N-Player Game in a Multiple Access Channel is Selfish

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Abstract

This paper studies behavior of players in a common exclusively-shared channel using a backoff protocol for resolving collisions. We show that when players have freedom to choose backoff parameters (or time to send a next packet), they behave selfishly. The system has an undesirable Nash equilibrium, where every player tries to grasp as much channel as possible. Since the channel is exclusively shared, no player would get a packet through (all packets will collide). Although the result is seemingly obvious, we were unable to find it in the literature. We also evaluate a simple incentive mechanism based on an arbiter model, which controls channel access by jamming misbehaving players.

1 Introduction

The backoff protocol is a scheduling protocol for simultaneous access to a multiple access channel where simultaneous transmissions collide. To deal with collisions, a backoff protocol was introduced and adopted in such protocols as Aloha [2], Ethernet [13] and IEEE 802.11 (Wi-Fi) [1]. As an example, Aloha protocol uses a constant backoff protocol, while IEEE 802.11 uses a truncated exponential binary backoff protocol.

Over past thirty years, the backoff protocol was analyzed by several researchers [3–8]. Furthermore, following the idea by Kwak et al. [11] we analyzed general backoff protocols [12]. We studied optimality of a general backoff function instead of a fixed function. The analysis showed that the choice of the optimal protocol parameters depends on the number of active stations in the network and may vary depending on the load of the network. Hence, permitting the stations to choose the backoff parameters depending on the channel load can increase throughput for individual stations and the network itself.

On the other hand, recent studies on game-theoretic aspects of the backoff protocols showed that the freedom to control backoff parameters leads to selfish behavior of individual players (stations) [10].

In this paper we consider what if we give freedom to manipulate general backoff parameters to each station in the network. In other words, if a station is free to use the channel at any time, what the resulting behavior would be?

Unlike in the backoff model, here we do not give the history of interaction to a station. Hence, the network model is a black box to the end station. A station does not know had the packet collided before the game is finished, stations know only the number of other stations (players) and that every player in the network wants to selfishly maximize its throughput. Unfortunately, we omit consideration of the previous history (backoff counter) because it makes the model very complex otherwise. We believe that the model still represents the choice of each player as with a general backoff network without restrictions on behavior. Under these conditions, we show that the game has undesirable Nash equilibrium.

Additionally, we modify the model using a known incentive mechanism — a common network arbiter, which jams the channel if some player transmits too much packets. We show that these incentives do not give a unique Nash equilibrium solution, and one of the possible equilibrium solutions still involves unde-

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sirable behavior.

2 Analysis

2.1 Model

Consider the following game model. N players try to send packets through a shared channel during time T. The whole time is divided into timeslots; the length of each timeslot is 1, hence there are T slots of length 1, which are synchronized and known to each player. During one timeslot player can send one packet. At the beginning of each game, every player chooses timeslots for sending packets. We assume that a player i decides to use k_i slots for transmission. Knowing the number of slots to be used, the exact slots for transmission are chosen randomly and uniformly among other possible. There are $\binom{T}{k_i}$ combinations to place k_i elements on T and probability for every combination is equal. For such a game we want to find which strategy (a number of packets to send) a player will choose.

A similar problem was studied by Kolchin et al. in the book "Random allocations" [9]. The difference is that the book did not consider a game problem, but used the same k_i for every player. Even for such problem, it is hard to analyze the collision probability. In our case, the probability that k_1 and k_2 will collide exactly in Δ slots equals to $\frac{\binom{k_1}{\Delta}\binom{T-k_1}{k_2-\Delta}}{\binom{T}{k2}}$.

2.2 Two-player game

Consider a particular case of the game above, when the number of players is two. The first player decides to send packets in k_1 slots, the second in k_2 slots. As in [9] consider the following random variable μ_r be the number of slots, during which r packets are sent ($0 \le r \le N$). In case of two-player game, there are at most two packets in a slot from both players. Now, let us calculate μ_2. If we define as q_i the event that two packets were sent in slot i, then $\mu_2 = \sum_{i=1}^{T} \mathbb{1}\{q_i\}$, where $\mathbb{1}\{A\}$ is an indicator function for event A. Taking expectation from the equation we get $E\mu_2 = TP\{q_i\}$, and for a two-player game it is equal to $E\mu_2 = T\frac{k_1}{T}\frac{k_2}{T} = \frac{k_1k_2}{T}$.

That value is exactly the expected number of collided packets. The expected number of successful packets for the first player is $k_1 - \frac{k_1k_2}{T}$ and for the second player is $k_2 - \frac{k_1k_2}{T}$. Hence, we have the income function $H_1(k1, k2) = k_1(1 - \frac{k_2}{T})$ for the first player and $H_2(k1, k2) = k_2(1 - \frac{k_2}{T})$ for the second. It is clear that unless one of players chooses T as a strategy, the best income for another player is to choose T as a strategy. This is a Nash equilibrium. Since we assumed that players behave similarly, we can assume that the Nash equilibrium is (T, T). Each of two players behaves selfishly.

2.3 N-player game

Now, consider an N-player game. It can be reduced to a two-player game, if we consider the first player as one player, and the rest of players as another player. Hence, if we define Δ as the number of slots taken by the rest of the players, then the income for the first player will be

$$H_1(k_1, k_2, \dots, k_N) =$$

$$\sum_{\Delta=0}^{T} k_1(1-\frac{\Delta}{T})P\{k_2, \dots, k_N \text{ occupies } \Delta\} = k_1(1-\frac{E\Delta}{T}).$$

Consider again, μ_r. Now we need to find μ_0, the number of free slots for players $\overline{2, N}$. Let q_i be an event that slot i is unoccupied by players $\overline{2, N}$. Then $\mu_0 = \sum_{i=1}^{T} \mathbb{1}\{q_i\}$. The expectation of this value is $E\mu_0 = TP\{q_i\} = T\prod_{i=2}^{N}(1 - \frac{k_i}{T})$. Thus, the expected number of free slots is $T(1 - \prod_{i=2}^{N}(1 - \frac{k_i}{T}))$, and hence the income function for the first player is $H_1(k_1, k_2, \dots, k_N) = k_1\prod_{i=2}^{N}(1 - \frac{k_i}{T})$. The income for player j is

$$H_j(k_1, k_2, \dots, k_N) = k_j \prod_{i=1, i\neq j}^{N}(1 - \frac{k_i}{T}).$$

From here, we again see that unless one of the other players chooses T as a strategy, any player is forced to choose T. Because of similarity and as players cannot know what other players choose, the expected Nash equilibrium for the game will be $(T, \dots, T)$. Hence, the N-player game leads to selfish behavior.

2.4 On optimality and improvement of the game

Using the equilibrium derived above, every player receives zero income. Consider the case when players behave equally. Every player chooses k as the strategy; let us find the maximum possible profit for a player. We need to find optimal points for function $k\prod_{i=1,i\neq j}^{N}(1-\frac{k}{T})$. The derivative for this function is equal to $(1-\frac{k}{T})^{N-2}(1-\frac{Nk}{T})$. The optimal point is $k=\frac{T}{N}$, and the income (if all players choose that as an optimal point) is $\frac{T}{n}(1-\frac{1}{n})^{n-1} \approx \frac{T}{n}e^{-1}$. That means that at most $\frac{T}{e}$ of the channel is divided equally (utilization e^{-1} of the channel is a well-known theoretical limit for shared channels). Now, to get that optimal behavior as a Nash equilibrium for all N players we need to change the income function to the following form $H_j(k_1,k_2,\ldots,k_N) = k_j\prod_{i=1,i\neq j}^{N}(1-\frac{k_i}{T})1\{k_j \le \frac{T}{N}\}$. It means that we give nothing to a player who tries to use more than $\frac{T}{N}$ of the channel. Unfortunately, this is hard to implement in practice. A known way is to add an arbiter station that jams the channel if some player uses more than it should. In that case, the income function will get the following form

$$H_j(k1,\ldots,k_N) = \begin{cases} k_j \prod_{i=1,i\neq j}^{N}(1-\frac{k_i}{T}) & k_j < \frac{T}{N}, \\ \frac{T}{N} \prod_{i=1,i\neq j}^{N}(1-\frac{k_i}{T}) & k_j \ge \frac{T}{N}. \end{cases}$$

Unfortunately this equation does not restrict $(\frac{T}{N},\ldots,\frac{T}{N})$ to be the only Nash equilibrium. A player i can choose any value between $\frac{T}{N}$ and T.

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