Biological Models
Population Dynamics
Predator-Prey

Some Well-known Population Dynamics Applications

- Population Dynamics of Single Population
- Predator-Prey Models (e.g. Foxes and Rabbits)

Population Dynamics of Single Population

- P – population size = number of individuals in a population
- $\dot{P}$ – population change rate, change per time unit
- g – growth factor of population (e.g. % births per year)
- d – death factor of population (e.g. % deaths per year)

$$growthrate = g \cdot P$$

$$deathrate = d \cdot P$$

$$\dot{P} = growthrate - deathrate$$

$$\dot{P} = (g - d) \cdot P$$

Exponentially increasing population if (g-d)>0

Exponentially decreasing population if (g-d)<0

Population Dynamics Model

- g – growth rate of population
- d – death rate of population
- P – population size

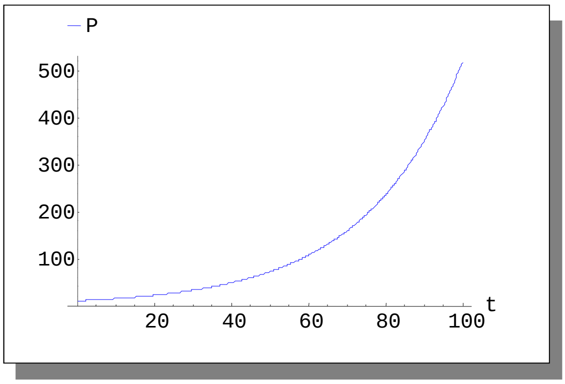
$$\dot{P} = growthrate - deathrate$$

```
class PopulationGrowth
  parameter Real g = 0.04     "Growth factor of population";
  parameter Real d = 0.0005   "Death factor of population";
  Real           P(start=10) "Population size, initially 10";
equation
  der(P) = (g-d)*P;
end PopulationGrowth;
```


Simulation of PopulationGrowth

```
simulate(PopulationGrowth, stopTime=100)
plot(P)
```

Exponentially increasing population if (g-d)>0



Population Growth Exercise!!

- Locate the PopulationGrowth model in DrModelica
- Change the initial population size and growth and death factors to get an exponentially decreasing population

```
simulate(PopulationGrowth, stopTime=100)
plot(P)
```

Exponentially decreasing population if (g-d)<0

```
class PopulationGrowth
  parameter Real g = 0.04     "Growth factor of population";
  parameter Real d = 0.0005   "Death factor of population";
  Real           P(start=10) "Population size, initially 10";
equation
  der(P) = (g-d)*P;
end PopulationGrowth;
```


Population Dynamics with both Predators and Prey Populations

- Predator-Prey models

Predator-Prey (Foxes and Rabbits) Model

- R = rabbits = size of rabbit population
- F = foxes = size of fox population
- $\dot{R}$ = der(rabbits) = change rate of rabbit population
- $\dot{F}$ = der(foxes) = change rate of fox population
- g_r = g_r = growth factor of rabbits
- d_f = d_f = death factor of foxes
- d_{rf} = d_rf = death factor of rabbits due to foxes
- g_{fr} = g_rf = growth factor of foxes due to rabbits and foxes

$$\dot{R} = g_r \cdot R - d_{rf} \cdot F \cdot R \qquad \dot{F} = g_{fr} \cdot d_{rf} \cdot R \cdot F - d_f \cdot F$$

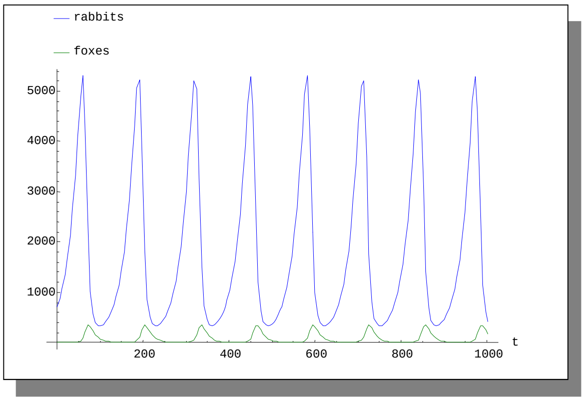
```
der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;
der(foxes)   = g_fr*d_rf*rabbits*foxes - d_f*foxes;
```


Predator-Prey (Foxes and Rabbits) Model

```
class LotkaVolterra
  parameter Real g_r =0.04    "Natural growth rate for rabbits";
  parameter Real d_rf=0.0005  "Death rate of rabbits due to foxes";
  parameter Real d_f =0.09    "Natural deathrate for foxes";
  parameter Real g_fr=0.1     "Efficency in growing foxes from rabbits";
  Real      rabbits(start=700) "Rabbits,(R) with start population 700";
  Real      foxes(start=10)    "Foxes,(F) with start population 10";
equation
  der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;
  der(foxes)   = g_fr*d_rf*rabbits*foxes - d_f*foxes;
end LotkaVolterra;
```


Simulation of Predator-Prey (LotkaVolterra)

```
simulate(LotkaVolterra, stopTime=3000)
plot({rabbits, foxes}, xrange={0,1000})
```



Exercise of Predator-Prey

- Locate the LotkaVolterra model in DrModelica
- Change the death and growth rates for foxes and rabbits, simulate, and observe the effects

```
simulate(LotkaVolterra, stopTime=3000)
plot({rabbits, foxes}, xrange={0,1000})
```

```
class LotkaVolterra
  parameter Real g_r =0.04    "Natural growth rate for rabbits";
  parameter Real d_rf=0.0005  "Death rate of rabbits due to foxes";
  parameter Real d_f =0.09    "Natural deathrate for foxes";
  parameter Real g_fr=0.1     "Efficency in growing foxes from rabbits";
  Real     rabbits(start=700) "Rabbits,(R) with start population 700";
  Real     foxes(start=10)    "Foxes,(F) with start population 10";
equation
  der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;
  der(foxes)   = g_fr*d_rf*rabbits*foxes - d_f*foxes;
end LotkaVolterra;
```

