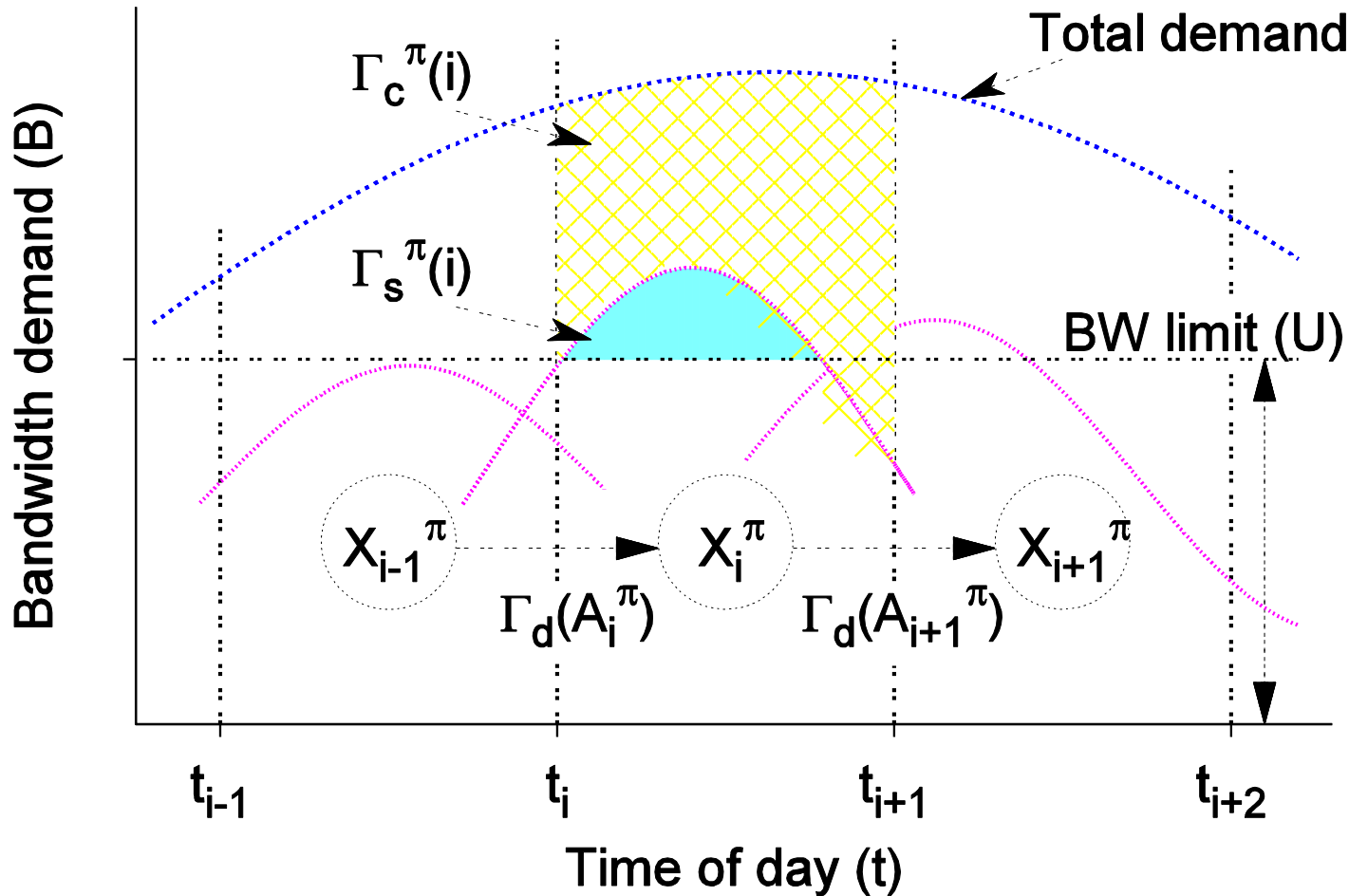
- How to get the best out of two worlds?
 - Improved workload models and prediction enables prefetching ...
- Dynamic content allocation
 - Utilize capped bandwidth (and storage) as much as possible
 - Use elastic cloud-based services to serve “spillover”
-

Dynamic Content Allocation Problem

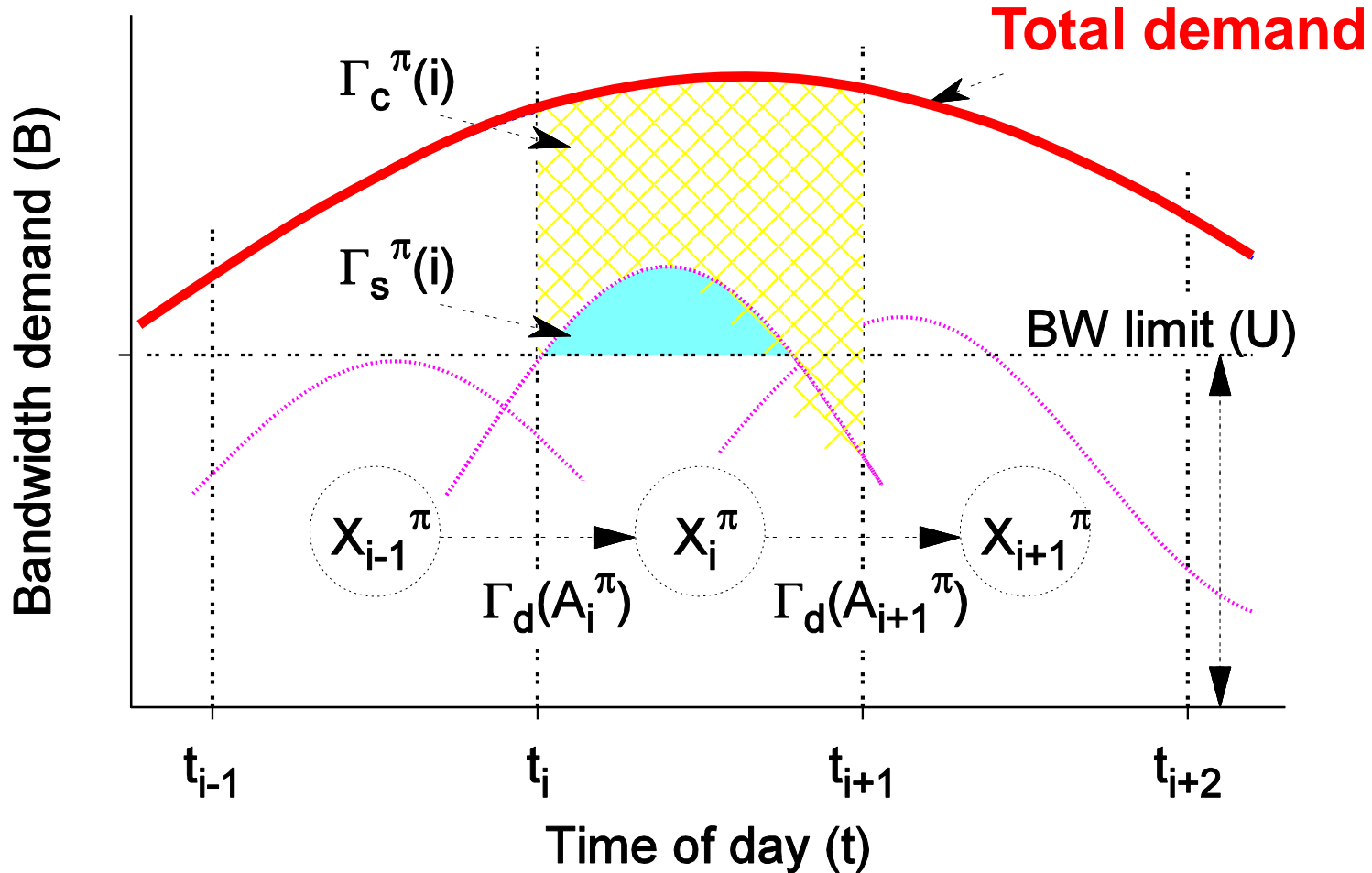


- Formulate as a finite horizon dynamic decision process problem
- Show discrete time decision process is good approximation
- Define exact solution as MILP
- Provide computationally feasible approximations (and prove properties about approximation ratios)
- Validate model and policies using traces from Spotify

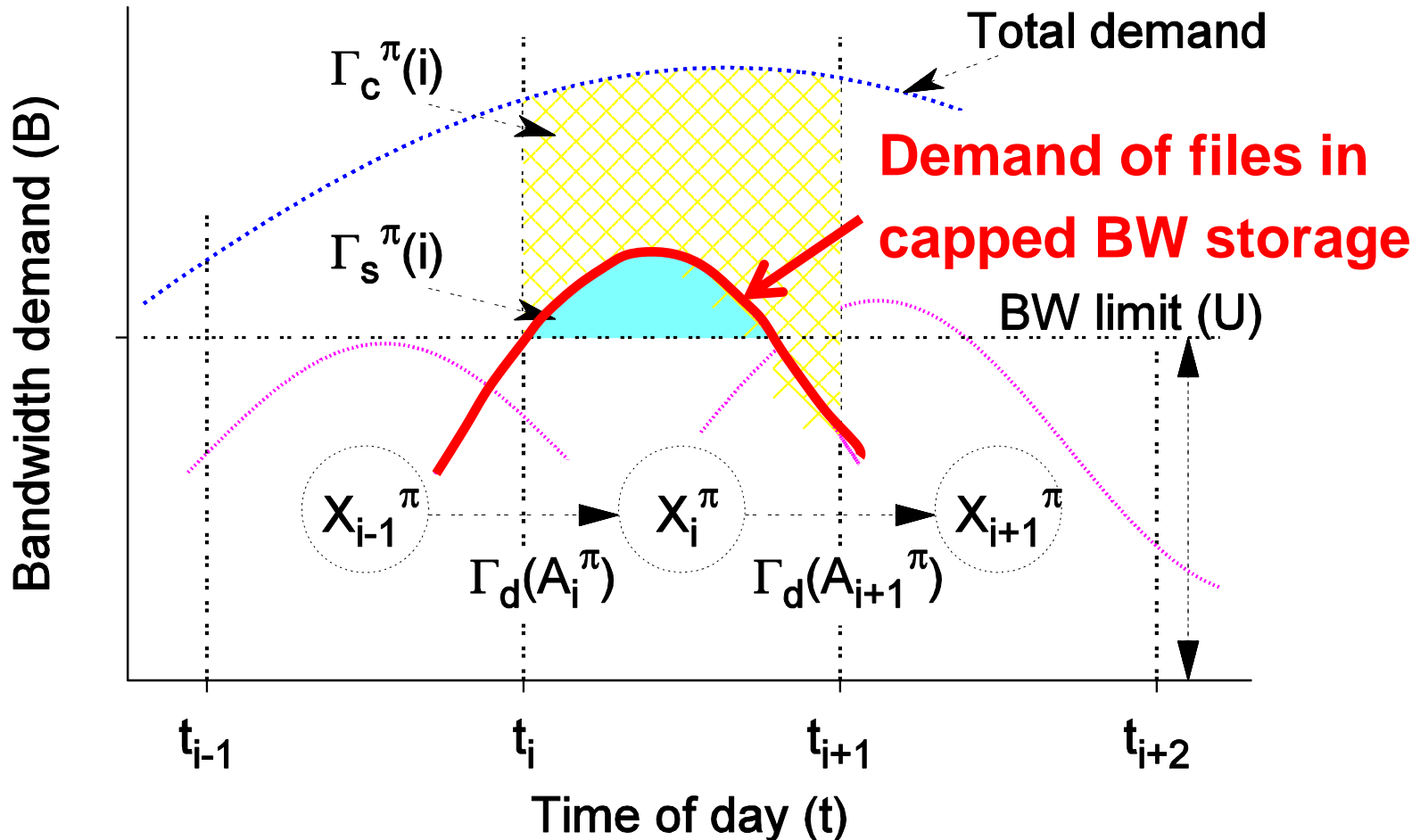
Cost minimization formulation



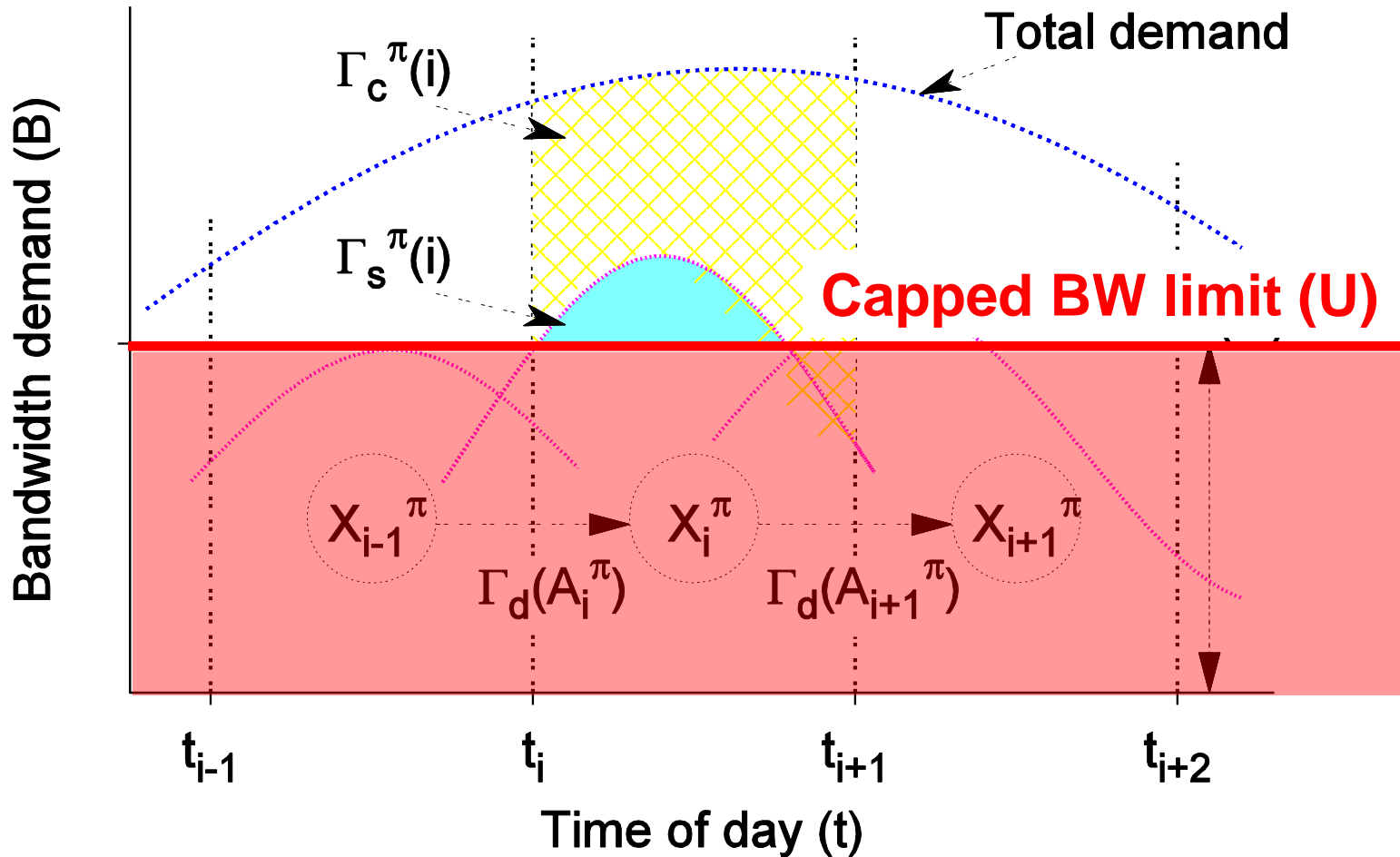
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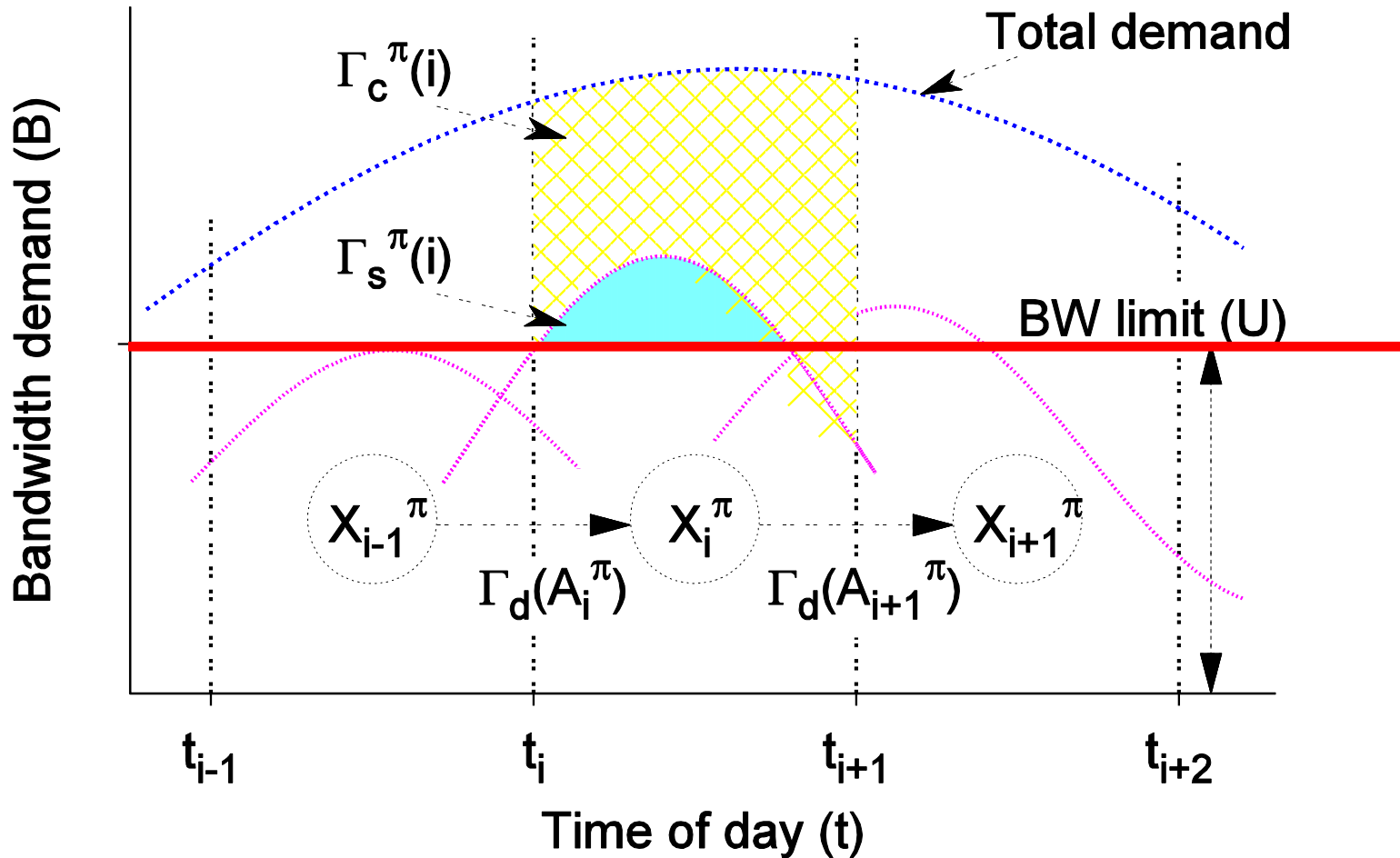
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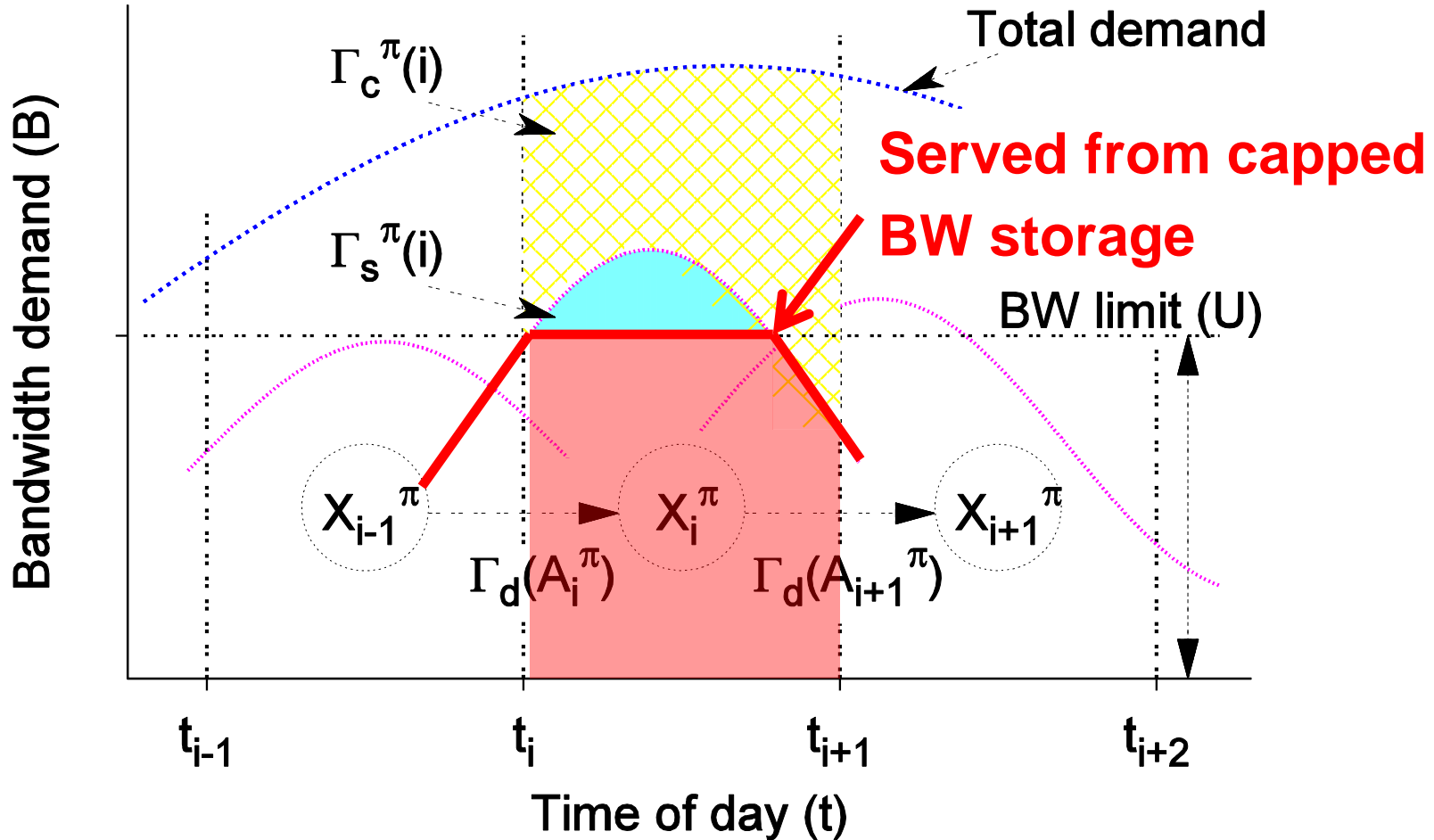
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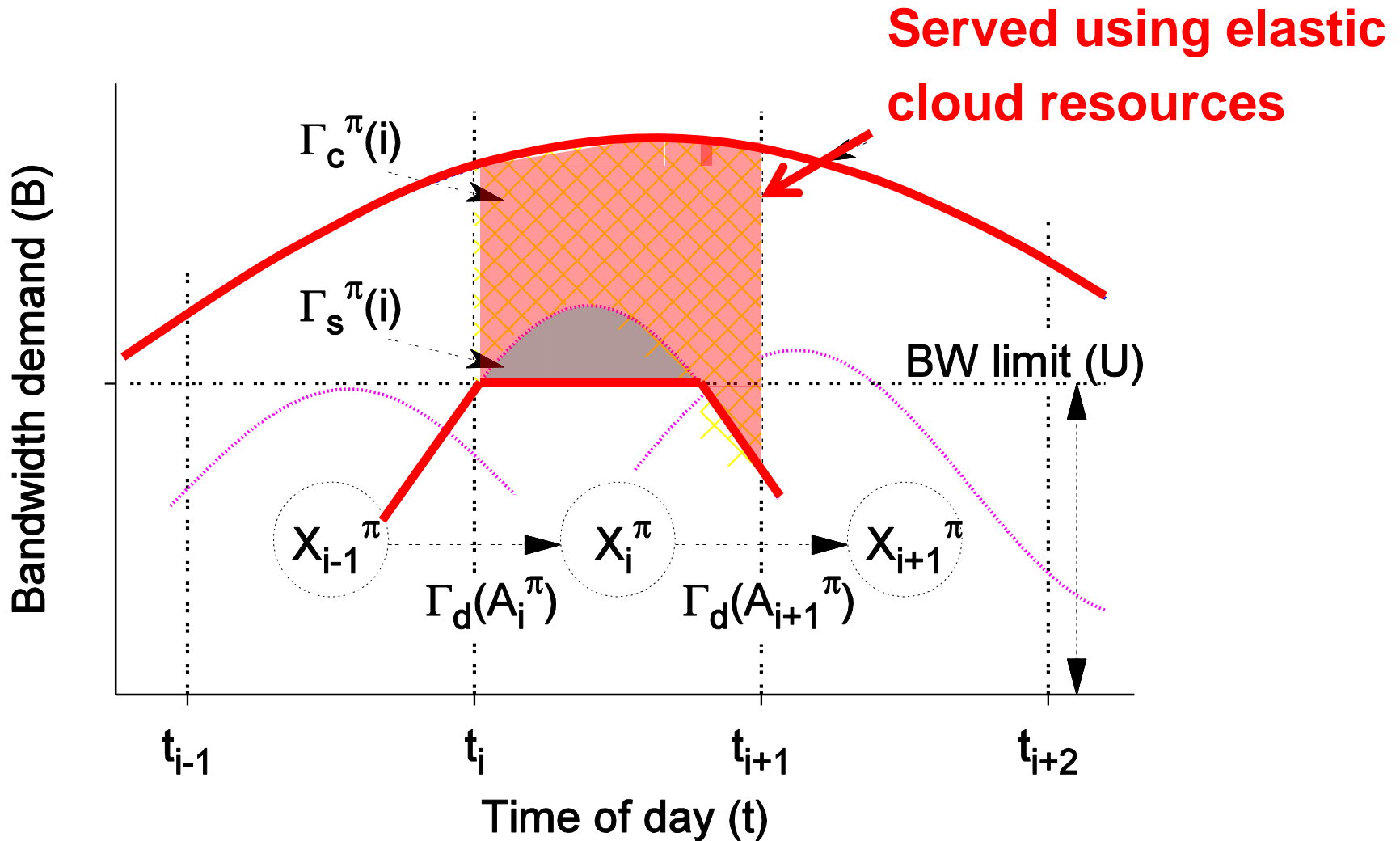
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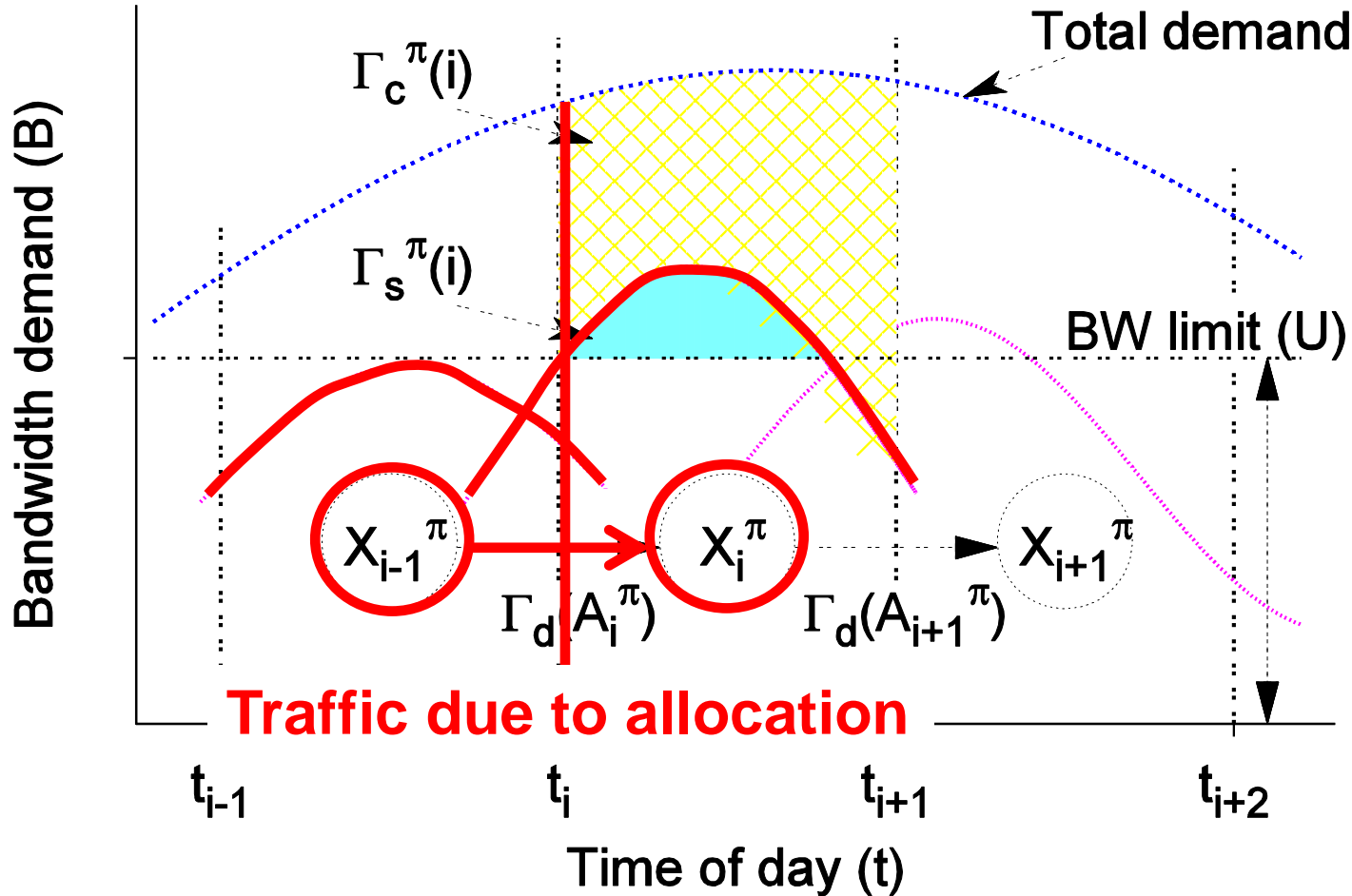
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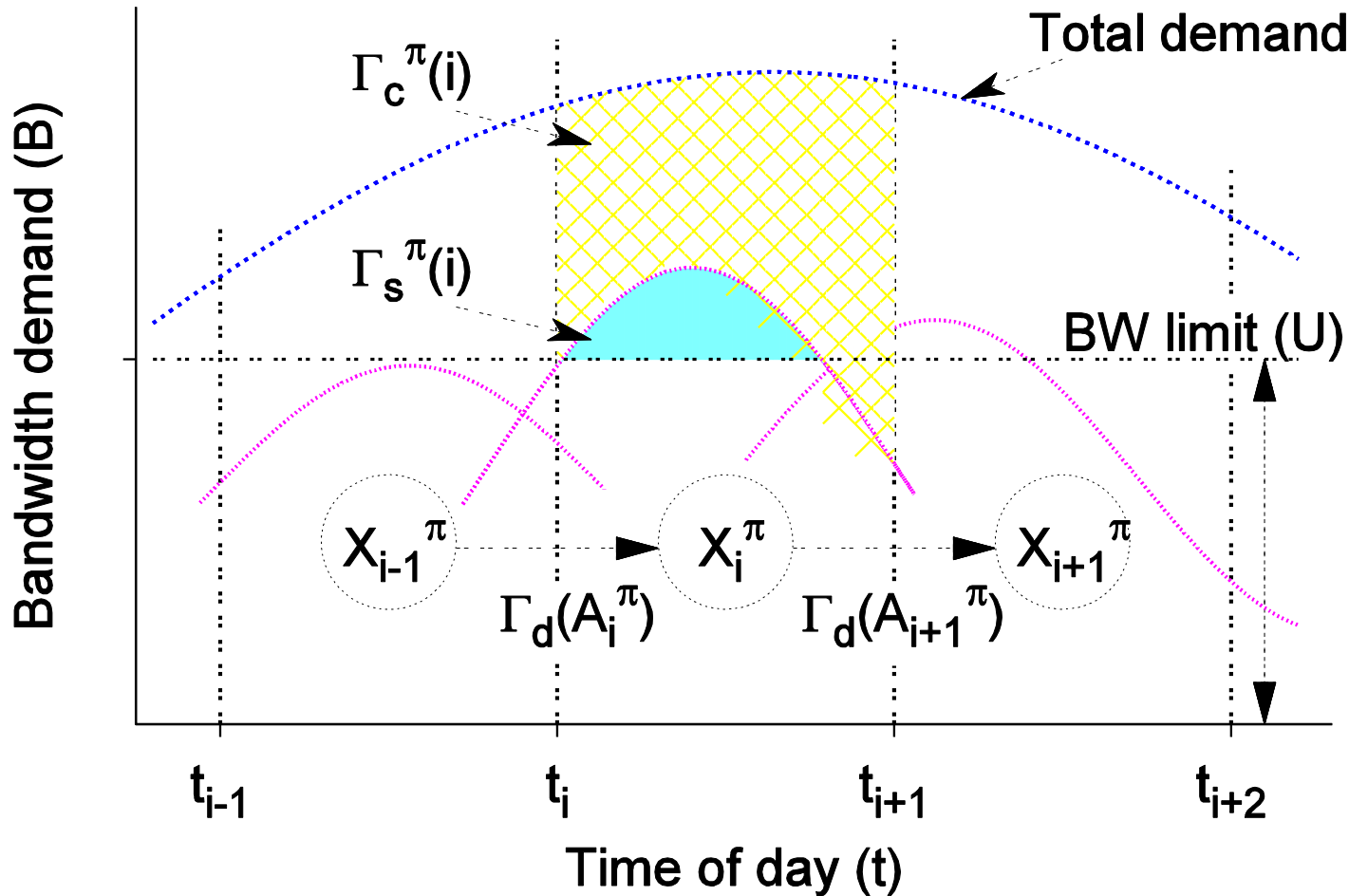
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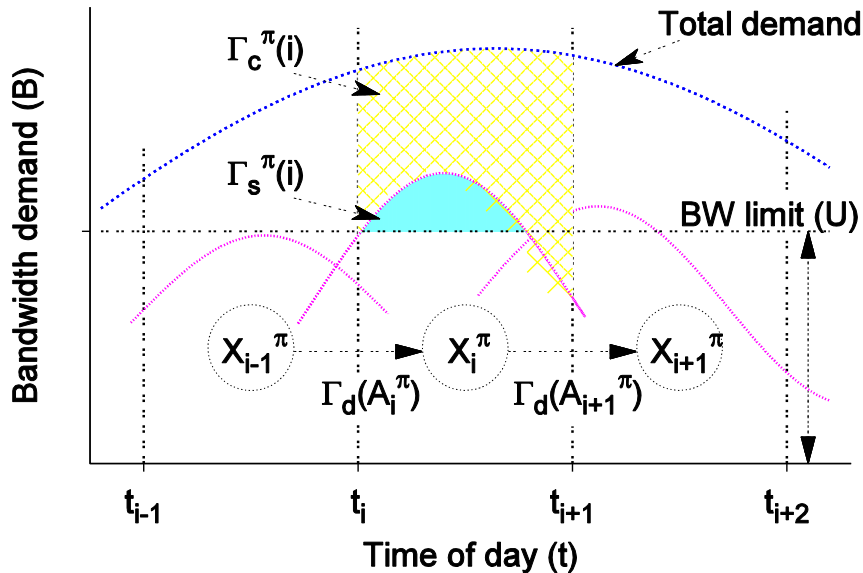
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- Traffic of files only in cloud

$$\Gamma_c^\pi(i) = E \left[\int_{t_i^\pi}^{t_{i+1}^\pi} \sum_{f \notin \mathcal{X}_i^\pi} B_f(t) \right]$$

- Spillover traffic

$$\Gamma_s^\pi(i) = E \left[\int_{t_i^\pi}^{t_{i+1}^\pi} \left(\sum_{f \in \mathcal{X}_i^\pi} B_f(t) - U \right)^+ dt \right]$$

- Traffic due to allocation

$$\Gamma_d^\pi(A_i^\pi) = \sum_{f \in A_i^\pi} L_f$$

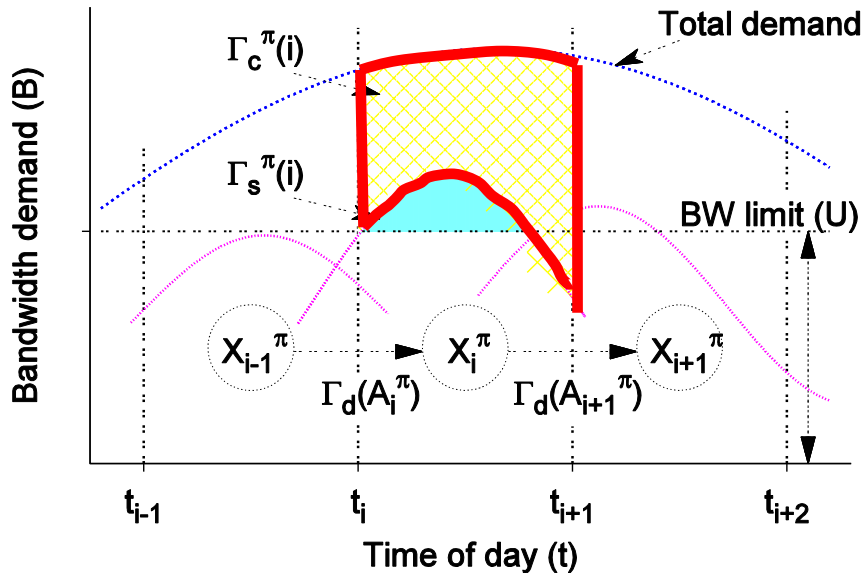
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$$J^\pi(T, \mathcal{X}_0) = \gamma \times \sum_{i=0}^{I^\pi} \{ \Gamma_d^\pi(A_i^\pi) + \Gamma_c^\pi(i) + \Gamma_s^\pi(i) \}$$

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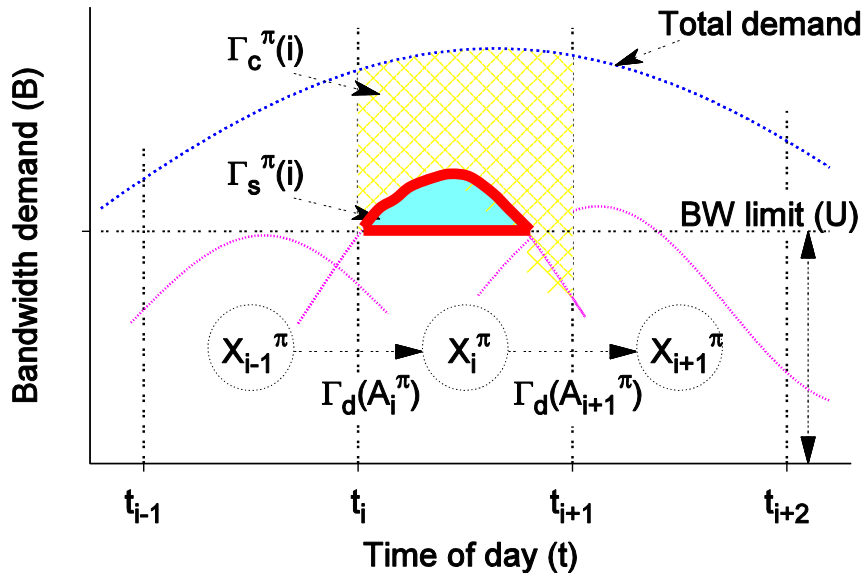
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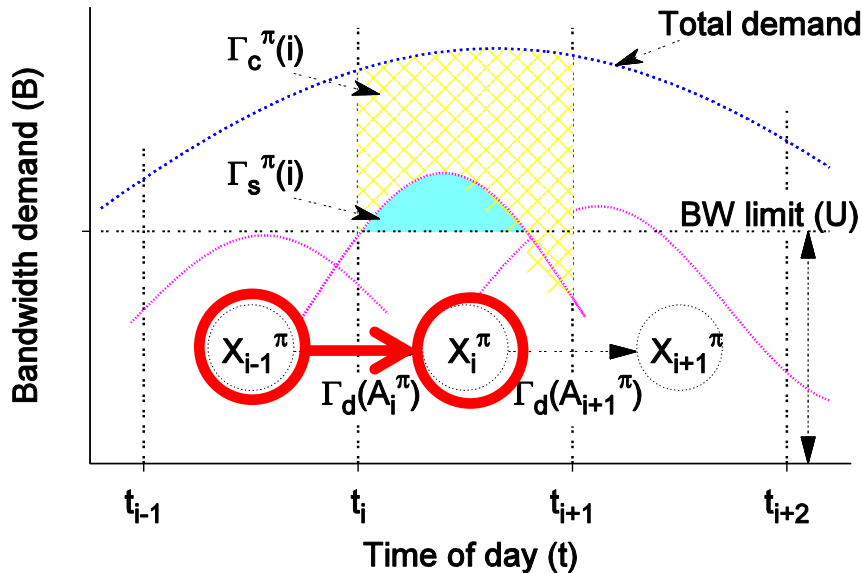
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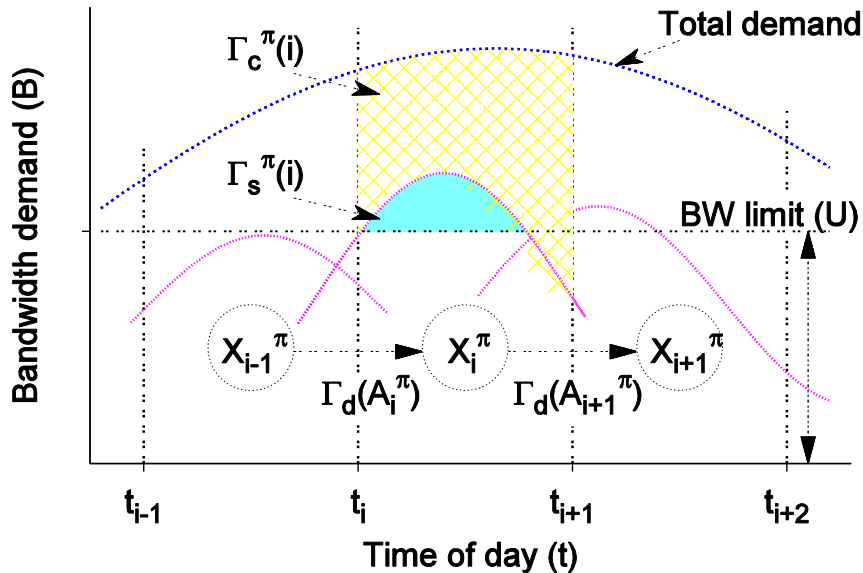
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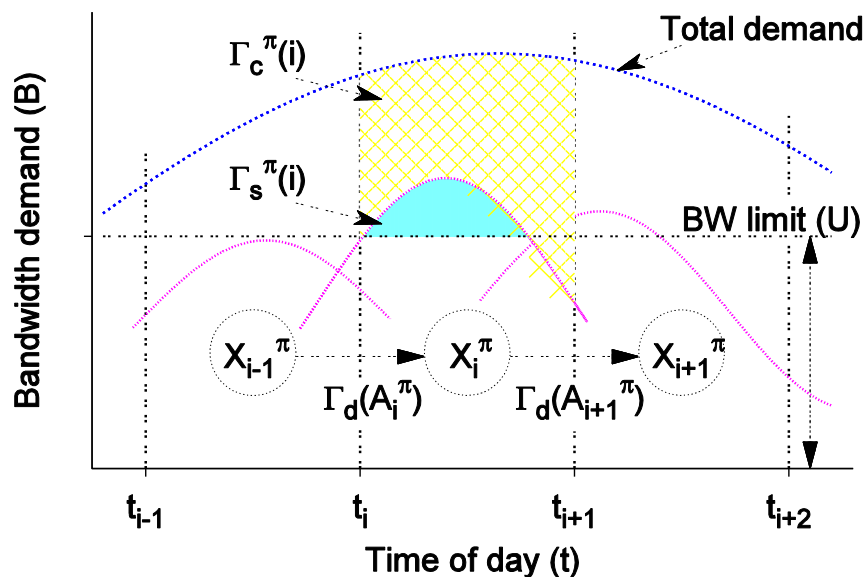
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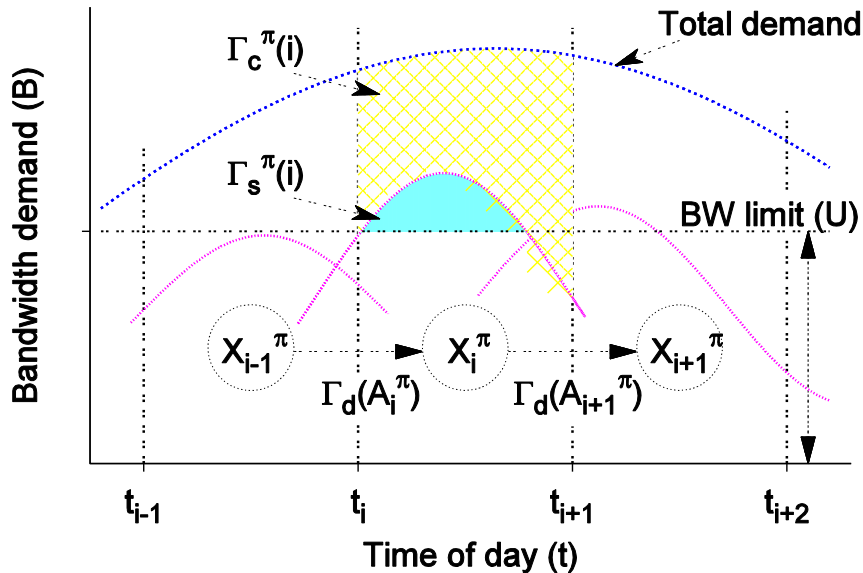
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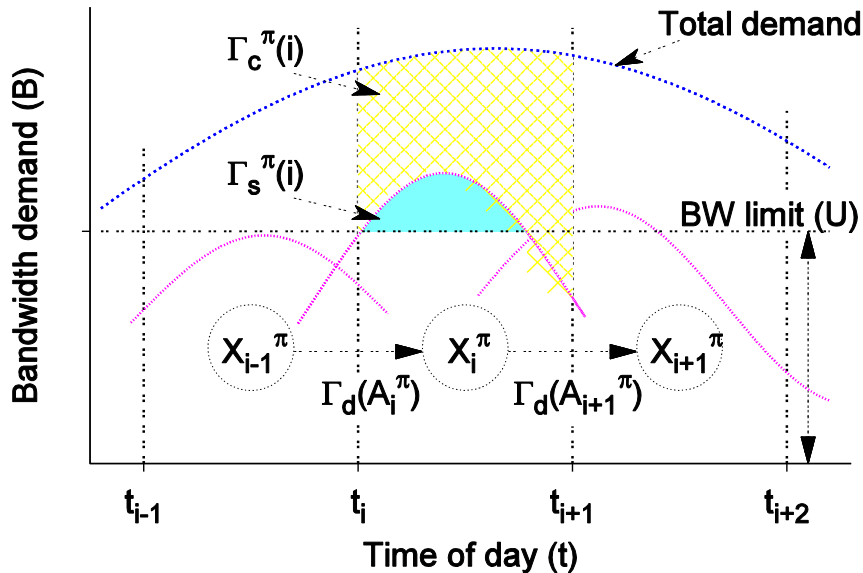
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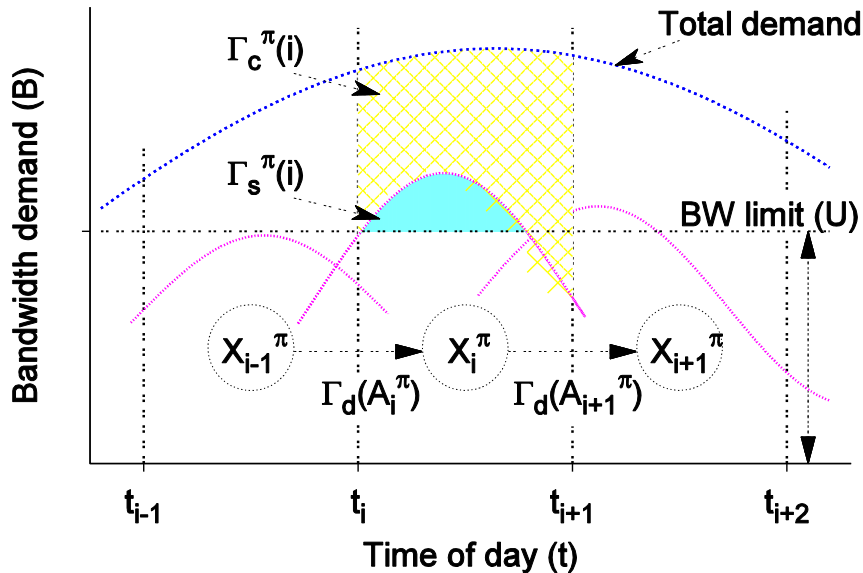
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Utilization maximization

~~Cost minimization formulation~~



Equivalent formulation

$$\bar{\Gamma}_s^\pi(i) = E \left[\int_{t_i^\pi}^{t_{i+1}^\pi} \min \left(U, \sum_{f \in \mathcal{X}_i^\pi} B_f(t) \right) dt \right]$$

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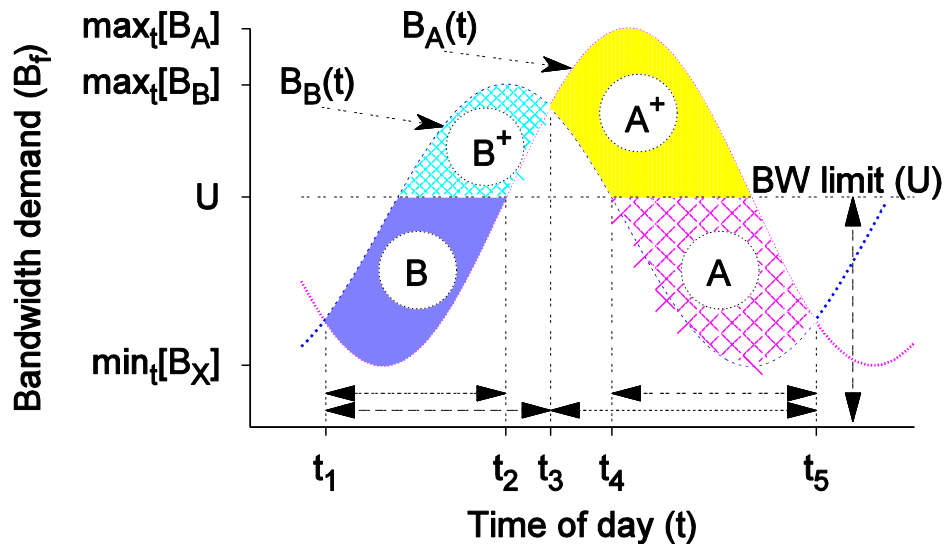
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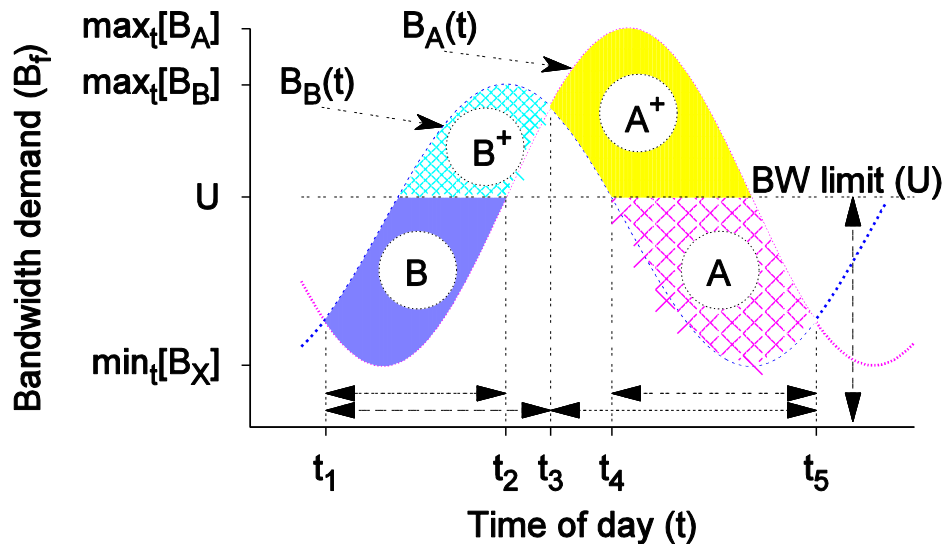
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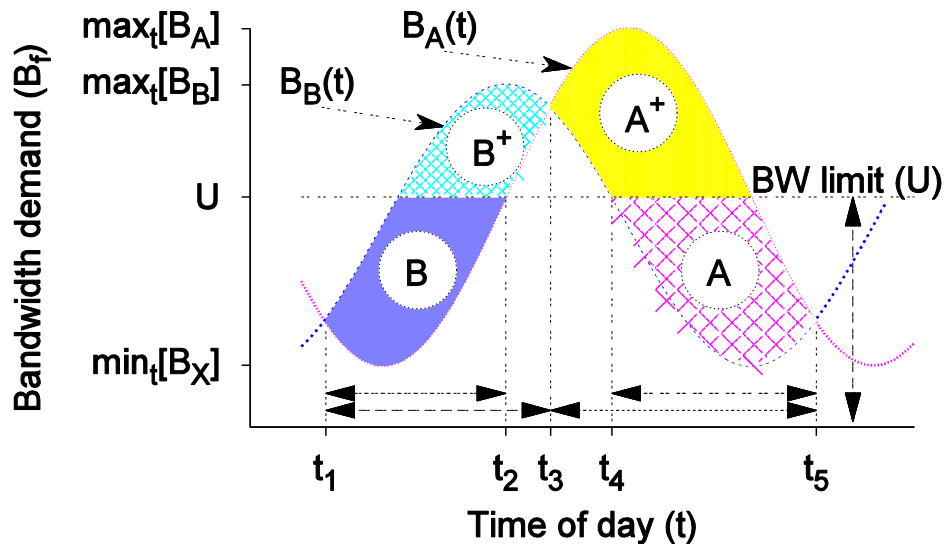
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Two file example



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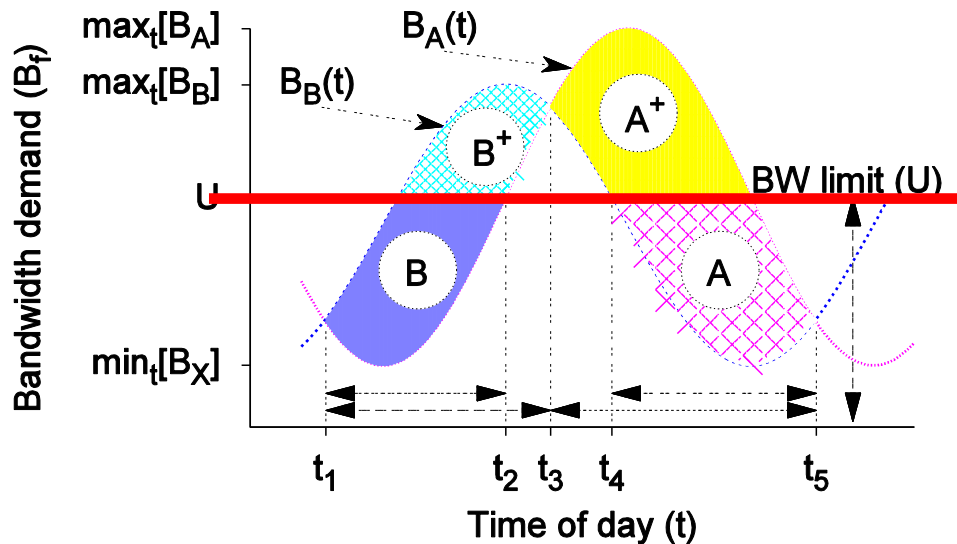
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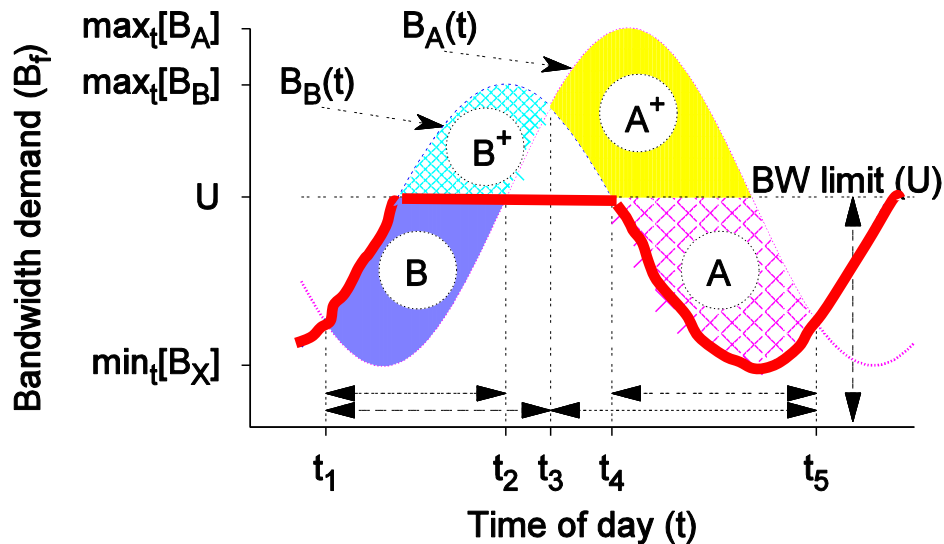
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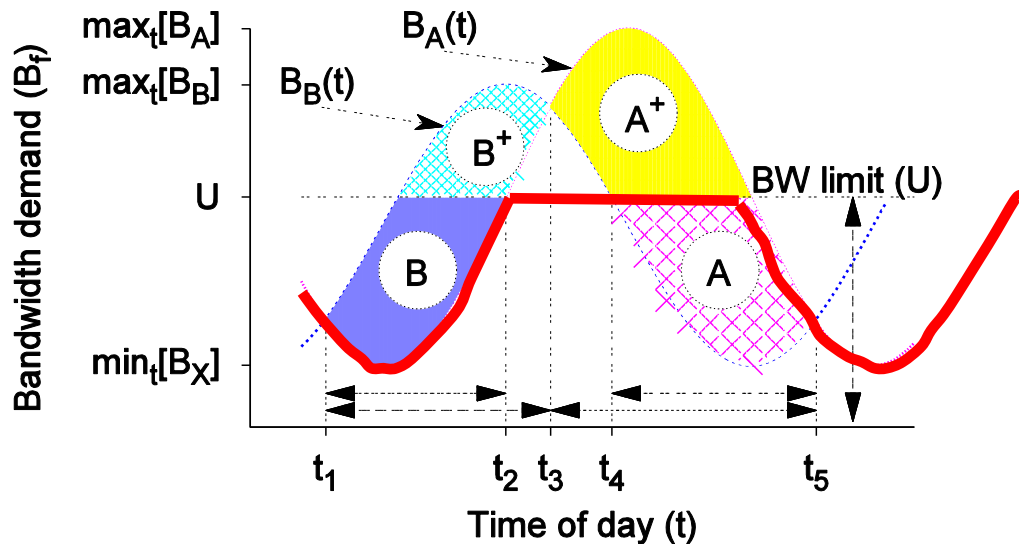
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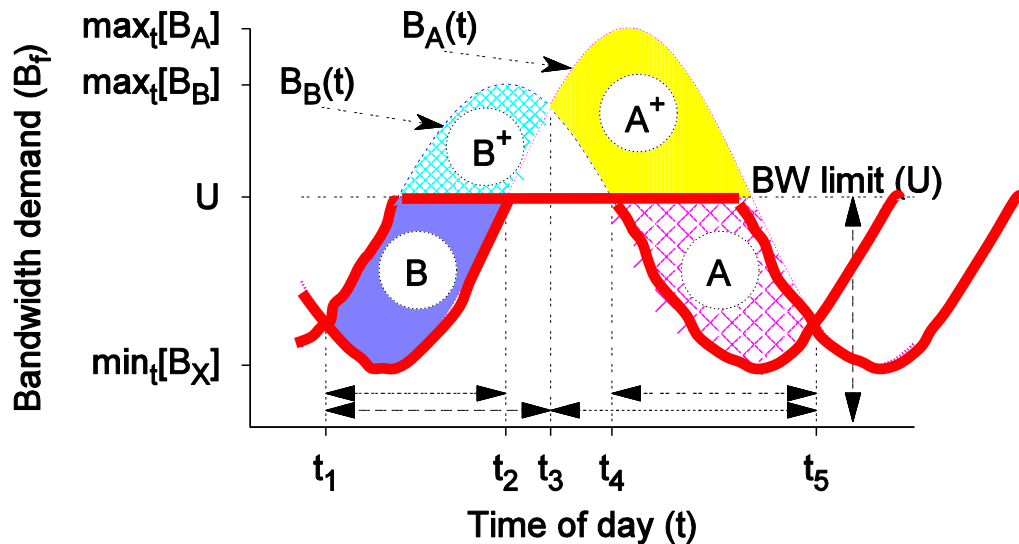
$$U^\pi(T, \mathcal{X}_0) = \gamma \times \sum_{i=0}^{I^\pi} \left\{ \bar{\Gamma}_s^\pi(i) - \Gamma_d^\pi(A_i^\pi) \right\}$$

$$\text{Optimal policy } \pi^* = \arg \max_{\pi \in \Pi} U^\pi(T, \mathcal{X}_0)$$

Utilization maximization

~~Cost minimization formulation~~

Two file example



- **Equivalent formulation**

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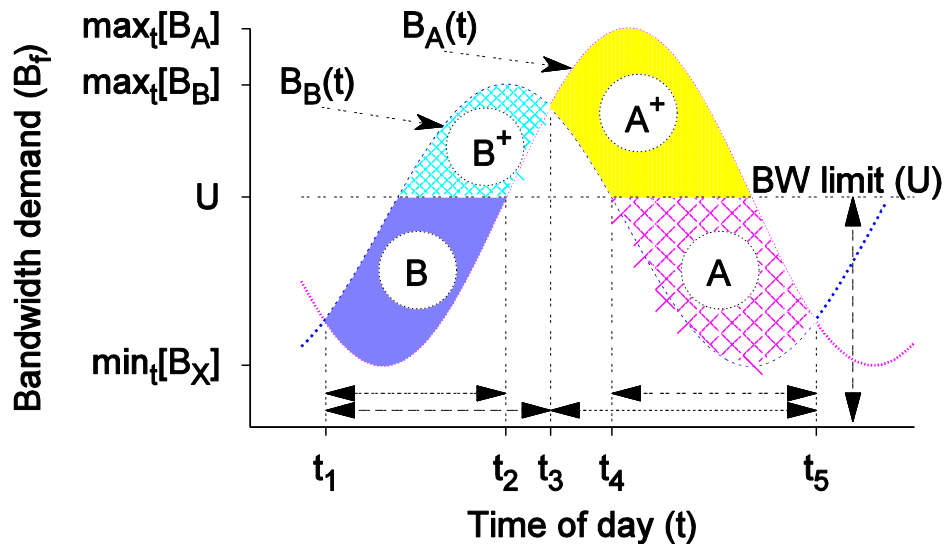
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Discrete-time Decision Problem



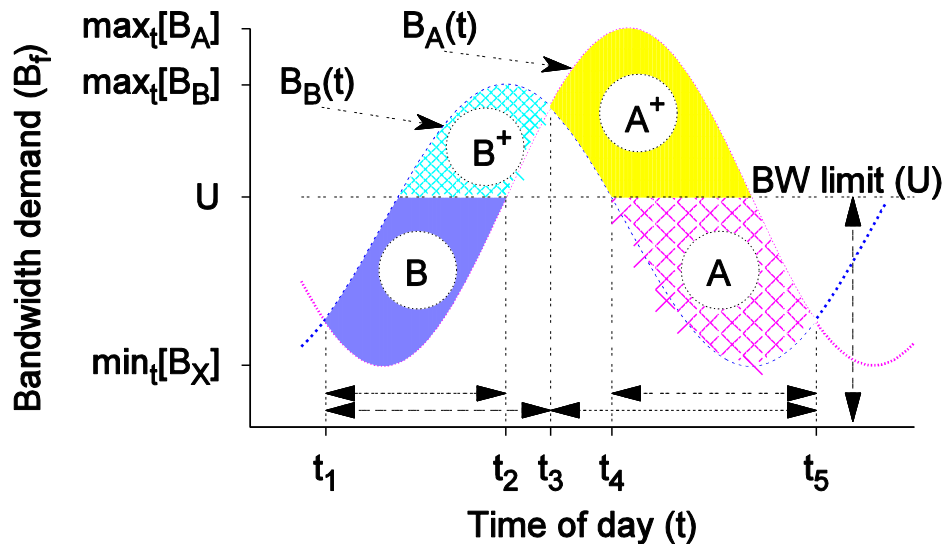
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Discrete-time Decision Problem



- Approximation

$$\sum_{f \in \mathcal{X}(t)} B_f(t) \approx \sum_{f \in \mathcal{X}_i} \bar{B}_f^i \text{ for } t_i \leq t < t_{i+1}$$

$P(\sum_{f \in \mathcal{X}} B_f(t) \leq U)$ decrease exponentially

- Finite horizon decision

$$U^{\pi^*}([t_i, t_{I+1}], \mathcal{X}_{i-1}) = \max_{\mathcal{X}_i} \{ \bar{\Gamma}_s(i) - \Gamma_d(A_i) + U^{\pi^*}([t_{i+1}, t_{I+1}], \mathcal{X}_i) \}$$

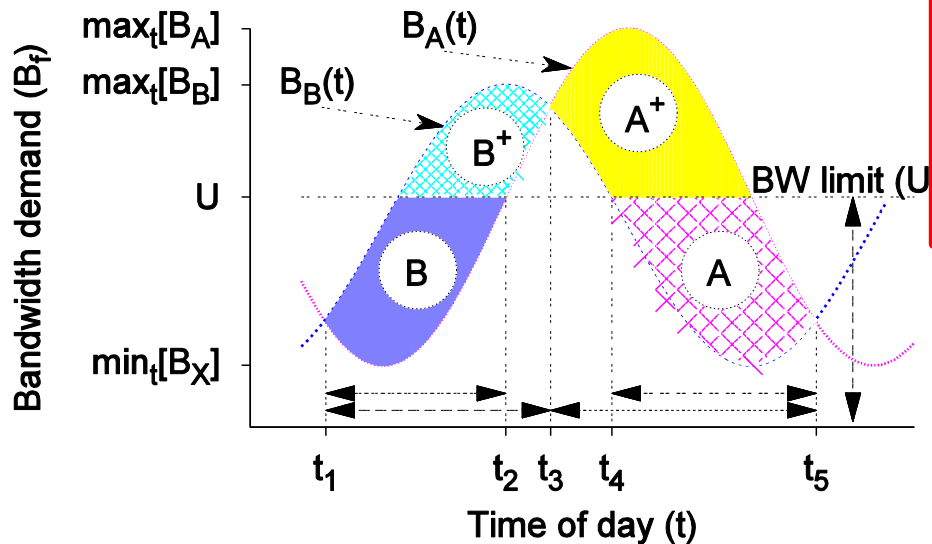
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Discrete-time Decision Problem



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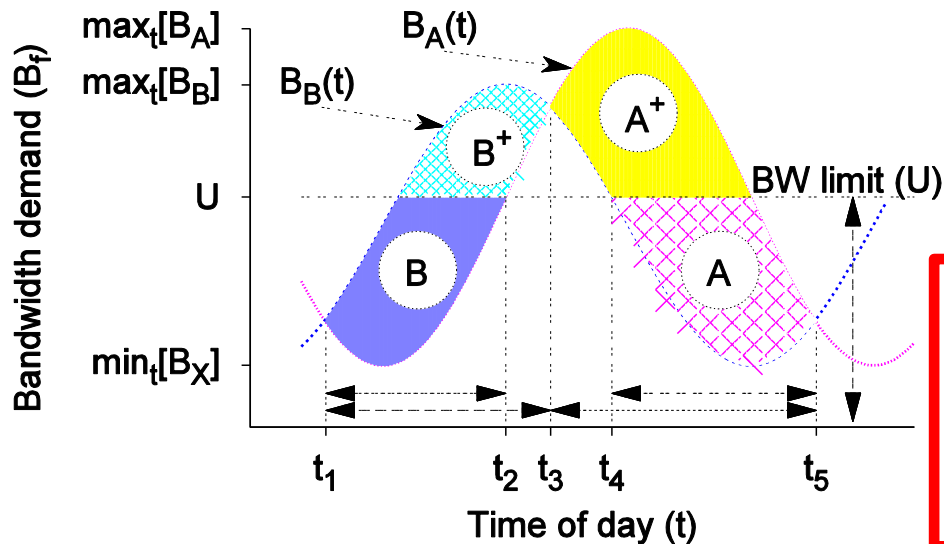
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Discrete-time Decision Problem



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- **Finite horizon decision**

$$U^{\pi^*}([t_i, t_{I+1}], \mathcal{X}_{i-1}) = \max_{\mathcal{X}_i} \{ \bar{\Gamma}_s(i) - \Gamma_d(A_i) + U^{\pi^*}([t_{i+1}, t_{I+1}], \mathcal{X}_i) \}$$

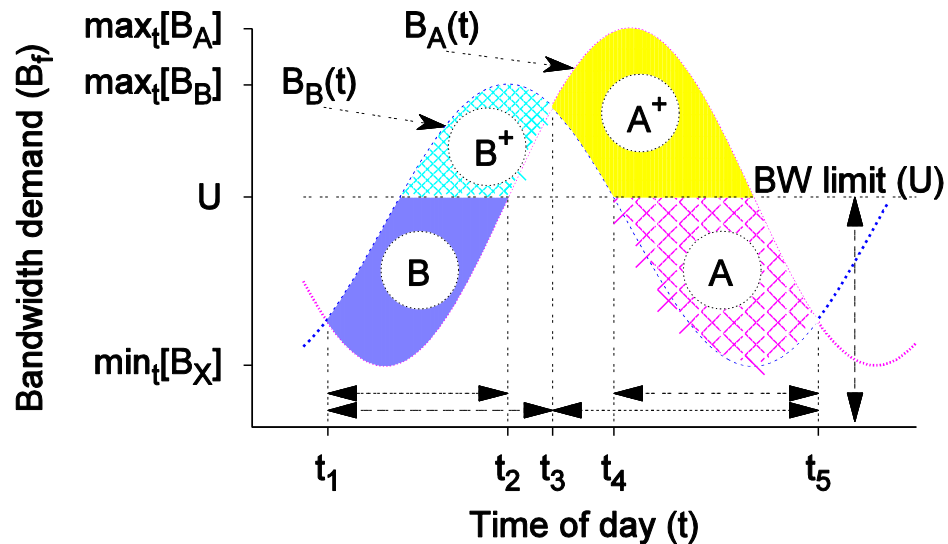
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Discrete-time Decision Problem



- Approximation

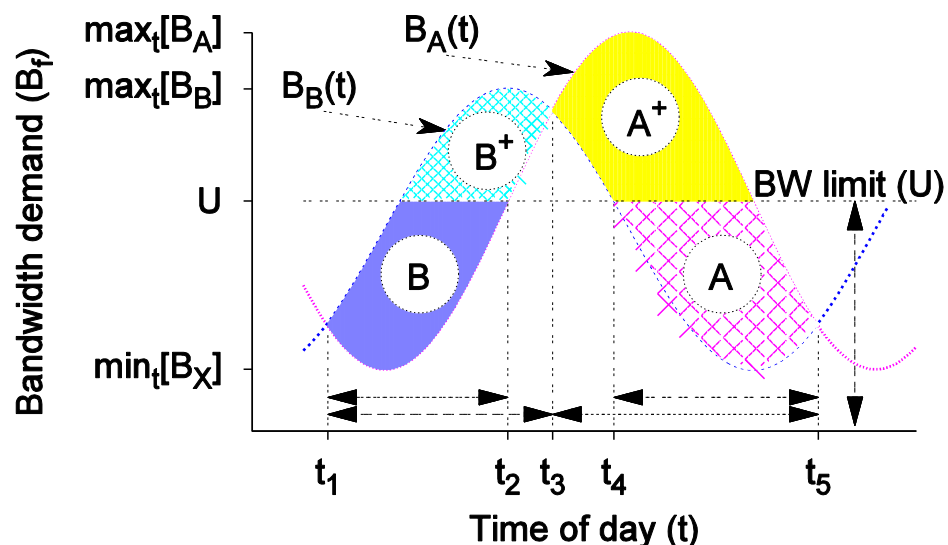
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Discrete-time Decision Problem



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Theorem: Exact solution as a MILP

Let $\Delta_i = t_{i+1} - t_i$. Every solution of the MILP

$$\max \sum_{i=1}^I \left\{ \Delta_i \left(\sum_{f \in \mathcal{F}} \bar{B}_f^i x_{i,f} - s_i \right) - \sum_{f \in \mathcal{F}} L_f b_{i,f} \right\}$$

$$\sum_{f \in \mathcal{F}} \bar{B}_f^i x_{i,f} - s_i \leq U, \quad \forall 1 \leq i \leq I \quad (1)$$

$$x_{i,f} - x_{i-1,f} - b_{i,f} \leq 0, \quad \forall 1 \leq i \leq I, f \in \mathcal{F} \quad (2)$$

$$\sum_{f \in \mathcal{F}} L_f x_{i,f} \leq S, \quad \forall 1 \leq i \leq I \quad (3)$$

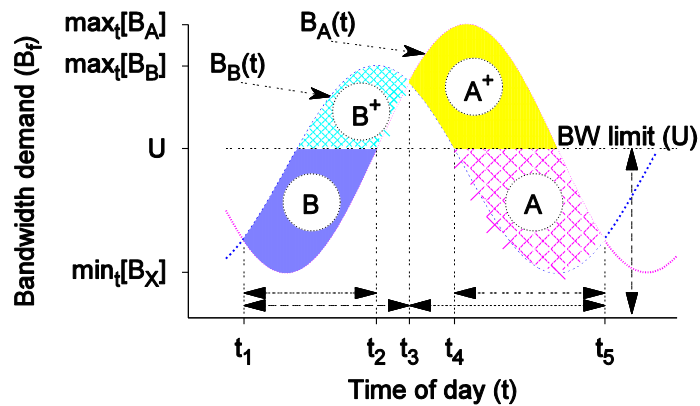
s.t.

$$b_{i,f} \geq 0, \quad x_{i,f} \in \{0, 1\}, \quad \forall 1 \leq i \leq I, f \in \mathcal{F} \quad (4)$$

$$s_i \geq 0, \quad \forall 1 \leq i \leq I, \quad (5)$$

is an optimal policy π^* .

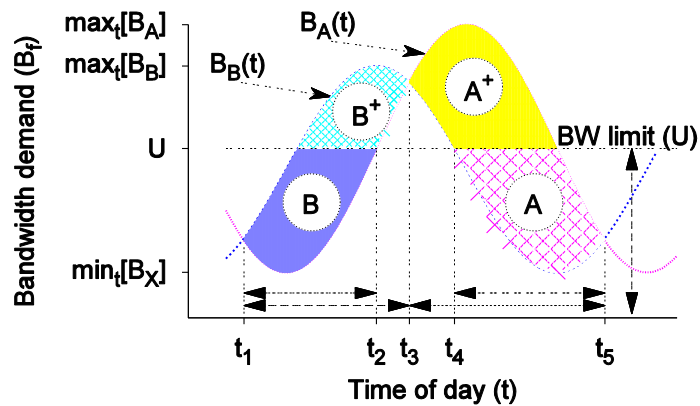
Policy: No Download Cost (NDC)



- Consider next interval only

$$\chi_i^{NDC} = \arg \max \chi_i \bar{\Gamma}_s^\pi(i)$$

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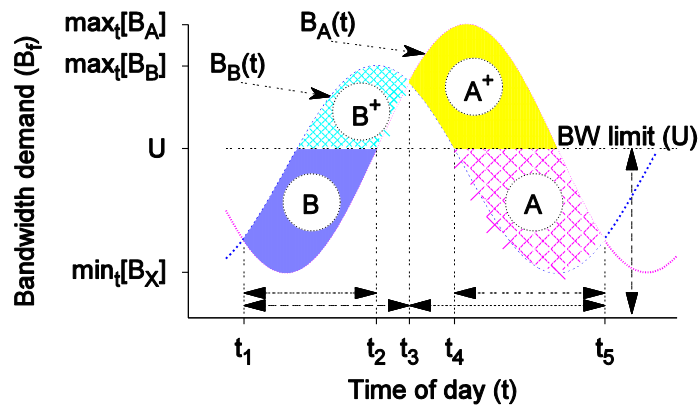
- Proposition 1: Unbounded approximation ratio

$$\frac{J^{NDC}}{J^{\pi^*}} = \frac{1+\epsilon}{1.5\epsilon} \Rightarrow \lim_{\epsilon \rightarrow 0} \frac{J^{NDC}}{J^{\pi^*}} = \infty$$

- Proposition 2: Approximation bound

The approximation ratio of NDC is $\frac{J^{NDC}}{J^{\pi^*}} \leq 1 + IS/J^{\pi^*}$.

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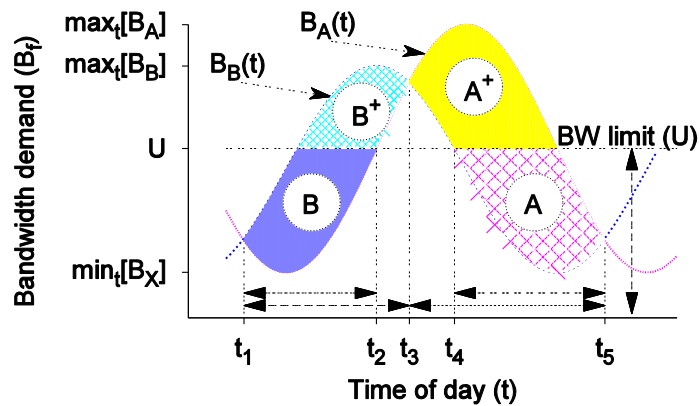
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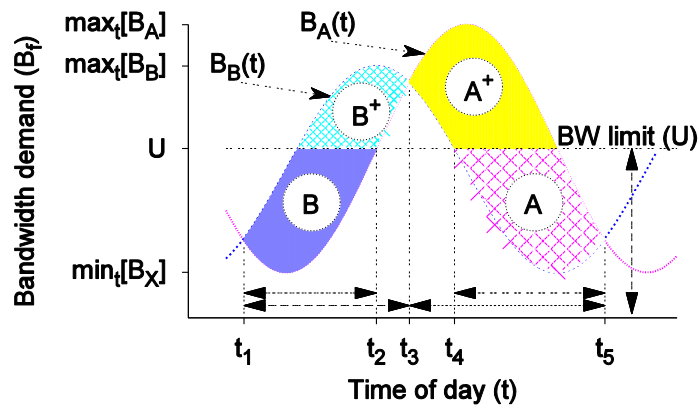
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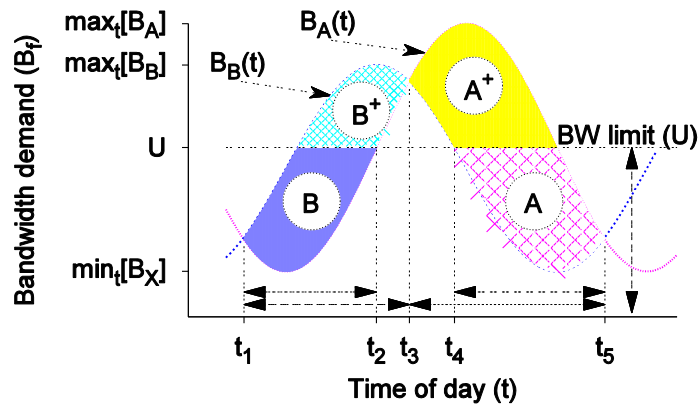
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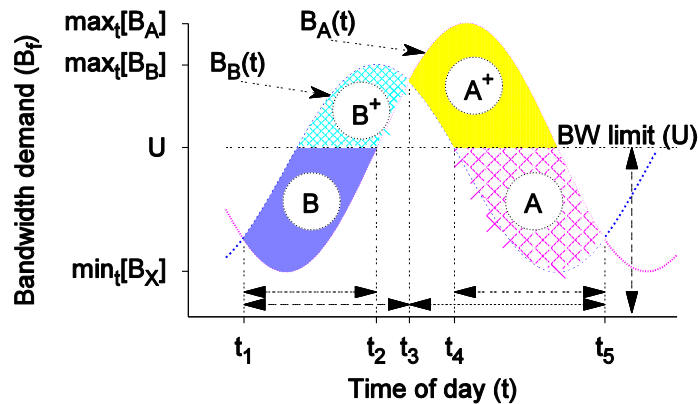
Policy: k-Step Look Ahead (k-SLA)



- Consider k next intervals

$$\chi_i^{1-SLA} = \arg \max \chi_i \left\{ \bar{\Gamma}_s^\pi(i) - \Gamma_d^\pi(A_i) \right\}$$

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$$\chi_i^{1-SLA} = \arg \max \chi_i \left\{ \bar{\Gamma}_s^\pi(i) - \Gamma_d^\pi(A_i) \right\}$$

- Proposition 3: Unbounded approximation ratio

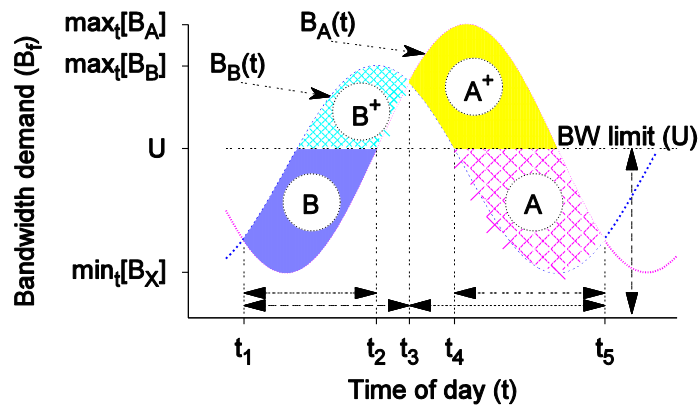
$$\frac{J^{1-SLA}}{J^{\pi^*}} = \frac{1+\epsilon}{3\epsilon} \Rightarrow \lim_{\epsilon \rightarrow 0} \frac{J^{1-SLA}}{J^{\pi^*}} = \infty$$

- Proposition 4: Approximation bound

Assume the average demand of each file inserted into the dedicated storage by an optimal policy π^* is lower bounded by a factor $\rho > 0$ such that the demand of each such file satisfies $\rho \frac{1}{I} \sum_{i=0}^I \bar{B}_f^i \Delta_i \geq L_f$. Then, for $k > \frac{\rho I}{I-\rho}$ the approximation ratio of k-SLA is

$$\frac{J^{k-SLA}}{J^{\pi^*}} \leq \frac{1}{1 - \frac{\rho}{k} \left(1 + \frac{k}{I}\right)}. \quad (6)$$

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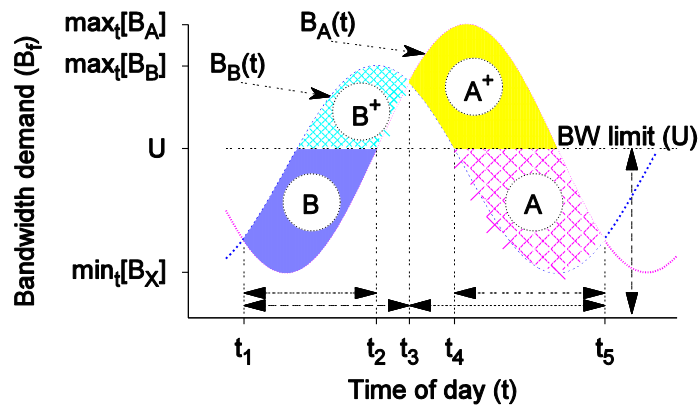
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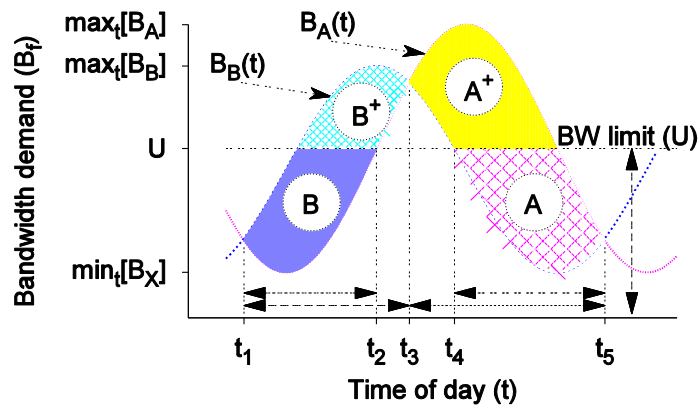
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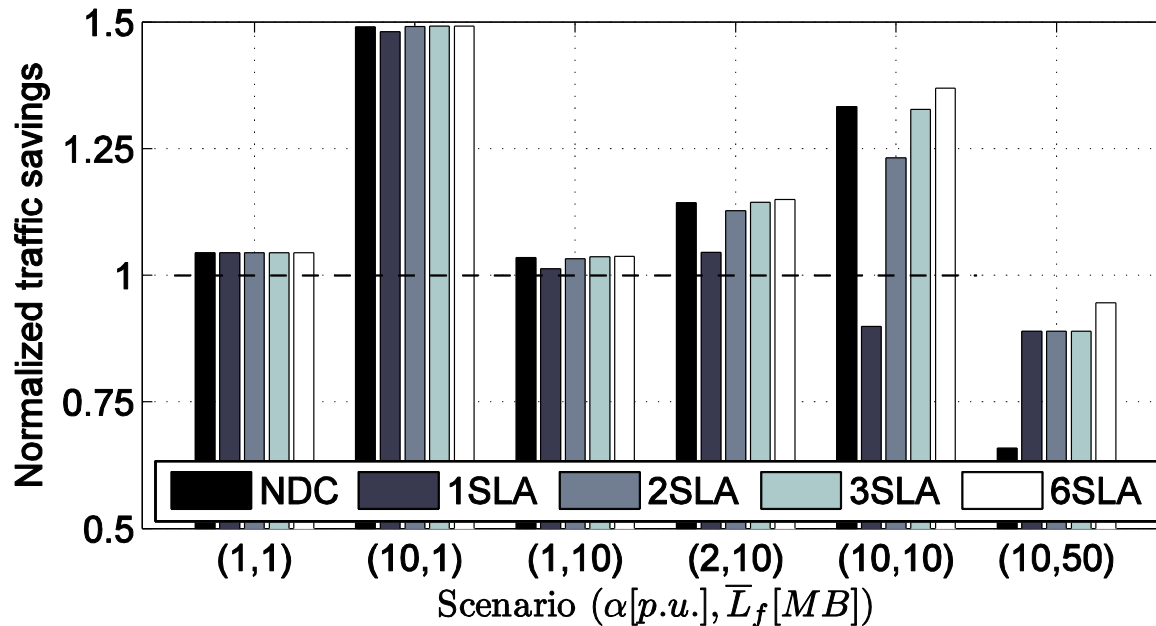
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Trace-based analysis (Synthetic)

- Normalized traffic savings

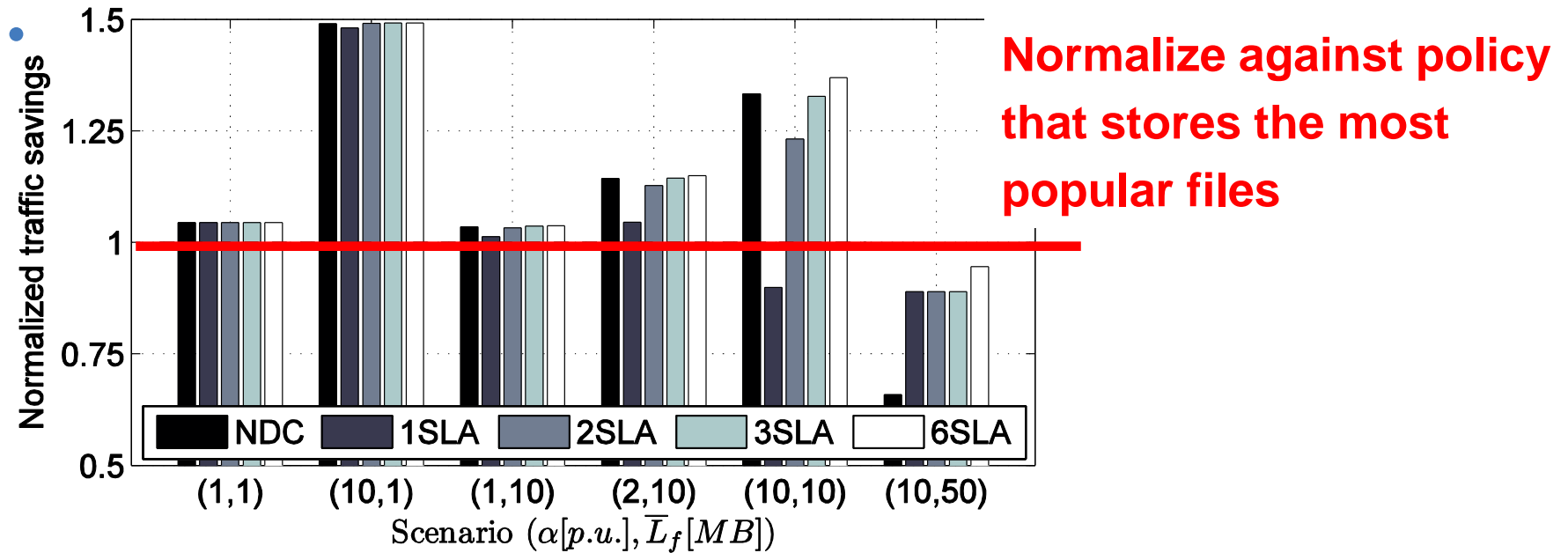


**Based on Spotify
trace characterization**

- Workload: 3 groups of 1000 files; peaks $N(0,2)$ offset by 8h for each group; sinusoid with 24h period; min/max ratio $N(0.075,0.075)$, file sizes $U(L/2,3L/2)$, bandwidth demand Bounded Pareto ($B_{\min}, B_{\max}, \alpha$)

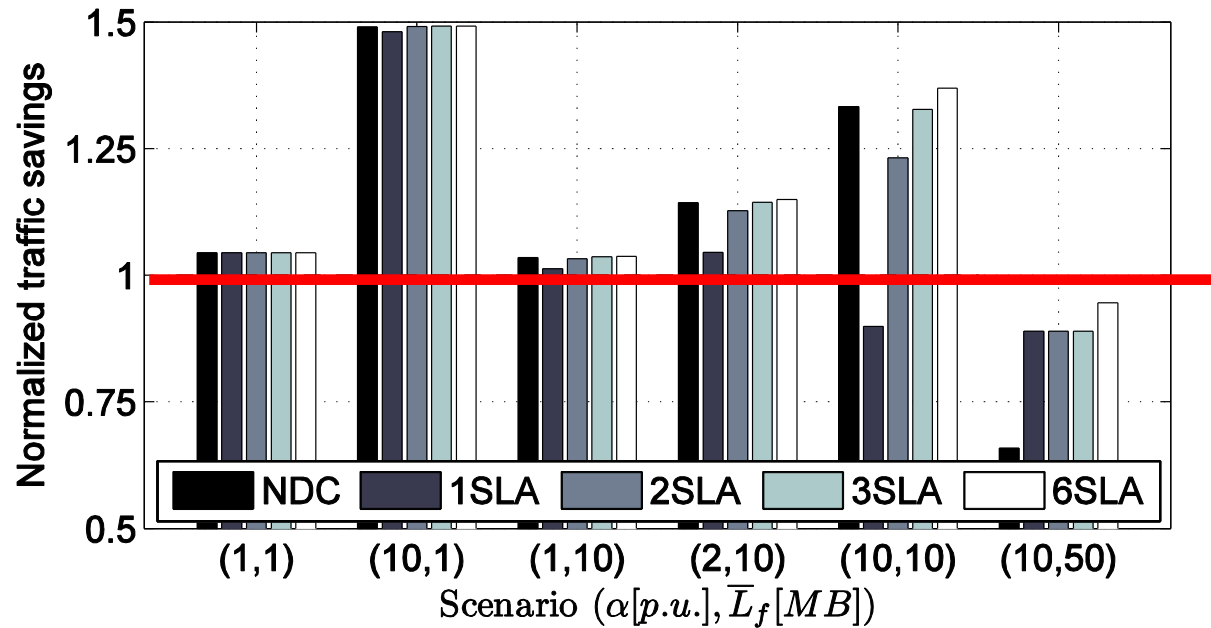
Trace-based analysis (Synthetic)

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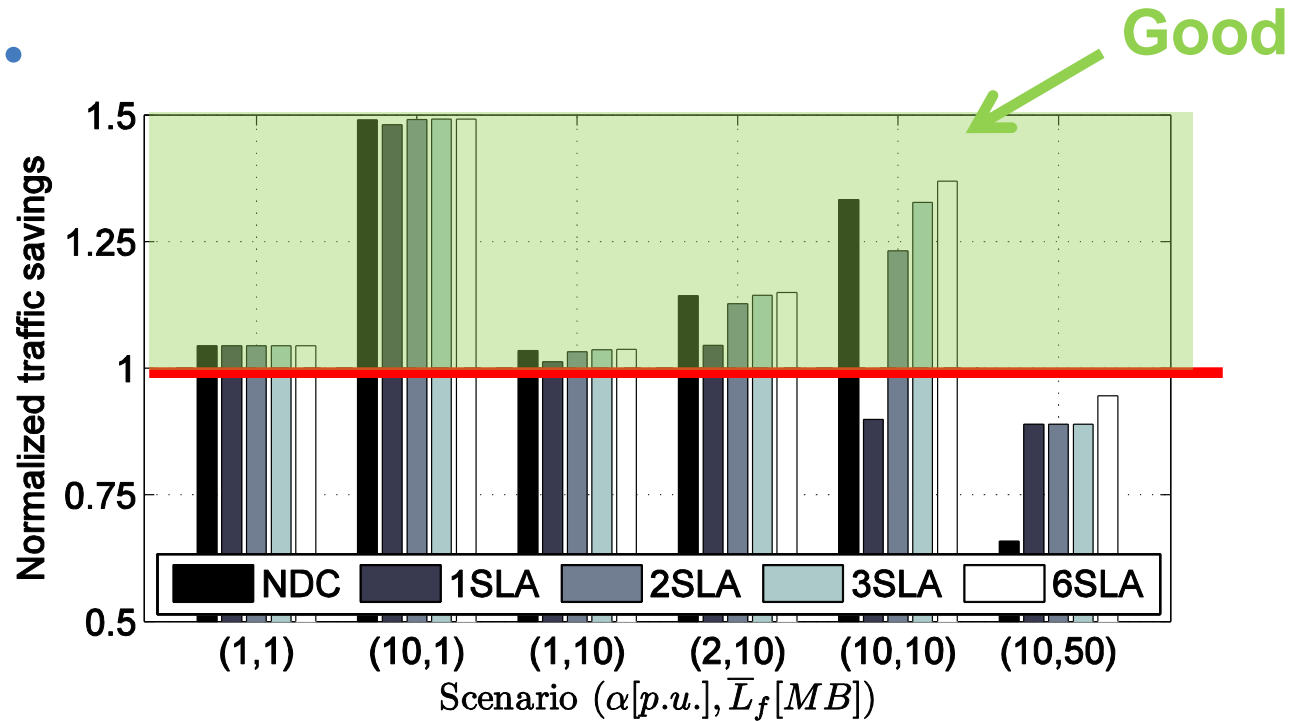
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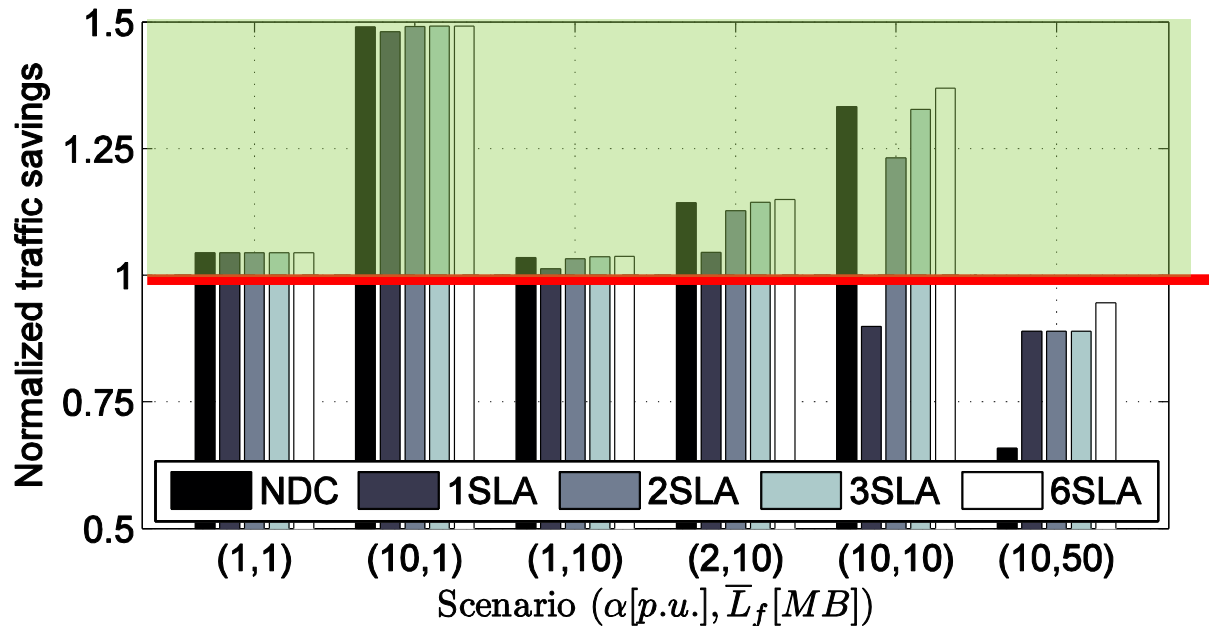
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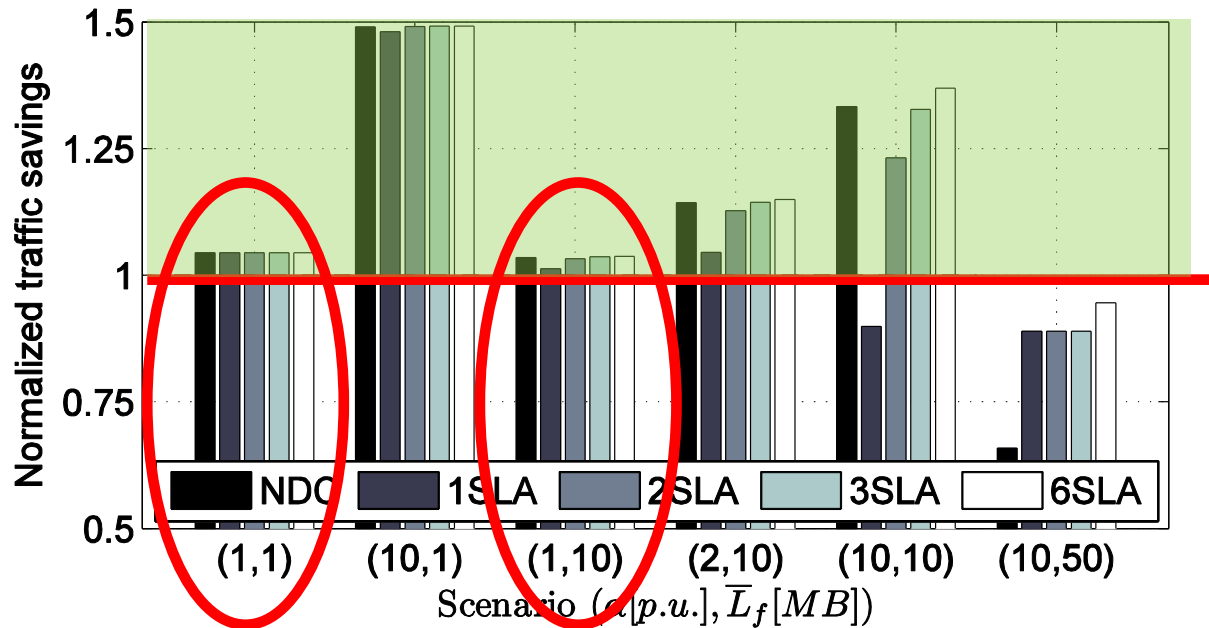
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- Modest gains when Zipf-like ($\alpha \approx 1$) rank popularity
- Significant gains when more uniform ($\alpha \approx 10$)
- NDC fails for large sizes (6-SLA still works well)

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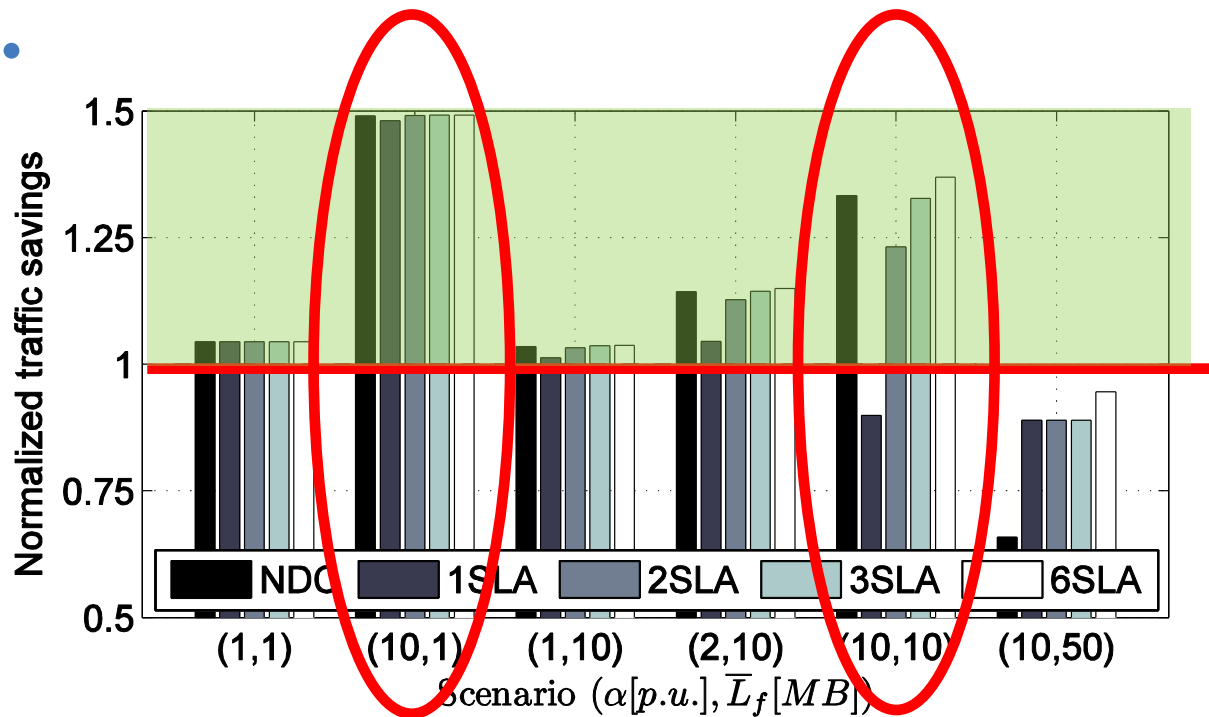
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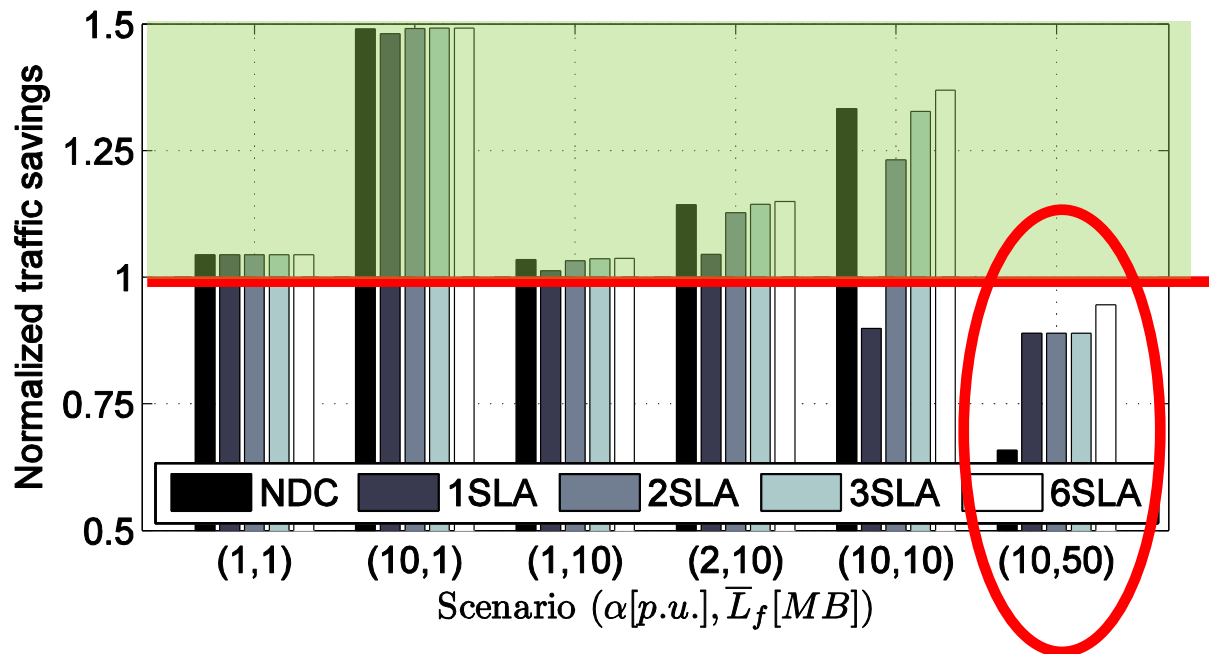
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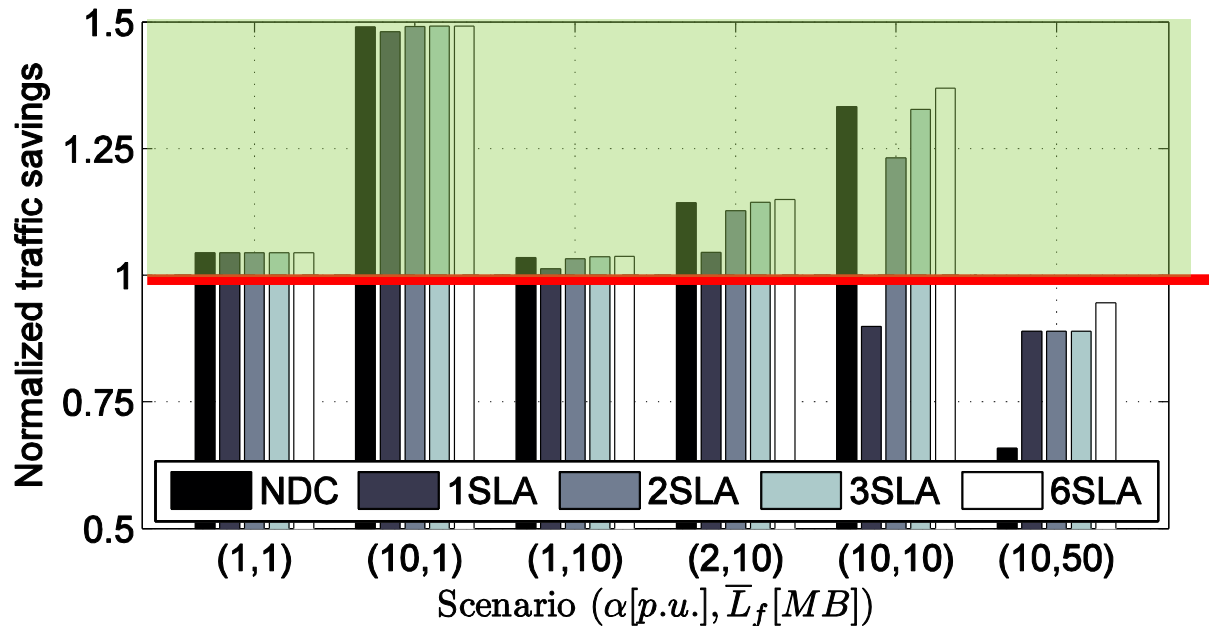
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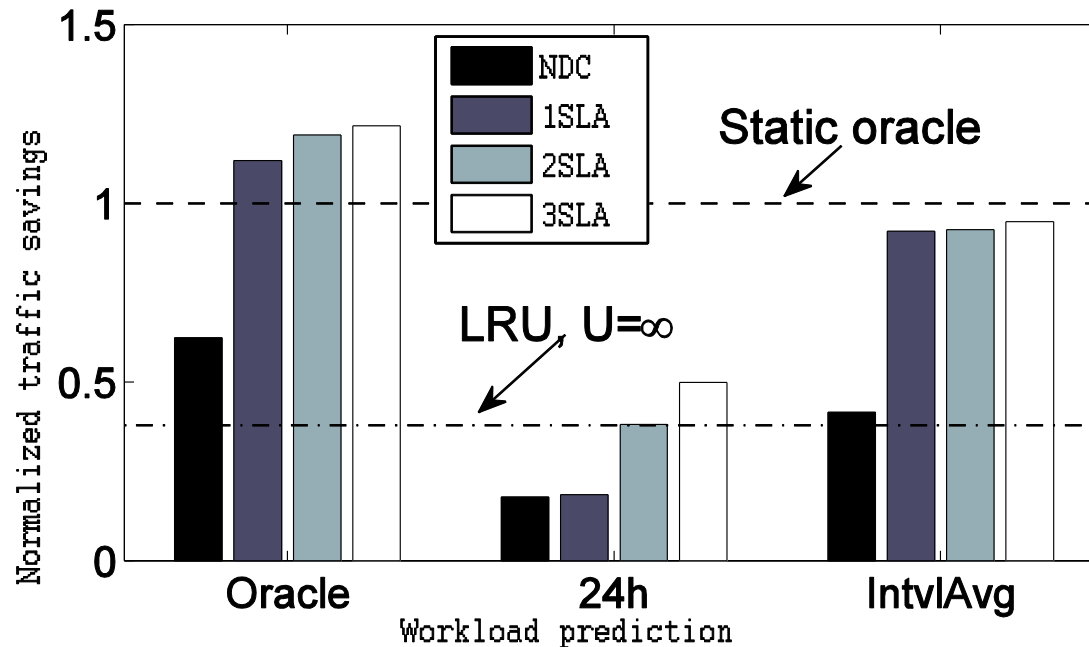
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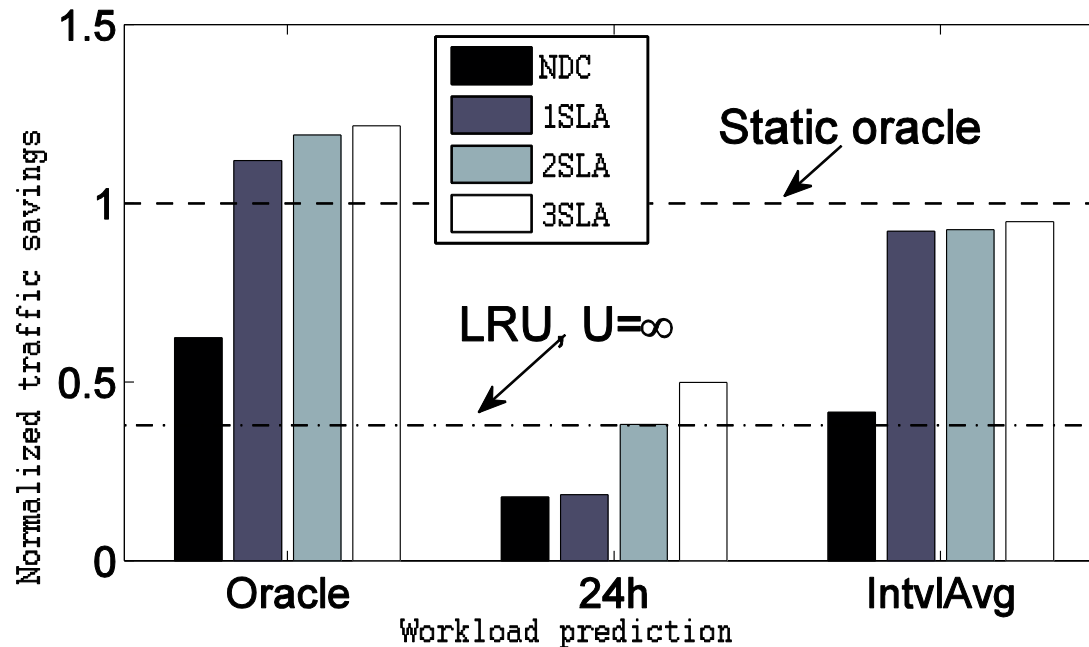
Trace-based Analysis

- Spotify traces (all requests for 1M random tracks; 1 week)
- Prediction policies: (i) “oracle”, (ii) 24h, (iii) interval average



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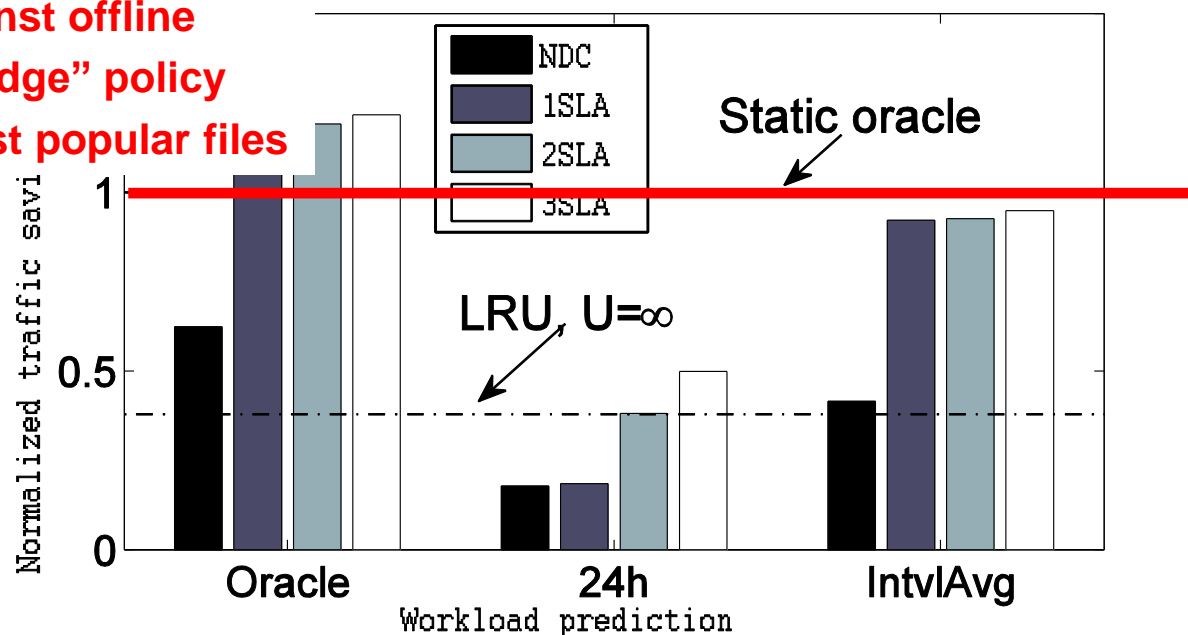


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- Dynamic allocation with k-SLA outperform LRU by far

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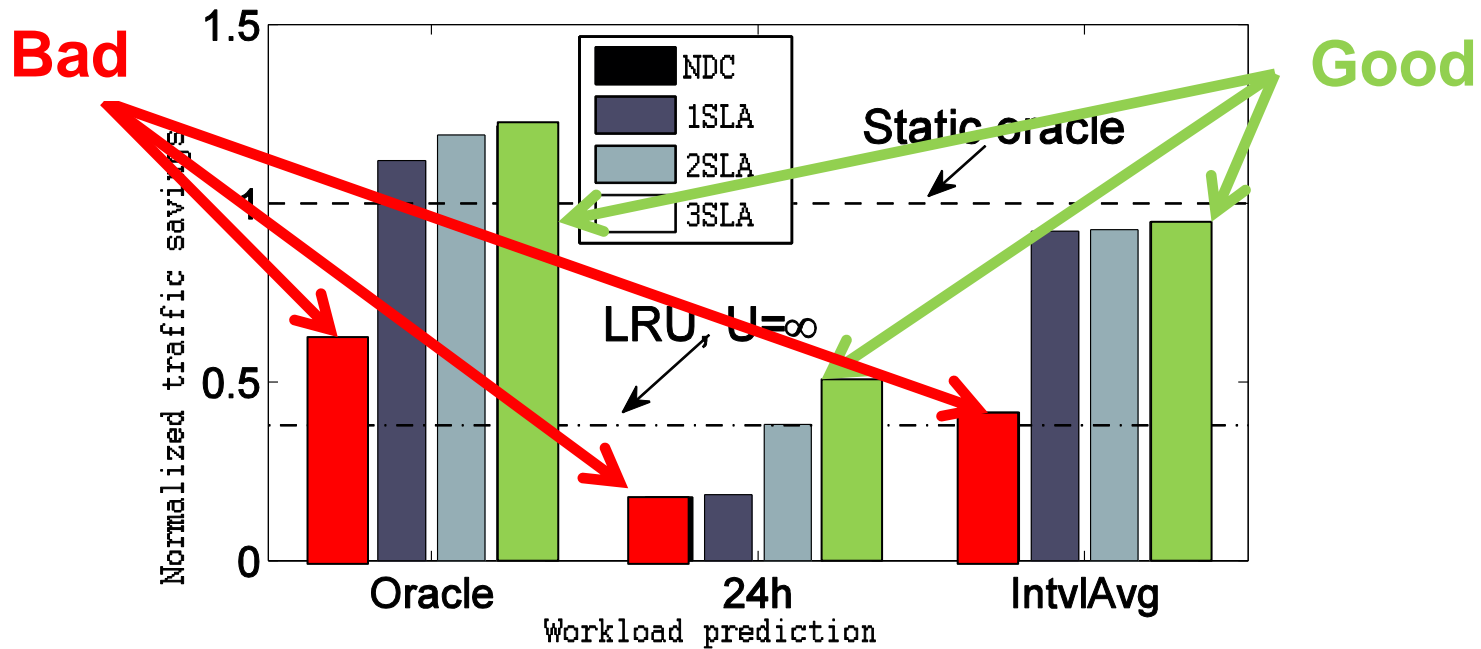
Normalize against offline
“global knowledge” policy
that stores most popular files



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Trace-based Analysis

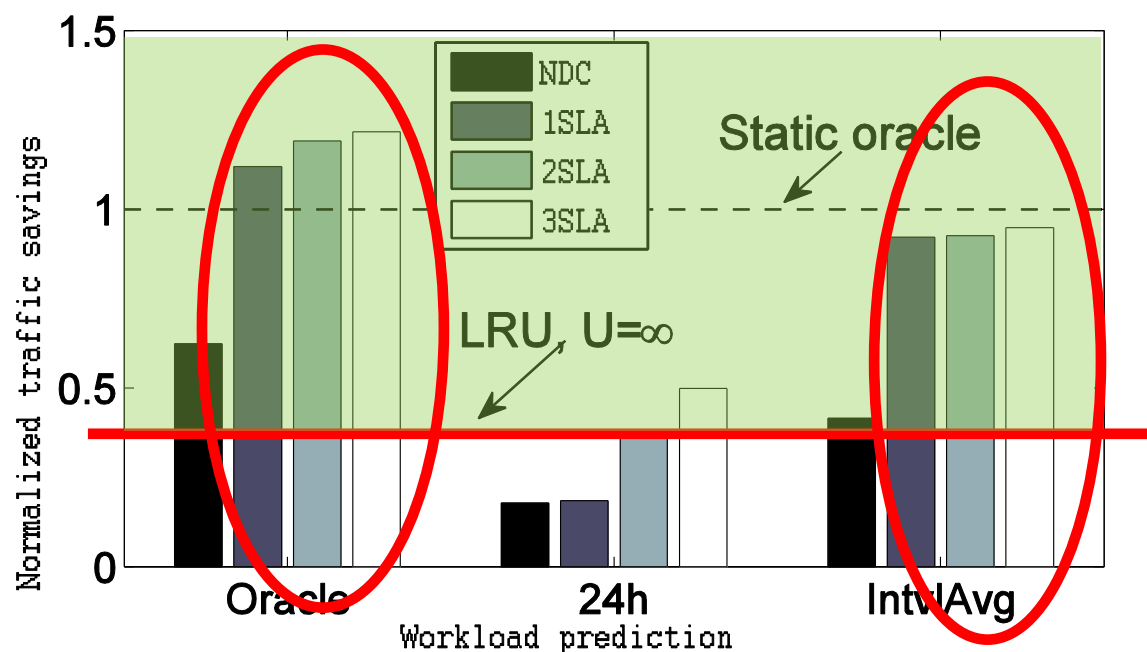
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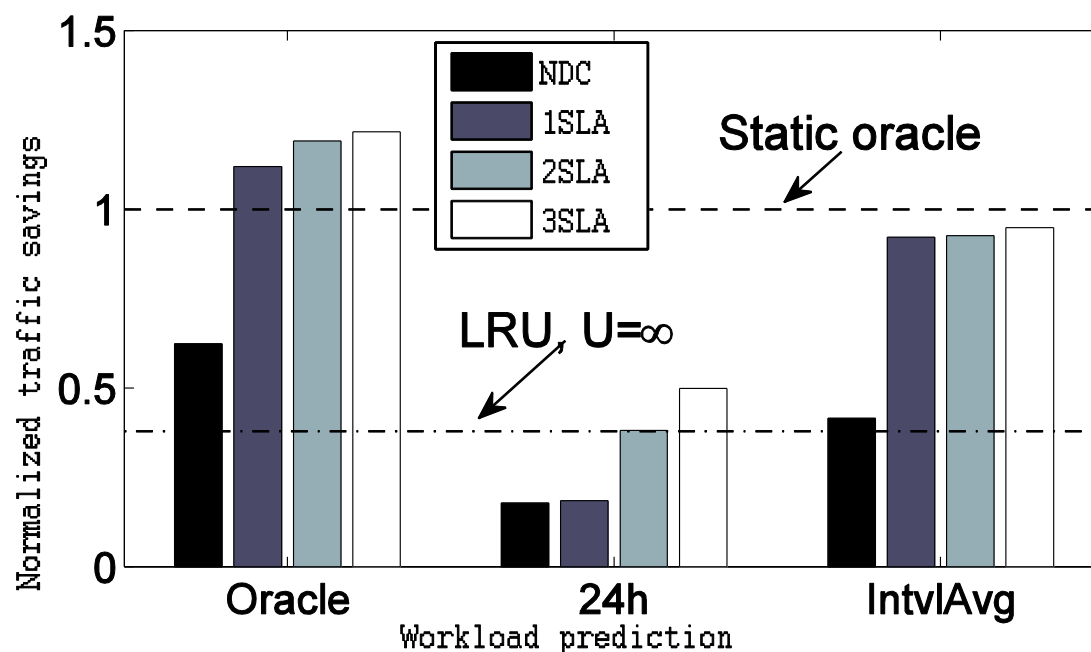
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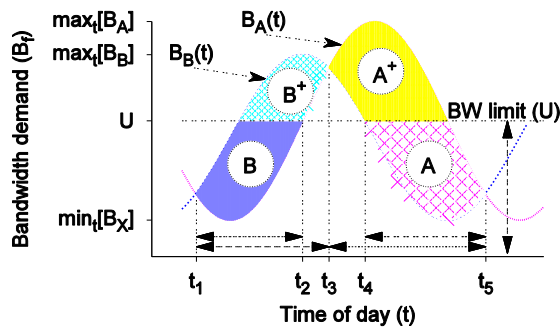
Trace-based Analysis

- Spotify traces (all requests for 1M random tracks; 1 week)
- Prediction policies: (i) “oracle”, (ii) 24h, (iii) interval average



- NDC fails; 3-SLA works reasonably well
- Dynamic allocation with k-SLA outperform LRU by far

Dynamic Content Allocation Problem



- Finite horizon dynamic decision problem
- Discrete mean-value approximation
- Exact solution as MILP
- Computationally feasible approximations (e.g., k-SLA) with performance bounds
- Validate model and policies using traces from Spotify

Dynamic Content Allocation for Cloud-assisted Service of Periodic Workloads

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Thank you!

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