Biological Models
Population Dynamics
Predator-Prey

Some Well-known Population Dynamics Applications

• Population Dynamics of Single Population

• Predator-Prey Models (e.g. Foxes and Rabbits)
Population Dynamics of Single Population

- \( P \) – population size = number of individuals in a population
- \( \dot{P} \) – population change rate, change per time unit
- \( g \) – growth factor of population (e.g. \% births per year)
- \( d \) – death factor of population (e.g. \% deaths per year)

\[
growthrate = g \cdot P \\
deathrate = d \cdot P
\]

Exponentially increasing population if \((g-d)>0\)

\[
\dot{P} = growthrate - deathrate \\
\dot{P} = (g-d) \cdot P
\]

Exponentially decreasing population if \((g-d)<0\)

Population Dynamics Model

- \( g \) – growth rate of population
- \( d \) – death rate of population
- \( P \) – population size

\[
\dot{P} = growthrate - deathrate
\]

class PopulationGrowth
    parameter Real g = 0.04  "Growth factor of population";
    parameter Real d = 0.0005  "Death factor of population";
    Real           P(start=10) "Population size, initially 10";
    equation     der(P) = (g-d)*P;
end PopulationGrowth;
Simulation of Population Growth

simulate(PopulationGrowth, stopTime=100)
plot(P)

Exponentially increasing population if \((g-d)>0\)

Population Growth Exercise!!

- Locate the PopulationGrowth model in DrModelica
- Change the initial population size and growth and death factors to get an exponentially decreasing population

simulate(PopulationGrowth, stopTime=100)
plot(P)

Exponentially decreasing population if \((g-d)<0\)

class PopulationGrowth
parameter Real g = 0.04 "Growth factor of population";
parameter Real d = 0.0005 "Death factor of population";
Real P(start=10) "Population size, initially 10";
equation
  der(P) = (g-d)*P;
end PopulationGrowth;
Population Dynamics with both Predators and Prey Populations

• Predator-Prey models

Predator-Prey (Foxes and Rabbits) Model

• $R = \text{rabbits} = \text{size of rabbit population}$
• $F = \text{foxes} = \text{size of fox population}$
• $\dot{R} = \text{der(rabbits)} = \text{change rate of rabbit population}$
• $\dot{F} = \text{der(foxes)} = \text{change rate of fox population}$
• $g_r = g_r = \text{growth factor of rabbits}$
• $d_f = d_f = \text{death factor of foxes}$
• $d_{rf} = d_{rf} = \text{death factor of rabbits due to foxes}$
• $g_{fr} = g_{fr} = \text{growth factor of foxes due to rabbits and foxes}$

\[
\dot{R} = g_r \cdot R - d_{rf} \cdot F \cdot R \\
\dot{F} = g_{fr} \cdot d_{rf} \cdot R \cdot F - d_f \cdot F
\]

\[
\text{der(rabbits)} = g_r \cdot \text{rabbits} - d_{rf} \cdot \text{rabbits} \cdot \text{foxes}; \\
\text{der(foxes)} = g_{fr} \cdot d_{rf} \cdot \text{rabbits} \cdot \text{foxes} - d_f \cdot \text{foxes};
\]
Predator-Prey (Foxes and Rabbits) Model

```modelica
class LotkaVolterra
  parameter Real g_r = 0.04  "Natural growth rate for rabbits";
  parameter Real d_rf = 0.0005  "Death rate of rabbits due to foxes";
  parameter Real d_f = 0.09  "Natural death rate for foxes";
  parameter Real g_fr = 0.1  "Efficiency in growing foxes from rabbits";
  Real rabbits(start=700)  "Rabbits, (R) with start population 700";
  Real foxes(start=10)  "Foxes, (F) with start population 10";
  equation
    der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;
    der(foxes) = g_fr*d_rf*rabbits*foxes - d_f*foxes;
end LotkaVolterra;
```

Simulation of Predator-Prey (LotkaVolterra)

```modelica
simulate(LotkaVolterra, stopTime=3000)
plot({rabbits, foxes}, xrange={0, 1000})
```

![Simulation graph showing the population dynamics of rabbits and foxes over time.](image-url)
Exercise of Predator-Prey

- Locate the LotkaVolterra model in DrModelica
- Change the death and growth rates for foxes and rabbits, simulate, and observe the effects

```modelica
simulate(LotkaVolterra, stopTime=3000)
plot({rabbits, foxes}, xrange={0,1000})
```

```modelica
class LotkaVolterra
  parameter Real g_r = 0.04  "Natural growth rate for rabbits";
  parameter Real d_rf=0.0005  "Death rate of rabbits due to foxes";
  parameter Real d_f = 0.09  "Natural deathrate for foxes";
  parameter Real g_fr=0.1  "Efficiency in growing foxes from rabbits";
  Real rabbits(start=700)  "Rabbits, (R) with start population 700";
  Real foxes(start=10)  "Foxes, (F) with start population 10";
  equation
    der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;
    der(foxes) = g_fr*d_rf*rabbits*foxes - d_f*foxes;
end LotkaVolterra;
```