A Local Equilibrium Model for P2P Resource Ranking

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1. PROBLEM

We consider the problem of peer-to-peer (P2P) rank computation. In particular, it applies to a P2P resource sharing system (RSS) where a node has to decide what resources it keeps locally, what external resources it consumes and through which nodes, what quality of service it provides to other nodes for local and transit resources. We contribute a mathematical model for local P2P resource ranking that optimizes these decisions for better performance.

Given the following RSS (Table 1), let $N = \{u, v, w, \ldots\}$ and $S = \{i, j, k, \ldots\}$ be nodes and resources, respectively. Each node u keeps $N_u \subseteq N$ neighbors to communicate with.

A node u acts as a client that consumes external resources $i \in P_u^+$, as a server that contributes local resources $j \in R_u^-$, and as a router that both consumes (P_{vu}^0, $v \in N_u$) and provides (R_{uv}^0, $v \in N_u$) transit resources $k \in Q_u$ by forwarding requests.

Although the same resource k appears in the system at different nodes, u may distinguish k according to which neighbor is used. For instance, if both v and w provide k then u considers it separately (k_v and k_w) since the performance depends on which node provides k. Assume that P_{vu}^+, P_{vu}^0, R_{uv}^-, and R_{uv}^0 are disjunctive as well as P_u^+ and R_u^-. Also we have $\bigcup_{v \in N_u} P_{vu}^0 = \bigcup_{w \in N_u} R_{uw}^0$, since for any transit resource k there exist nodes $v \neq w$ such that $k \in P_{vu}^0 \cap R_{uw}^0$.

A node u locally estimates parameters $a_{vi} \geq 0$ for $i \in P_{vu}$ and $b_{vj} \geq 0$ for $j \in R_{uv}$. The former is the interest of u to resource i, e.g., the better is the performance when consuming i from v, the higher is a_{vi}. The latter is the interest of v to resource j, e.g., the more requests v made to u for j, the higher is b_{vj}.

Let u compute resource ranks $s_i \geq 0$, $i \in S$ locally. Those induce the node ranks $r_v = \sum_{i \in P_{vu}} s_i$, i.e., v is of high rank when v contributes well-ranked resources. Different nodes may rank the same resource differently.

Table 1: Symbol notation of the model.

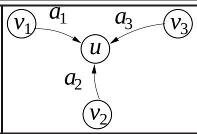
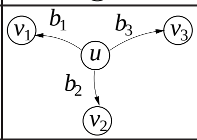
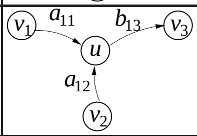
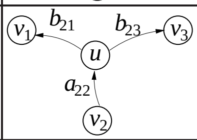
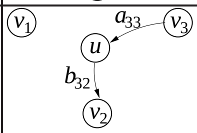
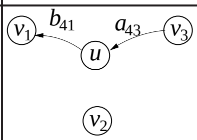
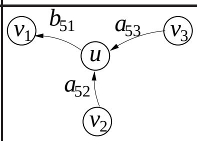
Notation	Description
N_u	All nodes that u communicates with.
P_{vu}^+, P_{vu}^0, P_{vu}	External resources (P_{vu}^+), transit resources (P_{vu}^0), and both ($P_{vu} = P_{vu}^+ \cup P_{vu}^0$) that u consumes from v.
R_{uv}^-, R_{uv}^0, R_{uv}	Local resources (R_{uv}^-), transit resources (R_{uv}^0), and both ($R_{uv} = R_{uv}^+ \cup R_{uv}^0$) that u provides to v.
P_u^+, P_u	All external resources ($P_u^+ = \bigcup_{v \in N_u} P_{vu}^+$) and all resources ($P_u = P_u^+ \cup Q_u$) that u consumes.
R_u^-, R_u	All local resources ($R_u^- = \bigcup_{v \in N_u} R_{uv}^-$) and all resources ($R_u = R_u^- \cup Q_u$) that u provides.
Q_u	All transit resources that u consumes and provides, $Q_u = \bigcup_{v \in N_u} P_{vu}^0 = \bigcup_{v \in N_u} R_{uv}^0$.
A^+, A^0, A	Non-negative matrices of u's interests in external resources (A^+), transit resources (A^0), and both ($A = (A^+ \| A^0)$). The larger is a_{vk}, the higher is the interest of u in consuming $k \in P_{vu}$ through v.
B^-, B^0, B	Non-negative matrices of other nodes' interests in local resources (B^-), transit resources (B^0), and both ($B = (B^0 \| B^-)$). The larger is b_{vk}, the higher is the interest of v in consuming $k \in P_{vu}$ through v.
x, y, z, s, r	External resource ranks (x_i, $i \in P_u$), local resource ranks (y_j, $j \in R_u$), transit resource ranks (z_k, $k \in Q_u$), all ranks $s = (x, y, z)^\mathsf{T}$, and node ranks ($r_v = \sum_{i \in P_{vu}} s_i$, $v \in N_u$).

Ranks are computed and applied iteratively. Given the rank values, u can apply many strategies to control its neighborhood. They provide feedback to the rank computation mechanism. The presence of low-ranked resources and nodes is reduced in cooperation. Using observations B, appropriate control of N_u, P_u, R_u, Q_u, and A implements this reduction and feeds the next iteration of rank computation.

In particular, u maintains local resources periodically adding high-rank and removing low-rank resources. Another application is selecting a node at u of the highest rank for requesting a given resource k.

Example 1. Let a node u know three neighbors, $N_u = \{v_1, v_2, v_3\}$. Table 2 shows the u's local consumption and contribution in RSS. External and local resources depend

Table 2: Example of u's neighborhood in RSS.

Resource	Description	Graph	Rank
i_1 i_2 i_3	Distinct external resources that u consumes from v_1, v_2, and v_3; $P^+_{vu} = \{i_v\}$, $a^+_1 = a^+_3 = 1$, $a^+_2 = 2$		x_1 x_2 x_3
j_1 j_2 j_3	Distinct local resources that u provides to v_1, v_2, and v_3; $R^-_{uv} = \{j_v\}$, $b^-_1 = b^-_2 = 1$, $b^-_3 = 2$		y_1 y_2 y_3
k_1	Transit resource from v_1 or v_2 to v_3; $a^0_{11} = 2$, $a^0_{12} = 1$, $b^0_{13} = 1$		z_1
k_2	Transit resource from v_2 to v_1 and v_3; $a^0_{22} = 3$, $b^0_{21} = 1$, $b^0_{23} = 3$		z_2
k_3	Transit resource from v_3 to v_2; $a^0_{33} = 1$, $b^0_{32} = 1$		z_3
k_4	Transit resource from v_3 to v_1; $a^0_{43} = 2$, $b^0_{41} = 3$		z_4
k_5	Transit resource from v_2 or v_3 to v_1; $a^0_{52} = 3$, $a^0_{53} = 4$, $b^0_{51} = 2$		z_5

on neighbors; u consumes a distinct resource i_v from v and provides a distinct resource j_v to v. A transit resource k may be consumed from or provided to several nodes.

Consider two particular scenarios. 1) v_1 starts consuming more from u, and the observations $b_{v_1 1}$ become higher. 2) v_1 worsens the performance, and u decreases the interests $a_{v_1 1}$. In both scenarios, the rank r_{v_1} must increase, and u may respond by sending more requests to v_1 or rejecting some requests from v_1 (up to removing v_1 from N_u).

2. MODEL

In this section, we introduce a model for the rank computation in RSS. Let $s \geq \mathbb{O}$ satisfy the linear equation system

$$\sum_{i \in P_{vu}} a_{vi} s_i = \sum_{j \in R_{uv}} b_{vj} s_j, \quad v \in N_u, \tag{1}$$

where s represents resource prices. The left-hand side $\sum_{i \in P_{vu}} a_{vi} s_i$ is the v's contribution through u. Similarly, $\sum_{j \in R_{uv}} b_{vj} s_j$ is the v's consumption. The contribution and consumption are observed at u, and (1) is the local barter balance according to which u defines ranks.

Property 1 (Barter balance). *System (1) states that the bilateral resource circulation between u and any node $v \in N_u$ is equal in terms of local resource prices.*

Note that for any solution s and constant $c \geq 0$, a vector cs is also a solution to (1). A normalized rank satisfies

$$\sum_{k \in S} s_k = 1. \tag{2}$$

If s' is a non-zero solution then $s = s' / \sum_{k \in R} s'_k$ is a normalized rank. The same is for node ranks, $\sum_{v \in N_u} r_v = 1$.

Consider a v-equation in (1) and define the ith and jth v-families of solutions ($i \in P_{vu}$, $j \in R_{uv}$):

$$\begin{aligned} \mathcal{H}_{vj} &= \Big\{ h = \begin{pmatrix} \delta \\ \mathbf{e}_j \end{pmatrix} \mid \sum_{i \in P_{vu}} a_{vi} \delta_i = b_{vj},\ \delta_i \geq 0 \Big\}, \\ \mathcal{H}_{vi} &= \Big\{ h = \begin{pmatrix} \mathbf{e}_i \\ \delta \end{pmatrix} \mid a_{vi} = \sum_{j \in R_{uv}} b_{vj} \delta_j,\ \delta_j \geq 0 \Big\}, \end{aligned} \tag{3}$$

where $\mathbf{e}_i$ and $\mathbf{e}_j$ are canonical unit vectors. Since v-equation does not necessarily contain all unknowns of the whole system, the corresponding components of h take arbitrary values. Hence $\mathcal{H}_{vj}$ contains such solutions that $s_j = 1$ and $s_k = 0$ for $k \in R_{uv} \setminus \{j\}$. Similarly, $\mathcal{H}_{vi}$ contains such s that $s_i = 1$ and $s_k = 0$ for $k \in P_{vu} \setminus \{i\}$.

Construct the following set of solutions to a v-equation.

$$\mathcal{H}_v = \Big\{ s = \sum_{k \in P_{vu} \cup R_{uv}} \lambda_k h^{(k)} \ \mid \ h^{(k)} \in \mathcal{H}_{vk},\ \lambda_k \geq 1 \Big\}. \tag{4}$$

A solution to (1) is a v-rational rank iff $s \in \mathcal{H}_v$. Intuitively, $s = s' + \lambda_k h^{(k)}$, and $h^{(k)}$ increases either the v's contribution by b_{vj} (if $k = j \in R_{uv}$) or the v's consumption by a_{vi} (if $k = i \in P_{vu}$). Proportions $b_{vj} : 1$ and $1 : a_{vi}$ define dependencies in ranks as the following property states.

Property 2 (Proportion). *For a v-rational rank, a unit variation of s_j for $j \in R_{uv}$ (s_i for $i \in P_{vu}$) affects the v's contribution (consumption) with b_{vj} (a_{vi}) units.*

Each v-family in (3) defines a set of possible proportions but according to (4) a v-rational rank includes exactly one proportion for each $k \in P_{vu} \cup R_{uv}$. That is, every resource k should affect the rank, and $\lambda_k \geq 1$ in (4) is a uniform lower bound for this effect. Note that (2) does not provide this separation, since (2) allows some $s_k = 0$.

Let v as a client increase its interest b_{vj} in external resource j through u. Then δ_i and hence s_i for $i \in P_{vu}$ grow in $\mathcal{H}_{vj}$ for fixed a_{vi}, and the node rank r_v becomes higher. In contrast, s_j for $j \in R_{uv}$ become lower (see δ_j in $\mathcal{H}_{vi}$). In the other case, let v as a client extend its consumption through u requesting a new resource j^*; $R_{uv} := R_{uv} \cup \{j^*\}$ and $b_{vj^*} > 0$. Then the ranks of resources $i \in P_{vu}$ increase at u; $s_i := s_i + \delta_i$ where $\sum_{i \in P_{vu}} a_{vi} \delta_i = b_{vj^*}$. The following reciprocation property captures both cases above.

Property 3. *At a node u, when v augments its consumption through u then r_v increases and r_u decreases.*

Let u augment its consumption through v increasing a_{vi} for $i \in P_{vu}$ or $|P_{vu}|$. Then δ_i and hence s_i tend to smaller values in $\mathcal{H}_{vj}$ for fixed b_{vj}. In contrast, s_j for $j \in R_{uv}$ become higher (see δ_j in $\mathcal{H}_{vi}$).

Property 4. *At a node u, when u augments its consumption through v then r_v decreases and r_u increases.*

A rank s is rational if it is v-rational for all $v \in N_u$. The set of all rational ranks is $\mathcal{H} \subseteq \bigcap_{v \in N_u} \mathcal{H}_v$. This representation is not suitable for computation. Let us approximate the model using linear programming (LP) techniques.

THEOREM 1. *A v-rational rank ($s \in \mathcal{H}_v$) is a solution to the following linear system*

$$\begin{cases} \sum_{i \in P_{vu}} a_{vi} s_i = \sum_{j \in R_{uv}} b_{vj} s_j, \\ s_k \geq 1, \quad k \in P_{vu} \cup R_{uv}. \end{cases} \tag{5}$$

Note that any solution to (5) can be presented as $s = \sum_{k \in P_{vu} \cup R_{uv}} \lambda_k h^{(k)}$, but not necessarily with $\lambda \geq \mathbf{1}$. Therefore, (5) is an approximation. In terms of Prop. 2, some solutions are ranks that affect the node contribution or consumption less than the bound $\lambda_k \geq 1$ dictates.

Let $\mathbf{1} = (1, \ldots, 1)^{\mathsf{T}}$. According to Theorem 1, rational ranks $\mathcal{H}$ are solutions to

$$\begin{cases} \sum_{i \in P_{vu}} a_{vi} s_i = \sum_{j \in R_{uv}} b_{vj} s_j, \quad v \in N_u, \\ s \geq \mathbf{1}. \end{cases} \tag{6}$$

There can be infinitely many solutions to (6). Recall that when the rank r_v is small, then u reduces its consumption through v. Hence, u considers the pessimistic case when

$$\sum_{v \in N_u} r_v = \sum_{v \in N_u} \sum_{i \in P_{vu}} s_i \to \min. \tag{7}$$

According to Table 1, let us rewrite balance equations (1) separating $s = (x, y, z)$ to external (x), local (y), and transit (z) resources:

$$\begin{gathered} \sum_{i \in P^+_{vu}} a^+_{vi} x_i + \sum_{k \in P^0_{vu}} a^0_{vk} z_k = \\ = \sum_{j \in R^-_{uv}} b^-_{vj} y_j + \sum_{k \in R^0_{uv}} b^0_{vk} z_k, \quad v \in N_u. \end{gathered} \tag{8}$$

EXAMPLE 2. Consider the u's local consumption and contribution from Table 2. The optimization model is

$$\begin{array}{lr} x_1 + x_2 + x_3 + 2z_1 + z_2 + z_3 + z_4 + 2z_5 \to \min & \\ x_1 + 2z_1 = y_1 + z_2 + 3z_4 + 2z_5 & (v_1) \\ 2x_2 + z_1 + 3z_2 + 3z_5 = y_2 + z_3 & (v_2) \\ x_3 + z_3 + 2z_4 + 4z_5 = 2y_3 + z_1 + 3z_2 & (v_3) \end{array}$$

Its optimal solution is $(x, y, z) = (1, 1, 1; 1, 10, 1; 3, 1, 1, 1, 1)$. The normalized resource ranks are (0.045, 0.045, 0.045; 0.045, 0.455, 0.045; 0.136, 0.045, 0.045, 0.045, 0.045). Most resources are equal in ranks (0.045). Resource j_2 has the highest rank (0.455) compensating the u's consumption from v_2. The z_3 value in v_2-equation cannot be high, otherwise v_3-equation is unbalanced. The z_1 value is high (0.146); it is not obvious, and our model captures this hidden fact.

The normalized node ranks are $r = (0.29, 0.43, 0.29)$, where $r_{v_1} = x_1 + z_2$, $r_{v_2} = x_2 + z_1 + z_2 + z5$, and $r_{v_3} = x_3 + z_3 + z_4 + z_4$. From this point of view, v_2 is the best node for sending requests to and rejecting requests from. In particular, let i_1 and i_2 present the same resource but from v_1 and v_2, respectively. Then u preferably uses v_1 as a server. Another example, let u be overloaded and receive several requests for k_5 from v_2 and v_3. Then u rejects requests from v_2 with higher likelihood.

3. ANALYSIS

LP methods require an initial solution for efficient computations. Let us analyze solvability of a particular case.

Consider the following assumption (the left square bracket stands for logical OR)

$$\left[\begin{array}{lll} |V| \leq \left|\bigcup_{v \in V} P^+_{vu}\right| & \forall V \subseteq N_u, & \text{(X)} \\ |V| \leq \left|\bigcup_{v \in V} R^-_{uv}\right| & \forall V \subseteq N_u. & \text{(Y)} \end{array} \right. \tag{9}$$

Bound (9.X) states that the number of distinct resources u consumes is not less than the number of their providers. Bound (9.Y) is similar but for the number of distinct resources u provides to their consumers.

The following theorem shows that (9) is sufficient for existence of a rational rank s, i.e., $\mathcal{H} \neq \varnothing$ for (6). Moreover, such s can be constructed explicitly.

THEOREM 2. *If (9) holds then (8) has a solution $s = (x, y, z)$ such that $x, y \geq \mathbf{1}$ and $z = \mathbf{1}$.*

Table 3 shows our preliminary experiments with the rank model (7)–(8) where some parameters are varied. In the initial model (Table 2 and Example 2), the normalized node ranks are r =(0.29,0.43,0.29). The numerical results complement the model properties from the previous sections.

Experiments 1 and 2. If u's interest in resources from v_1 grows (a^+_1 for x_1 and a_{11} for z_1), the node rank converges. Since v_1 contributes well, the model does not discriminate v_1 but makes r_{v_1} lower than r_{v_2} and v_3.

Experiments 3 and 5. If v_1's consumption through u grows then r_{v_1} increases (up to $0.4 \ldots 0.6$ for $b \sim 10 \ldots 15$).

Experiment 4. The u's and v_3's interests in transit resource k_1 grows equally. As a result, the rank r_{v_1} decreases (v_1 is a k_1 provider).

Table 3: Experimental trends in normalized node ranks $r = (r_{v_1}, r_{v_3}, r_{v_3})$.

#	Balance and Variation	Trend
1	$a^+_1 x_1 + 2z_1 = y_1 + z_2 + 3z_4 + 2z_5$ $2x_2 + z_1 + 3z_2 + 3z_5 = y_2 + z_3$ $x_3 + z_3 + 2z_4 + 4z_5 = 2y_3 + z_1 + 3z_2$ $a^+_1 = 1, 2, \ldots$	converging r to (0.2, 0.4, 0.4)
2	$x_1 + a^0_{11} z_1 = y_1 + z_2 + 3z_4 + 2z_5$ $2x_2 + z_1 + 3z_2 + 3z_5 = y_2 + z_3$ $x_3 + z_3 + 2z_4 + 4z_5 = 2y_3 + z_1 + 3z_2$ $a^0_{11} = 2, 3, \ldots$	converging r to (0.2, 0.4, 0.4)
3	$x_1 + 2z_1 = b^+_1 y_1 + z_2 + 3z_4 + 2z_5$ $2x_2 + z_1 + 3z_2 + 3z_5 = y_2 + z_3$ $x_3 + z_3 + 2z_4 + 4z_5 = 2y_3 + z_1 + 3z_2$ $b^+_1 = 1, 2, \ldots$	increasing r_{v_1}; decreasing r_{v_2} and r_{v_3}
4	$x_1 + a^0_{11} z_1 = y_1 + z_2 + 3z_4 + 2z_5$ $2x_2 + z_1 + 3z_2 + 3z_5 = y_2 + z_3$ $x_3 + z_3 + 2z_4 + 4z_5 = 2y_3 + b^0_{31} z_1 + 3z_2$ $a^0_{11} = 2, 3, \ldots$; $b^0_{31} = 1, 2, \ldots$	decreasing r_{v_1}; increasing r_{v_2} and r_{v_3}
5	$x_1 + 2z_1 = y_1 + z_2 + 3z_4 + 2z_5$ $2x_2 + z_1 + 3z_2 + 3z_5 = y_2 + z_3$ $x_3 + z_3 + 2z_4 + 4z_5 = 2y_3 + b^0_{31} z_1 + 3z_2$ $b^0_{31} = 1, 2, \ldots$	increasing r_{v_1}; decreasing r_{v_2} and r_{v_3}

APPENDIX

Proof of Theorem 1

PROOF. Let $s \in \mathcal{H}_v$, where $\mathcal{H}_v$ is defined in (4). Then s is a solution to the v-equation in (5) since

$$s = \sum_{i \in P_{vu}} \lambda_i h^{(i)} + \sum_{j \in R_{uv}} \lambda_j h^{(j)},$$

where $h^{(i)} \in \mathcal{H}_{vi}$ and $h^{(j)} \in \mathcal{H}_{vj}$ are solutions to the v-equation, see (3).

Take arbitrary $i \in P_{vu}$. The definition of $\mathcal{H}_{vi}$ leads to $h_i^{(i)} = 1$, $h_k^{(i)} = 0$ for $k \in P_{vu} \setminus \{i\}$, and

$$s_i = \lambda_i + \sum_{j \in R_{uv}} \lambda_j h_i^{(j)},$$

Applying $\lambda \geq \mathbf{1}$ results to $s_i \geq 1$.

Symmetrically, taking arbitrary $j \in R_{uv}$ yields $s_j \geq 1$. Therefore, arbitrary taken $s \in \mathcal{H}_v$ satisfies (5). □

Proof of Theorem 2

PROOF. Let no transit resource appear in (8):

$$\sum_{i \in P_{vu}^+} a_{vi}^+ x_i = \sum_{j \in R_{uv}^-} b_{vj}^- y_j \quad v \in N_u. \tag{10}$$

Hence, unknowns are disjunctive sets x and y.

Let (9.X) be true; it allows applying Hall's theorem (the marriage theorem) to find a distinct representative x_{i_v} for each v-equation of (10). Set all other x_i to 1. That is, (10) is split to independent v-equations

$$a_{vi_v}^+ x_{i_v} = \sum_{j \in R_{uv}^-} b_{vj}^- y_j - \sum_{i \in P_{vu}^+} a_{vi}^+,$$

which can be solved with $x, y \geq \mathbf{1}$ for large enough y.

If (9.Y) is true then the case is symmetrical to the previous one, and distinct representatives are selected among y_j.

Return to the general case (8) and assumption (9.X). Selecting distinct representatives x_{i_v} as above and setting $z = \mathbf{1}$, we yield the system

$$a_{vi_v}^+ x_{i_v} = \sum_{j \in R_{uv}^-} b_{vj}^- y_j + \sum_{k \in R_{uv}^0} b_{vk}^0 - \sum_{k \in P_{vu}^0} a_{vk}^0 - \sum_{i \in P_{vu}^+} a_{vi}^+,$$

where

$$C_v = \sum_{k \in R_{uv}^0} b_{vk}^0 - \sum_{k \in P_{vu}^0} a_{vk}^0 - \sum_{i \in P_{vu}^+} a_{vi}^+ = \text{const.}$$

If $C_v < 0$ then taking large enough $y \geq \mathbf{1}$ results to a positive value in the right-hand side that is bigger than $a_{vi_v}^+$ and hence $x_{vi_v} \geq 1$.

Therefore, we found a solution to (8) with $x, y \geq \mathbf{1}$ and $z = \mathbf{1}$. Moreover, such a solution can be constructed straightforwardly based on equations in the proof. □