Automatic library generation

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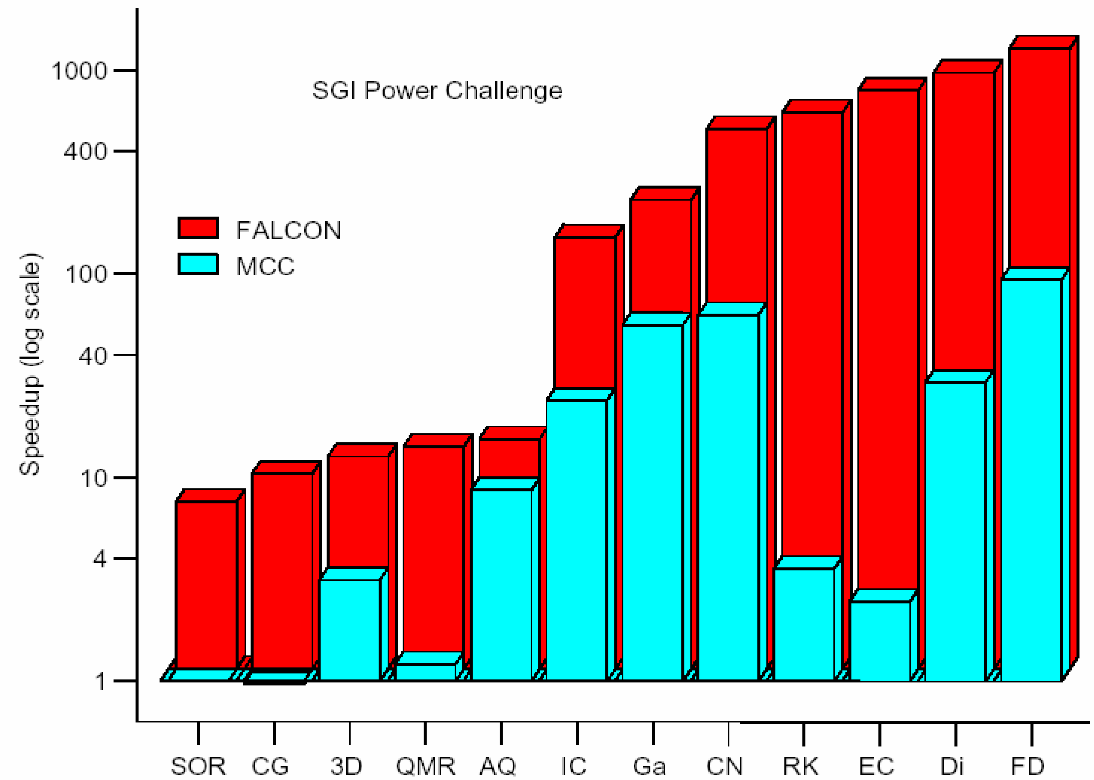
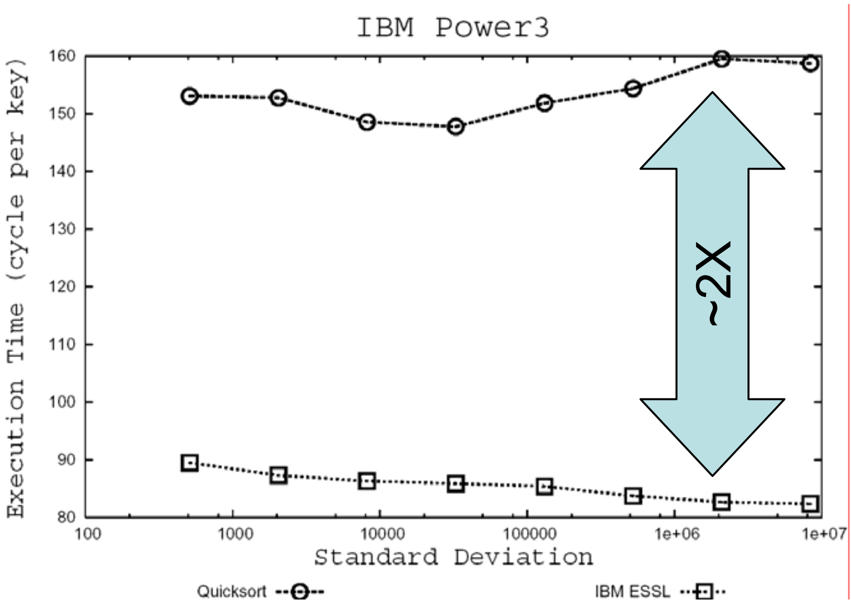
Libraries and Productivity

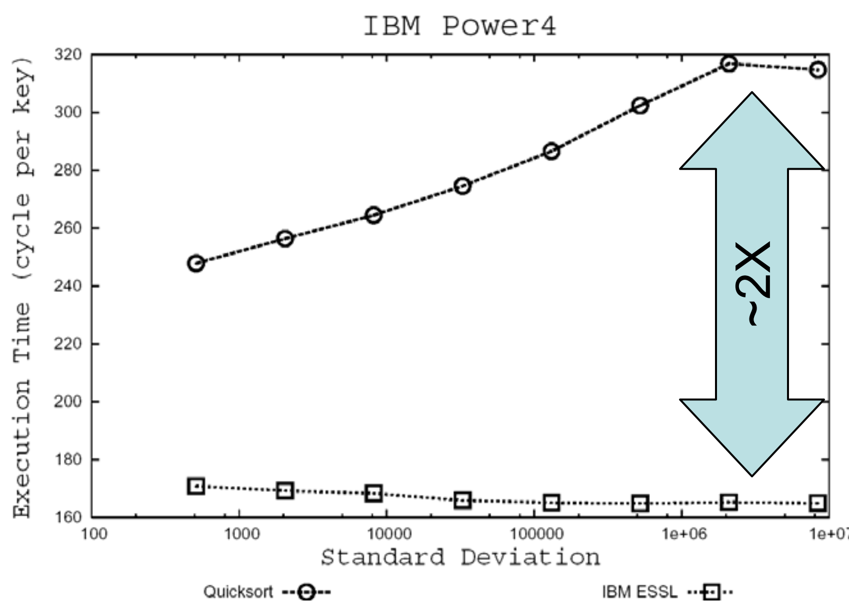
- Libraries help productivity.
- But not always.
  - Not all algorithms implemented.
  - Not all data structures.
- In any case, much effort goes into highly-tuned libraries.
- Automatic generation of libraries libraries would
  - Reduce cost of developing libraries
  - For a fixed cost, enable a wider range of implementations and thus make libraries more usable.

An Illustration based on MATLAB of the effect of libraries on performance

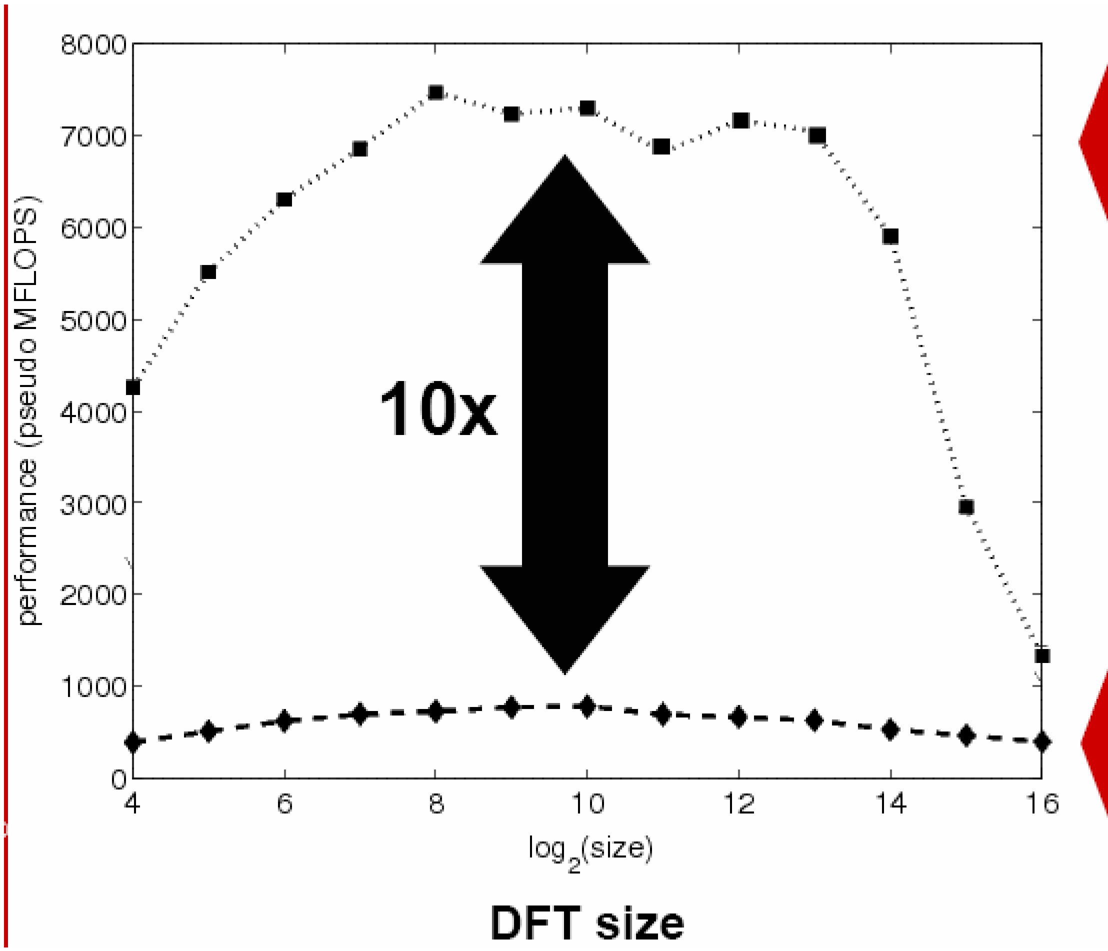


Compilers vs. Libraries in Sorting





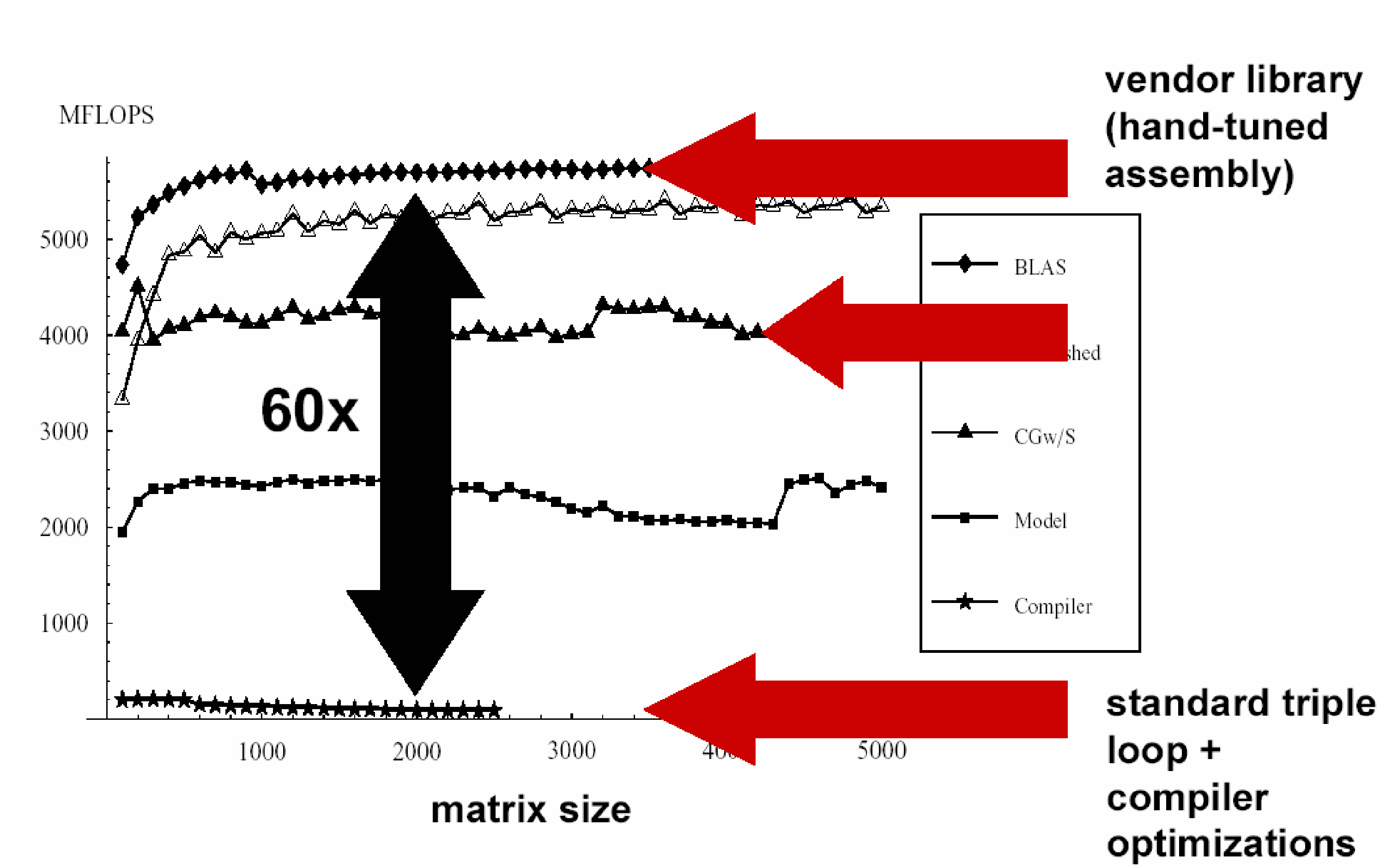
Compilers versus libraries in DFT



vendor library (hand-tuned assembly)
but also
FFTW (adaptable library)
SPIRAL (generated code)

reasonable implementation (Numerical recipes. GNU scientific library)

Compilers vs. Libraries in Matrix-Matrix Multiplication (MMM)



Library Generators

- Automatically generate highly efficient libraries for a class platforms.
- No need to manually tune the library to the architectural characteristics of a new machine.

Library Generators (Cont.)

- Examples:
  - In linear algebra: ATLAS, PhiPAC
  - In signal processing: FFTW, SPIRAL
- Library generators usually handle a fixed set of algorithms.
- Exception: SPIRAL accepts formulas and rewriting rules as input.

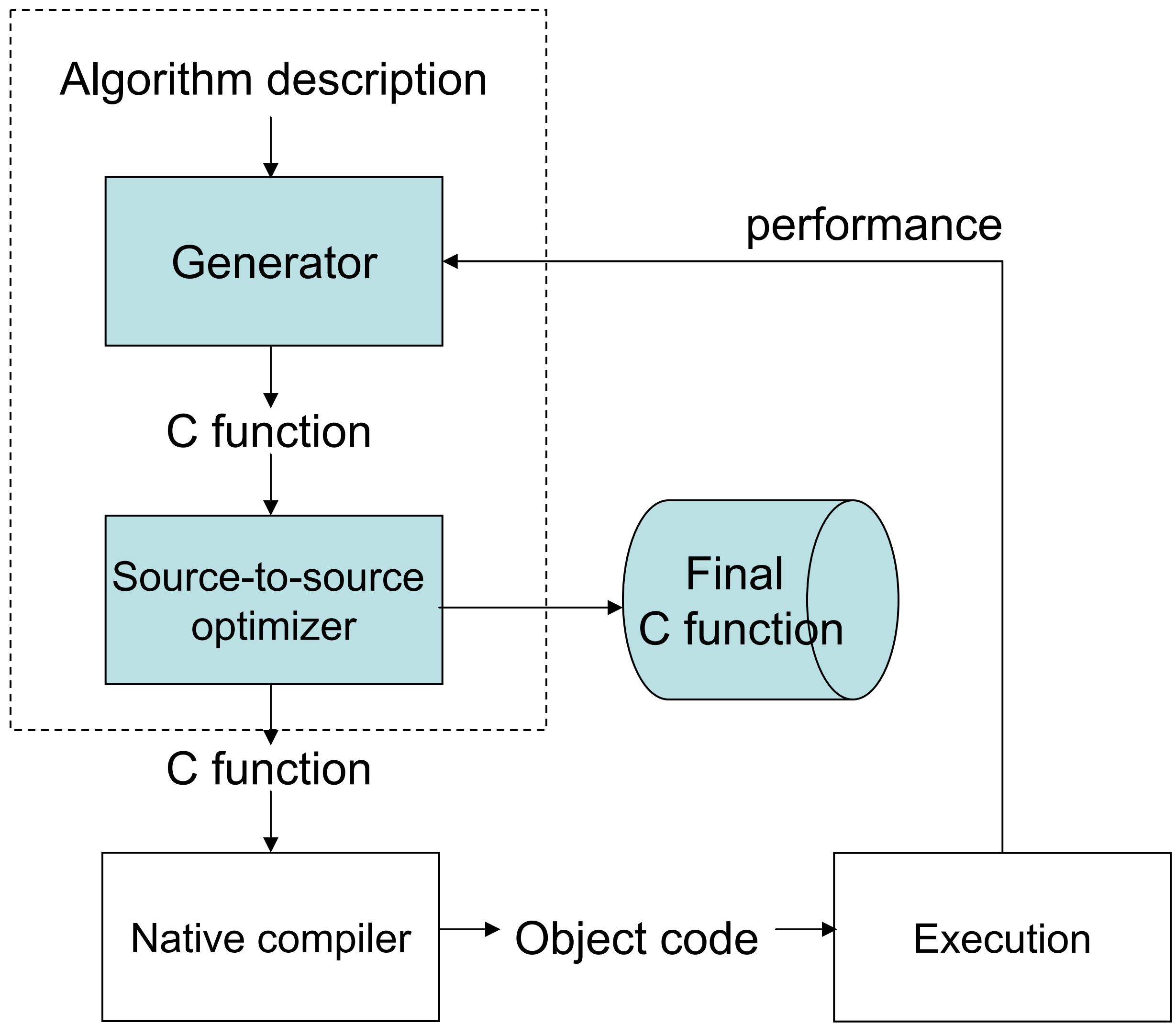
Library Generators (Cont.)

- At installation time, LGs apply empirical optimization.
  - That is, search for the best version in a set of different implementations
  - Number of versions astronomical. Heuristics are needed.

Library Generators (Cont.)

- LGs must output C code for portability.
- Unenven quality of compilers =>
  - Need for source-to-source optimizers
  - Or incorporate in search space variations introduced by optimizing compilers.

Library Generators (Cont.)



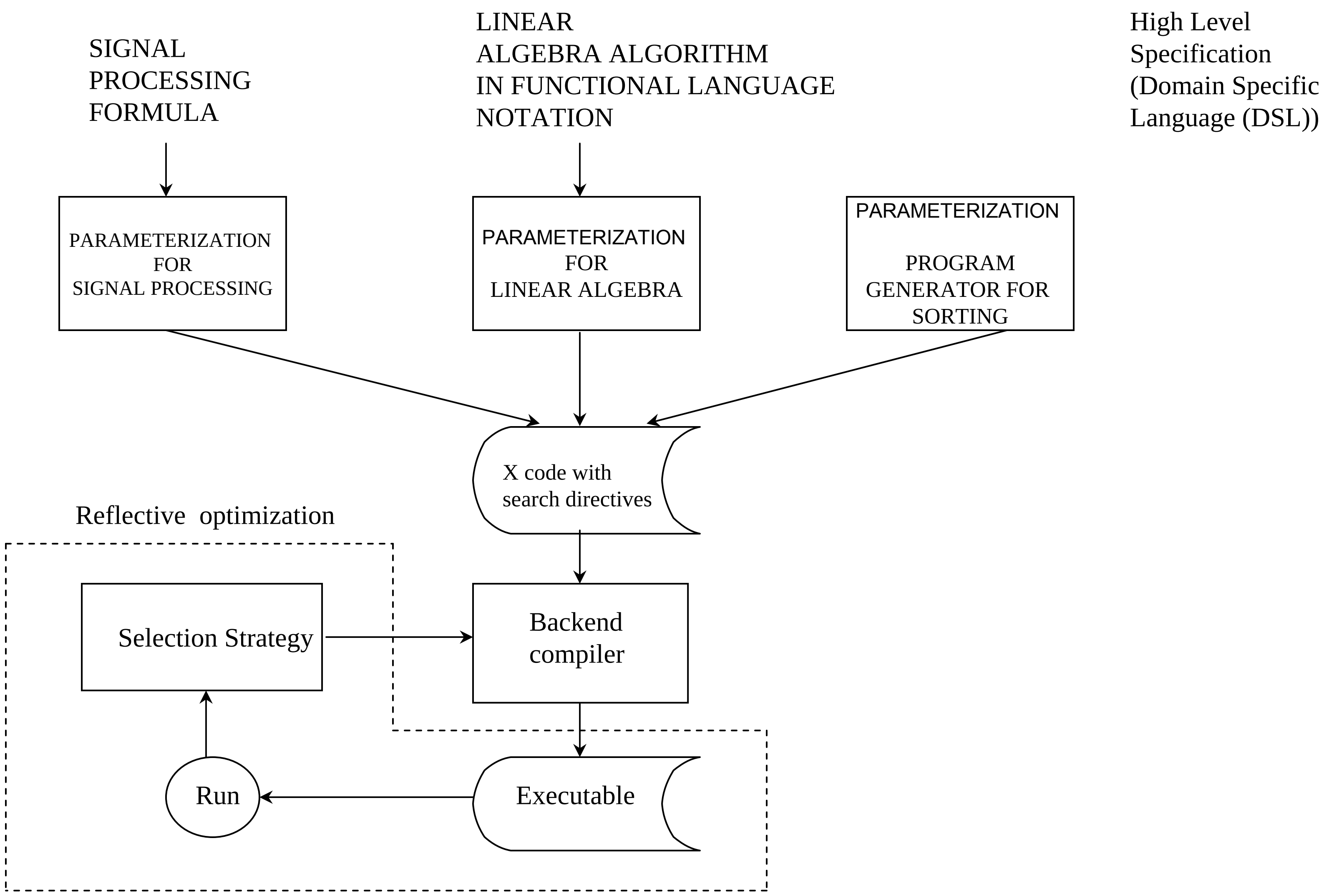
Important research issues

- Reduction of the search space with minimal impact on performance
- Adaptation to the input data (not needed for dense linear algebra)
- More flexible of generators
  - algorithms
  - data structures
  - classes of target machines
- Tools to build library generators.

Library generators and compilers

- LGs are a good yardstick for compilers
- Library generators use compilers.
- Compilers could use library generator techniques to optimize libraries in context.
- Search strategies could help design better compilers -
  - Optimization strategy: Most important open problem in compilers.

Organization of a library generation system



Three library generation projects

1. Spiral and the impact of compilers
2. ATLAS and analytical model
3. Sorting and adapting to the input

PROCEEDINGS OF THE IEEE

VOLUME 93, NUMBER 2 FEBRUARY 2005

Special Issue on:

PROGRAM GENERATION, OPTIMIZATION, AND PLATFORM ADAPTATION

Papers on:

Design & Implementation of FFTW3 • SPIRAL: Code Generation for DSP Transforms • Synthesis of Parallel Programs for *Ab Initio* Quantum Chemistry Models • Self-Adapting Linear Algebra Algorithms & Software • Parallel VSIPL++: An Open Standard for Parallel Signal Processing • Parallel MATLAB: Doing it Right • Broadway: Exploiting the Domain-Specific Semantics of Software Libraries • Is Search Really Necessary to Generate High-Performance BLAS? • Telescoping Languages: Automatic Generation of Domain Languages • Efficient Utilization of SIMD Extensions • Intelligent Monitoring for Adaptation in Grid Applications • Design & Engineering of a Dynamic Binary Optimizer • A Survey of Adaptive Optimization in Virtual Machines

plus

Scanning Our Past: Electrical Engineering Hall of Fame: Alexander Graham Bell

IEEE

Spiral: A code generator for digital signal processing transforms

Joint work with:

Jose Moura (CMU),

Markus Pueschel (CMU),

Manuela Veloso (CMU),

Jeremy Johnson (Drexel)

SPIRAL

- The approach:
  - Mathematical formulation of signal processing algorithms
  - Automatically generate algorithm versions
  - A generalization of the well-known FFTW
  - Use compiler technique to translate formulas into implementations
  - Adapt to the target platform by searching for the optimal version

DSP Algorithms: Example 4-point DFT

Cooley/Tukey FFT (size 4):

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fourier transform

Diagonal matrix (twiddles)

$$DFT_4 = (DFT_2 \otimes I_2) \cdot T_2^4 \cdot (I_2 \otimes DFT_2) \cdot L_2^4$$

Kronecker product

Identity

Permutation

- product of structured sparse matrices
- mathematical notation

Fast DSP Algorithms As Matrix Factorizations

- Computing $y = F_4 x$ is carried out as:

  $t_1 = A_4 x$ (permutation)

  $t_2 = A_3 t_1$ (two F_2's)

  $t_3 = A_2 t_2$ (diagonal scaling)

  $y = A_1 t_3$ (two F_2's)
- The cost is reduced because A_1, A_2, A_3 and A_4 are structured sparse matrices.

General Tensor Product Formulation

Theorem

$$F_{rs} = (F_r \otimes I_s) T_s^{rs} (I_r \otimes F_s) L_r^{rs}$$

T_s^{rs} is a diagonal matrix

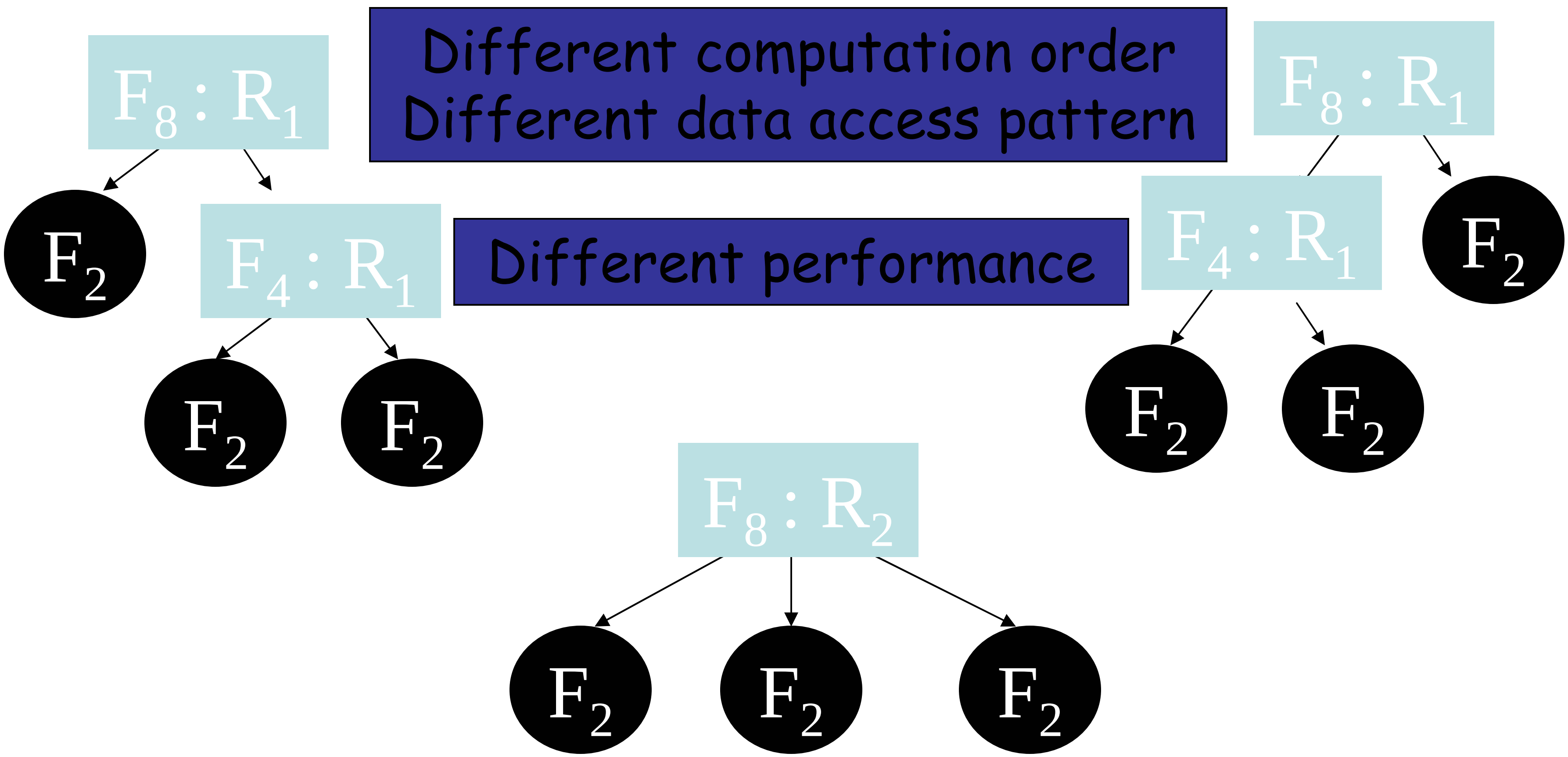
L_r^{rs} is a stride permutation

Example

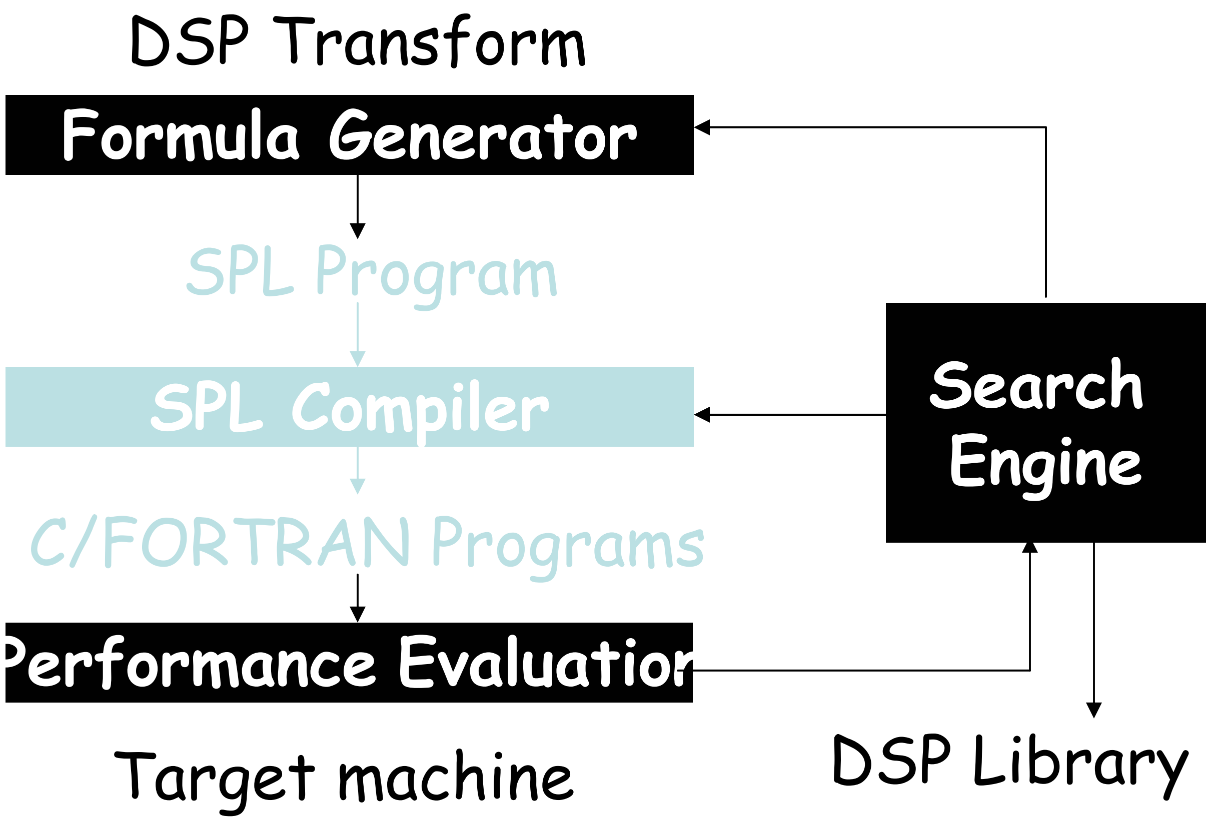
$$F_4 = (F_2 \otimes I_2) T_4^4 (I_2 \otimes F_2) L_2^4$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

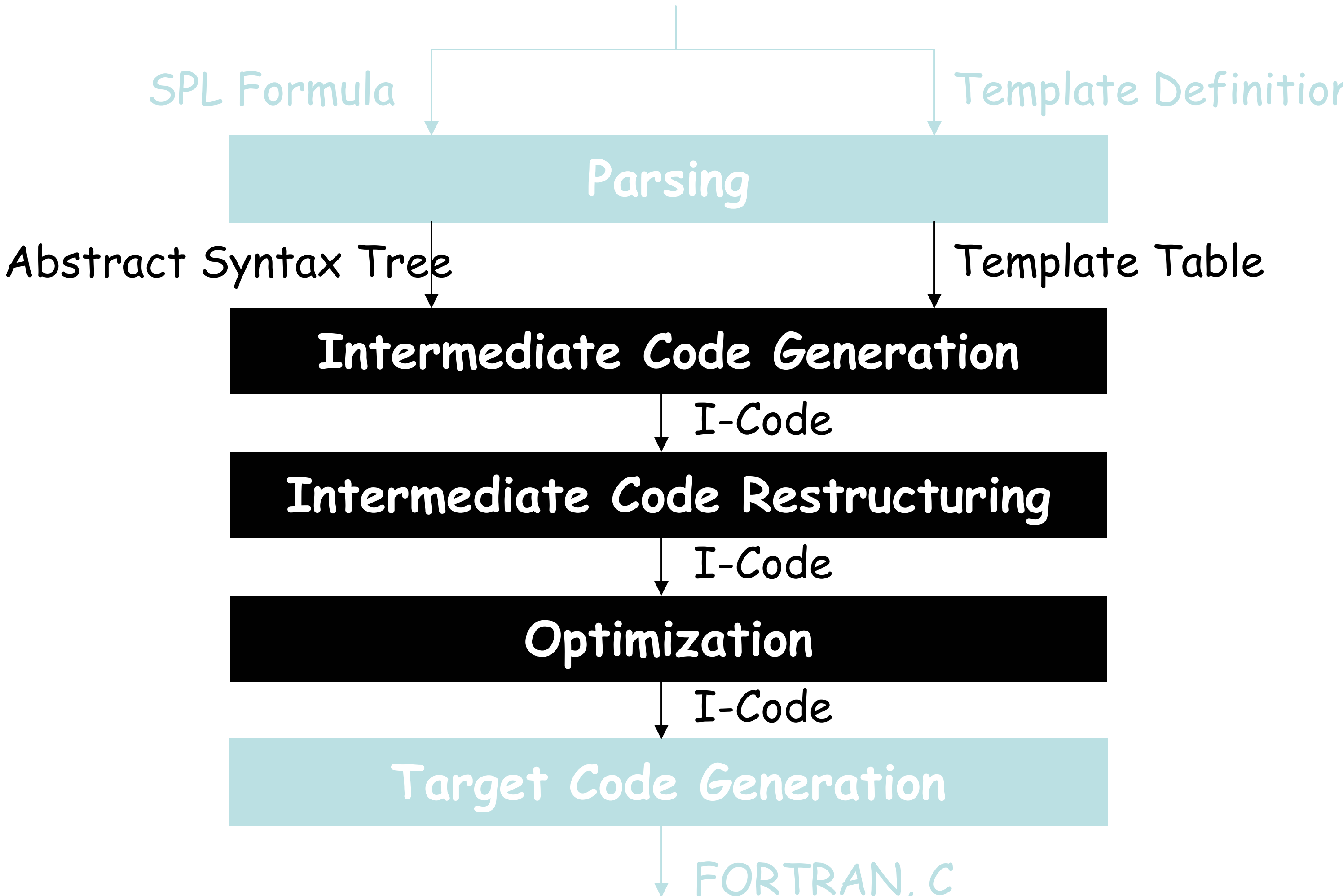
Factorization Trees



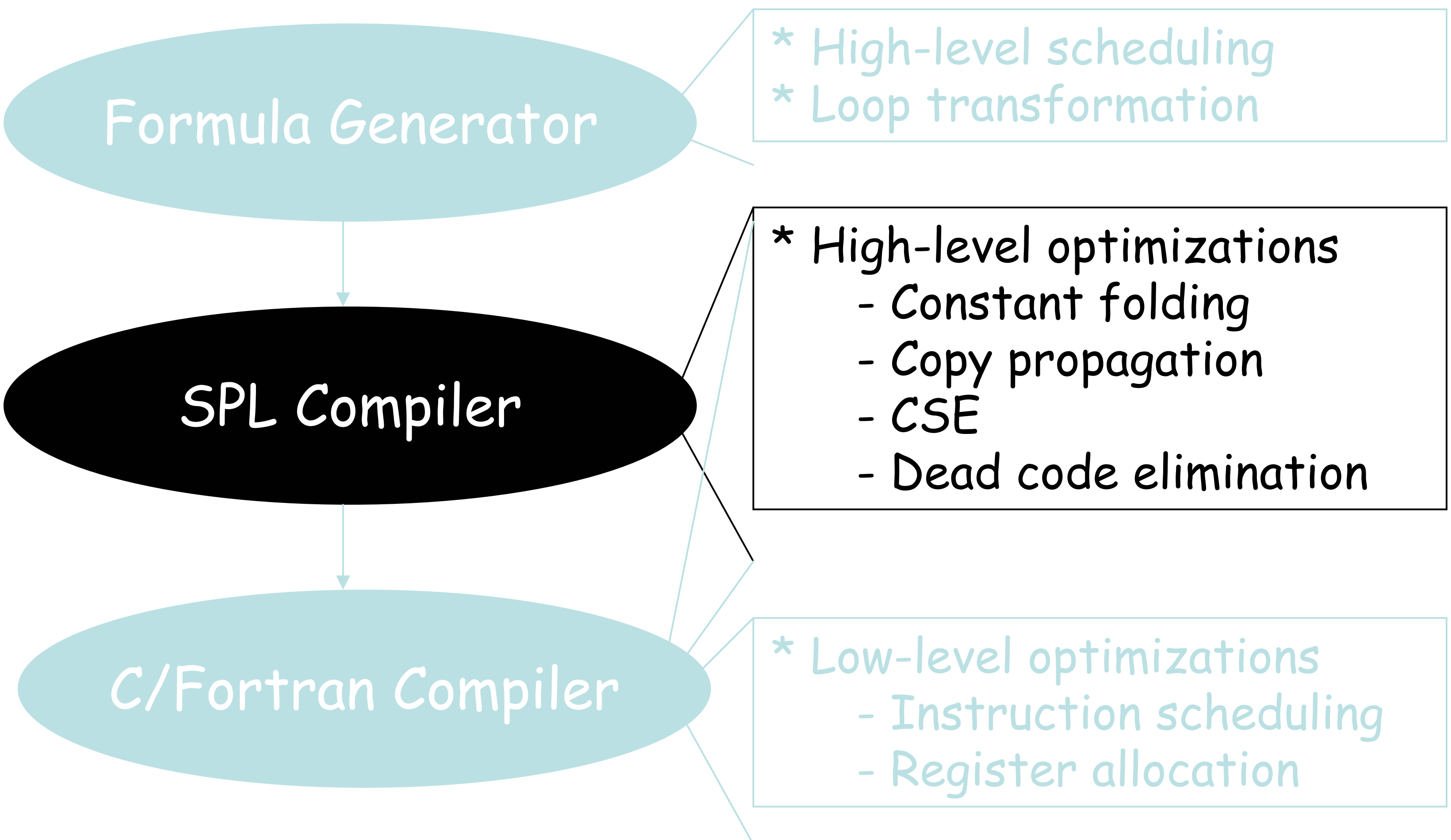
The SPIRAL System



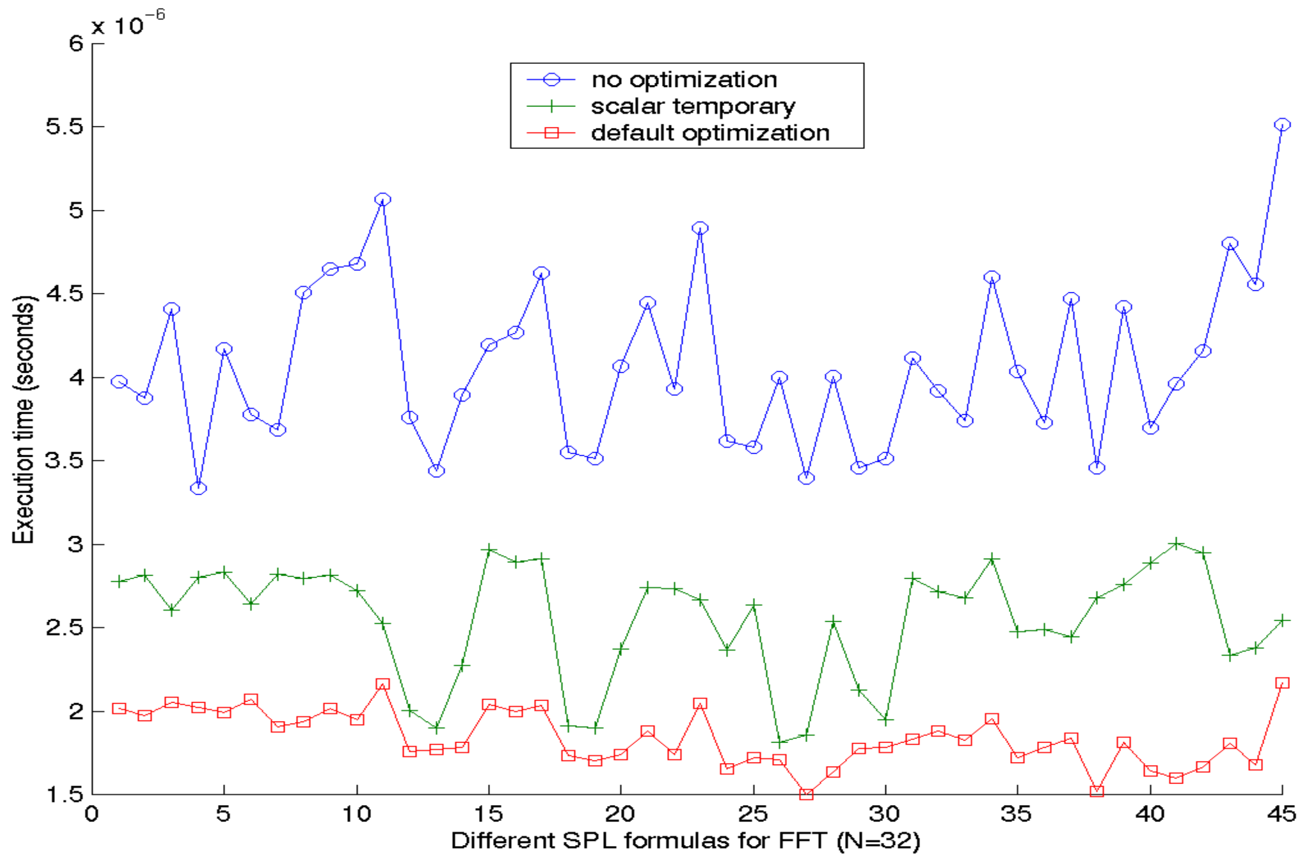
SPL Compiler



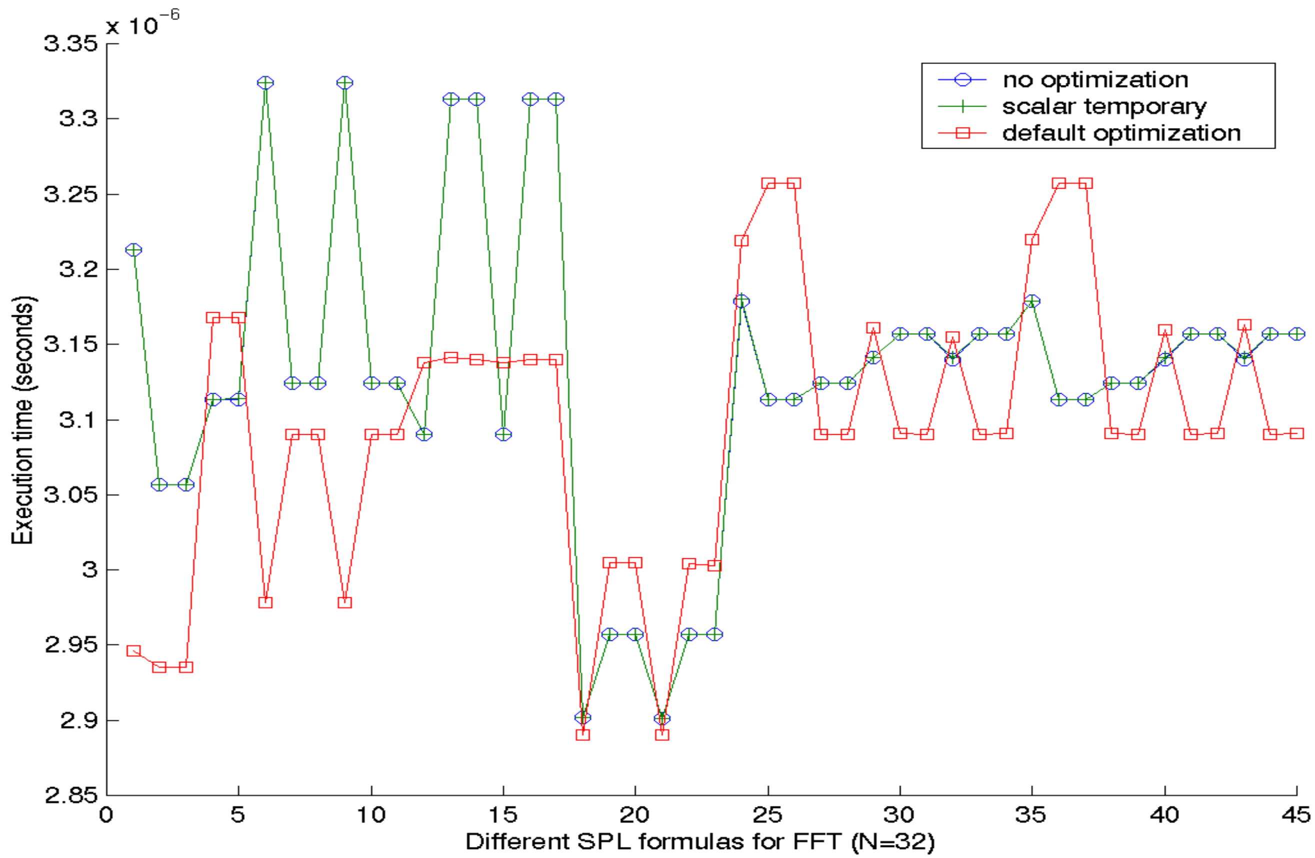
Optimizations



Basic Optimizations
(FFT, N=2^5, SPARC, f77 -fast -O5)

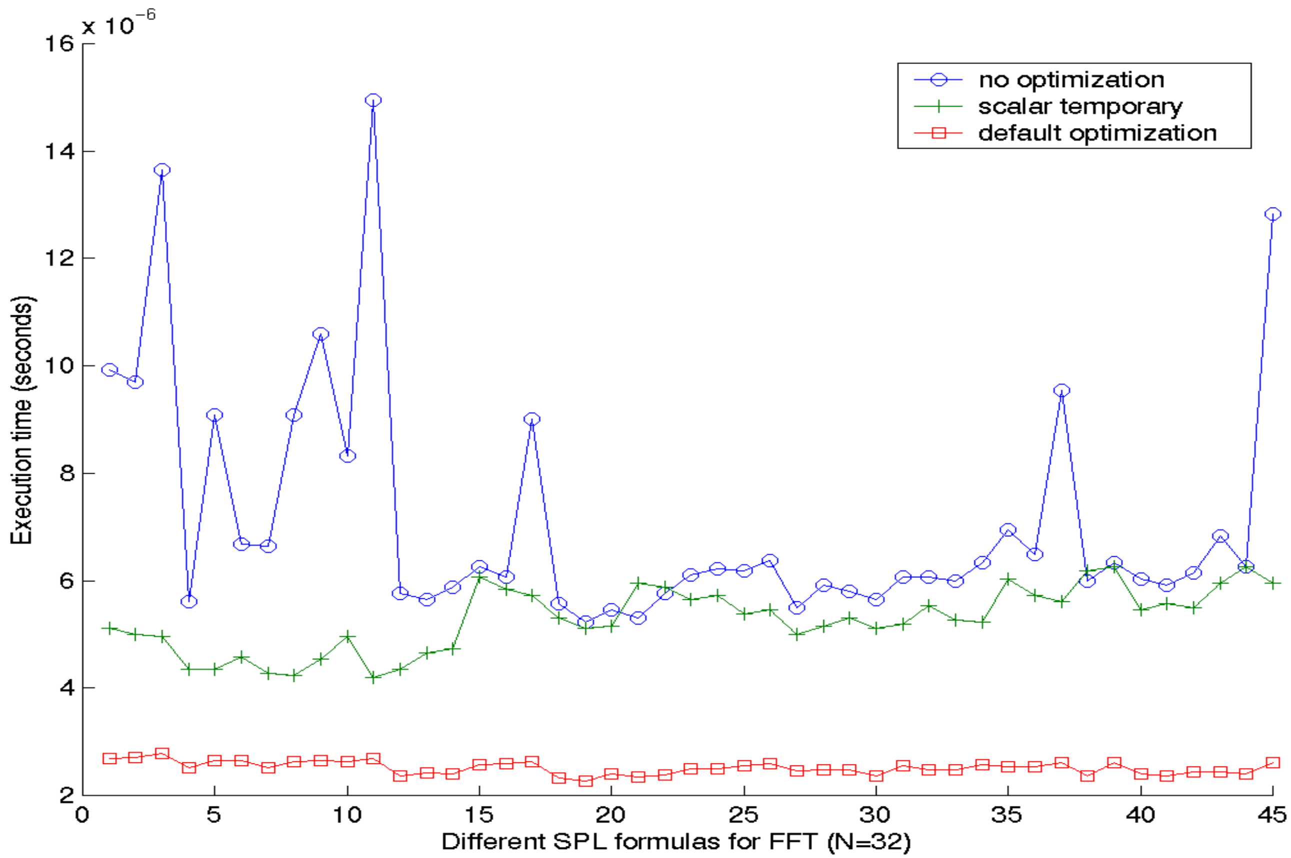


Basic Optimizations
(FFT, N=2^5, MIPS, f77 -O3)

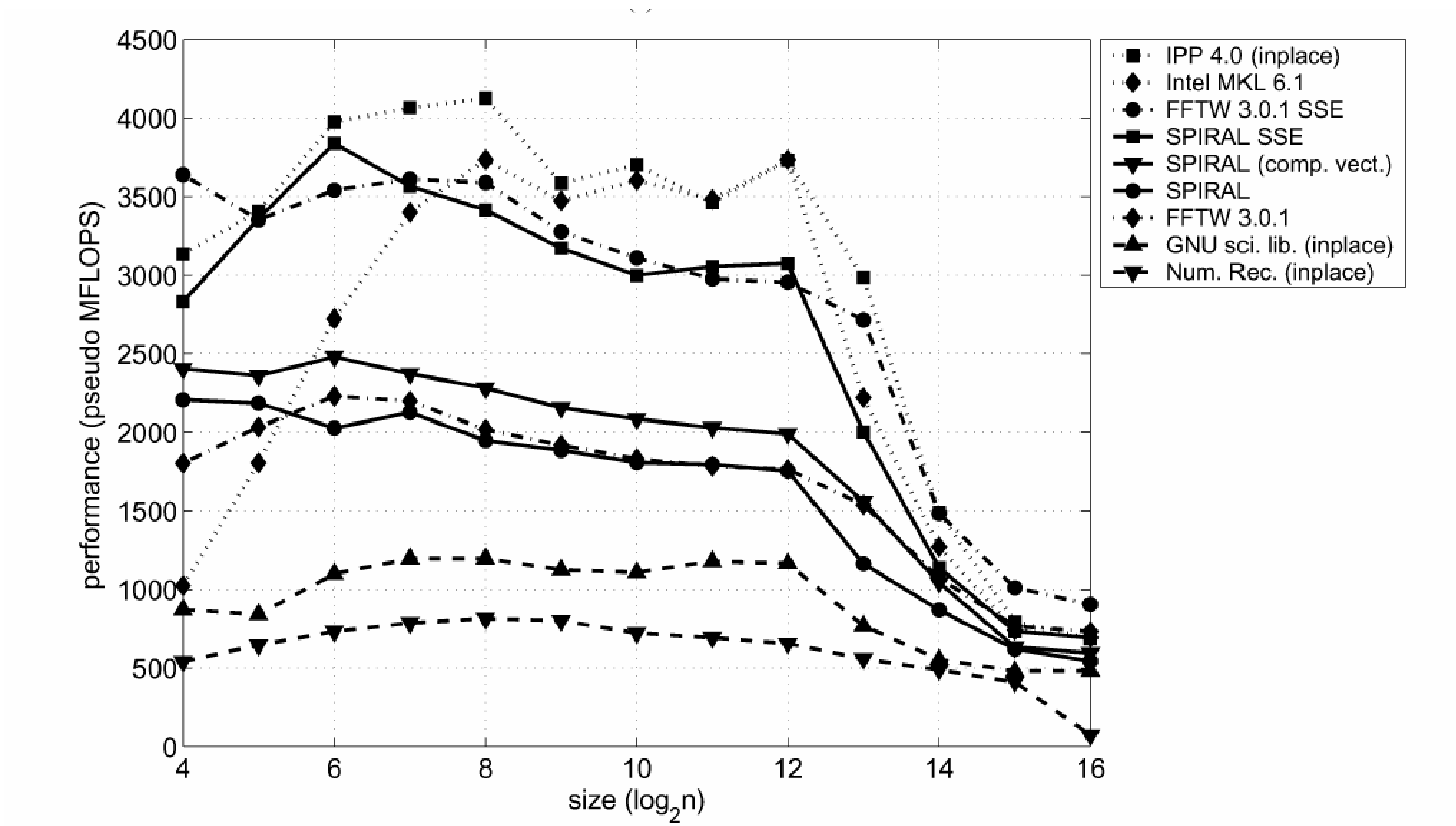


Basic Optimizations

(FFT, N=2^5, PII, g77 -O6 -malign-double)



Overall performance



An analytical model for ATLAS

Joint work with
Keshav Pingali (Cornell)
Gerald DeJong
Maria Garzaran

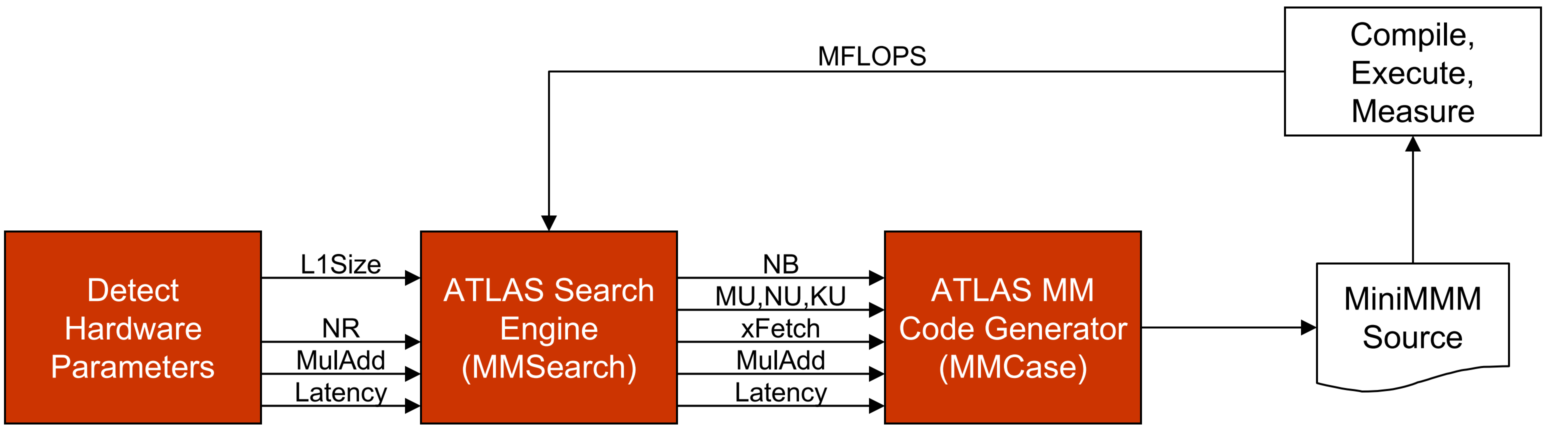
ATLAS

- ATLAS = Automated Tuned Linear Algebra Software,

  developed by R. Clint Whaley, Antoine Petite and Jack Dongarra, at the University of Tennessee.

- ATLAS uses empirical search to automatically generate highly-tuned Basic Linear Algebra Libraries (BLAS).
  - Use search to adapt to the target machine

ATLAS Infrastructure

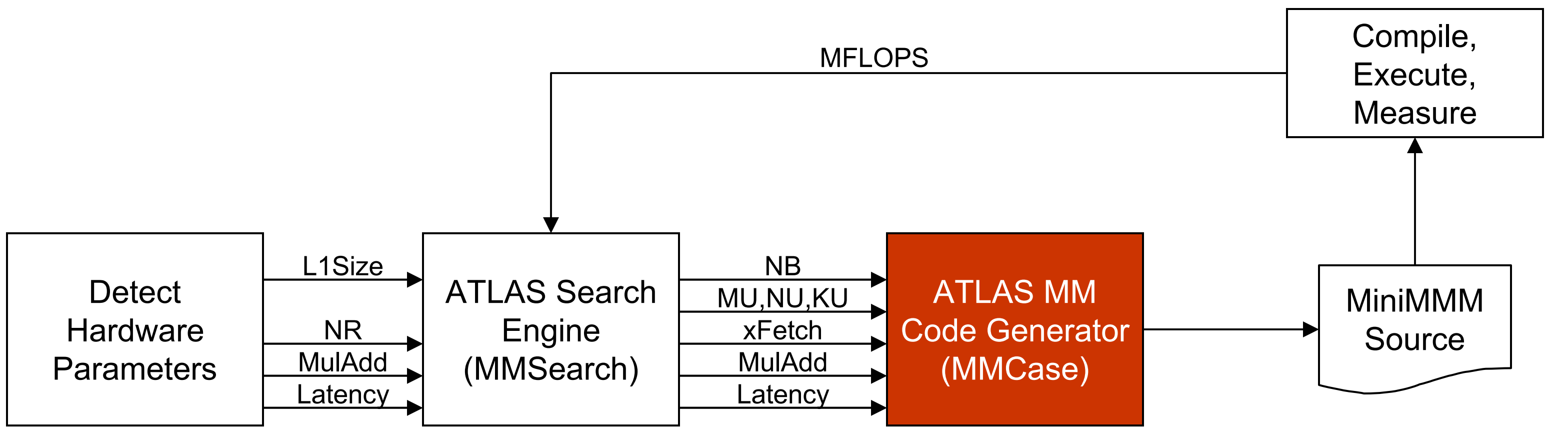


Detecting Machine Parameters

- Micro-benchmarks
  - L1Size: L1 Data Cache size
    - Similar to Hennessy-Patterson book
  - NR: Number of registers
    - Use several FP temporaries repeatedly
  - MulAdd: Fused Multiply Add (FMA)
    - “c+=a*b” as opposed to “c+=t; t=a*b”
  - Latency: Latency of FP Multiplication
    - Needed for scheduling multiplies and adds in the absence of FMA

Compiler View

- ATLAS Code Generation



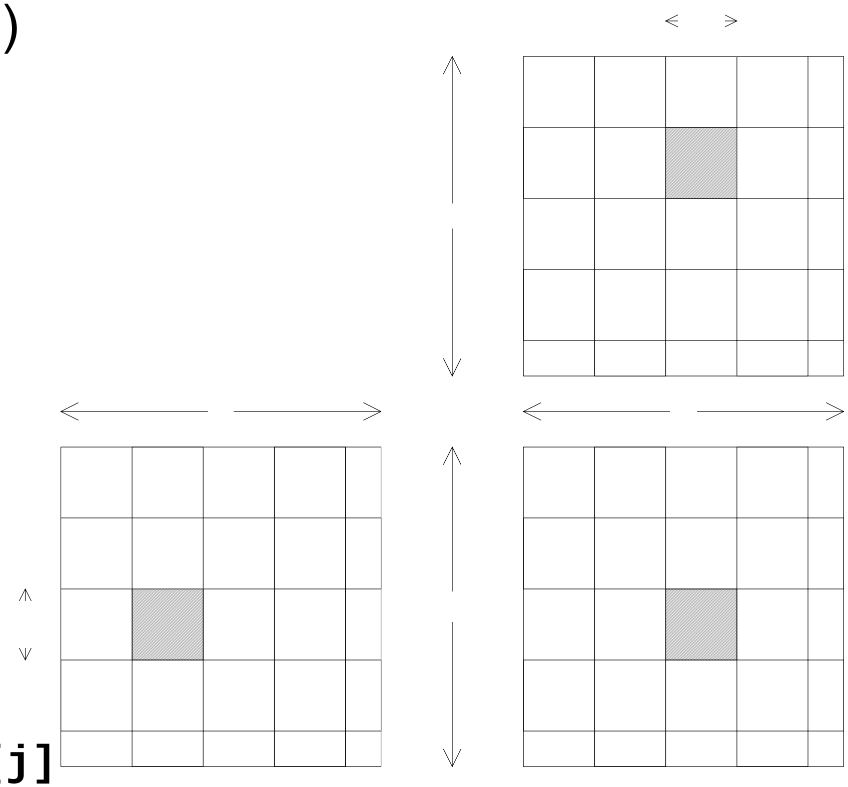
- Focus on MMM (as part of BLAS-3)
  - Very good reuse $O(N^2)$ data, $O(N^3)$ computation
  - No “real” dependecies (only input / reuse ones)

Adaptations/Optimizations

- Cache-level blocking (tiling)
  - Atlas blocks only for L1 cache
- Register-level blocking
  - Highest level of memory hierarchy
  - Important to hold array values in registers
- Software pipelining
  - Unroll and schedule operations
- Versioning
  - Dynamically decide which way to compute

Cache-level blocking (tiling)

- Tiling in ATLAS
  - Only square tiles (NBxNBxNB)
  - Working set of tile fits in L1
  - Tiles are usually copied to continuous storage
  - Special “clean-up” code generated for bounderies



- Mini-MMM

```
for (int j = 0; j < NB; j++)
 for (int i = 0; i < NB; i++)
  for (int k = 0; k < NB; k++)
    C[i][j] += A[i][k] * B[k][j]
```

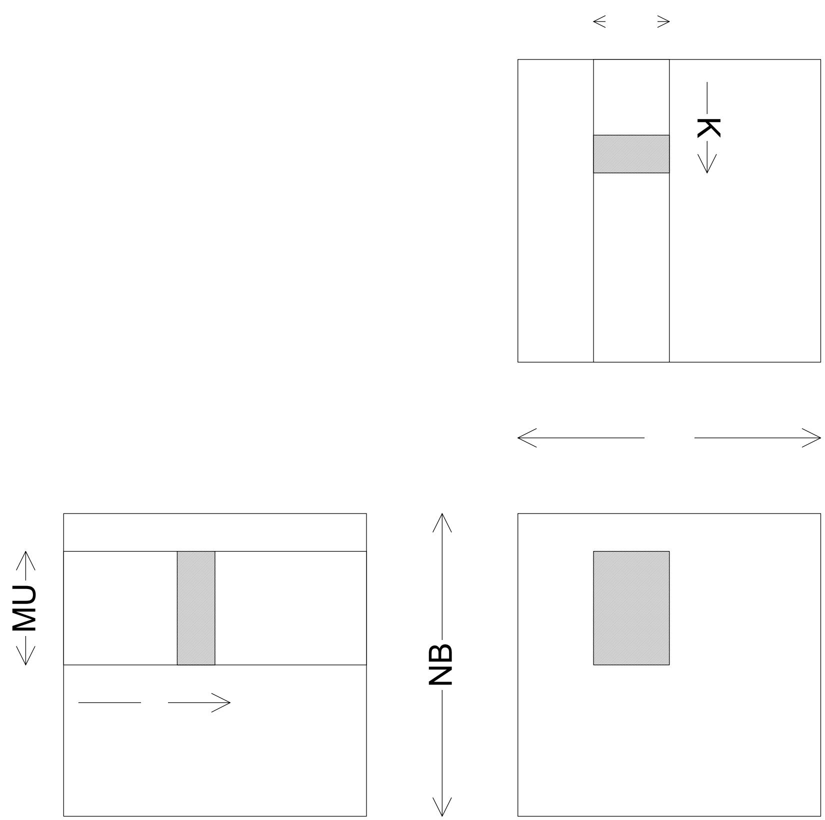
- NB: Optimization parameter

Register-level blocking

- Micro-MMM
  - MUx1 elements of A
  - 1xNU elements of B
  - MUxNU sub-matrix of C
  - MU*NU + MU + NU ≤ NR
- Mini-MMM revised

```
for (int j = 0; j < NB; j += NU)
  for (int i = 0; i < NB; i += MU)
    load C[i..i+MU-1, j..j+NU-1] into
    registers
    for (int k = 0; k < NB; k++)
       load A[i..i+MU-1,k] into registers
       load B[k,j..j+NU-1] into registers
       multiply A's and B's and add to C's
    store C[i..i+MU-1, j..j+NU-1]
```

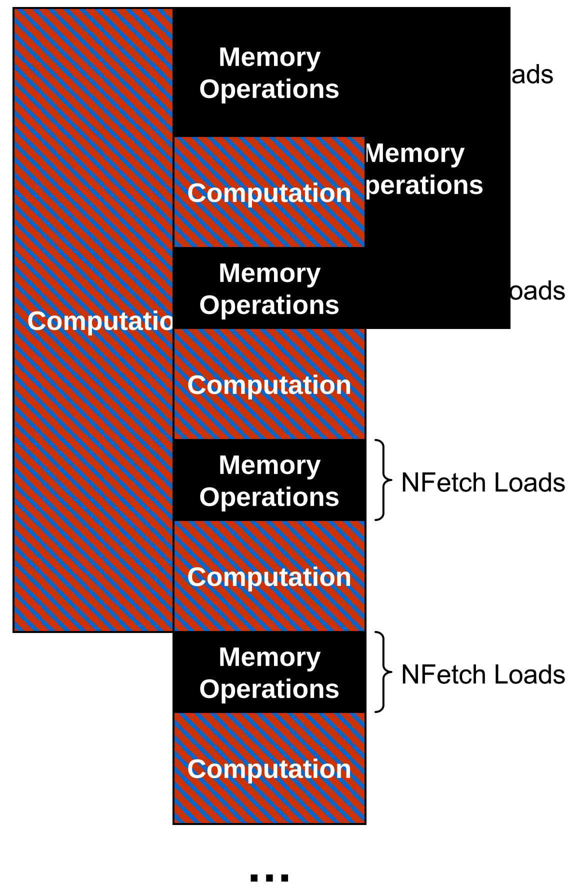
- Unroll K look KU times
- MU, NU, KU: optimization parameters



Scheduling

- FMA Present?
- Schedule Computation
  - Using Latency
- Schedule Memory Operations
  - Using FFetch, IFetch, NFetch
- Mini-MMM revised

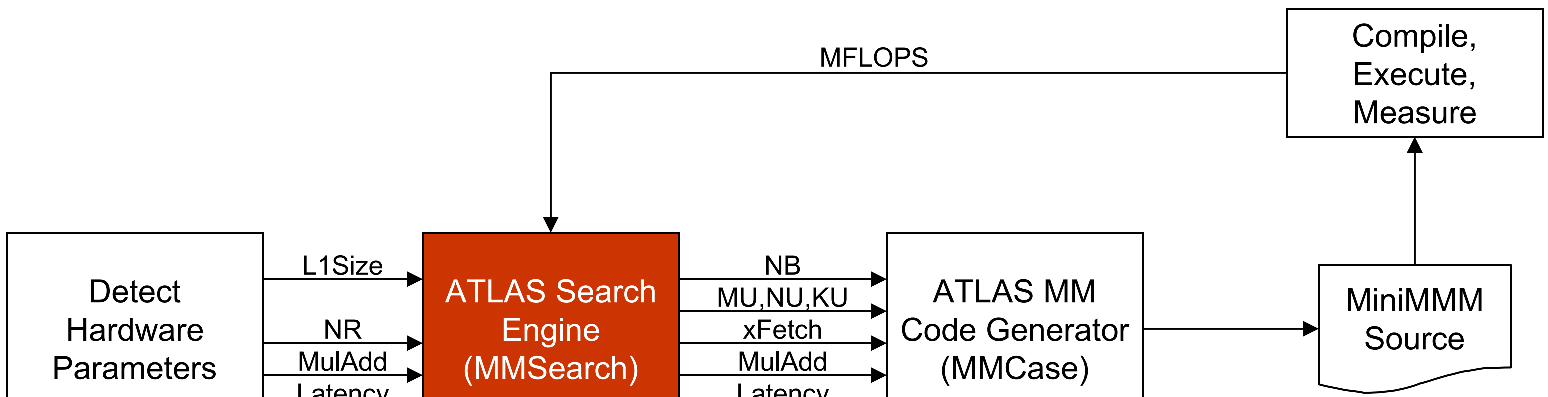
```
for (int j = 0; j < NB; j += NU)
  for (int i = 0; i < NB; i += MU)
   load C[i..i+MU-1, j..j+NU-1] into
   registers
KU times
   for (int k = 0; k < NB; k += KU)
      load A[i..i+MU-1,k] into registers
      load B[k,j..j+NU-1] into registers
      multiply A's and B's and add to C's
      ...
      load A[i..i+MU-1,k+KU-1] into
   registers
      load B[k+KU-1,j..j+NU-1] into
   registers
      multiply A's and B's and add to C's
       store C[i..i+MU-1, j..j+NU-1]
```



- Latency, xFetch: optimization parameters

Searching for Optimization Parameters

- ATLAS Search Engine



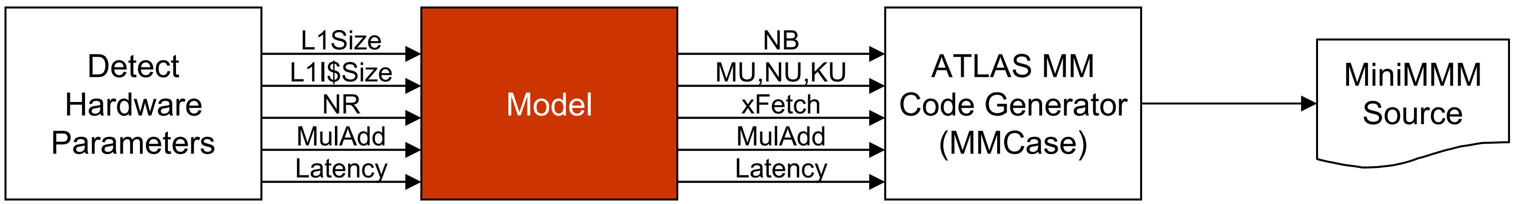
- Multi-dimensional search problem
  - Optimization parameters are independent variables
  - MFLOPS is the dependent variable
  - Function is implicit but can be repeatedly evaluated

Search Strategy

- Orthogonal Range Search
  - Optimize along one dimension at a time, using reference values for not-yet-optimized parameters
  - Not guaranteed to find optimal point
  - Input
    - Order in which dimensions are optimized
      - NB, MU & NU, KU, xFetch, Latency
    - Interval in which search is done in each dimension
      - For NB it is $16 \leq NB \leq \min(\sqrt{L1Size}, 80)$, step 4
    - Reference values for not-yet-optimized dimensions
      - Reference values for KU during NB search are 1 and NB

Modeling for Optimization Parameters

- Our Modeling Engine



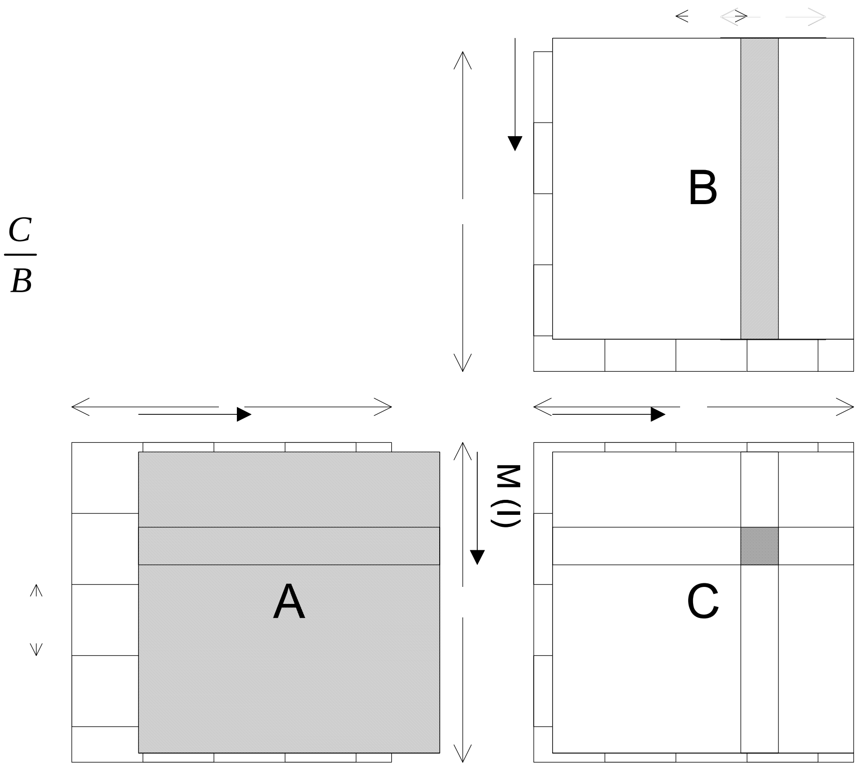
$$MU * NU + MU + NU + Latency \leq NR$$

- Optimization parameters
  - NB: Hierarchy of Models (later)
  - MU, NU:
  - KU: maximize subject to L1 Instruction Cache
  - Latency, MulAdd: from hardware parameters
  - xFetch: set to 2

Modeling for Tile Size (NB)

- Models of increasing complexity
  - $3*NB^2 \leq C$
    - Whole work-set fits in L1
  - $NB^2 + NB + 1 \leq C$
    - Fully Associative
    - Optimal Replacement
    - Line Size: 1 word
  - $\left\lceil \frac{NB^2}{B} \right\rceil + \left\lceil \frac{NB}{B} \right\rceil + 1 \leq \frac{C}{B}$ or $\left\lceil \frac{NB^2}{B} \right\rceil + NB + 1 \leq \frac{C}{B}$
    - Line Size > 1 word
  - $\left\lceil \frac{NB^2}{B} \right\rceil + 2\left\lceil \frac{NB}{B} \right\rceil + \left(\left\lceil \frac{NB}{B} \right\rceil + 1 \right) \leq \frac{C}{B}$ or

    $\left\lceil \frac{NB^2}{B} \right\rceil + 3NB + 1 \leq \frac{C}{B}$
    - LRU Replacement



Experiments

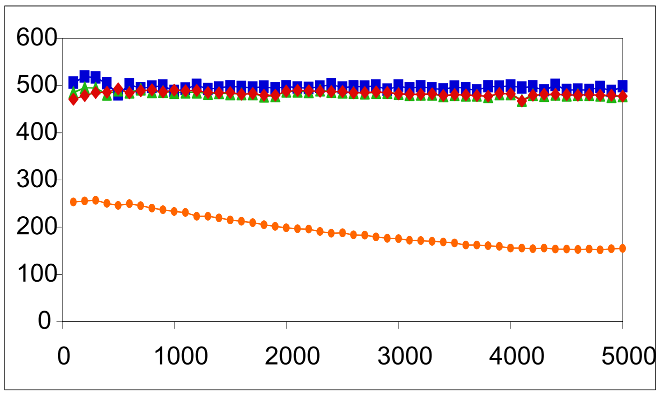
- Architectures:
  - SGI R12000, 270MHz
  - Sun UltraSPARC III, 900MHz
  - Intel Pentium III, 550MHz
- Measure
  - Mini-MMM performance
  - Complete MMM performance
  - Sensitivity to variations on parameters

MiniMMM Performance

- SGI
  - ATLAS: 457 MFLOPS
  - Model: 453 MFLOPS
  - Difference: 1%
- Sun
  - ATLAS: 1287 MFLOPS
  - Model: 1052 MFLOPS
  - Difference: 20%
- Intel
  - ATLAS: 394 MFLOPS
  - Model: 384 MFLOPS
  - Difference: 2%

MMM Performance

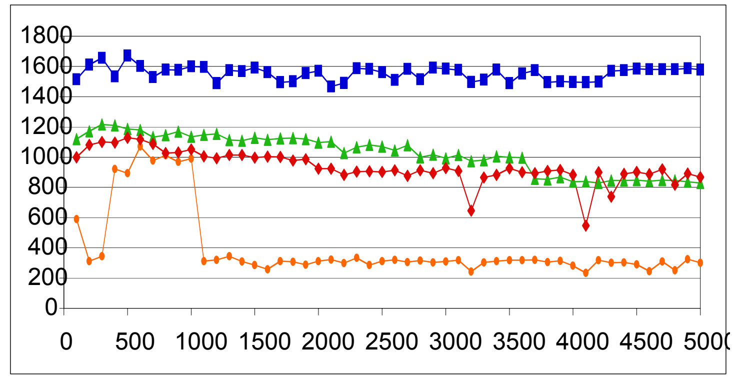
- SGI



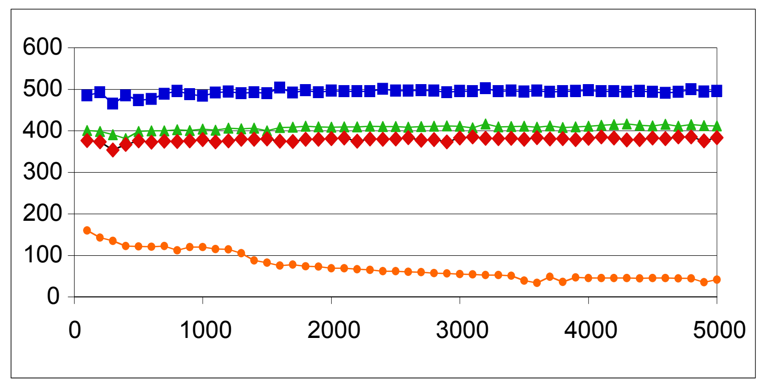
BLAS COMPILER

ATLAS MODEL

- Sun

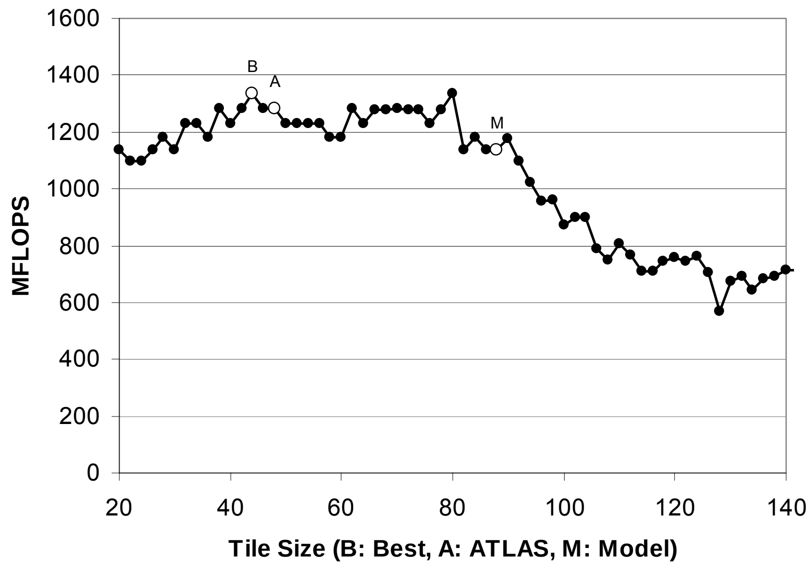


- Intel

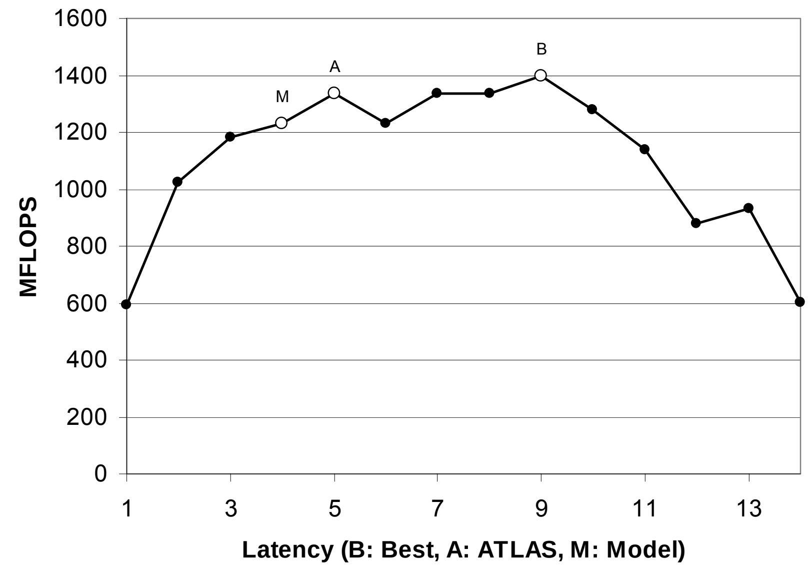


Sensitivity to NB and Latency on Sun

- Tile Size (NB)

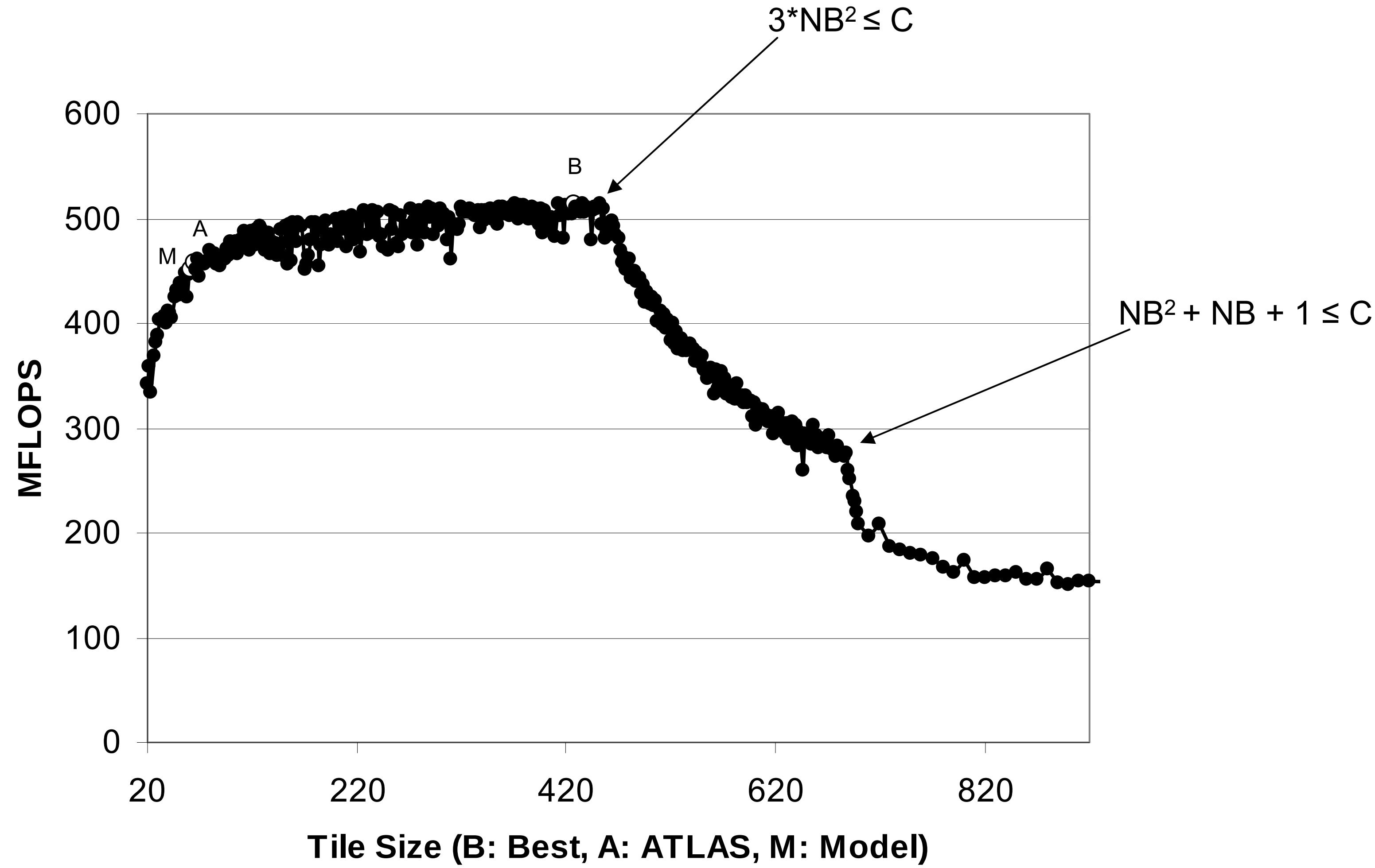


- Latency



- MU & NU, KU, Latency, xFetch for all architectures

Sensitivity to NB on SGI



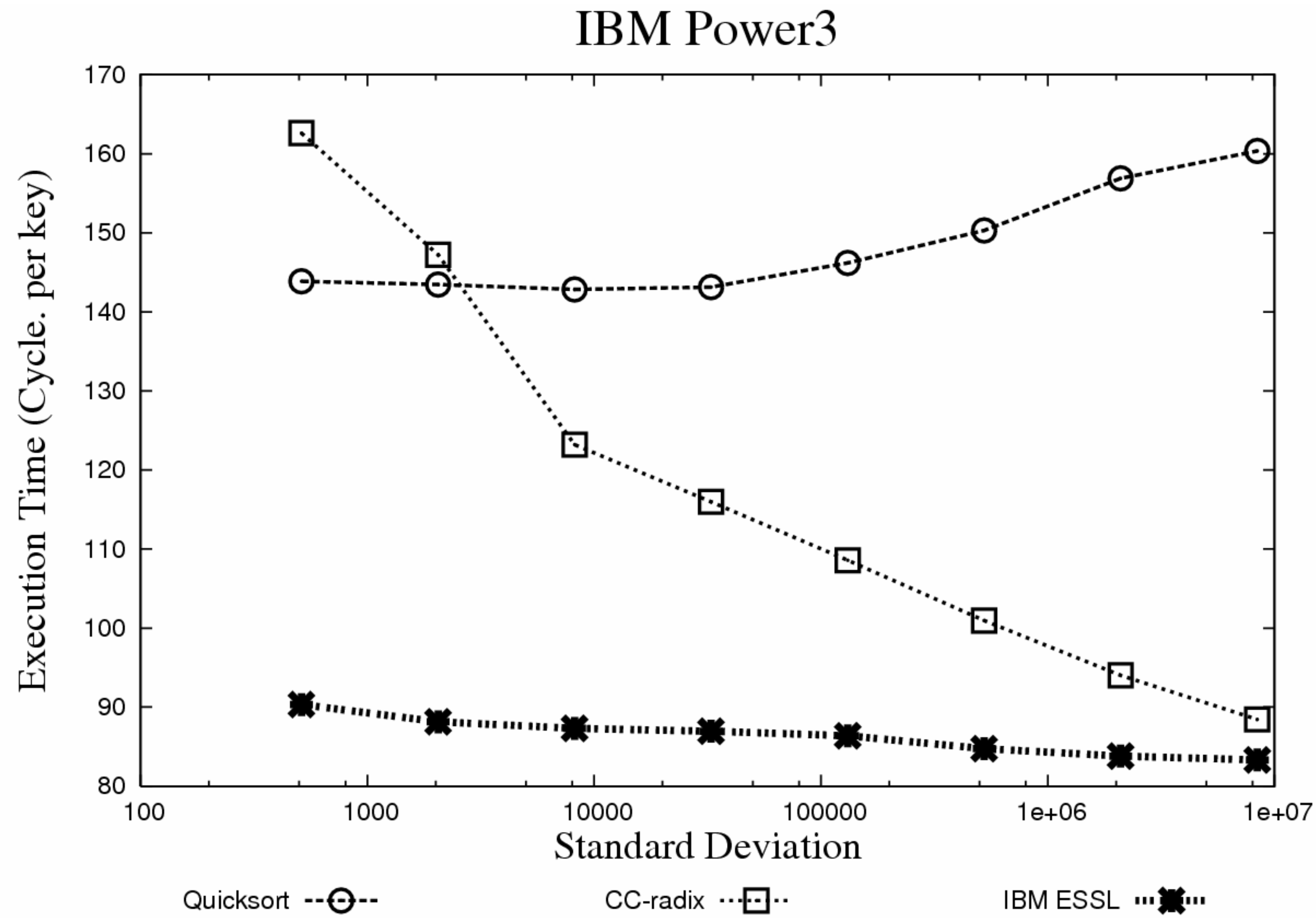
Sorting

Joint work with

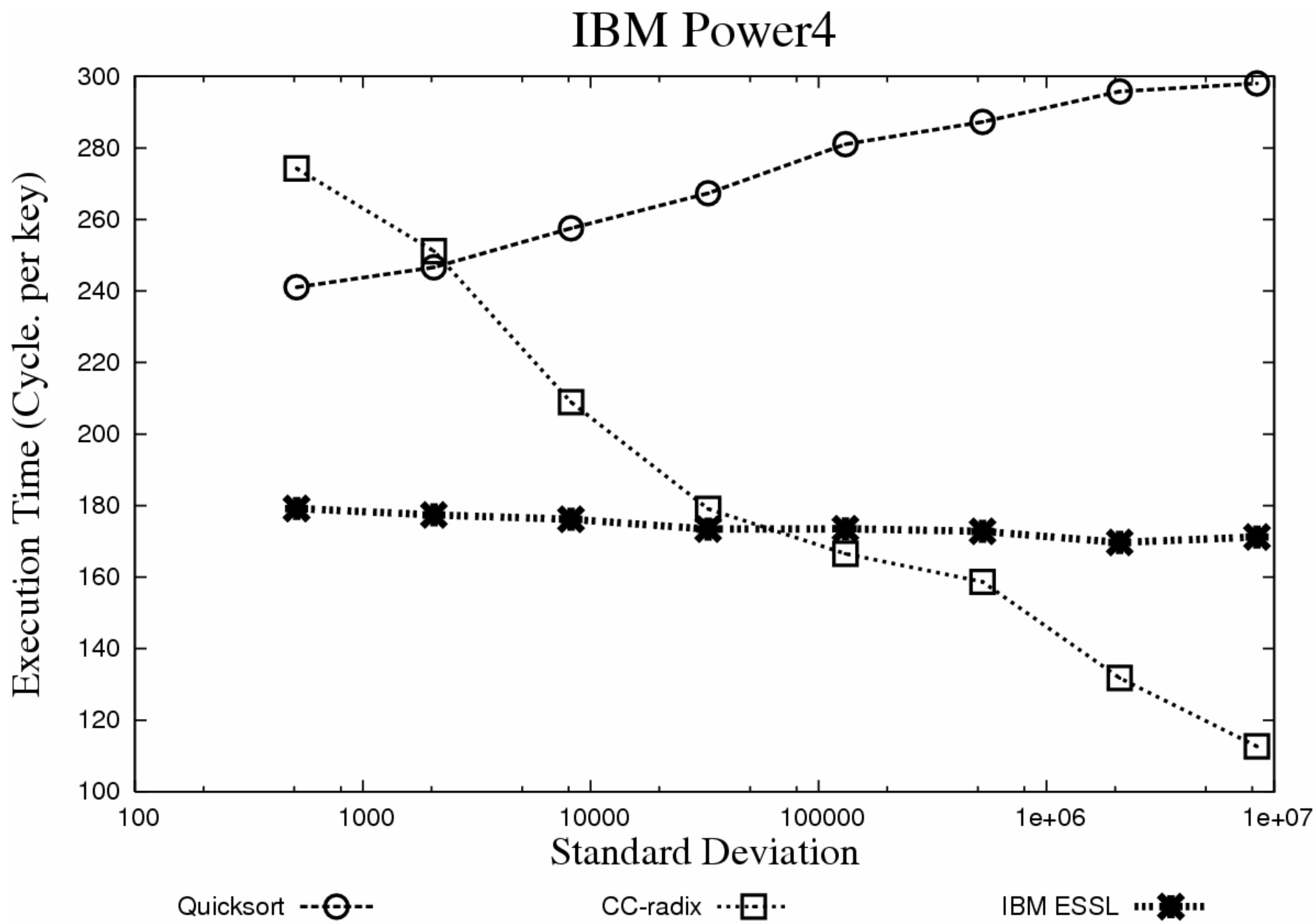
Maria Garzaran

Xiaoming Li

ESSL on Power3



ESSL on Power4



Motivation

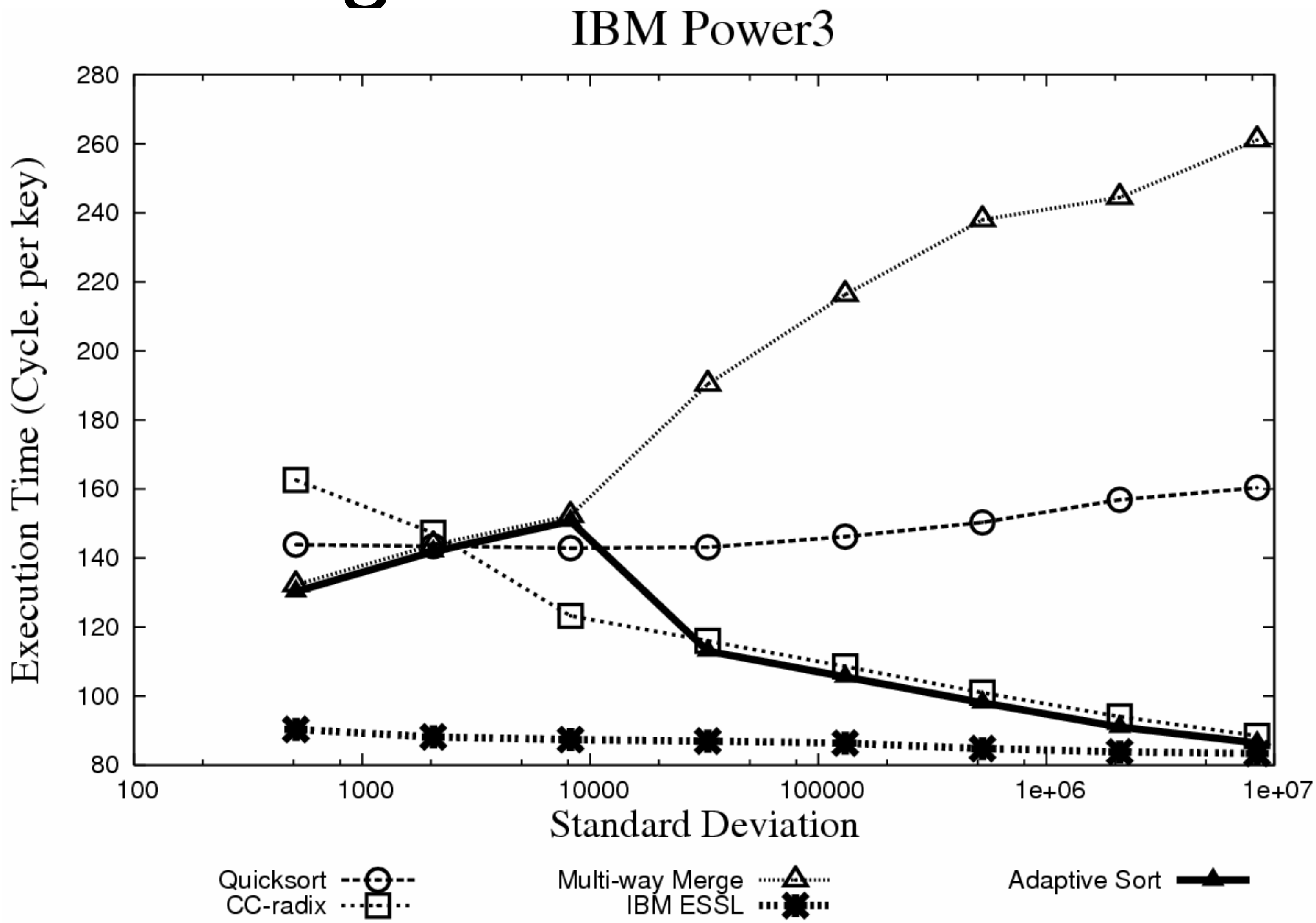
- No universally best sorting algorithm
- Can we automatically GENERATE and tune sorting algorithms for each platform ?
- Performance of sorting depends not only on the platform but also on the input characteristics.

A firs strategy: Algorithm Selection

- Select the best algorithm from Quicksort, Multiway Merge Sort and CC-radix.

- Relevant input characteristics: number of keys, entropy vector.

Algorithm Selection



A better Solution

- We can use different algorithms for different partitions
- Build Composite Sorting algorithms
  - Identify primitives from the sorting algorithms
  - Design a general method to select an appropriate sorting primitive at runtime
  - Design a mechanism to combine the primitives and the selection methods to generate the composite sorting algorithm

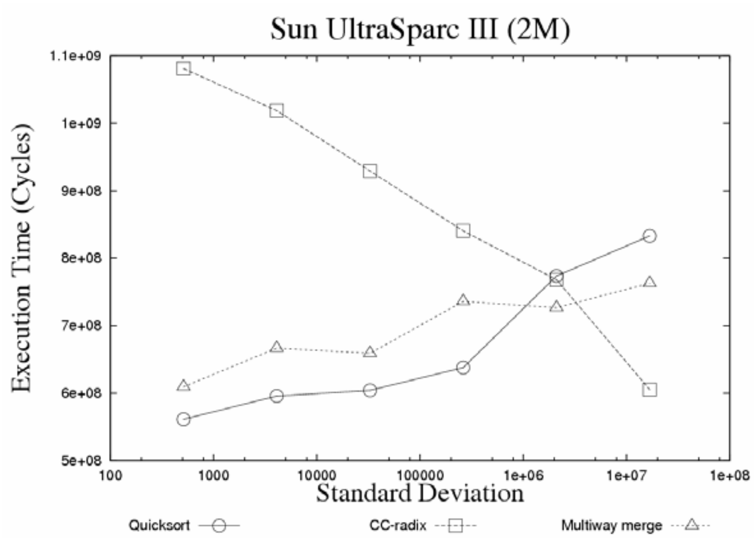
Sorting Primitives

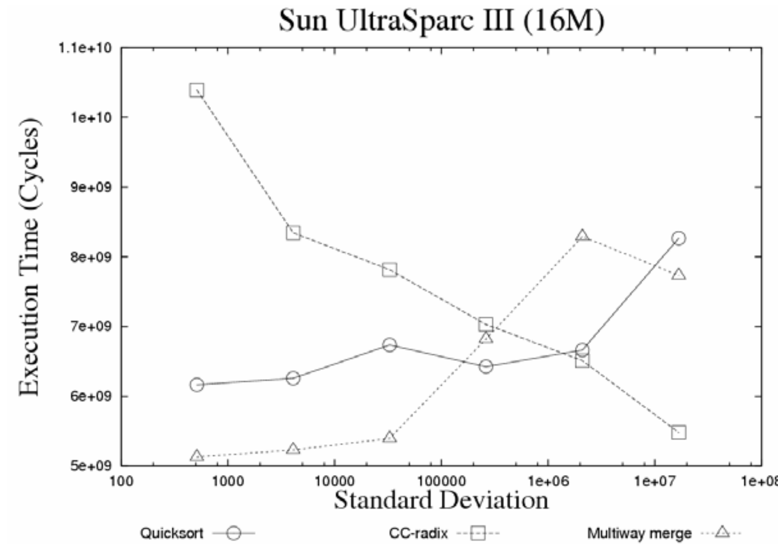
- Divide-by-Value
  - A step in Quicksort
  - Select one or multiple pivots and sort the input array around these pivots
  - Parameter: number of pivots
- Divide-by-Position (DP)
  - Divide input into same-size sub-partitions
  - Use heap to merge the multiple sorted sub-partitions
  - Parameters: size of sub-partitions, fan-out and size of the heap

Sorting Primitives

- Divide-by-Radix (DR)
  - Non-comparison based sorting algorithm
  - Parameter: radix (r bits)

Selection Primitives





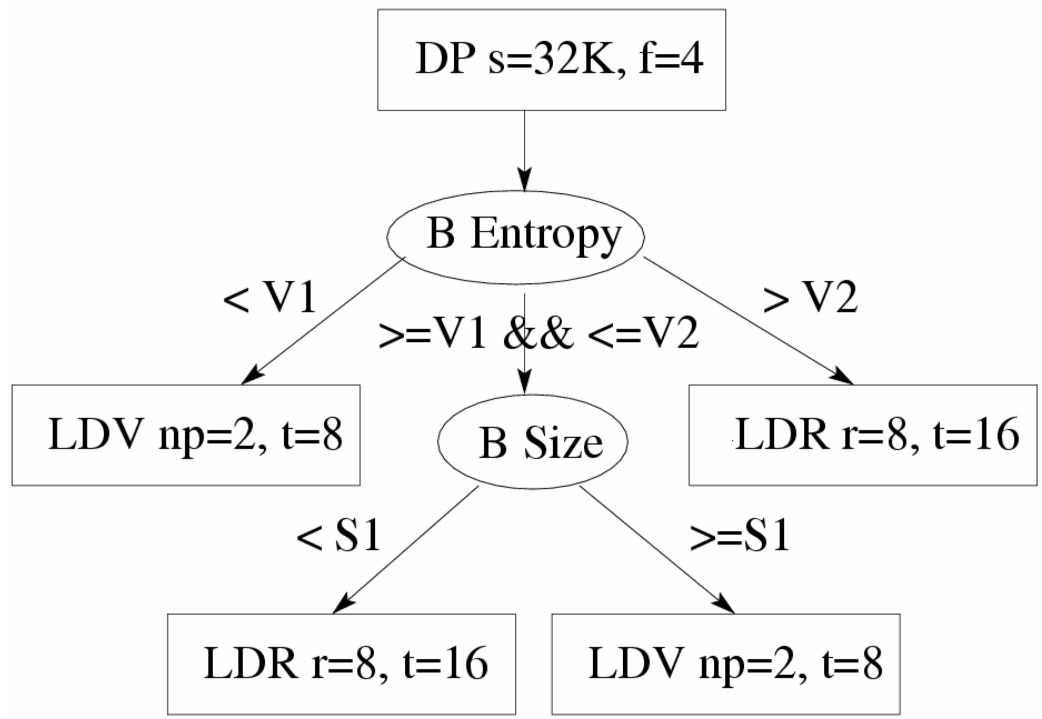
- Branch-by-Size
- Branch-by-Entropy
  - Parameter: number of branches, threshold vector of the branches

Leaf Primitives

- When the size of a partition is small, we stick to one algorithm to sort the partition fully.
- Two methods are used in the cleanup operation
  - Quicksort
  - CC-Radix

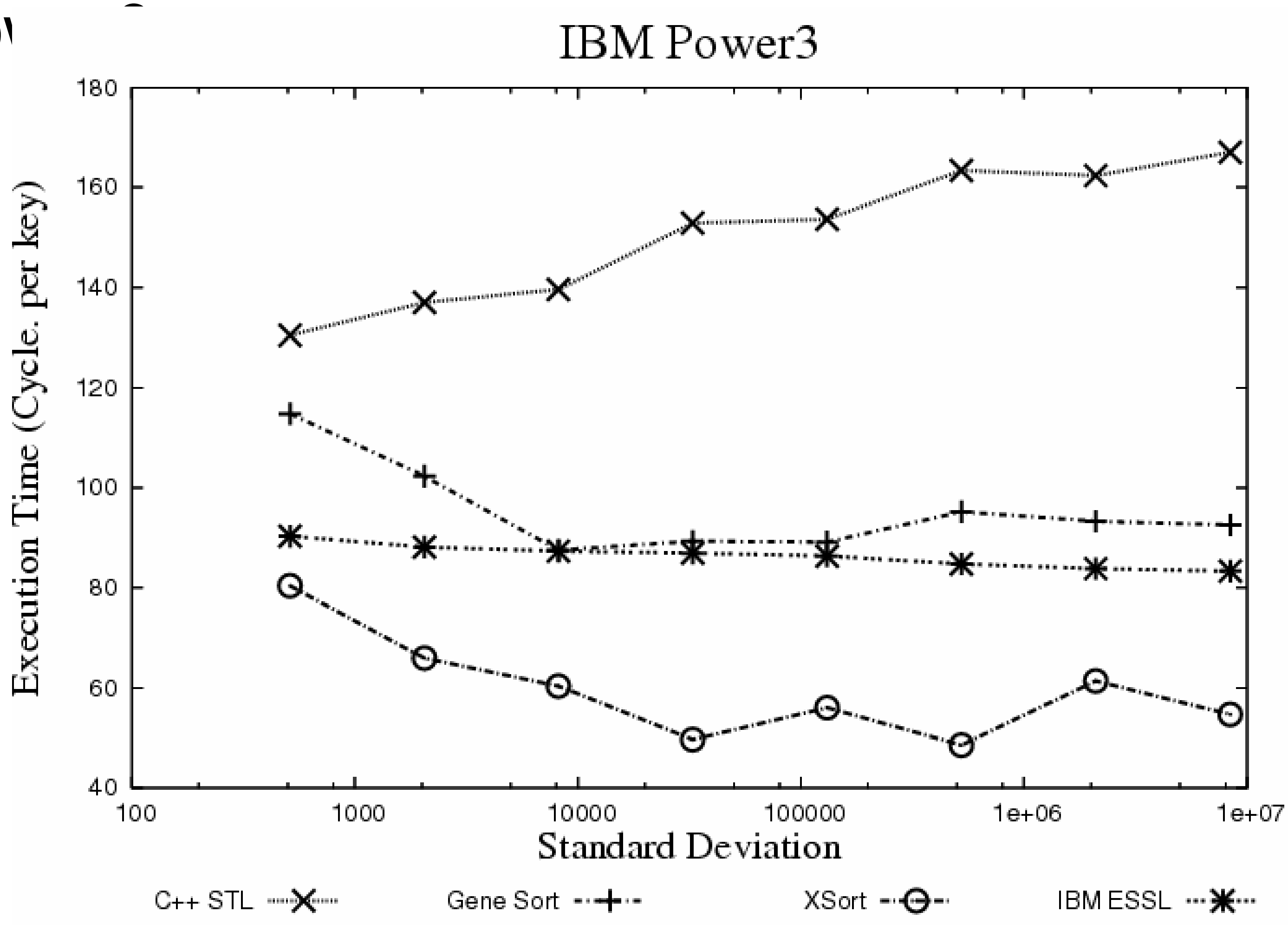
Composite Sorting Algorithms

- Composite sorting algorithms are built with these primitives.
- Algorithms are represented as trees.

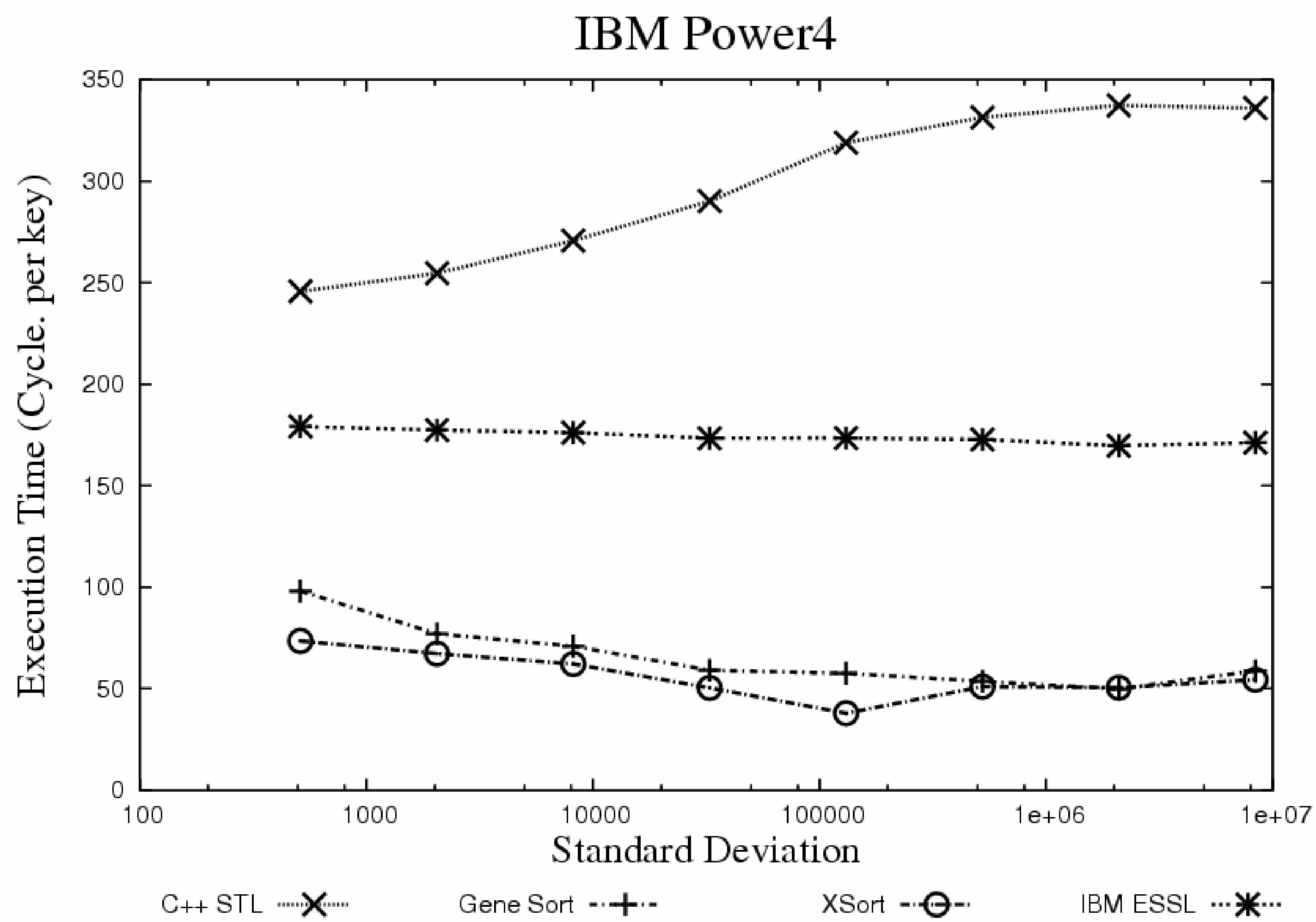


Performance of Classifier Sorting

- Po



Power4


IBM Power4
Execution Time (Cycle. per key)
350
300
250
200
150
100
50
0
100
1000
10000
100000
1e+06
1e+07
Standard Deviation
C++ STL
Gene Sort
XSort
IBM ESSL