We consider the three possible options below.

is feasible. Assume the contrary. Then, some of the conditions in Figure 1 cannot hold. Resulting connectivity component \( L \) are done. Let \( A \) \((\text{resp.} \ H) \) denote the rightmost connectivity component in \( O_H \) st some connectivity component to the left of \( L \) in \( O_H \) is to the right of the connectivity component immediately to the right of \( L \) in \( O_G \). Let \( R \) denote the connectivity component immediately to the right of \( L \) in \( O_G \). Note that \( R \) is to the left of \( L \) in \( O_H \). We first show that merging \( L \) and \( R \) in \( G \) is feasible. Assume the contrary. Then, some of the conditions in Figure 2 cannot hold. Note that \( \text{deg}(L) \cap \text{pa}_G(R) = \emptyset \). Then, condition 3 in Figure 2 holds. This leaves us with the following three options.

- **Condition 1 in Figure 2** does not hold because there are two nodes \( A \in \text{ch}_G(L) \cap R \) and \( B \in L \) st \( B \rightarrow A \) is not in \( G \) and \( B \leftrightarrow C \rightarrow A \) with \( C \in L \) is an induced subgraph of \( G \). Then, \( G \) does not have an unshielded collider between \( B \) and \( A \) over \( C \). Then, neither does \( H \) since \( G \) and \( H \) are Markov equivalent. However, \( R \) is to the left of \( L \) in \( O_H \), which means that \( B \leftrightarrow C \leftarrow A \) is an induced subgraph of \( H \). This is a contradiction.

- **Condition 1 in Figure 2** does not hold because there are two nodes \( A \in \text{ch}_G(L) \cap R \) and \( B \in \text{pa}_G(L) \) st \( B \rightarrow A \) is not in \( G \). Note that the previous bullet allows us to assume without loss of generality that \( L \subseteq \text{pa}_G(A) \). Then, \( B \rightarrow C \rightarrow A \) with \( C \in L \) is an induced subgraph of \( G \). Then, \( G \) does not have an unshielded collider between \( B \) and \( A \) over \( C \). Then, neither does \( H \) since \( G \) and \( H \) are Markov equivalent. However, \( R \) is to the left of \( L \) in \( O_H \), which means that \( B \rightarrow C \leftarrow A \) is an induced subgraph of \( H \). Note that \( B \rightarrow C \) is in \( H \) due to how \( L \) was selected. This is a contradiction.

- **Condition 2 in Figure 2** does not hold because there are three nodes \( B \in \text{pa}_G(R) \cap L \) and \( A, C \in \text{ch}_G(B) \cap R \) st \( A \leftrightarrow C \) is not in \( G \). Then, \( A \leftrightarrow B \rightarrow C \) is an induced subgraph of \( G \). Then, \( G \) does not have an unshielded collider between \( B \) and \( A \) over \( C \). Then, neither does \( H \) since \( G \) and \( H \) are Markov equivalent. However, \( R \) is to the left of \( L \) in \( O_H \), which means that \( A \rightarrow B \leftarrow C \) is an induced subgraph of \( H \). This is a contradiction.

In summary, merging \( L \) and \( R \) in \( G \) is feasible. Let us perform the merging and call the resulting connectivity component \( L \cup R \). We now show that splitting \( L \cup R \) in \( G \) into \( R \) and \( L \) is feasible. Assume the contrary. Then, some of the conditions in Figure 1 cannot hold. We consider the three possible options below.

- **Condition 1 in Figure 1** does not hold because there are two nodes \( A \in \text{sp}_G(R) \cap L \) and \( B \in R \) st \( B \leftrightarrow A \) is not in \( G \) and \( B \leftrightarrow C \leftrightarrow A \) with \( C \in R \) is an induced subgraph.

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of $G$. Then, $G$ has an unshielded collider between $B$ and $A$ over $C$. Then, so does $H$ since $G$ and $H$ are Markov equivalent. However, $R$ is to the left of $L$ in $O_H$, which means that $B \leftrightarrow C \rightarrow A$ is an induced subgraph of $H$. This is a contradiction.

- Condition 2 in Figure 1 does not hold because there are two nodes $A \in sp_G(R) \cap L$ and $B \in pa_G(R)$ st $B \rightarrow A$ is not in $G$. Note that the previous bullet allows us to assume without loss of generality that condition 1 in Figure 1 holds. Then, $B \rightarrow C \leftrightarrow A$ with $C \in R$ is an induced subgraph of $G$. Then, $G$ has an unshielded collider between $B$ and $A$ over $C$. Then, so does $H$ since $G$ and $H$ are Markov equivalent. However, $R$ is to the left of $L$ in $O_H$, which means that $B \rightarrow C \rightarrow A$ is an induced subgraph of $H$. This is a contradiction.

- Condition 3 in Figure 1 does not hold because there are three nodes $B \in sp_G(L) \cap R$ and $A, C \in sp_G(B) \cap L$ st $A \leftrightarrow C$ is not in $G$. Then, $A \leftrightarrow B \leftrightarrow C$ is an induced subgraph of $G$. Then, $G$ has an unshielded collider between $B$ and $A$ over $C$. Then, so does $H$ since $G$ and $H$ are Markov equivalent. However, $R$ is to the left of $L$ in $O_H$, which means that $A \leftarrow B \rightarrow C$ is an induced subgraph of $H$. This is a contradiction.

In summary, splitting $L \cup R$ in $G$ into $R$ and $L$ is feasible. Let us perform the split and restart the proof. This iterative process will end when $O_G = O_H$, which means that $G = H$.

Theorem 1. Let $G$ and $H$ denote two Markov equivalent CGs. Then, there is a sequence of feasible splits and mergings that transforms $G$ into $H$.

Proof. Recall from Lemma 1 in the main text that $G$ and $H$ can be transformed via two sequences of feasible splits into two Markov equivalent CGs $G'$ and $H'$ that have exactly the minimal set of bidirected edges for their Markov equivalence class. Note that feasible splits and mergings are inverse operations and, thus, there is a sequence of feasible mergings that transforms $H'$ into $H$. Now, if $G' = H'$ then we are done, else note that $G'$ can be transformed into $H'$ via a sequence of feasible splits and mergings by Lemma 1 above.