Multipath multiuser scheduling game for elastic traffic

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Our multipath multiuser routing optimization problem is based on Wardrop model [1, 2, 3] of splittable traffic routing. Minimization of the end-to-end traffic delay for each user is the criterion of optimality.

The problem is considered as the game $\Gamma = \langle n, m, w, f \rangle$, where n users send their traffic through m parallel routes from the source s to destination t. Each user i wants to send traffic of the amount w_i from s to t. Each path e has a characteristic $\alpha_{ie} > 0$.

Users act selfish and choose routes to minimize their maximal traffic delay. They can split their traffic and send it on several or all paths simultaneously. User's i strategy is $x_i = \{x_{ie} \geq 0\}$, where x_{ie} is the traffic amount that he sends on the path e so that $\sum_{e=1}^{m} x_{ie} = w_i$. Then $x = (x_1, \ldots, x_n)$ is users strategy profile. Denote for the original profile x the new profile $(x_{-i}, x'_i) = (x_1, \ldots, x_{i-1}, x'_i, x_{i+1}, \ldots, x_n)$ where the user i changes his strategy from x_i to x'_i and all other users keep their strategies the same as in x.

The load of the path e is a function $\delta_e(x)$ that is continuous and non-decreasing by x_{ie}. A continuous traffic delay function $f_{ie}(x) = f_{ie}(\delta_e(x))$ is defined for each user i and each route e. It is non-decreasing by the path load and hence by x_{ie}.

Function $PC_i(x)$ defines an individual i-th user's costs. Each user i tries to minimize his individual costs – the maximal traffic delay among the routes that he uses

$$PC_i(x) = \max_{e: x_{ie} > 0} f_{ie}(x).$$

A strategy profile x is a Wardrop equilibrium iff for each i holds: if $x_{ie} > 0$ then $f_{ie}(x) = \min_{l} f_{il}(x) = \lambda_i$ and if $x_{ie} = 0$ then $f_{ie}(x) \geq \lambda_i$.

Social costs are the total costs of the system as a result of using parallel routes of the network:

$$SC(x) = \sum_{i=1}^{n} \sum_{e=1}^{m} x_{ie} f_{ie}(x).$$

A social optimum is a solution of a minimization problem $SC(x) \to \min_{x \text{ is a strategy profile}}$. Price of Anarchy is a ratio of equilibrium social costs in the worst case equilibrium and optimal social costs.

$$PoA(\Gamma) = \max_{x \text{ is an equilibrium}} \frac{SC(x)}{SC_{opt}}.$$

In this work we consider a routing game with traffic delay functions $1 - e^{-\alpha_{ie}\delta_e}$ in case where for each path e its traffic delay is the same for each user. Experimental modeling confirms an adequacy of such delay function and explains a sence of parameters α. Wardrop Equilibria and their properties in this model are objects of the research. We obtain that a Wardrop equilibrium is any situation where loads are distributed by routes as follows:

$$\sum_{i=1}^{n} x_{ie} = \delta_e(x) = \frac{W}{\alpha_e \sum_{e=1}^{m} \frac{1}{\alpha_e}} \text{ for each } e \in \{1, \ldots, m\},$$

and the equilibrium social costs are

$$SC(x) = W\left(1 - e^{-\frac{W}{\sum_{e=1}^{m} \frac{1}{\alpha_e}}}\right).$$

Also we prove, that the Price of Anarchy is about 1.3 for this model.

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References

[1] Gairing M., Monien B., Tiemann K. *Routing (Un-) Splittable Flow in Games with Player-Specific Linear Latency Functions* // Proceedings of the 33rd International Colloquium on Automata, Languages and Programming (ICALP 2006), LNCS 4051, 2006, 501–512.

[2] Wardrop J.G. *Some theoretical Aspects of Road Traffic Research* // Proceedings of the Institute of Civil Engineers, 1952, II, 1, 325–278.

[3] Mazalov V., Monien B., Schoppmann F., Tiemann K. *Wardrop Equilibria and Price of Stability for Bottleneck Games with Splittable Traffic* // Proceedings of 2nd international Workshop on Internet & Network Economics (WINE 2006, Patras / Greece), 2006, to appear in Lecture Notes in Computer Science.

[4] Julia V. Chuiko, Vladimir V. Mazalov. *Nash Equilibrium in Splittable Traffic Routing Problem* // Proceedings of Kosen Workshop MTE2008 - Mathematics, Technology and Education - Ibaraki National College of Thechnology, Hitachinaka, Ibaraki, Japan. 2008. Pp. 13-18.

[5] A.Gurtov, T. Polishchuk, *Secure Multipath Transport for Legacy Internet Applications* // Proceedings of BROADNETS'09, Madrid, Spain, September 2009.