An Adaptive Backoff Protocol With Markovian Contention Window Control

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Abstract

The backoff protocol is widely used for sharing a common channel among several stations in communication networks. The Binary Exponential Backoff (BEB) improves the system throughput but increases the capture effect, permitting a station to seize the channel for a long time. In this paper, we introduce and analyze a new class of adaptive backoff protocols where a station changes its contention window after a successful transmission differently than BEB. The transitions between the contention window states are determined by a stochastic matrix which describes a finite, irreducible, aperiodic Markov chain. We derive a stationary distribution of an associated embedded Markov chain in an explicit form and then find the stationary distribution of the basic Markov chain explicitly. Preliminary simulation results show that our backoff protocol can reduce the capture effect in Ethernet and wireless networks significantly.

1 Introduction

The Binary Exponential Backoff protocol (BEB) is often used when sharing of a resource among several stations is needed. When two stations attempt to

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transmit a packet simultaneously on a shared channel the resulting collision leads to data loss. Each collision requires subsequent attempts to retransmit a data frame after a uniformly distributed delay. The maximum delay is doubled after each unsuccessful transmission attempt, and represents the *contention window* (CW).

Perhaps the most prominent application of BEB is Medium Access Control in Ethernet [Metcalfe and Boggs(1976)] and Wireless LANs [IEEE(2009)]. The BEB is also used by Internet transport protocols, including TCP, during timeouts. Thus, even a small improvement in backoff performance could have significant impact on real-life applications.

For several reasons, most systems today implement BEB rather than a generic backoff protocol (GB), where stations increase the size of the CW before a next transmission attempt by other factor than two. It is simple to implement in computers with a registry shift operation. However, BEB does not perform optimally in all scenarios as we have showed [Lukyanenko and Gurtov(2008)].

BEB was analyzed extensively in the related work [Aldous(1987), Bianchi(2000), Hastad et al.(1987)Hastad, Leighton, and Rogoff, Kwak et al.(2005)Kwak, Song, and Miller]. A few attempts have been made to develop an analytical model of backoff behavior. However, no explicit solution has been obtained due to complexity of the analysis. In this paper, we present and analyze a general model of an adaptive backoff protocol with Markovian control of CW size. First of all, following [Bianchi(2000)], we assume that the collision probability p_c is constant and does not depend on a station. This basic assumption, called *fixed point equation* (FPE), was analyzed in [Kumar et al.(2007)Kumar, Altman, Miorandi, and Goyal] and is now widely recognized and applied for instance, in related work [Bianchi(2000), Kwak et al.(2005)Kwak, Song, and Miller]. We note that the FPE assumption is critical to develop a detailed and tractable analysis of general adaptive backoff protocol. A survey of FPE assumption can be found in [Cho and Jiang(2009)]; in this paper we do not challenge this basic underlying assumption. Nevertheless, we note that FPE is quite natural for a stationary and homogeneous network with a large number of stations N, when a role of each station becomes negligible. However, for small N, a difference between stations may imply a great variability of collision probability p_c. It is the average of the collision probabilities, conditioned on the stations and current states of CW.

Introducing GB improves the performance especially in scenarios with many stations [Lukyanenko and Gurtov(2008)]. Unfortunately, it also increases the so-called *capture effect*, where a station can utilize the medium after a successful transmission. Although GB can increase the overall system throughput, it does not achieve a fair channel allocation among stations.

In this paper we introduce and analyze a new class of protocols, called *matrix adaptive backoff protocols* (MAB). We modify the behavior of the backoff counter (BC) that counts successive collisions during transmission attempts. The BC does not return to initial (*minimal*) state after a successful transmission. Instead, we use a matrix to choose the new starting state of BC. In other words, a station waits for a random number of timeslots depending on the current state of CW instead of transmitting immediately.

Introducing the matrix mechanism for changing of the CW allows to develop a Markov model that describes the dynamics of the transmission process at an arbitrary station. We analyze and derive explicit solution for such a model using embedded Markov chains. First we consider a particular case of MAB, called *direct MAB protocol* (DMAB), which corresponds to known schemes of adaptive backoff protocol. Based on DMAB, we produce a more detailed analysis of the stationary distribution describing the states of the CW.

However, our preliminary simulation shows that DMAB has a high capture effect and, to resolve this problem, we introduce a modification of MAB, called the *reverse MAB protocol* (RMAB). This modification of the model with a suitable matrix mechanism allows to reduce or eliminate the capture effect, while retaining higher throughput provided by GB. Moreover, this modification of MAB is easy to analyze in the same way as DMAB.

The rest of the paper is organized as follows. In Section 2, we give background material on GB. A general model of MAB is introduced in Section 3. In Section 4, we analyze sequentially MAB, DMAB and RMAB models using embedded Markov chains. In Section 5, preliminary simulations of MAB and RMAB demonstrate a considerable decrease of the capture effect when RMAB is applied. Section 6 contains a summary of the main results.

2 Description of Matrix Adaptive Backoff Protocol

Before giving a detailed description of the new model, we discuss in brief some known forms of GB. Backoff protocols differ in how the CW size is changed after a collision. For instance, in BEB the CW size is doubled upon a collision. The BEB also has the upper bound; that is, after the CW value reaches 1024 no further increase is happening. That rule holds in our simulation model, but is not mentioned during analysis, as CWs are not used in computation directly.

Using a backoff protocol, a station attempts to transmit a data frame uniformly within the current CW. Each collision implies an increase of the CW size to decrease the chance of further collisions. In most models, the frame is discarded after a finite number of collisions, that is after $M + 1$ unsuccessful attempts to transmit a packet. Thus, the CW sizes caused by successive collisions constitute an increasing sequence $CW_0 < \cdots < CW_M$, where CW_i is the CW size after i collisions, $i = 0, \ldots, M$.

Most backoff protocols upon a successful transmission, reduce CW to initial (minimal) fixed value CW_0. Previous works [Aldous(1987), Bianchi(2000), Hastad et al.(1987)Hastad, Leighton, and Rogoff, Lukyanenko and Gurtov(2008)] had focused on this scenario. Furthermore, in the paper [Song et al.(2005)Song, Kwak, Song, and Miller], the *exponential increase exponential decrease* (EIED) algorithm was introduced and analyzed, in which the current CW size increases/decreases by some fixed multiplier. In that work, EIED is compared to MILD (Multiple Increase Linear Decrease) algorithm [Bharghavan et al.(1994)Bharghavan, Demers, Shenker, and Zhang], where the decrease of the CW size is linear. It is shown, that EIED has better performance than MILD and BEB. However, the capture effect and the fairness of EIED protocol was not discussed in this work. There are also schemes which, to some extent, are similar to EIED. For instance, different forms of additive increase multiple decrease (AIMD) protocols for TCP can be found in [Kesselman and Mansour(2003), Yang and Lam(2000)].

To realize the new mechanism for the initial value of CW_0, we introduce and investigate a general MAB protocol. It is a general adaptive scheme for the change of CW_0 after transmission/loss of the data frame. The idea to change CW_0 is not new and is used, for instance, in [Song et al.(2003)Song, Kwak, Song, and Miller]. Moreover, the protocols AIMD, EIED and MILD also belong to a class of *adaptive backoff* protocols, where reduction of the window depends on the current state of the backoff counter (BC). The MAB protocol also belongs to the class of adaptive backoff protocols. In particular, when a collision occurs in MAB protocol, then the current CW size is changed to one of states formed by an *increase factor* a (for instance, in standard BEB $a = 2$.) In fact, these states form a finite Markov chain with state space $\mathcal{M} = \{0, \dots, M\}$.

Now we discuss in brief a difference between algorithms of MAB and above mentioned adaptive backoff protocols. In MAB, each state $i+1$ of the Markov chain, describing the status of the BC, is achieved from the state i by one collision. The transition/discarding of a frame returns the Markov chain to a state belonging to the state space $\mathcal{M}$. At the same time, EIED protocol allows to jump down on a new state which is not visited during extension of the CW. For instance, it is shown in [Song et al.(2003)Song, Kwak, Song, and Miller], that the optimal increase factor is $a = 2$, while the optimal *decrease factor* is $\sqrt{2}$. Hence, after a successful transmission at state i, the CW_i size reduces to $CW_i/\sqrt{2}$. Moreover, for instance MILD, has 1024 states, while the increase procedure gives 10 states only.

The MAB protocol allows to eliminate a disadvantage in widely used backoff protocols. However, simulation shows that the decrease speed of CW in EIED and in MILD is too slow. The congested stations remain congested (in a large loaded network) for a long time and are unable to return directly to the initial state with the minimal value CW_0. The main novelty of our approach is that the control mechanism of CW_0 can be used to *decrease the capture effect, and by this to increase fairness of the protocol.* Indeed, as simulation shows, the RMAB protocol allows to achieve this goal.

Now we describe the MAB protocol in detail. It is assumed that each station has a BC counting consecutive collisions related to a frame being transmitted by the station. The next state of the BC is determined by its current state and one of possible *independent events:* collision, successful transmission or discarding. Thus BC dynamics can be described by a Markov chain $Y = \{Y_n\}$ on the state space $\mathcal{M}$. More exactly, the event $\{Y_n = i\}$, or equivalently, $\{BC = i\}$, corresponds to i successful collisions during transmission of a frame. Recall that each state i corresponds to a unique CW value CW_i, and $CW_0, \dots, CW_M$ is an increasing sequence. A new aspect of the considered model is that after a successful transmission in state i, BC restarts in a state j with a given probability $p_{i,j} = \mathsf{P}(Y_{n+1} = j | Y_n = i)$. We denote the transition matrix $\mathsf{P} = ||p_{i,j}||$. If $p_{i,j} = 0$ for all $j \geq i$ and $p_{0,0} = 1$, then this general model corresponds to DMAB protocol. Moreover, if there exists $j > i$ such that $p_{i,j} > 0$, then the model describes an RMAB protocol.

Thus, a transition matrix P allows not only a decrease but also *an increase* of the CW after transmission/discarding of a frame, and is assumed to be given. It is evident that the properties of the protocol are mainly determined by the matrix P. Although all described adaptive protocols use a Markovian mechanism to change CW but, by the matrix mechanism, MAB may considerably extend a choice of the initial value CW_0. We note that the mechanism of CW extension adopted in DMAB is widely used in various forms. The idea to use RMAB is motivated by the necessity to decrease the capture effect. Although the main purpose of this paper is to study RMAB, it is easier to analyze DMAB first and then to apply this analysis to RMAB.

We assume that the finite Markov chain Y is *irreducible and aperiodic*. Moreover, to study Y it is enough to analyze an embedded Markov chain $X = \{X_n\}$ formed by the states of BC *just after a jump down* (that is after a change of CW caused by a successful transmission or discarding of a frame), or an embedded Markov chain $X^* = \{X_n^*\}$, formed by the states *before a jump down*. Of course, these chains are closely connected and it is easy to verify that they are also aperiodic and irreducible. Thus the weak limits $X_n^* \Rightarrow X_\infty^*$, $X_n \Rightarrow X_\infty$ exist, and we denote by $\pi^* = \{\pi_0^*, \dots, \pi_M^*\}$ and $\pi = \{\pi_0, \pi_1, \dots, \pi_M\}$ the corresponding stationary distributions of the chains X^*, X, respectively. Thus, $\pi_k = \mathsf{P}(X_\infty = k)$, $\pi_k^* = \mathsf{P}(X_\infty^* = k)$ for each k.

3 Analysis of Matrix Adaptive Backoff Protocol

In this section, we present mathematical analysis of embedded Markov chains corresponding to MAB. We start from analysis of a general MAB protocol. This analysis is automatically applied to RMAB and DMAB, which are particular cases of the MAB protocols. Then we obtain in an explicit form a stationary distribution of the embedded Markov chain describing DMAB.

3.1 MAB protocol

First, we construct the transition $(M+1) \times (M+1)$ matrix $Q = ||q_{i,j}||$ connecting the starting state of CW extension and its final state when a successful transmission (or discarding) occurs. Obviously, $q_{i,j} = (1-p_c)p_c^{j-i}$, if $0 \le i \le j < M$, and $q_{i,j} = 0$, if $0 \le j < i \le M$. Moreover, because at state M only a successful transmission or discarding occurs, then $q_{i,M} = p_c^{M-i}$, $i = 0, \dots, M$. Hence, corresponding transition matrix is

$$\mathsf{Q} = \begin{pmatrix} (1-p_c) & (1-p_c)p_c & (1-p_c)p_c^2 & \dots & p_c^M \\ 0 & (1-p_c) & (1-p_c)p_c & \dots & p_c^{M-1} \\ 0 & 0 & (1-p_c) & \dots & p_c^{M-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}. \tag{1}$$

Also we write a given $(M+1) \times (M+1)$ transition matrix $\mathsf{P} = ||p_{i,j}||$ corresponding to a general MAB (including RMAB):

$$\mathsf{P} = \begin{pmatrix} p_{0,0} & p_{0,1} & p_{0,2} & \cdots & p_{0,M} \\ p_{1,0} & p_{1,1} & p_{1,2} & \cdots & p_{1,M} \\ p_{2,0} & p_{2,1} & p_{2,2} & \cdots & p_{2,M} \\ \dots & \dots & \dots & \dots & \dots \\ p_{M,0} & p_{M,1} & p_{M,2} & \cdots & p_{M,M} \end{pmatrix}. \tag{2}$$

For the DMAB, this matrix takes the following specific form:

$$\mathsf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & 0 & 0 & \dots & 0 \\ p_{3,0} & p_{3,1} & p_{3,2} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ p_{M,0} & p_{M,1} & p_{M,2} & p_{M,3} & \dots & 0 \end{pmatrix}. \tag{3}$$

It is obvious that vectors π and π^* are connected as

$$\pi^* = \pi \mathsf{Q}, \quad \pi = \pi^* \mathsf{P}, \tag{4}$$

or $\pi^* = \pi^* \mathsf{PQ}$. In order to solve the equation, we need to find a kernel of matrix $(\mathsf{PQ} - I)^T$, where I is the identity matrix and $(\cdot)^T$ denotes transposition. (We use the transposed matrix equation for easy calculations.) Denote

$K_{i,j} = (1-p_c)\sum_{k=0}^{j} p_c^{j-k} p_{i,k}$, $i,j = 0,\dots,M$. Then it follows that

$$\mathsf{PQ} = \begin{pmatrix} K_{0,0} & K_{0,1} & \dots & K_{0,M-1} & \frac{K_{0,M}}{1-p_c} \\ K_{1,0} & K_{1,1} & \dots & K_{1,M-1} & \frac{K_{1,M}}{1-p_c} \\ \dots & \dots & \dots & \dots & \dots \\ K_{M,0} & K_{M,1} & \dots & K_{M,M-1} & \frac{K_{M,M}}{1-p_c} \end{pmatrix}. \tag{5}$$

After a simple algebra we obtain the following matrix

$$\mathcal{K} = \begin{pmatrix} K_{0,0}-1 & K_{1,0} & K_{2,0} & \dots & K_{M-1,0} & K_{M,0} \\ K_{0,1} & K_{1,1}-1 & K_{2,1} & \dots & K_{M-1,1} & K_{M,1} \\ K_{0,2} & K_{1,2} & K_{2,2}-1 & \dots & K_{M-1,2} & K_{M,2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ K_{0,M} & K_{1,M} & K_{2,M} & \dots & K_{M-1,M} & K_{M,M}-1+p_c \end{pmatrix}, \tag{6}$$

which differs from $(\mathsf{PQ}-I)^T$ only in that the last row in K is multiplied by $1-p_c$. Note that the matrices $(\mathsf{PQ}-I)^T$ and $\mathcal{K}$ have the same kernel. Because the following relation

$$K_{i,j+1} - p_c K_{i,j} = (1-p_c)p_{i,j+1}$$

hold for all i,j, then after some algebra, the kernel can be written as the following system of equations:

$$\begin{cases} \pi_0^* = (1-p_c)\sum_{k=0}^{M} p_{k,0}\pi_k^*, & \\ \pi_i^* = p_c\pi_{i-1}^* + (1-p_c)\sum_{k=0}^{M} p_{k,i}\pi_k^*, & 1 \le i \le M-1, \\ \pi_M^* = p_c\pi_{M-1}^* + p_c\pi_M^* + (1-p_c)\sum_{k=0}^{M} p_{k,M}\pi_k^*. & \end{cases} \tag{7}$$

An evident connection $\pi_i = \sum_{k=0}^{M} \pi_k^* p_{k,i}$ between the two embedded Markov chains gives the following relation between the distributions π^* and π,

$$\begin{cases} \pi_0^* = (1-p_c)\pi_0, & \\ \pi_i^* = p_c\pi_{i-1}^* + (1-p_c)\pi_i, & 1 \le i \le M-1, \\ \pi_M^* = p_c\pi_{M-1}^* + p_c\pi_M^* + (1-p_c)\pi_M, & \end{cases} \tag{8}$$

which can be rewritten as

$$\begin{cases} \pi_0 = \frac{\pi_0^*}{1-p_c}, & \\ \pi_i = \frac{\pi_i^* - p_c\pi_{i-1}^*}{1-p_c}, & 1 \le i \le M-1, \\ \pi_M = \pi_M^* - \frac{p_c}{1-p_c}\pi_{M-1}^*. & \end{cases} \tag{9}$$

In the paper [Lukyanenko and Gurtov(2008)] a stationary distribution $\bar\pi = \{\bar\pi_i\}$ of the basic Markov chain Y was obtained for the DMAB. That is $\bar\pi_i = \mathsf{P}(Y_\infty = i)$, where Y_∞ is a weak limit, $Y_n \Rightarrow Y_\infty$. It is easy to see that for a general MAB protocol a distribution $\bar\pi$ satisfies the following system of equations

$$\begin{cases} \bar\pi_0 = (1-p_c)\sum_{k=0}^M p_{k,0}\bar\pi_k, \\ \bar\pi_i = p_c\bar\pi_{i-1} + (1-p_c)\sum_{k=0}^M p_{k,i}\bar\pi_k, \qquad 1 \le i \le M-1, \\ \bar\pi_M = p_c\bar\pi_{M-1} + (1-p_c)\sum_{k=0}^M p_{k,M}\bar\pi_k. \end{cases} \tag{10}$$

Then it follows from (7) that the following connection between two distributions π^* and $\bar\pi$ holds:

$$\begin{cases} \bar\pi_i = \frac{\pi_i^*}{1-p_c\pi_M^*}, 0 \le i \le M-1, \\ \\ \bar\pi_M = \frac{(1-p_c)\pi_M^*}{1-p_c\pi_M^*}. \end{cases} \tag{11}$$

Thus, we have found connections between stationary distributions of the embedded Markov chains X, X^* and the basic Markov chain Y. To find an explicit expression, in the next section we will first focus on the DMAB protocol.

3.2 DMAB protocol

We emphasize that the analysis above holds for MAB protocol, including DMAB and RMAB. Now we focus on DMAB. Because in this case $p_{i,j} = 0$ for all $i \le j$ and $p_{0,0} = 1$, then relations (7) become

$$\begin{cases} \pi_0^* = (1-p_c)\sum_{k=0}^M p_{k,0}\pi_k^*, \\ \pi_i^* = p_c\pi_{i-1}^* + (1-p_c)\sum_{k=i+1}^M p_{k,i}\pi_k^*, \quad 1 \le i \le M-1, \\ \pi_M^* = p_c\pi_{M-1}^* + p_c\pi_M^*. \end{cases} \tag{12}$$

To find the distribution π^* in an explicit form, we denote

$$a_{i,i} = -1/p_c,\ d = (1-p_c)/p_c,\ a_{i,j} = dp_{i,j} \text{ for all } i > j.$$

Also we introduce the following $(M-1)\times(M+1)$ matrix

$$A = \begin{pmatrix} 1 & a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1} & a_{5,1} & \dots & a_{M-2,1} & a_{M-1,1} & a_{M,1} \\ 0 & 1 & a_{2,2} & a_{3,2} & a_{4,2} & a_{5,2} & \dots & a_{M-2,2} & a_{M-1,2} & a_{M,2} \\ 0 & 0 & 1 & a_{3,3} & a_{4,3} & a_{5,3} & \dots & a_{M-2,3} & a_{M-1,3} & a_{M,3} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & a_{M-1,M-1} & a_{M,M-1} \\ & & & & & & & & & \end{pmatrix}, \tag{13}$$

which is formed by a subset of the equations (12) with $i = 1, \dots, M-1$, and hence $A\pi^* = 0$. To find a *unique solution* π^*, we will use two remaining equations of the system (12) and the normalizing condition. Let

$$\alpha_0(i) = 1,\ \alpha_1(i) = \frac{1}{p_c} = -a_{i,i},$$

and for $0 \le i \le M-2$ define recursively

$$\alpha_k(i) = -\sum_{j=0}^{k-1} a_{i+k-1,i+j}\alpha_j(i), \quad 1 \le k \le M-2.$$

After some algebra we obtain that the kernel of matrix (13) is

$$\pi_i^* + \pi_{M-1}^* \sum_{j=i+1}^{M-1} \alpha_{j-i-1}(i+1) a_{M-1,j} + \pi_M^* \sum_{j=i+1}^{M-1} \alpha_{j-i-1}(i+1) a_{M,j} = 0,\ i = 0, \dots, M-2. \tag{14}$$

It follows from (12) that $\pi_{M-1}^* = d\,\pi_M^*$, and thus we obtain recursively from (14) the following expression

$$\pi_i^* = -\pi_M^* \sum_{j=i+1}^{M-1} (d\,a_{M-1,j} + a_{M,j})\,\alpha_{j-i-1}(i+1),\ \ 0 \le i \le M-2. \tag{15}$$

Normalization condition $\sum_{i=0}^{M} \pi_i^* = 1$ gives π_M^* in an explicit form,

$$\pi_M^* = \frac{1}{1 + d - \sum_{j=1}^{M-1} (d\,a_{M-1,j} + a_{M,j}) \sum_{i=0}^{j-1} \alpha_{j-i-1}(i+1)}. \tag{16}$$

Now we obtain for $0 \le i \le M-2$,

$$\pi_i^* = \frac{-\sum_{j=i+1}^{M-1} (d\,a_{M-1,j} + a_{M,j})\,\alpha_{j-i-1}(i+1)}{1 + d - \sum_{j=1}^{M-1} (d\,a_{M-1,j} + a_{M,j}) \sum_{i=0}^{j-1} \alpha_{j-i-1}(i+1)}, \tag{17}$$

and moreover,

$$\pi^*_{M-1} = \frac{d}{1 + d - \sum_{j=1}^{M-1} (d\, a_{M-1,j} + a_{M,j}) \sum_{i=0}^{j-1} \alpha_{j-i-1}(i+1)}. \tag{18}$$

Thus, distribution π^* is obtained in an explicit form. Hence, by (9), (10) distributions π and $\bar{\pi}$ are also found in an explicit form.

Note that for the DMAB, a connection between the distributions π^* and π can be simplified in comparison with general case (8), namely:

$$\begin{cases} \pi^*_0 = (1-p_c)\pi_0, \\ \pi^*_i = p_c \pi^*_{i-1} + (1-p_c)\pi_i, \quad 1 \le i \le M-1, \\ \pi^*_M = p_c \pi^*_{M-1} + p_c \pi^*_M. \end{cases} \tag{19}$$

The analysis above is based on the embedded Markov chains instead of studying the basic Markov chain Y describing the dynamics of the MAB protocol in detail. Such approach develops the analysis of a much simpler geometrical BP [Lukyanenko and Gurtov(2008)], in which a current state of BC increases with probability p_c and decreases to a fixed initial state CW_0 with probability $1-p_c$. Note that the knowledge of the stationary distribution $\bar{\pi}$ (of the chain Y) allows to calculate various *stationary performance measures* related to the state of MAB protocol at an arbitrary timeslot. At the same time, the stationary distribution π (or π^*) allows to calculate *performance cyclic measures* describing characteristics of MAB protocol over a full cycle of the CW dynamics (that is during full time of a frame transmission). For instance, using π one can calculate the expected full transmission as $\mathsf{E}S = \sum \pi_k \mathsf{E}_k S$, where $\mathsf{E}_k S$ is the full expected transmission time of a frame given the initial CW value CW_k.

A key feature of RMAB protocol is that when the Markov chain X^* is at the *good* states (being close to the state with minimal CW size), then transitions to *worst* states (with large CW size) are allowed. It can be treated as a way to establish a balance between stations being at good states and stations being at worst states, respectively. It is consistent with intuition, and as simulations in Section 5 shows, this model in some scenarios indeed demonstrates significant decrease of the capture effect.

4 Simulations

In this section, we describe simulations of adaptive backoff protocol for wireless networks containing $N = 5, 10, 20, 40$ stations, respectively. For simulations, we use the ns-3 simulator, with necessary modifications in the backoff protocol. The data observed at each station are: the amount of traffic sent successfully during the whole simulation time; the number of collided frames; the number of discarded frames. All simulation runs have been carried out for 60 seconds over a 11 Mbps link.

We use an IEEE 802.11 model from ns-3 simulator, with RTS/CTS control packets always turned on. The use of RTS/CTS makes results independent of the frame sizes. The parameters $CW_0 = 16$ and $M = 6$ are used, as in standard BEB for IEEE 802.11 [IEEE(2009)]. The maximum size of CW in that case is 1024. All packets are generated continuously at every station with speed of 12 Mbps. Hence, any station itself can fill the whole bandwidth to the access point (AP). All stations have been placed on the distance from 10 to 15 meters away from the AP.

To simulate the standard BEB, the following values of the increase factor $a = 1.1, \ldots, 2.9$ (with step 0.1) are used. The main goal is to increase throughput and to reduce the capture effect for any station. We define the average throughput as the amount of successfully transmitted data divided by the time needed for the transmission. Moreover, to measure capture effect, we use the standard deviation

$$s_N = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2}, \text{ where , } \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i,$$

based on calculation of the amount of transmitted data x_i for a station i per a minute, for each $i = 1, \ldots, N$.

Simulation shows that in some cases MAB, allowing only jumps down upon transmission, has the same throughput and a higher deviation than BEB. For example, in Figure 1 we compare a backoff protocol with $a = 1.1$ (we call it IEEE802.11 BEB because it has the same form as IEEE802.11 BEB but with $a = 1.1$), and MAB protocol with the same a and the following

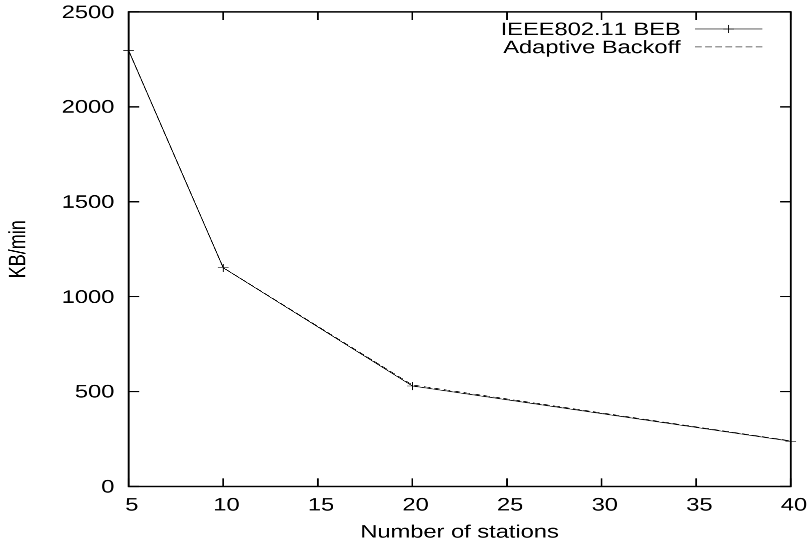

Figure 1: Average throughput at saturation during 1 min.

(7×7) – transition matrix

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{20}$$

Every state i of the chain (that is the state $\{BC = i\}$) corresponds to CW sizes $15 \times (1.1)^i$. It follows from (20) that the protocol returns from the *maximal state* $\{BC = 6\}$ to the *minimal state* $\{BC = 0\}$, while the intermediate states allow to return to states $\{BC = 2, 3\}$. In other words, the stations which are close to being congested, receive access to the medium faster than stations that are much less congested. However, Figure 2 shows that this MAB protocol has greater deviation per station than BEB. We interpret it as follows. The reason for a capture effect (which is shown by the deviation) is that after a successful transmission in a heavily loaded channel, the station returns to the initial window CW_0 and with a high probability gets an access to the channel again, while other stations are congested.

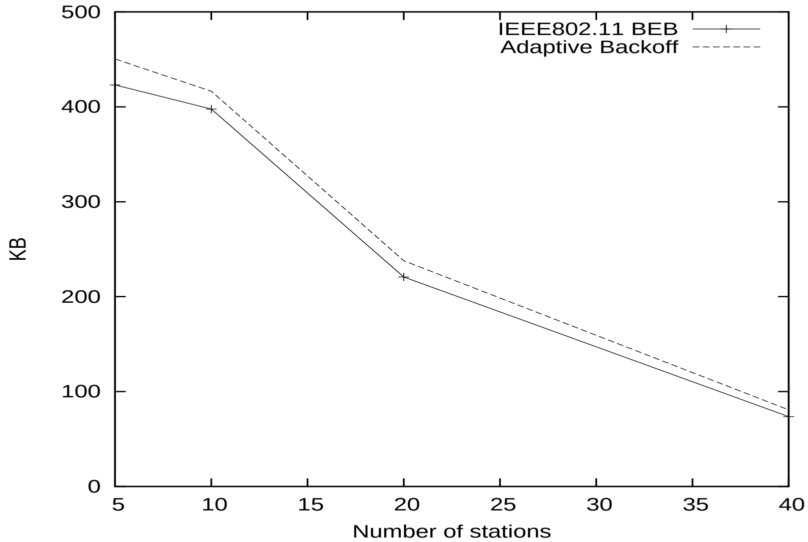

Figure 2: Standard deviation for stations during 1 min.

In wireless environment, stations that are located closer to AP have an advantage. The signals from those stations are stronger and dominate the signals from the distant sources (that is the signal-to-noise ratio is high). Hence, the stations that are closer to AP may stay in state $\{BC = 0\}$ for a long time, and thus *capture the channel for a long time with a high probability*. At the same time, situation for other stations becomes worse since after a successful transmission, they return not to initial (minimal) state, but to an intermediate state. Thus, such a MAB protocol can increases the capture effect.

As we mentioned above, to understand better the behavior of EIED protocol, a MAB protocol can be used to simulate *EIED-like protocols*. More exactly, assume that the current state of BC is i. Then if a transition occurs at this state, then the BC returns to state $i \cdot b$, where the multiplier b is chosen as $b = 0.1, \ldots, 0.9$. Simulation shows that EIED-like protocols behave similarly to MAB protocols. Indeed, some of EIED-like protocols ensure better throughput, but increase the capture effect. Thus, we argue with results of [Song et al.(2003)Song, Kwak, Song, and Miller] that EIED protocols can attain better throughput only by decreasing fairness. These observations show that to reduce the capture effect, a control mechanism must be used to prevent a capture of the channel for a long time by the stations staying at the good (or close to good) states. This idea is behind the introduction of RMAB protocol, which allows to achieve the balance between different stations.

To simulate RMAB, we have used the following 8×8 – transition matrix

$$\hat{P} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix}. \tag{21}$$

Thus, in our experiment (at every station), initial state $\{BC = 0\}$ transits to the maximal state $\{BC = 7\}$. The resulting average throughput and

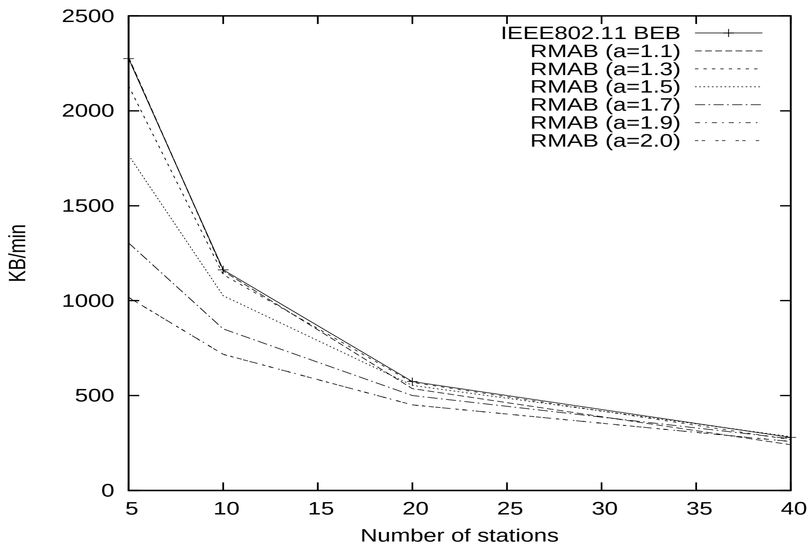

Figure 3: Average throughput for RMAB during 1 min.

deviation of throughput per stations for RMAB protocols with different a are shown in Figure 2, 4. Figure 2 shows that for a small number of stations, the performance of RMAB protocol with $a = 1.1$ is identical to the performance of IEEE802.11, and the RMAB protocol with $a = 1.3$ shows almost the same behavior as IEEE802.11. However, for a large number of stations ($N = 40$), the RMAB protocol (21) with $a = 2.0$ has the best performance. Moreover,

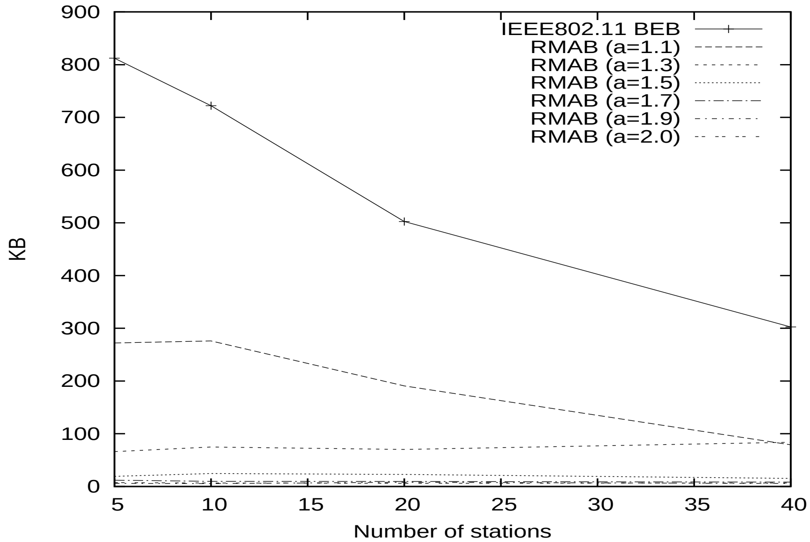

Figure 4: Standard deviation for RMAB during 1 min.

Figure 4 shows that RMAB protocols have much smaller throughput deviation than the standard BEB. At the same time, the RMAB protocols with $a = 1.1, 1.3$ have deviation in the range [50, 300] KB, while $a = 2.0$ implies deviation in interval [0, 10] KB. It means that the latter protocol gives all stations almost identical throughput independently of their states, and *the capture effect is almost eliminated*.

5 Conclusion

In this paper, a new model describing the behavior of a CW in general (not necessary exponential) backoff protocol as an irreducible, aperiodic, finite Markov chain is suggested. A new feature of this model is that an adaptive mechanism is used to extend flexibility of the CW behavior after a successful transition. This control mechanism is described by a transition matrix where, unlike standard settings, transitions (after a transmission) implying extension of CW are allowed.

The main motivation of the introduction of such a mechanism is to decrease the so-called capture effect, or in other words, to increase the *fairness* of the service provided by a network. To analyze this matrix adaptive backoff (MAB) protocol, the original Markov chain is replaced by an embedded Markov chain, whose stationary distribution is then obtained in an explicit form.

A preliminary simulation of a reverse (RMAB) protocol, which is a particular case of MAB protocol shows that in some cases it has almost the same average throughput as standard BEB, but decreases the capture effect in Ethernet and wireless networks significantly. The detailed analytical study of performance measures describing MAB protocols and their further verification by an extended simulation is the goal of our future research.

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