Linnéuniversitetet Kalmar Växjö

Static Single Assignment (SSA) Form

Construction - Analyses - Optimizations

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Outline

- Introduction to SSA
  - Motivation
  - Value Numbering
  - Definition, Observations
- Construction, Destruction
  - Theoretical, Pessimistic, Optimistic Construction
  - Destruction
  - Memory SSA,
  - Interprocedural analysis based on Memory SSA: example P2SSA
  - How to capture analysis results
- Optimizations

Intermediate Representations

- Intermediate representations (like BB, SSA graphs) separate compiler front-end (source code related representation) from back-end (target code related representation)
- Analyses and optimizations can be performed independently of the source and target languages
- Tailored for analyses and optimizations

What is an IR tailored for analyses and optimizations?

- Represents dependencies of operations in the program
  - Control flow dependencies
  - Data dependencies
- Only essential dependencies (approximation)
  - A dependency $s;s'$ of operations is essential *iff* execution $s';s$ changes observable behavior of the program
  - Computation of essential dependencies is not decidable
- Compact representation
  - … of dependencies
  - No (few) redundant expression representation

Static Single Assignment - SSA

- Goal:
  - increase efficiency of inter/intra-procedural analyses and optimizations
  - speed up dataflow analysis
  - represent def-use relations explicitly
- Idea:
  - Represent program as a directed graph of operations τ
  - Represent triples as a (single) assignment $t := \tau\, t'\, t''$ with t, t', t'' a variable/label/register/edge connecting operations
- SSA-Property: there is only one single (static) position in a program/procedure defining t
  - Does not mean t computed only once (due to iterations the program point is in general executed more than once, possibly each time with different values)
  - But, there is no doubt which static variable definition is used in arguments of operations

Avoid redundant computations

- Assign each (partial) expression a unique number.
  - Good optimization in itself as values can be reused instead of recomputed
  - Known as value numbering
  - Basic idea for SSA
- Values that are provable equivalent get the same number
- How to find equal values?
- Can be computed by data flow analysis (forward, must)

Equivalent Values

- Two expressions are semantically equivalent, *iff* they compute the same value - Not decidable
- Two expressions are syntactically equivalent, *iff* the operator is the same and the operands are the same or syntactically equivalent
- Generalization towards semantic equivalence using algebraic identities, e.g., $a+a = 2*a$
- In practice, provable equivalence (conservative approximation): two expressions are congruent, *iff* they are syntactically equivalent or algebraically identical

Idea of Value Numbering

- Congruent values get the same value number
- Values are defined by operations and used by other operations
- Values computed only once and then reused (referring to their value number)
- Algorithmic idea to prove equality of expression values at different program points (congruence of tuples):
  - Basic case: constants are easy to proof equivalent
  - Induction: see definition of syntactic equivalence: if inputs of two operations equal and the operator is equal the computed values are also equal
  - Also apply algebraic identities to prove congruence
- Problems (postponed):
  - Alias/Points-to problem: Addresses, hence address content, is not exactly computable. Where are values stored to and loaded from? Not decidable.
  - Meets in control flow: which definition holds?

Value Numbering

- Type of value numbers:
  - *INT* for integer constants; *BOOL* for Boolean constants etc.
  - Use ids (labels): $\{t_1, \ldots, t_n\}$ otherwise
- Data structure:
  - Mapping (hash table) gets value number *vn* of operations defining the values, i.e., for tuples $t := \tau\, t'\, t''$, a lookup of τ *vn* (t') *vn* (t'') gets the value number of t,
  - Mapping of operation labels t to tuples $\tau\, t'\, t''$ (implicit),
  - τ is an operator symbol,
  - *vn* (t') *vn* (t'') are value numbers of tuples labeled t', t'', i.e. t', t'' have hash table entries or are constants.
- For a first try:
  - Computation basic block local
  - One hash table per basic block.

Value Numbering

with Local Variables without Alias Problem

(1) Initially: value number *vn*(constant)=constant; *vn*(*t*) = *void* for all tuples *t*.
(2) for all tuple *t* in program order:
case (a) t = *ST >local< t'*
vn(*t*) := *vn* (*ST >local< vn* (*t'*))
if *vn* (*t*) = *void* then
vn (*LD <local >*):= *vn* (*t'*),
vn (*t*):= new value number,
generate: *vn* (*t*): *ST >local< vn* (*t'*)
(b) t = *LD <local>*
vn(*t*) := *vn* (*LD <local>*).
if *vn* (*t*) = *void* then
vn (*t*):= new value number,
generate: *vn* (*t*): *LD <local>*
(c) $t = \tau\, t'\, t''$
vn(*t*) := *vn*(τ *vn* (*t'*) *vn* (*t''*))
if *vn* (*t*) = *void* then
vn (*t*):= new value number,
generate: *vn* (*t*): τ *vn* (*t'*) *vn* (*t''*).
(d) t = *call proc t' t'' ...* -- *analog* (c) *with* τ = *call proc*

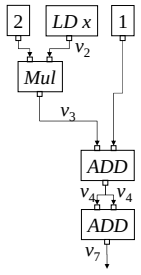
Example

Original				Result			
t_1: *ST*	>a<	2		v_1: *ST*	>a<	2	
t_2: *LD*	<a>						
t_3: *LD*	<x>			v_2: *LD*	<x>		
t_4: *MUL*	t_2	t_3		v_3: *MUL*	2	v_2	
t_5: *ADD*	t_4	1		v_4: *ADD*	v_3	1	
t_6: *ST*	>b<	t_5		v_5: *ST*	>b<	v_4	
t_7: *LD*	<x>						
t_8: *MUL*	2	t_7					
t_9: *ST*	>a<	t_8		v_6: *ST*	>a<	v_3	
t_{10}: *LD*	<a>						
t_{11}: *ADD*	t_{10}	1					
t_{12}: *LD*	<b>						
t_{13}: *ADD*	t_{11}	t_{12}		v_7: *ADD*	v_4	v_4	
t_{14}: *ST*	>c<	t_{13}		v_8: *ST*	>c<	v_7	

Value Number Graph of Basic Block

Result

v_1: *ST*	>a<	2	
v_2: *LD*	<x>		
v_3: *MUL*	2	v_2	
v_4: *ADD*	v_3	1	
v_5: *ST*	>b<	v_4	
v_6: *ST*	>a<	v_3	
v_7: *ADD*	v_4	v_4	
v_8: *ST*	>c<	v_7	



Value Numbering
with Global Variables without Alias Problem

- Case (a') as before case (a) for local variables
  case (a') $t = ST >global< t'$
  $vn(t) := vn\ (ST >global< vn(t'))$
  if $vn\ (t) = void$ then
  $vn\ (LD <global >) := vn\ (t')$,
  $vn(t)$:= new value number,
  generate: $vn\ (t)$: $ST >global< vn(t')$
- Procedures:
  - Case (d) as before, but if *global* (potentially) redefined in *proc*, set value number for tuple $ST >global<$, $LD <global>$ and transitive depending tuples to *void*
  - Easy implementation: set all value numbers to *void*
- Optimization for non-recursive leaf procedures:
  - New case (d): analyze procedure as if it was inlined
  - Not trivial if *proc* has more than one basic block

General Value Numbering

-- *t' is an address with unknown value (no compile time constant address, no name)*
-- *computed in an operation with value number vn (t')*
Case (e) $t = ST\ t'\ t''$
$vn\ (t) := vn\ (ST\ vn\ (t')\ vn\ (t''))$
if $vn\ (t) = void$ then
$vn\ (LD\ vn\ (t')) := vn\ (t')$,
$vn\ (t)$:= new value number,
Generate: $vn\ (t)$: $ST\ vn\ (t')\ vn\ (t'')$
if t' may be semantically equivalent to tt: -- *Points-to analysis*
$vn\ (ST\ vn\ (tt)\ ...) := void$,
$vn\ (LD\ vn\ (tt)) := void$,
value number of transitively depending tuples := *void*
Case (f) $t = LD\ t'$
$vn\ (t) := vn\ (LD\ vn\ (t'))$
if $vn\ (t) = void$ then
$vn\ (t)$:= new value number,
Generate: $vn\ (t)$: $LD\ vn\ (t')$

Remarks

- Initially all value numbers are set to *void* . By knowing the values of predecessor basic blocks, this can be relaxed (initializations over basic block solved by SSA, next issue)
- Each new entry in the hash table generates a new value number. After *call proc*, entries depending on non-local variables get *void*
- A store operation $ST >a< t'$ sets *void* all $vn\ (LD <a'>)$ and $vn\ (ST <a'> t')$ if it is not clear, whether $a = a'$ or $a \neq a'$ (alias-problem). Special case: arrays with index expressions
- Value number graph of a basic block
  - No (provable) unnecessary dependencies
  - No (provable) redundant computation

Value number graph → SSA

- SSA-Property: there is only one position in a program/procedure defining t
- Half way to SSA representation due to value numbering, i.e. value number graph is SSA graph of a basic block
- Problem: What to do with variables having assignments on more then one position?
- E.g.

```
if … then i:=1 else i:=2 end; x:=i

i:=0; while …loop …; i:=i+1; … end; x:=i
```


ϕ-Functions

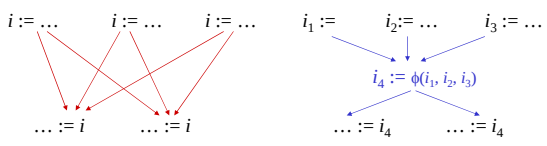
- Solution:
  - Each assignment to a variable a defines a new version a_i,
  - This version is actually the value number of the assigned expression
  - At meets in the control flow, we just add a pseudo expression selecting a value defining a new version (value number) of that variable
    $a_3 := \phi(a_1, a_2)$
- E.g.

```
if … then i1:=1 else i2:=2 end; i3:=ϕ(i1,i2); x:=i3
i1:=0; while i3:=ϕ(i1,i2); … loop … ; i2:=i3+1; … end; x:=i3
```

- ϕ–functions
  - always occur at the beginning of a block
  - are all are evaluated simultaneously for a block
  - guarantee that there is exactly one definition/assignment for each use of a variable
- Assignment $i_0 := \phi(i_1,\ldots,i_k)$ in a basic block indicates that the block has k direct predecessors in the control flow

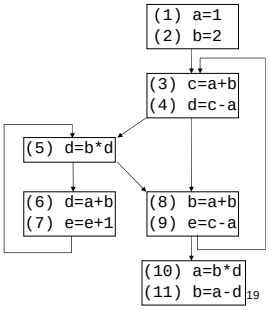
Compact representation of dependencies

- Previous: #def x #use dependency edges
- Now: #def + #use dependency edges



Example Program and Basic Block Graph

```
(1) a=1;
(2) b=2;
    while true{
(3)   c=a+b;
(4)   if (d=c-a)
(5)      while (d=b*d){
(6)          d=a+b;
(7)          e=e+1;
         }
(8)   b=a+b;
(9)   if (e=c-a) break;
    }
(10)a=b*d;
(11)b=a-d;
```



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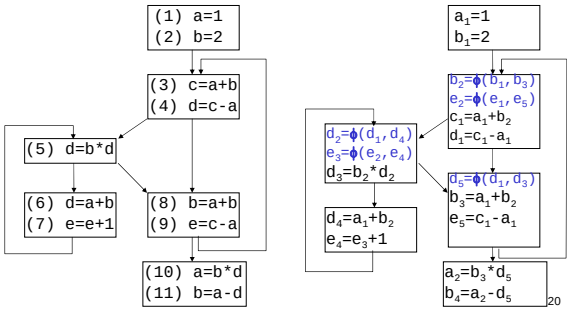
Basic Block and SSA Graph



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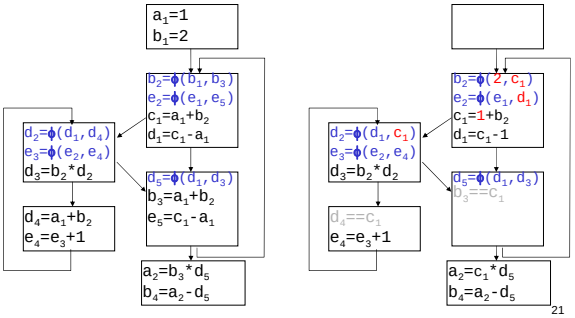
SSA-Graph before and after Constant and Copy Propagation



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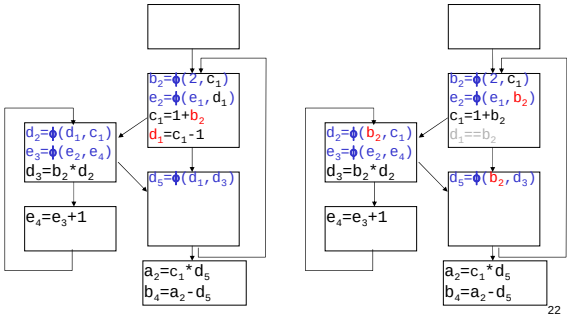
SSA-Graph before and after using Algebraic Identities



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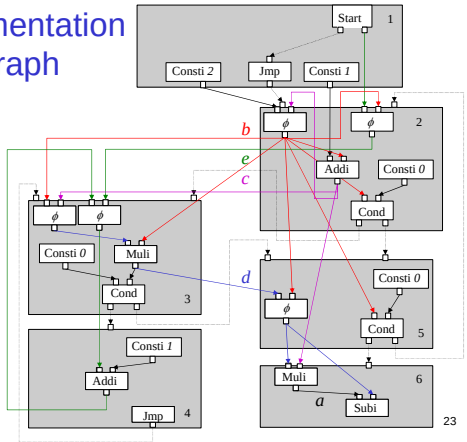
Implementation SSA graph



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SSA properties

P1: Typed in-/output of nodes, in- and output of operation node connected by edges have the same type.
P2: Operation nodes and edges of a basic block are a DAG.
Note: correspondence to value number graphs and expression trees
P3: Input of ϕ-operations have same type as their output.
P4: *i*-th operand of a ϕ-operation is available at the end of the *i*-th predecessor BB.
P5: A start node *Start* dominates all BBs of a procedure, an end node *End* post-dominates all nodes of a procedure.
P6: Every block has exactly one of nodes *Start*, *End*, *Jump*, *Cond*, *Ret*
P7: If operation x in a BB B_x defines a operand of operation y in a BB B_y then there is a path $B_x \rightarrow^+ B_y$.
P7a: (Special case of P7) operation y is a ϕ-operation and x in $B_x = B_y$ then there is a cyclic path $B_y \rightarrow^+ B_y$.

P8: Let X,Y be BBs each with a definition of a that may reach a use of a in BB Z. Let Z' be the first common BB of execution paths $X \rightarrow^+ Z$, $Y \rightarrow^+ Z$. Then Z' contains a ϕ-operation for a.

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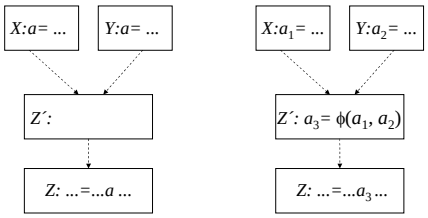
Property P8 revisited

Let X,Y be BBs each with a definition of a that may reach a use of a in BB Z.
Let Z' be the first common BB of execution paths $X \rightarrow^+ Z$, $Y \rightarrow^+ Z$.
Then Z' contains a ϕ-operation for a.



Dominance

Dominance: $X \geq Y$

On each path from the starting node S in the basic block graph to Y, X before Y.

$\geq$ is reflexive: $X \geq X$.

Strict Dominance: $X > Y$

$X > Y \Leftrightarrow X \geq Y \wedge X \neq Y$

Direct Dominance: $ddom(X)$

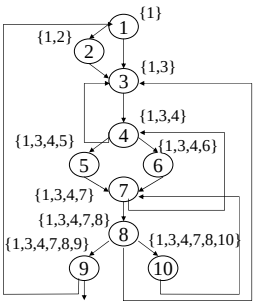
$X = ddom(Y) \Leftrightarrow X > Y \wedge \neg\exists\, Z\colon X > Z > Y$.

Post-Dominance: $X \leq Y$

On each path from the end node E in the basic block graph to Y, X before Y.

Definitions of strict and direct post dominance analogously.

Example



(Iterated) Dominance Frontiers

- Dominance Frontier $DF(n)$
  - Set of nodes just not strictly dominated by n any more
  - $DF(n) = \{m \mid n \not> m \wedge \exists b \in pre(m)\colon n \geq b\}$.
- Dominance Frontiers of a Set M of nodes $DF(M)$
  - $DF(M) = \bigcup_{n \in M} DF(n)$
- Iterated Dominance Frontier $DF^+(M)$
  - minimum fix point of:

$DF_0 = DF(M)$,

$DF_{i+1} = DF(\bigcup_{k \in 0..i} DF_k)$.

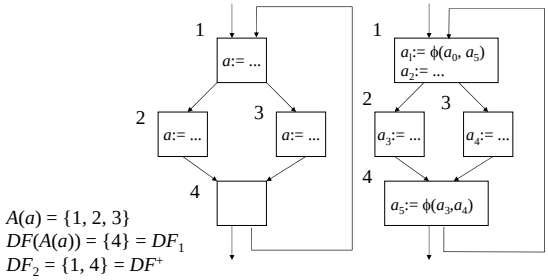
Discussion P8

- Defines, where to position ϕ-functions
- Let X,Y be the only BBs containing a definition of a. Both may reach a use of a in block Z. Then there are path $X \rightarrow^+ Z$, $Y \rightarrow^+ Z$ in the BB graph (P7). Let Z' be the first node common to both path then:
  - $a_3 := \phi(a_1, a_2)$ is assigned to Z'
  - $Z' \geq Z$ otherwise X,Y are not the only BBs (a not initialized)
  - Z' dominates Z, actually Z' is the first common successor of X and Y dominating Z
  - $Z' \in DF(X,Y)$ i.e. Z' is in the dominance frontier of X and Y
- Since $a_3 := \phi(a_1, a_2)$ itself is a definition of a, the dominance frontiers $DF(X,Y,Z')$ must contain ϕ functions.
- Fixed point iteration required,
- Observation: ϕ-functions in the iterated dominance frontiers $DF^+(...)$ of the BB defining a variable

Proof (idea) of this observation

1. $a_3 := \phi(a_1, a_2)$ cannot be defined before Z'
2. X and Y, resp., must dominate all direct predecessors of Z', otherwise there would be a use of a without previous definition. Hence $Z' \in DF(X,Y)$
3. $a_3 := \phi(a_1, a_2)$ should not (but may) be inserted later in other dominators of Z dominated by Z' since on the path $Z' \rightarrow^+ Z$ there is no change (new definition) of a.
4. Iteration $DF^+(...)$ is required, as there is now a further definitions of a in a block Z' and $DF(X,Y) \subseteq DF(X,Y,Z')$

Example Property E8



$A(a) = \{1, 2, 3\}$
$DF(A(a)) = \{4\} = DF_1$
$DF_2 = \{1, 4\} = DF^+$

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Construction Theory

- Program of size n may contain $O(n)$ variables
- In the worst case there are $O(n)$ ϕ–function (for the variables) in $O(n)$ BBs, hence worst case complexity is $\Omega(n^2)$
- Previous discussion gives a straight forward implementation:
  - Perform a value numbering and update BBs accordingly
  - For any used variable that is used but not locally (in BB) defined compute set of definition points (data flow analysis)
  - Compute iterated dominance frontiers of definition points
  - Insert ϕ–function and rename variables accordingly
- In practice easier constructions possible

Remainder Value Numbering

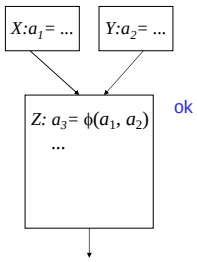
(1) Initially: value number *vn*(constant)=constant; *vn*(*t*) = *void* for all tuples *t*.
(2) for all tuple *t* in program order:
case
(a) *t* = *ST* >*local*< *t´*
vn(*t*) := *vn* (*ST* >*local*< *vn* (*t´*))
if *vn* (*t*) = *void* :
vn (*LD* <*local* >):= *vn* (*t´*),
vn (*t*):= new value number,
generate: *ST* >*local*< *vn* (*t´*)
(b) *t* = *LD* <*local*>
vn(*t*) := *vn* (*LD* <*local*>).
if *vn* (*t*) = *void* :
vn (*t*):= new value number,
generate : *vn* (*t*): *t*
(c) *t* = τ *t´ t´´*
vn(*t*) := *vn*(τ *vn* (*t´*) *vn* (*t´´*))
if *vn* (*t*) = *void* :
vn (*t*):= new value number,
generate : *vn* (*t*): τ *vn* (*t´*) *vn* (*t´´*).
(d) *t* = *call proc t´ t´´* ... -- *analog* (c) *with* τ = *call proc*

Extended Initialization

(1) Initialization for current block *Z*
(A) always: $vn(\text{constant})_Z$ =constant;
(B) if *Z* = start block: *vn*(*t*) = *void* for all tuples *t*.
(C) else: let *Pred*={*X*, *Y* , …} be the predecessors of *Z* in basic block graph
for all variables *t* used in current block *Z*:
if $vn(t)_X \neq vn(t)_Y$
$vn(t)_Z$:= new value number
generate: $vn(t)_Z := \phi(vn(t)_X\ vn(t)_Y)$
if $vn(t)_X = vn(t)_Y$
$vn(t)_Z := vn(t)_X$

(2) for all tuple *t* in program order:
-- *as before*

Extended Value Numbering



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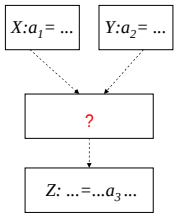
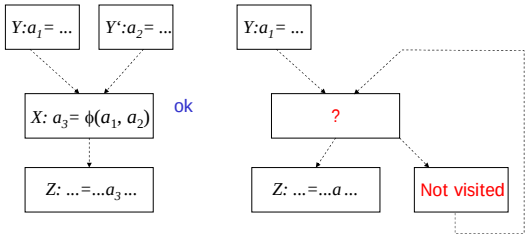


Extended Initialization

(1) Initialization for current block Z
- (A) always: $vn(\text{constant})_Z$ =constant;
- (B) if Z = start block: $vn(t)$ = *void* for all tuples t.
- (C) else: let $Pred=\{X, Y, \ldots\}$ be the predecessors of Z in basic block graph
  for all variables t used in current block Z:
  - if $vn(t)_{X,Y \in Pred}$ = *undefind*
    - recursively, initialize block X $vn(t)_{X,Y}$ with (1)
  - if $vn(t)_X \neq vn(t)_Y$
    - $vn(t)_Z$:= new value number
    - generate: $vn(t)_Z := \phi(vn(t)_X\ vn(t)_Y)$
  - if $vn(t)_X = vn(t)_Y$
    - $vn(t)_Z := vn(t)_X$

(2) for all tuple t in program order:
-- *as before*

Extended Value Numbering



Extended Initialization

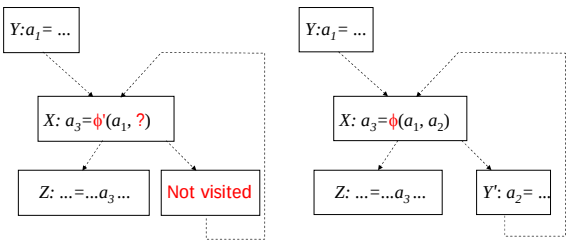
(1) Initialization for current block Z
- (A) always: $vn(\text{constant})_Z$ =constant;
- (B) if Z = start block: $vn(t)$ = *void* for all tuples t.
- (C) else: let $Pred=\{X, Y, \ldots\}$ be the predecessors of Z in basic block graph
  for all variables t used in current block Z:
  - if $X \in Pred$ = *unvisited*
    - $vn(t)_X$ = new special value number (guessed number)
  - if $vn(t)_{X,Y \in Pred}$ = *undefind*
    - recursively, initialize block X $vn(t)_{X,Y}$ w'ith (1)
  - if $vn(t)_X \neq vn(t)_Y$
    - $vn(t)_Z$:= new value number
    - generate: $vn(t)_Z := \phi(vn(t)_X\ vn(t)_Y)$
  - if $vn(t)_X$ or $vn(t)_Y$ is guessed
    - generate: $vn(t)_Z := \phi'(vn(t)_X\ vn(t)_Y)$
  - if $vn(t)_X = vn(t)_Y$
    - $vn(t)_Z := vn(t)_X$

(2) for all tuple t in program order:
-- *as before*

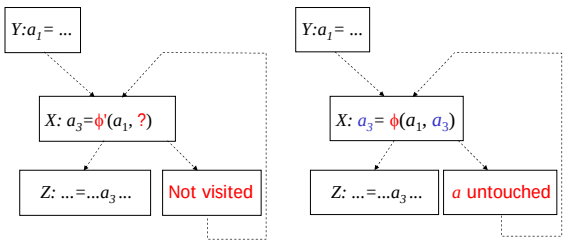
Eliminate/Mature ϕ′-Functions

- After value numbering is finished for each block X:
  - replace special value numbers in ϕ'-functions of X by last valid real value numbers in $pre(X)$
  - replace ϕ'-functions by mature ϕ-functions using real value numbers
  - delete: $vn(t)_Z := \phi(vn(t)_Y\ vn(t)_Z)$
    if t not changed in previously unvisited blocks, no ϕ function required
  - replace then use of $vn(t)_Z$ by $vn(t)_Y$
- Insight:
  - deletion could prove some other ϕ-functions unnecessary
  - iterative deletion till fix point

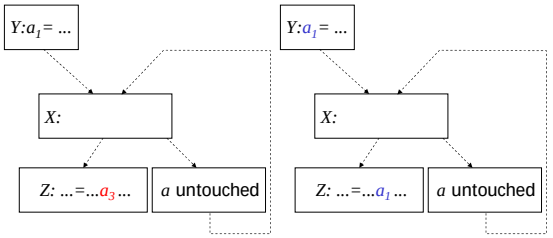
Example I: Mature ϕ′-Functions



Example II: Mature ϕ′-Functions



Example III: Mature ϕ′-Functions



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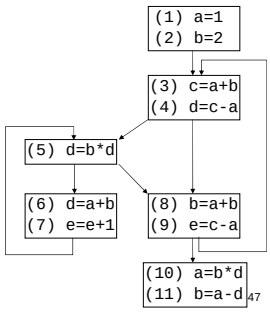
Example Program and BB Graph

```
(1) a=1;
(2) b=2;
    while true{
(3)   c=a+b;
(4)   if (d=c-a)
(5)      while (d=b*d){
(6)         d=a+b;
(7)         e=e+1;
          }
(8)   b=a+b;
(9)   if (e=c-a) break;
    }
(10)a=b*d;
(11)b=a-d;
```



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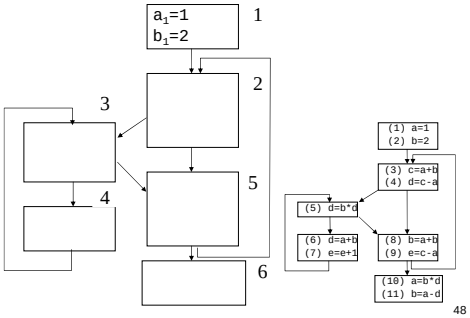
SSA Construction Block 1



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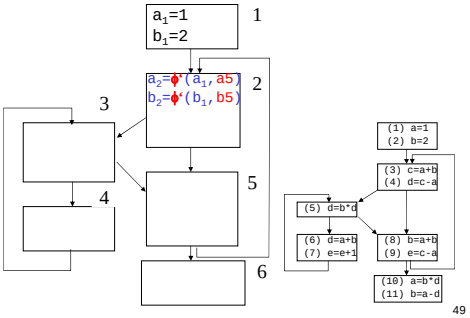
SSA Construction Block 2 - Initialization



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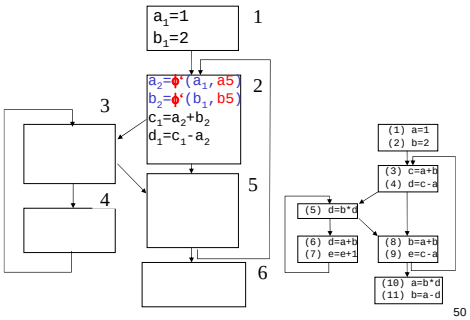
SSA Construction Block 2



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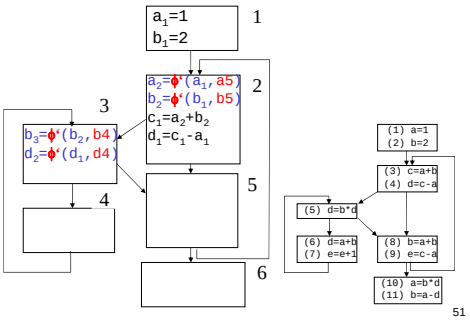
SSA Construction Block 3 - Initialization



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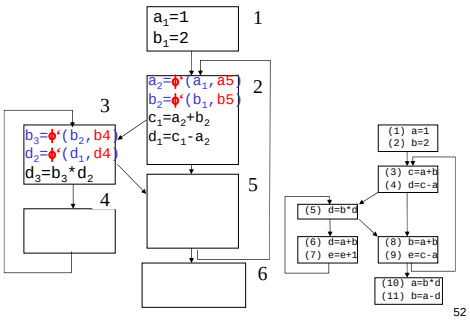
SSA Construction Block 3



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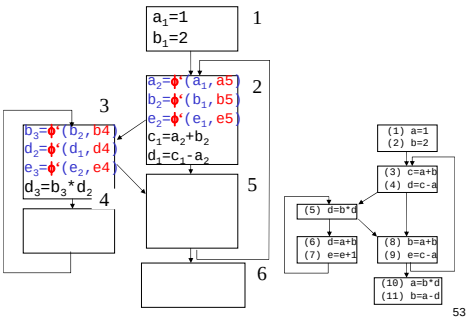
SSA Construction Block 4 - Initialization



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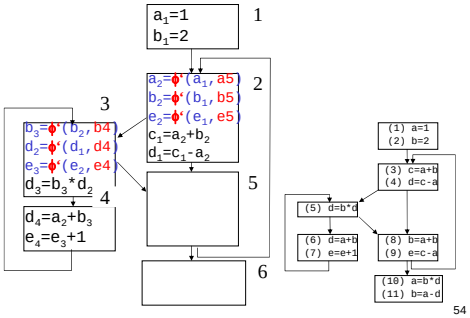
SSA Construction Block 4



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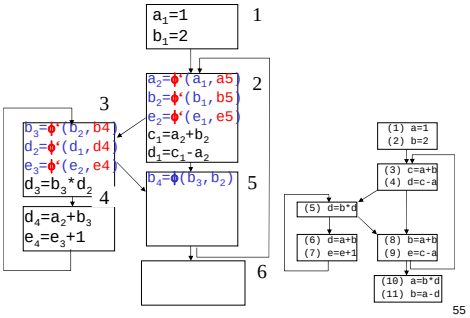
SSA Construction Block 5 - Initialization



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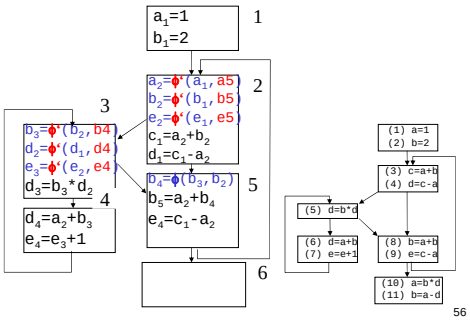
SSA Construction Block 5



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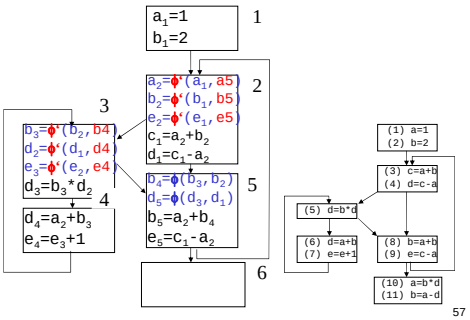
SSA Construction Block 6 - Initialization



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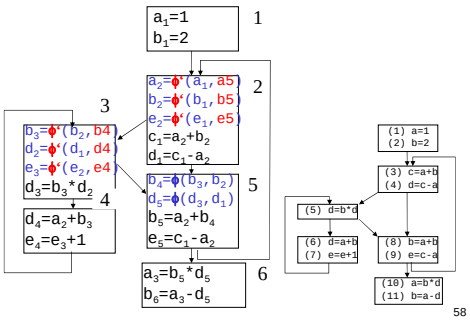
SSA Construction Block 6



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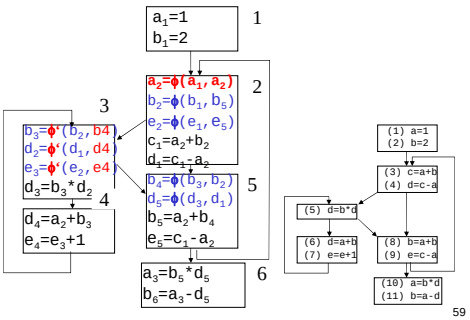
SSA Mature Block 2



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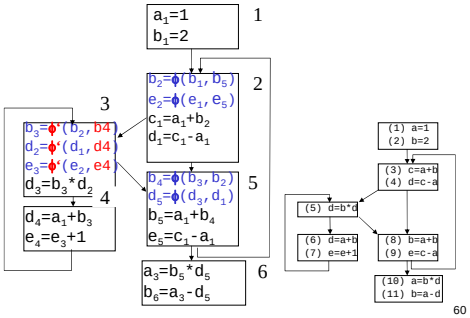
SSA Mature Block 2



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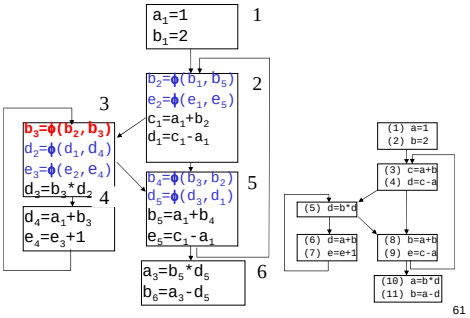
SSA Mature Block 3



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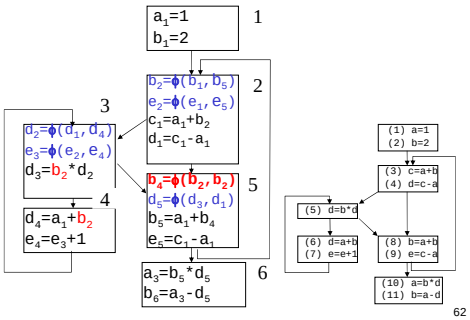
SSA Mature Block 3



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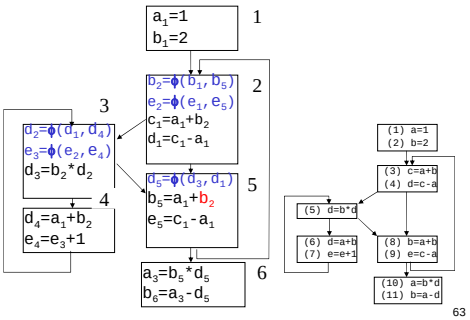
SSA Mature Block 3



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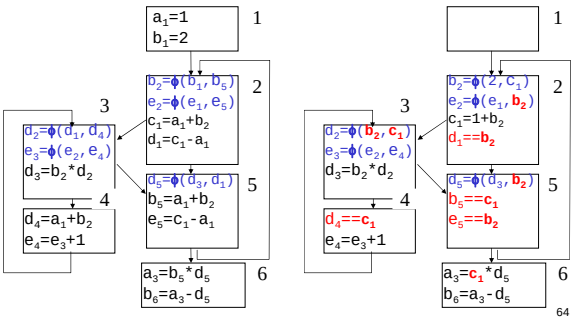
Final Simplifications



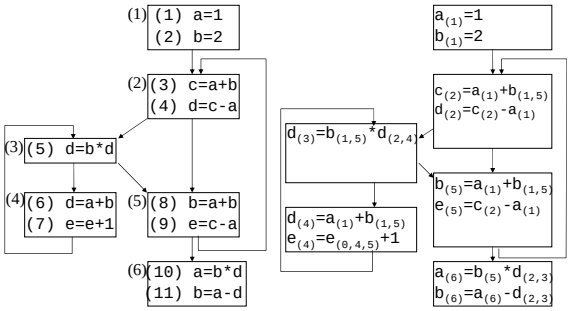
Optimistic SSA Construction

- Idea:
  - all values (value numbers) are equal until the opposite is proven
  - opposite is proven by:
    - Values are different constants
    - Values are generated form syntactical different operations
    - Values are generated form syntactical equivalent operations with proven different values as operands
- Advantage:
  - Detects sometimes congruence that are not detected by pessimistic construction
  - No ϕ–functions to mature
- Disadvantage:
  - Detects sometimes congruence not that are detected by pessimistic construction (e.g. algebraic identities)
  - Requires Definition-Use-Analyses on BB graph on construction
  - Requires computation of iterated dominance frontiers to position ϕ–functions

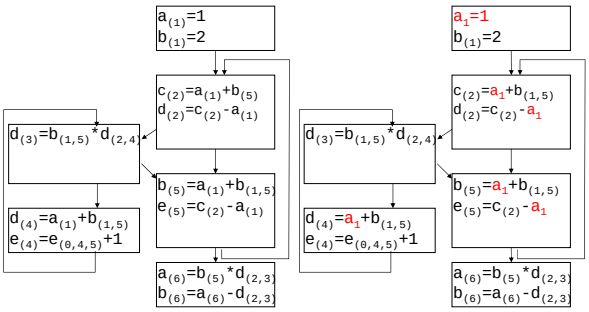
Construction Algorithm

- Generate BB graph and perform Definition-Use-Analysis (data flow analysis) for all variables. Notations:
  - $v_{(i)}$= ... Variable v in statement (i) defined
  - $u_{(i)}$= ...$v_{(x,y,z,...)}$... Variable v in statement (i) used with may reaching definitions in statements (x,y,z,...)
  - statements can be abstracted to blocks
- Set $v_{(i)} \equiv u_{(j)}$ for all $v_{(i)}$, $u_{(j)}$ in the program
- Iterate until a fixed point over:
  - Set $v_{(i)} \not\equiv u_{(j)}$ for:
    - $v_{(i)} = c$ and $u_{(j)} \neq c$
    - $v_{(i)} = op_1(...)$ and $u_{(j)} \neq op_1(...)$
    - $v_{(i)} = op_1(x_1,y_1)$ and $u_{(j)} = op_1(x_2,y_2)$ and $x_1 \not\equiv x_2$ or $y_1 \not\equiv y_2$
- Find a unique value number for each equivalence class
- Replace variables consistently by value number for each equivalence class
- Insert, if necessary, ϕ-functions (also possible during iteration)

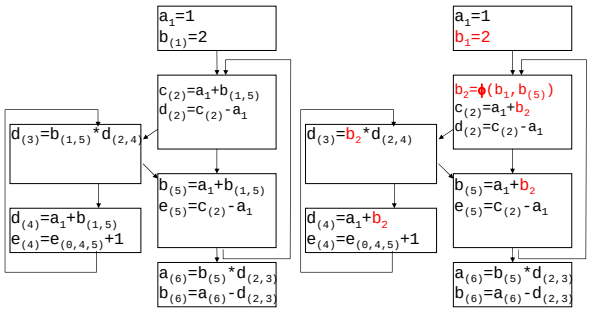
Optimistic SSA Construction



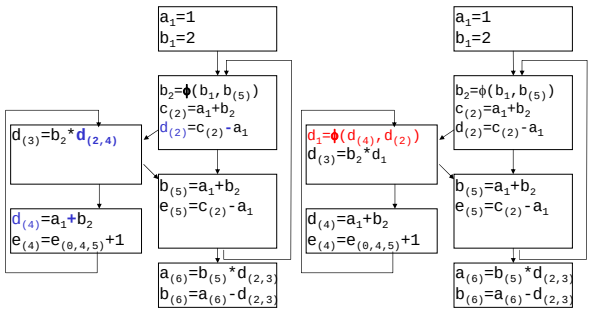
Optimistic SSA Construction



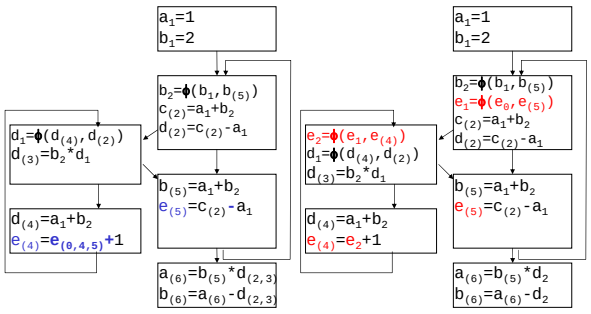
Optimistic SSA Construction



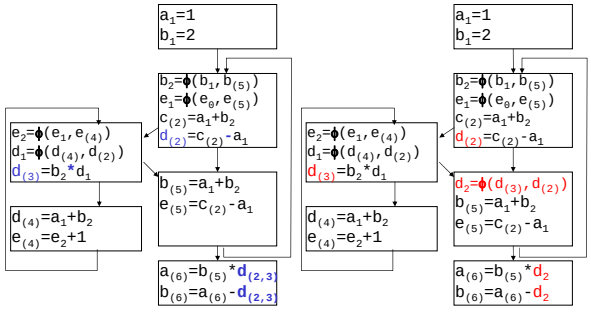
Optimistic SSA Construction



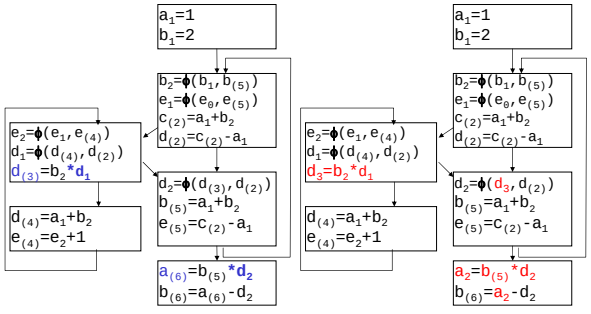
Optimistic SSA Construction



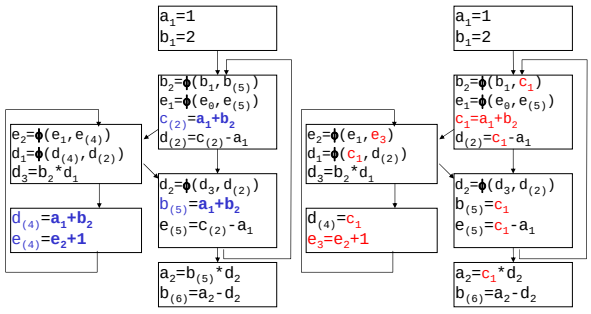
Optimistic SSA Construction



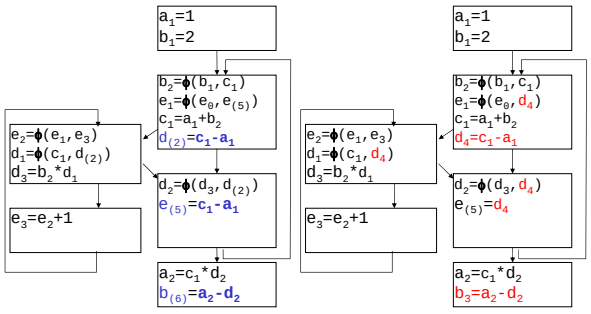
Optimistic SSA Construction



Optimistic SSA Construction

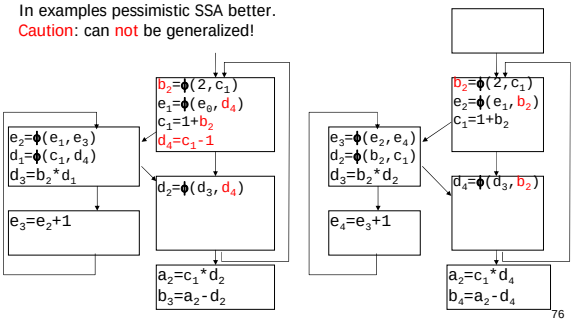


Optimistic SSA Construction



Optimistic vs. Pessimistic SSA

In examples pessimistic SSA better.
Caution: can not be generalized!



Optimal (good) Policy

- Generate pessimistic SSA
  - Program size reduced
  - Definition-Use-Information computed
  - No immature ϕ-functions
- Set all value(-number)s congruent
- Compute optimistic SSA
- Iteration (pessimistic-optimistic-pessimistic- ...)
  - until fix point possible
  - in practice only pessimistic SSA or pessimistic SSA with only one subsequent optimistic SSA computation (no iteration)

Minimal SSA-Form

- Insight:
  - ϕ-functions guarantee that for each use of a variable there is exact one definition
  - "Variable" means program- or auxiliary variable Solution of the (may) Reaching-Definitions-Problem
  - Problems with array elements and indirectly addressed variables retain (discussed and solved later)
- Minimal *SSA*-form: set ϕ-function $a_0 := \phi(a_1, a_2,...)$ in block B iff value a_0 is live in B.
  - Use data flow analysis $liveIn(B)$ and check $a \in liveIn(B)$.
  - Better: generate value numbers only on demand (integration in construction algorithms).

SSA – Construction from AST

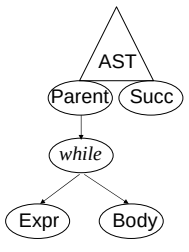
- Left-Right Traversal (1. Round):
  - compute for each syntactic expression its basic block number
  - compute precedence relation on basic blocks
  - generate expression triples into the BBs
- Right-Left Traversal (2. Round):
  - compute, for each live (beginning with the results of a procedure) expression, the value numbers (contains ϕ') using the data structures known from value numbering
- Left-Right Traversal (3. Round):
  - Mature ϕ'-functions
  - generate SSA for non empty blocks
- Further eliminations on SSA graph

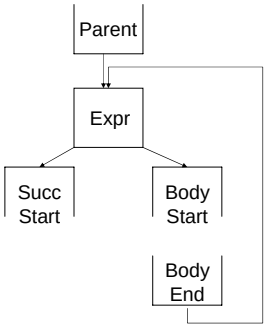
SSA from AST

- Rounds 1+2 on one left-right tree traversal if liveliness is ignored,
  - Construct BBs
  - Construct SSA code for the basic blocks (value number graphs)
  - Construct control flow between BBs
- For each statement type (AST node type) there is different set of actions when visiting the nodes of that type including:
  - Assignment to local variables and expressions: like local value numbering in a left-to-right traversal
  - Procedure calls like any other operation expressions
  - While, If, Exception, … on the fly introduce new BBs and control flow

SSA from AST

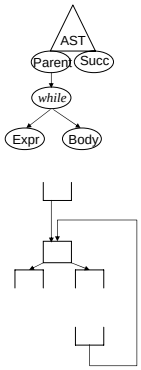
- *while* AST and BB graph





SSA from AST

- *while* actions
  - Finalize current block B(Parent)
  - Create a new current block B(Expr)
  - Add control flow B(Paret) to B(Expr)
  - Recursively, generate code for Expr computing value numbers locally
  - Finalize current block B(Expr)
  - Create a new current block B_{start}(Body)
  - Add control flow B(Expr) to B_{start}(Body)
  - Recursively, generate code for Body
  - After return current block is B_{end}(Body), finalize it
  - Add control flow B_{end}(Body) to B(Expr)
  - Create a new current block B_{start}(Succ)
  - Add control flow B(Expr) to B_{start}(Succ)
  - Return with B_{start}(Succ) as current block

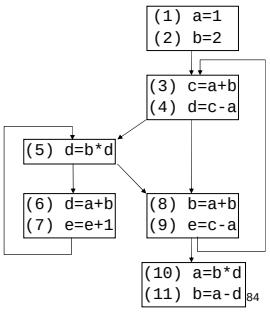


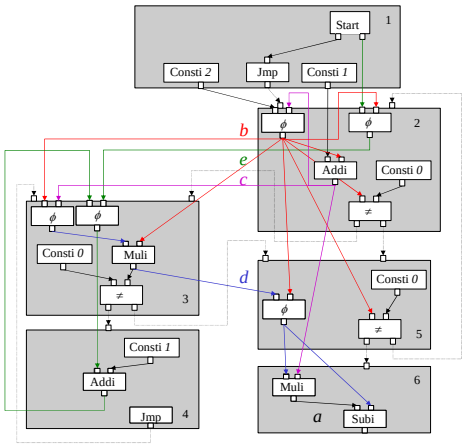
Deconstruction of SSA

- Serialize the SSA graph
- Replace data dependency edges by variables
- Remove ϕ-functions:
  - Define a new variable
  - Copy from value in predecessor basic blocks (requires possibly new blocks on some edges)
  - Perform copy propagation to avoid unnecessary copy operations
- Allocate registers for variables
  - Fixed number of registers
  - More variables than registers
  - Idea: assign variables with non overlapping lifetimes to the same register

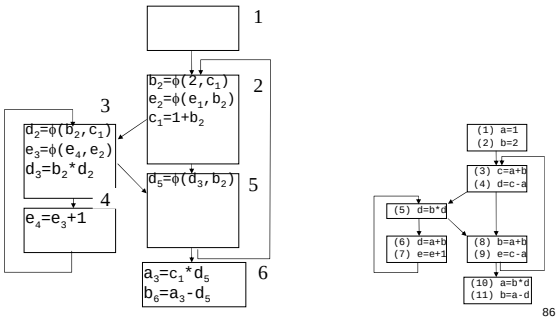
Example Program and BB Graph

```
(1) a=1;
(2) b=2;
    while true{
(3)   c=a+b;
(4)   if (d=c-a)
(5)      while (d=b*d){
(6)         d=a+b;
(7)         e=e+1;
         }
(8)   b=a+b;
(9)   if (e=c-a) break;
    }
(10)a=b*d;
(11)b=a-d;
```

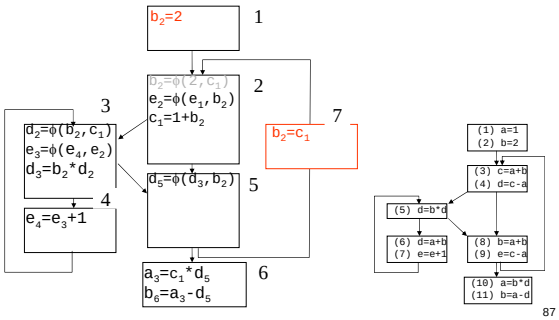




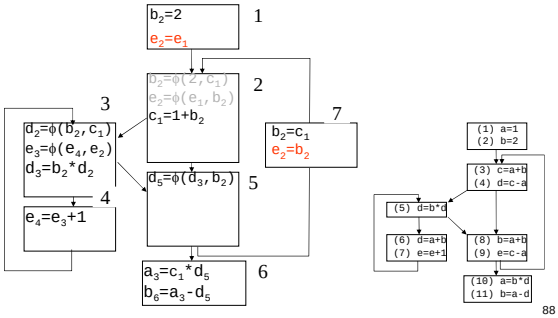
Introduce Variables for Edges



Remove ϕ-functions



Remove ϕ-functions



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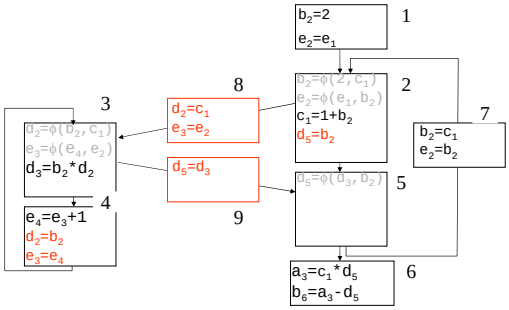
Remove ϕ-functions



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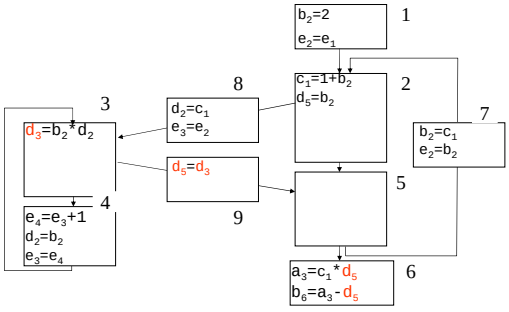
Copy Propagation



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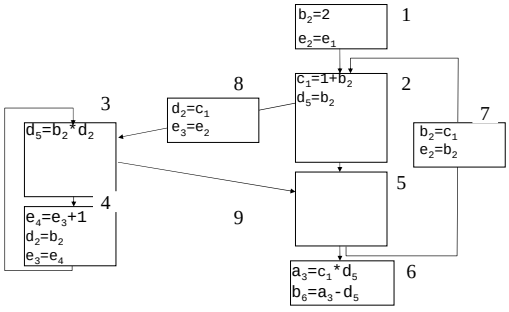
Remove Empty Block



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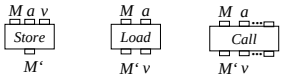
Memory SSA

- By now we can only handle simple variables
- Extension:
  - Node: memory changing operations
  - Edges:
    - Data- and control flow.
    - Anti- / out dependencies between memory changing operations
- Functional modeling of memory changing operations



M, M‘	Memory state
a	Address
v	Value

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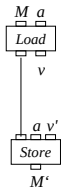
Why *Load* Defines Memory?



Anti-depending memory operations:
Read an address essentially before
Redefine the value

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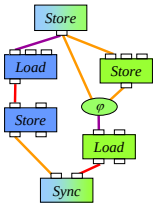
Memory SSA

- To capture only essential dependencies distinguish disjoint memory fragments
  - In general not decidable
  - Approximated by analyses
  - Initial distinctions e.g.
    - Heap vs. Stack
    - Different arrays on the stack
    - Heap partitions for different object types
- Distinction often only locally possible
  - Union necessary
  - Sync operation unifies disjoint memory fragments
  - Like ϕ- functions but sync is strict



Properties of Memory SSA

P1-P8: as before
P9: New! Lifetime of memory states do not overlap if they define different values of the same memory slot

- Otherwise we would need to keep two versions of the memory alive
- Memory does not fit into a register (usually)
- Would be make the programs non-implementable

- Note: if we only have to analyze the program, P9 could be ignored

Reduced SSA Representations

- Assume goal is analysis not compilation as in many software tech. applications in IDEs
  - Rename ids
  - Go back in debugging
  - Move code
  - …
- Not the whole program is directly relevant for an analysis
  - Certain data types are uninteresting, e.g., Int, Bool in call graph construction
  - Consequently, operation nodes consuming/defining values of these types and edges connecting them can be removed
- More compact program representation
  - Faster in analysis
  - Still SSA properties hold
- Example Points-To SSA capturing only reference information necessary for Points-To analysis (ignoring basic types and operations)

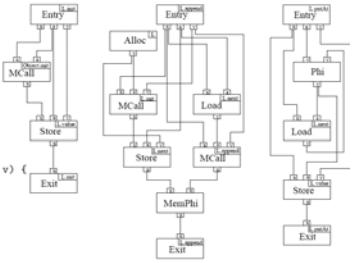
Example Points-to-SSA

```
public class List {
  Object value = null;
  List next = null;

  public List (Object v) {
    value = v;
  }

  public void append(Object v) {
    if (next == null)
      next = new List(v);
    else
      next.add(v);
  }

  public void putAt(int n,Object v) {
    int count = 0;
    List l = this;
    while (count < n) {
      l = l.next;
      count++;
    }
    l.value = v;
  }
}
```



Cliffhanger from the first day

- Inter-Procedural analysis
- Call graph construction (fast)
- Call graph construction (precise)
- Call graph construction (fast and precise)

Recall: Call graph construction

- Points-to Analysis (P2A) computes reference information:
  - Which abstract objects may a variable refer to.
  - Which are possible abstract target objects of a call.
  - In general: for any expression in a program, which are the abstract objects that are possibly bound to it in an execution of that program.
- Call graph construction is a simple client analysis of P2A

Recall: Precise call graph construction

- Construction of a Points-to Graphs (P2G):
  - Node for objects and variables,
  - Edges for assignments and calls
- Propagate objects along edges, i.e., data-flow analysis on that graph

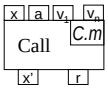
- The baseline P2G approach is locally and globally flow-insensitive; it focuses on data-dependencies over variables and ignores the control flow
  - An analysis is flow-sensitive if it takes into account the order in which statements in a program are executed
  - In principle, additional def-use analysis avoids this problem
  - Using global def-use analysis does not scale
- The baseline P2G approach is context-insensitive
  - An analysis is context-sensitive if distinguishes different contexts in which methods are called
  - Object-sensitivity distinguish methods by the abstract objects they are called on – can be understood as copies of the method's graph
  - Scales but is quite slow

Fast and Precise P2A

1. Data values
   - allocation site abstract from objects `O`
   - abstract heap memory: `O×F⟶O` (`F` set of fields) heap size: `Int`
2. Data-flow graph: Points-to-SSA graph for each method (constructor)
   - Nodes with ports represent operations relevant for P2A, ports correspond to operands, special ϕ-nodes for merge points in control flow
   - Edges represent intra-procedural control- and data-flow
3. Transfer function:
   - update the heap according to the abstract semantics of the
   - special ϕ-nodes: $\cup$ on `O` values and max on `Int` values, resp.
4. Initialization: ϕ for `O` ports and 0 for `Int` ports, resp.
5. Simulated execution

Recall: Simulated Execution

- Interleaving of process method and update call nodes' transfer function
- Processes a method:
  - Starts with main,
  - propagates data values analog the edges in P2-SSA graph
  - updates the heap and the data values in the nodes according to their transfer functions
  - If node type is a call then …
- Call nodes' transfer function; if input changes:
  - Interrupts the processing of a caller method
  - Propagates arguments $Pt(v_1 \ldots v_n)$ to the all callees $Pt(a)$
  - Processes the callees (one by one) completely (iterate in case of recursive calls)
  - Propagates back and merges the results $Pt(r)$ of the callees
  - Updates heap size size(x')
  - Continue processing the caller method …



Example transfer functions

if input changes, update as below else skip:

$Pt(v)=\{o_i\}$ (unique i)

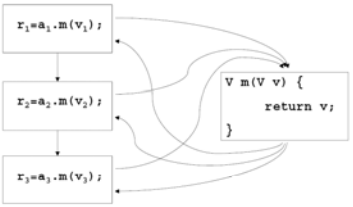
$Pt(v)=\cup_{oi \in Pt(a)}$ heap(o_i,f)

$\forall o_i \in Pt(a)$: heap(o_i,f) $\cup = Pt(v)$
size(x') = heap.size

Flow-sensitivity

- The Points-to-SSA approach has two features that contribute to flow-sensitivity:
  1. Locally flow-sensitive: We have SSA edges imposing the correct ordering among all operations (calls and field accesses) within a method.
  2. Restricted globally flow-sensitive: We have the simulated execution technique that follows the inter-procedural control-flow from one method to another.
- Effect:
  - An access $a_1.x$ will never be affected by another $a_2.x$ that we process after $a_1.x$.
  - Each return only contains contributions from previously processed calls, i.e. reduced mixing of values returned by calls targeting the same method.
  - But: information is accumulated in method arguments to guarantee scalability.

Example: Global flow-sensitivity



Flow-insensitive Result

$$Pt(r_1) = Pt(r_2) = Pt(r_3) = Pt(v_1) \cup Pt(v_2) \cup Pt(v_3)$$

Flow-sensitive Result

$$\begin{aligned} Pt(r_1) &= Pt(v_1), \\ Pt(r_2) &= Pt(v_1) \cup Pt(v_2), \\ Pt(r_3) &= Pt(v_1) \cup Pt(v_2) \cup Pt(v_3) \end{aligned}$$

Context-sensitivity

- Context-insensitive analysis
  - Actual arguments of calls targeting the same method were mixed in the formal arguments.
  - Advantage scalability: ensures termination for recursive call sequences and reaches a fix point quickly
  - Disadvantage accuracy: inaccurate due to mixed return values
- Context-sensitive analysis
  - Divide calls targeting a given method `a.m(v ...)` into a finite number of different categories
  - Analyze them separately – as if they defined different copies of that method.
  - We can define contexts as an abstraction of call stack situations:
    context: `[m, call id, a, v1,… , vn]` ⟶ [c_1, … , c_p]
  - Often it is just one context per call stack situations
    context: `[m, call id, a, v1,… , vn]` ⟶ c_i

Context-sensitive call handling

Call(m, call, x_{in}, a, v_1, … , v_n] ⟶ [x_{out}, r]
  [x_{out}, r] = [0,⊥]
  for all c ∈ *context* `[m, call id, a, v1,… , vn]` **do**
    if [x_{in}, a, v_1, … , v_n] ⊑ previous arguments (m, c) **then**
      r = previous return (m, c)
      x_{out} = x_{in}
    else
      args = previous args (m, c) ⊔ [x_{in}, a, v_1, … , v_n]
      args previous (m, c) = args
      [x_{out}, r] = [x_{out}, r] ⊔ processMethod(m, args)
      previous return (m, c) = r
    end if
  end for
  return [x_{out}, r]

Examples: Context abstractions

Object-sensitivity
- A context is given by a pair `(m,o)`
- where o ∈ O is a unique abstract object in the points-to value analyzed for the call target variable this.
- Linear (in program size) many contexts,
- In practice slightly more precise than This-sensitivity.

This-sensitivity
- A context is given by a pair `(m,this)`
- where `this` ∈ 2^o is the unique points-to value (set of abstract objects) analyzed for the call target variable this.
- Exponentially many contexts (in practice ok),
- In practice an order of magnitude faster than Object-sensitivity.

Examples: Precision

In favor of Object-sensitivity
- Method definitions:
  m() {field = this; }
  V n() {return field; }
- Call 1:
  a_1 = {o_1 , o_2}
  a_1.m()
- Call 2:
  a_2 = {o_1 }
  r_2 = a_2.n()

- Object-sensitivity: Pt(r_2) = {o_1 }.
- This-sensitivity: Pt(r_2) = {o_1 , o_2 }.

In favor of This-sensitivity
- Method definition:
  V m(V v) {return v; }
- Call 1:
  a_1 = {o_1 }, v_1 = {o_3 }
  r_1 = a_1.m(v_1)
- Call 2:
  a_2 = {o_1 , o_2 }, v_2 = {o_4 }
  r_2 = a_2.m(v_2)

- Object-sensitivity: Pt(r_2) = {o_3, o_4}.
- This-sensitivity: Pt(r_2) = {o_4 }.

Results

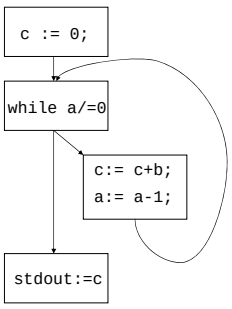
- Fast and accurate P2A
  - Points-to SSA ⟹ locally flow sensitive PTA
  - Simulated execution ⟹ globally flow-sensitive PTA, fast
  - Context-insensitive in the first presented baseline version
  - More accurate (in theory and practice ca. 20%) and 2× as fast compared to classic flow- and context-insensitive P2A
  - Fast: < 1 min on `javac` with > 300 classes.
- Context-sensitive variant this sensitivity even more accurate
  - As fast and up to 3× as precise compared to classic flow- and context-insensitive P2A
  - As precise and 10× as fast compared to the best known context-sensitive variant (object sensitivity) of classic P2A
- Shows in clients analyses like synchronization removal and static garbage collection (escape and side effect analysis)

Assignment I

Compute the SSA graph for:

```
1:      c := 0;
2:      while a/=0 do
3:              c := c+b;
4:              a := a-1;
        od;
5:      stdout := c
```

- Define the program with immature ϕ' functions and the final version.
- Draw the graph (replacing variables by edges). Note that `stdout := c` is actually a function call.



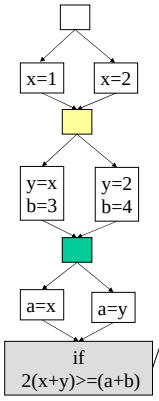
Assignment II

- Leave the SSA graph for the program from Assignment I
  - Remove ϕ functions
  - Perform copy propagation
  - Compare the produced program to the original one
- Perform register allocation for 3 registers
  - Do it mechanically using live analysis and graph coloring
  - Why is it immediately clear that 3 registers are sufficient but 2 are not.

Context-sensitive analysis

- Distinguish different call context of a method
- In general: Distinguish different execution path to a program point
  - Exponentially (in program size) many path in a sequential program
  - Exponentially many analysis values
- (Let away the problem of analysis) How to capture the results efficiently?

Context-insensitive Data flow analysis

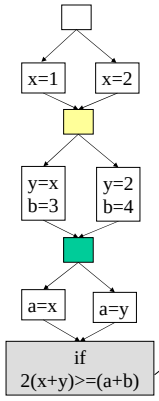


x+y = {2,3,4}

a+b = {4,5,6}

2(x+y) >= a+b ?

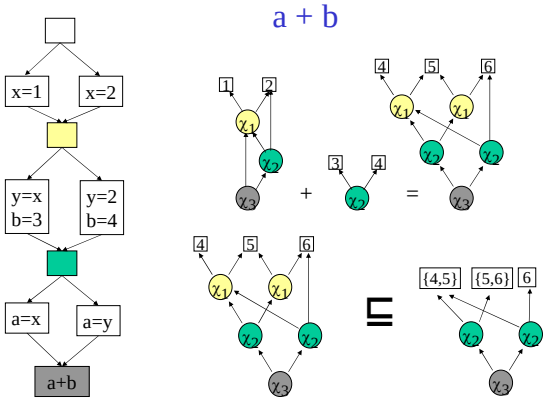
Context-sensitive Data flow analysis



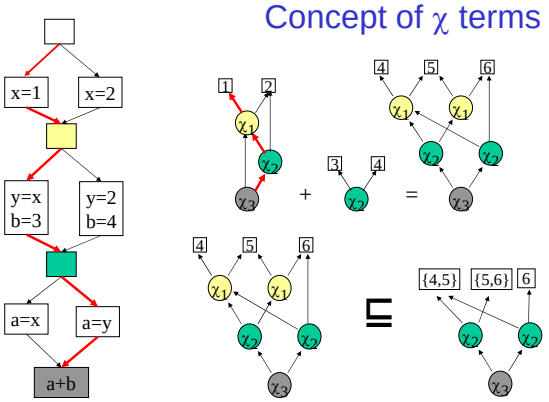
	000	001	010	011	100	101	110	111
x	1	1	1	1	2	2	2	2
y	1	1	2	2	2	2	2	2
a	1	1	1	1	2	2	2	2
b	3	3	4	4	3	3	4	4

Representation of context in decision tables
Too large! Too much redundancy!

a + b



Concept of χ terms



Advantages of χ-Terms

- Compact representation of context sensitive information
- Delayed widening (abstract interpretation) of terms until no more memory: no unnecessary loss of information

  $\chi_3(\chi_2(\chi_1(4,5),\chi_1(5,6)),\chi_2(\chi_1(4,5),6)) \sqsubseteq \chi_3(\chi_2(\{4,5\},\{5,6\}),\chi_2(\{4,5\},6)) \sqsubseteq \chi_3(\{4,5,6\},\{4,5,6\}) = \{4,5,6\} \sqsubseteq [4,6] \sqsubseteq \top$
- χ-Terms are implementation of decision diagrams.
- Adaptation of OBDD implementation techniques.
- Symbolic computation with χ-terms (simplification of terms),
- Transition of the idea of SSA ϕ-function to data-flow values
- Especially interesting for address values to distinguish memory partitions as long as possible

Data-Flow Analyses (DFA) on SSA

- Each SSA node type n has a concrete semantics $[n]$,
  - On execution of n, map inputs i to outputs o and $o = [n](i)$
  - inputs i and outputs o are records of typed values (P1)
  - $[n] :: \text{type}(i_1) \times \ldots \times \text{type}(i_k) \to \text{type}(o_1) \times \ldots \times \text{type}(o_l)$
- Each static data-flow analysis abstracts from concrete semantics and values
  - Abstract semantics T_n is called transfer function of node type n
  - On analysis of n, map abstract analysis inputs $a(i)$ to abstract analysis outputs $a(o)$ and $a(o) = T_n(\, a(i)\,)$
  - $T_n :: \text{type}(a(\, i_1)) \times \ldots \times \text{type}(a(\, i_k)) \to \text{type}(a(\, o_1)) \times \ldots \times \text{type}(a(\, o_l))$
- For each abstract semantics type $A = \text{type}(a(\bullet))$ – analysis universe – there is a partial order relation $\subseteq$ and a meet operation $\cup$ (supremum),
  - $\subseteq :: A \times A$
  - $\cup :: A \times A \to A$ with $\cup = T_+$
  - $(A, \subseteq)$ defines a complete partial order

DFA on SSA (cont.)

- Provided that transfer functions are monotone: $x \le y \Rightarrow T_n(x) \le T_n(y)$ with $x,y \in A_1 \times \ldots \times A_k$ and ≤ defined element-wise with $\subseteq$ of the respective abstract semantics type
- Following iteration terminates in a unique fixed point
  - Initialize the input of each node the SSA graph with the smallest (bottom) element of the Lattice/CPO corresponding its abstract semantics type
  - Initialize the *start* nodes of the SSA graph with proper abstract values of the corresponding its abstract types
  - Attach each node with its corresponding transfer function
  - Uniform randomly compute abstract output values
  - (Fewer updates use SCC and interval analysis to determine an traversal strategy, backward problems analyzed analogously)

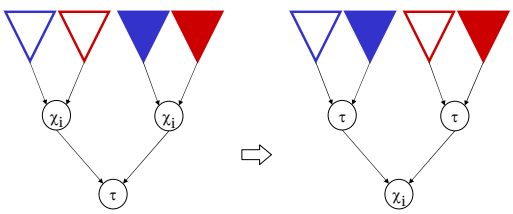
Generalization to χ-terms

- Given such a context-insensitive analysis (lattices for abstract values, set of transfer functions, initialization of *start* node) we can systematically construct a context-sensitive analysis
- χ-term algebras X over abstract semantic values $a \in A$ introduced
  - $a \in A \Rightarrow a \in \mathrm{X}_A$
  - $t_1, t_2 \in \mathrm{X} \Rightarrow \chi_i(t_1, t_2) \in \mathrm{X}_A$
  - Induces sensitive CPO for abstract values $(\mathrm{X}_A, \sqsubseteq)$ and for $a_1, a_2 \in A$ and $t_1, t_2, t_3, t_4 \in \mathrm{X}$:
    $a_1 \subseteq a_2 \Rightarrow a_1 \sqsubseteq a_2$
    $\chi_i(a_1, a_2) \sqsubseteq a_1 \cup a_2$
    $t_1 \sqsubseteq t_3,\ t_2 \sqsubseteq t_4 \Rightarrow \chi_i(t_1, t_2) \sqsubseteq \chi_i(t_3, t_4)$
- New transfer functions induced

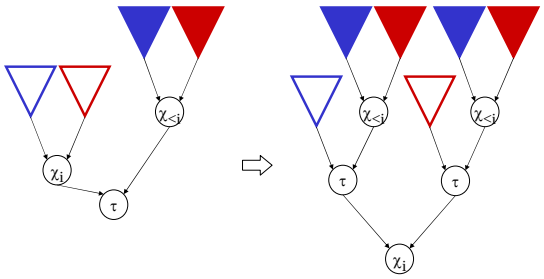
New transfer functions

- ϕ-node's transfer functions:
  - Insensitive: $T_\phi = \cup = a(\, i_1) \cup \ldots \cup a(\, i_k)$
  - Sensitive: $S_\phi = \sqcup_b = a'(\, i_1) \sqcup_b \ldots \sqcup_b a'(\, i_k)$ with b block number of ϕ-node and for $a_1, a_2 \in A$ and $t_1, t_2, t_3, t_4 \in \mathrm{X}$:
    $a_1 \sqcup_b a_2 = \chi_b(a_1, a_2)$
    $\chi_x(t_1, t_2) \sqcup_b \chi_y(t_3, t_4) = \chi_b(t_1 \cup t_2\ ,\ t_3 \sqcup_b t_4)$ iff $x{=}b$ (cases $y = b$ analog)
    $\chi_x(t_1, t_2) \sqcup_b \chi_y(t_3, t_4) = \chi_b(t_1 \sqcup_b t_2\ ,\ t_3 \sqcup_b t_4)$ otherwise
- Ordinary operation's (τ node's) transfer functions:
  - Insensitive (w.l.o.g. binary operation): $T_\tau :: A_a \times A_b \to A_c$
  - Sensitive: $S_\tau :: \mathrm{X}_{A_a} \times \mathrm{X}_{A_b} \to \mathrm{X}_{A_c}$ and for $a_1, a_2 \in A_a$, A_b and $t_1, t_2 \in \mathrm{X}_a$, X_b:
    $S_\tau(a_1, a_2) = T_\tau(a_1, a_2)$
    $S_\tau(\chi_x(t_1, t_2), \chi_y(t_3, t_4)) = \chi_k(S_\tau(\text{cof}(\chi_x(t_1,t_2),\chi_k,1), \text{cof}(\chi_y(t_3,t_4),\chi_k,1)),$
    $S_\tau(\text{cof}(\chi_x(t_1,t_2),\chi_k,2), \text{cof}(\chi_y(t_3,t_4),\chi_k,2)))$
    with k larger of x,y and cof is the co-factorization

Sensitive Transfer Schema (case a)



Sensitive Transfer Schema (case b)



Co-factorization

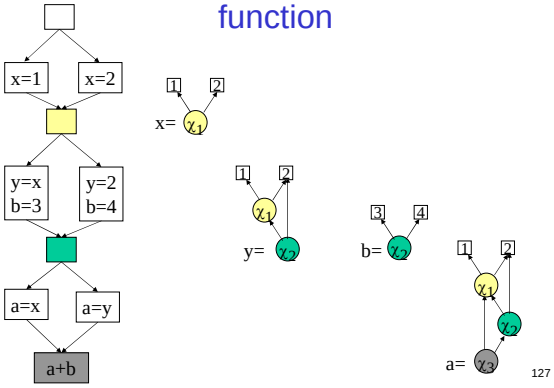
- cof($\chi_x(t_1,t_2),\chi_k,i$) selects the i-th branch of a χ-term if $\chi_x=\chi_k$ and returns the whole χ-term, otherwise

- cof($\chi_x(t_1,t_2),\chi_k,i$) = $\chi_x(t_1,t_2)$ iff $k > x$
- cof($\chi_x(t_1,t_2),\chi_k,1$) = t_1 iff $k = x$
- cof($\chi_x(t_1,t_2),\chi_k,2$) = t_2 iff $k = x$

Example revisited: insensitive

- SSA node ⊕
- Semantic
  - $[\oplus] :: \text{Int} \times \text{Int} \to \text{Int}$
  - $[\oplus](a,b) = a+b$
- Abstract Int values { ⊥, 1,2, …, maxint,⊤}
- Context-insensitive transfer function:
  - $T_\oplus(\bot, x) = T_\oplus(x, \bot) = \bot$
  - $T_\oplus(\top, x) = T_\oplus(x, \top) = \top$
  - $T_\oplus(a,b) = [\oplus](a,b) = a+b$ for $a,b \in$ Int
- Context-insensitive meet function
  - $T_\phi(\bot, x) = T_\phi(x, \bot) = x$
  - $T_\phi(\top, x) = T_\phi(x, \top) = \top$
  - $T_\phi(x, x) = x$
  - $T_\phi(x, y) = \top$

Context-sensitive ϕ-node transfer function

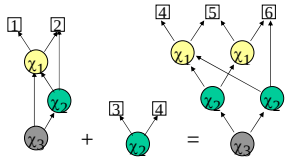


Context-sensitive a ⊕ b

$S_\oplus(\ \chi_3(\chi_1(1,2),\chi_2(\chi_1(1,2),2))\ ,\ \chi_2(3,4)\ )$
$= \chi_3(\ S_\oplus(\text{cof}(\chi_3(\chi_1(1,2),\chi_2(\chi_1(1,2),2)),\ \chi_3,1),\text{cof}(\chi_2(3,4),\ \chi_3,1)),$
$\quad S_\oplus(\text{cof}(\chi_3(\chi_1(1,2),\chi_2(\chi_1(1,2),2)),\ \chi_3,2),\text{cof}(\chi_2(3,4),\ \chi_3,2))\ )$
$= \chi_3(\ S_\oplus(\chi_1(1,2),\chi_2(3,4)),$
$\quad S_\oplus(\chi_2(\chi_1(1,2),2),\ \chi_2(3,4))\ )$
$= \chi_3(\ \chi_2(S_\oplus(\text{cof}(\ \chi_1(1,2),\ \chi_2,1),\text{cof}(\chi_2(3,4),\ \chi_2,1))\ ,$
$\quad S_\oplus(\text{cof}(\ \chi_1(1,2),\ \chi_2,2),\text{cof}(\chi_2(3,4),\ \chi_2,2))\ ),$
$\quad \chi_2(S_\oplus(\text{cof}(\chi_2(\chi_1(1,2),2),\ \chi_2,1),\text{cof}(\chi_2(3,4),\ \chi_2,1))\ ,$
$\quad S_\oplus(\text{cof}(\chi_2(\chi_1(1,2),2),\ \chi_2,2),\text{cof}(\chi_2(3,4),\ \chi_2,2))\ ))$
$= \chi_3(\ \chi_2(S_\oplus(\chi_1(1,2),\ 3)\ ,$
$\quad S_\oplus(\chi_1(1,2),4)),$
$\quad \chi_2(S_\oplus(\chi_1(1,2),3),$
$\quad S_\oplus(2,4)\ ))$

Context-sensitive a ⊕ b (cont.)

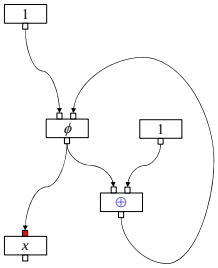
$S_\oplus(\ \chi_3(\chi_1(1,2),\chi_2(\chi_1(1,2),2))\ ,\ \chi_2(3,4)\ ) = \ldots$
$= \chi_3(\ \chi_2(S_\oplus(\chi_1(1,2),\ 3),\ S_\oplus(\chi_1(1,2),4)\ )\ ,\ \chi_2(S_\oplus(\chi_1(1,2),3),\ S_\oplus(2,4)\ ))$
$= \chi_3(\ \chi_2(S_\oplus(\chi_1(1,2),\ 3),\ S_\oplus(\chi_1(1,2),4)\ )\ ,\ \chi_2(S_\oplus(\chi_1(1,2),3),\ 6\ ))$
$= \ldots$
$= \chi_3(\ \chi_2(\chi_1(4,5),\ \chi_1(5,6)\ )\ ,\ \chi_2(\chi_1(4,5),\ 6\ ))$



Assignment III

Given the SSA fragment on the left

- Perform context-insensitive data-flow analysis (using the definitions on the previous slides). What is the the value at the entry of node *x*?
- Perform context-sensitive data-flow analysis (using the definitions on the previous slides). What is the the value at the entry of node *x*?
- Why is the former less precise than the latter?
- Construct a scenario where you could take advantage of that precision in an optimization!



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