Wardrop Equilibria and Price of Anarchy in Multipath Routing Games with TCP Traffic

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Abstract

We consider equilibrium in multiuser multipath routing optimization problem in Wardrop model where selfish players distribute their TCP traffic in the shared multipath network. Minimization of the end-to-end traffic delay over all paths for each user is the criterion of optimality.

We discover that in the game with latency function $f_e(\delta) = 1 - e^{-\alpha_e \delta}$ approximating the TCP congestion control over the paths, the price of anarchy is bounded, leading to the conclusion that non-cooperative selfish users can safely coexist in the multipath network and successfully achieve a good performance if each adheres to the equilibrium flow splitting strategy.

Key words: TCP, selfish user, multipath routing, Wardrop equilibrium, price of anarchy

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1 Introduction

Nowadays many communication systems are based on the principle of sharing a common resource among different users. One of the examples is Internet, where TCP traffic comprises a major share. TCP is traditionally the conventional networking scheme [8], [20], where multiple users share the same communication links and buffering capabilities of the network routers.

One of the main objectives of the communication protocol is to establish a number of rules guaranteeing that the common resources are fairly shared among all the users. TCP-friendliness [21] and TCP-fairness [12] have emerged as measures of correctness in Internet congestion control. A congestion control mechanism should restricts non-TCP flows from exceeding the bandwidth of a conforming TCP running under comparable conditions.

Recent studies has demonstrated the benefits of multipath data transfer in obtaining high available bandwidth, better loss patterns and bounded delay in the best-effort Internet environment [4], [5], [11]. However, new multipath mechanisms are often accused in unfriendliness when they share network resources with traditional flows. Proper per-flow congestion control is required to limit aggressiveness of the proposed multipath solutions.

In the prior work [11] we showed how a single user could maximize his TCP throughput if he is given a control over multiple end-to-end paths simultaneously. Nonetheless, as the same paths are shared between many users in the Internet, how much throughput a single user can achieve depends not only on his own decisions but also on decisions of the other users of the same network. Greedy behavior can be not optimal.

In this paper we analyze a scenario where multiple selfish users route their traffic in a shared multipath network between the common source and destination. The users send TCP traffic and are not allowed to change transport protocol parameters, as, for example, in a Linux kernel the end users could have no permission to tune TCP congestion control settings or choose transport protocol for data transmission. Flow growth in each path is also controlled by a TCP-like function. We take game-theoretic approach to answer the following questions: do multiple users with selfish objectives each exploiting similar scheduling techniques share multipath network fairly? Is such a network sharing optimal or needs to be improved by applying some global congestion controllers?

The main contributions of this work include mathematical modeling, which quantifies the effect of selfish behavior of multiple independent users sharing a multipath network. We define the traffic delay function $f_e(\delta) = 1 - e^{-\alpha_e \delta}$, which approximates the dependency between end-to-end delay of the TCP traffic and total path load. Wardrop equilibrium in such a model coincides with the optimal scheduling strategy proposed in our prior work, proving correctness of our choice of the latency function.

We discover that the proposed optimal traffic splitting policy leaves a very small room for fairness improvement. The price of anarchy is bounded by the value of 1.3, leading to the conclusion that selfish users can successfully achieve their personal goals without cooperation, and the resulting unfairness is rather moderate and could be tolerated.

The rest of the paper is organized as follows. Section 2 summarizes the related work. Model description are given in Section 3. Analytical results for Wardrop equilibrium and the price of anarchy are presented in Section 4, and followed by the experimental evaluation in Section 5. Section 6 concludes the paper and presents the ideas for future work.

2 Related Work

The game theoretic frameworks are powerful in describing and analyzing competitive decision problems. Game theory has been used to study various communication and networking problems including routing, service provisioning, flow-rate controlling by formulating them as either cooperative or non-cooperative games. The authors of [2] summarized different modeling and solution concepts of networking games, as well as a number of different applications in telecommunication technology.

Networking games have been studied in the context of road traffic since 1950, when Wardrop proposed his definition of a stable traffic flow on a transportation network [22]. Both Wardrop and Nash [18] equilibria are traditionally used to give and idea on the fair resource sharing between the players [6], [9], [15]. However, they do not optimize social costs of the system. In 1999 the concept of the price of anarchy was proposed by Koutsoupias and Papadimitriou to solve this problem. In [13] network routing was modeled as a non-cooperative game and the worst-case ratio of the social welfare, achieved by a Nash equilibrium and by a socially optimal set of strategies. This concept has recently received considerable attention and is widely used to quantify the degradation in network performance due to unregulated traffic [14], [19].

In the conventional TCP/IP networking [20] multiple users share communication links and buffering capabilities of the network routers. When users do not cooperate and do not respect the protocol rules, it is possible that unfair or unstable behaviors emerge in the system. This problem of the TCP protocol has already been addressed in the networking literature using a game-theoretic perspective. For example, Nagle [17] and Garg et al. [10] proposed solutions based on creating incentive structures in the systems that discourage evil behavior and show the potential applications of Game Theory within the problem of congestion control and routing in packet networks.

An excellent analysis of TCP behavior in the context of Game Theory has been proposed by Akella et al. [1]. In this work, a combination of analyses and simulations is carried in an attempt to characterize the performance of TCP in the presence of selfish users. Our results for multipath networks presented in this paper agree with one of the main conclusions for the traditional unipath networks from [1]: when the users use TCP New Reno loss-recovery [8] in combination with drop-tail queue management the equilibrium strategies of the users are quite efficient for fair resource allocation.

3 Multiuser Multipath Network Routing Game

3.1 Model Description and Notations

First we formulate the problem as a non-cooperative static routing game and construct a Wardrop equilibrium model with splittable traffic. The amount of flow to route through the network is a variable whose value is set optimally, simultaneously with the routes, as a function of network characteristics and the users demand. Minimization of the end-to-end traffic delay for each user is the criterion of optimality.

The problem is modeled as the game $\Gamma = \langle n, m, w, f \rangle$, where n users send their TCP traffic through m parallel routes from the source S to destination D as shown in Figure 1.

Each user of the network is multihomed, which gives him the ability to deliver his traffic along multiple paths simultaneously. The global TCP congestion window grows and shrinks according to the TCP New Reno AIMD (additive increase multiplicative decrease) policy. The change in the window size, which occurs when a new acknowledgement message is received by the source from the receiver, represents a step in the decision-making process. On each step a user makes identical decisions how to split the given amount w_i of

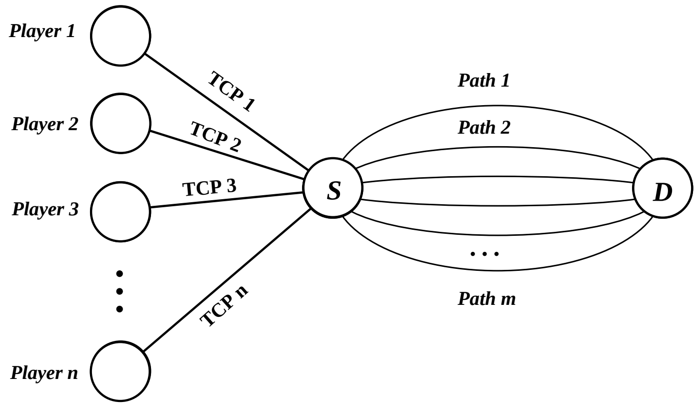

Figure 1: Multiuser multipath network model

his TCP traffic flow among the available paths.

The users act selfish and choose routes to minimize their maximal traffic delay. User's i strategy is $x_i = \{x_{ie} \geq 0\}$, where x_{ie} is the traffic amount that he sends on the path e so that $\sum_{e=1}^{m} x_{ie} = w_i$. Then $x = (x_1, \ldots, x_n)$ is users' strategy profile. Denote for the original profile x the new profile $(x_{-i}, x'_i) = (x_1, \ldots, x_{i-1}, x'_i, x_{i+1}, \ldots, x_n)$ where the user i changes his strategy from x_i to x'_i and all other users keep their strategies the same as in x.

Each path e has some characteristics, which depends on the end-to-end path parameters, such as propagation delay D_e and the bottleneck link bandwidth B_e. The total load of the path e is a function $\delta_e(x)$ that is continuous and non-decreasing by x_{ie}. A continuous traffic delay function $f_{ie}(x) = g_{ie}(\delta_e(x))$ is defined for each user i and each route e. It is non-decreasing by the path load and hence by x_{ie}.

Function $PC_i(x)$ defines an individual i-th user's costs. Each user i tries to minimize his individual costs – the maximal traffic delay among the routes that he uses $PC_i(x) = \max\limits_{e: x_{ie} > 0} f_{ie}(x)$.

Social costs depend on the users' traffic volume $w = (w_1, \ldots, w_n)$, characteristics of the paths and users' strategies. Here social costs are the total traffic delay on the paths of the network [9]:

$$SC(x) = \sum_{i=1}^{n} \sum_{e=1}^{m} x_{ie} f_{ie}(x).$$

3.2 Nash and Wardrop Equilibria

Remark 1. A strategy profile x is a Nash equilibrium iff for each user i for any profile $x' = (x_{-i}, x'_i)$ holds $PC_i(x) \leq PC_i(x')$.

Remark 2. A strategy profile x is a Wardrop equilibrium iff for each i: if $x_{ie} > 0$ then $f_{ie}(x) = \min\limits_{l} f_{il}(x) = \lambda_i$ and if $x_{ie} = 0$ then $f_{ie}(x) \geq \lambda_i$.

Nash and Wardrop equilibria definitions are not always equivalent. It depends on the type of traffic delay functions defined in the model.

Theorem 1. *If the strategy profile x is a Wardrop equilibrium then x is a Nash equilibrium.*

Theorem 2. *If all delay functions $f_e(x)$ in the model are strictly increasing by all x_{ie} then in this model any Nash equilibrium is a Wardrop equilibrium.*

A property in the theorem 2 means that it is always possible to redistribute some small user's traffic amount from any of routes to the less loaded routes in order to decrease traffic delay on this route for this user.

4 Routing Game with Traffic Delay Function $1 - e^{-\alpha_e \delta_e(x)}$

The amount of time needed to traverse a single path of a network is typically load-dependent, that is, the traffic latency in a path increases as it becomes more congested. Basing on a series of simulations of TCP traffic with variable path characteristics, conducted with the use of ns-2 simulator [16], we choose a traffic delay function $f_{ie}(\delta) = 1 - e^{-\alpha_{ie}\delta}$ to approximate the dependency between the end-to-end delay of the TCP traffic controlled by New Reno loss-recovery [8] in combination with drop-tail queue management, and the total path load δ. TCP regulates the load by relying on the packet loss and reduces the rate in response to that. When path load is large, packet loss on the path is large too, so it prevents an infinite growth of the delay.

In the model with the traffic delay function $f_{ie}(x) = 1 - e^{-\alpha_{ie}\delta_e(x)}$, where $\delta_e(x) = \sum\limits_{i=1}^{n} x_{ie}$, Nash and Wardrop equilibria are obviously coincident, because the theorem 2 property holds.

The social costs are $SC(x) = W - \sum_{i=1}^{n}\sum_{e=1}^{m} x_{ie}e^{-\alpha_{ie}\delta_e(x)}$, where $W = \sum_{i=1}^{n} w_i$ – is a total traffic in the network.

Now we suppose that traffic delay on a path e is the same for each user and equals $f_e(x) = 1 - e^{-\alpha_e\delta_e(x)}$, resulting in $SC(x) = W - \sum_{e=1}^{m} \delta_e(x)e^{-\alpha_e\delta_e(x)}$.

4.1 Wardrop Equilibrium

Let a profile x be a user's profile in a Wardrop equilibrium. By definition if $x_{ie} > 0$ then $f_e(x) = \min_l f_l(x) = \lambda_i$ and if $x_{ie} = 0$ then $f_e(x) \geq \lambda_i$. Since traffic delay on the path e is equal for all users, for each i, such that $x_{ie} > 0$, $\lambda_i = \lambda$. Delays on the unused routes are equal to zero, that is why in the Nash equilibrium each path must be used by at least one user. Moreover, if for some user i on the path e the traffic load is $x_{ie} = 0$, then traffic delay on this path must not be less than delays on the paths which he uses, i.e. $1 - e^{-\alpha_e\delta_e(x)} \geq \lambda > 0$. It means that there is at least one user k, such that $x_{ke} > 0$, hence the traffic delay on this path is exactly equal to λ. So, we have: in the Wardrop equilibrium traffic delays on each route equal to λ and for all $e \in \{1, \ldots, m\}$ holds $\delta_e(x) = -\frac{\ln(1-\lambda)}{\alpha_e}$.

Summing these expressions by e we get

$$W = -\ln(1-\lambda)\sum_{e=1}^{m}\frac{1}{\alpha_e}, \text{ and } \lambda = 1 - e^{-\frac{W}{\sum_{e=1}^{m}\frac{1}{\alpha_e}}}.$$

Substituting λ into the expression for $\delta_e(x)$ we obtain that in a Wardrop equilibrium loads are distributed by routes as follows:

$$\sum_{i=1}^{n} x_{ie} = \delta_e(x) = \frac{W}{\alpha_e\sum_{e=1}^{m}\frac{1}{\alpha_e}} \text{ for each } e \in \{1, \ldots, m\},$$

4.2 The Price of Anarchy

Price of Anarchy is a ratio of equilibrium social costs in the worst case equilibrium and optimal social costs

$$PoA(\Gamma) = \max_{x \text{ is an equilibrium}} \frac{SC(x)}{SC_{opt}}.$$

Here the social optimum SC_{opt} is a solution of a minimization problem $SC(x) \to \min_{x \text{ is a strategy profile}}$.

Then the equilibrium social costs are $SC(x) = W\left(1 - e^{-\frac{W}{\sum\limits_{e=1}^{m}\frac{1}{\alpha_e}}}\right)$.

The value is the same for any Wardrop equilibrium providing the Price of Anarchy cannot be infinite.

4.2.1 Stationary Point

Next we find a socially optimal situation. A strategy profile x is a social optimum if it provides a minimum of social costs by all the profiles. Social costs function is not convex, and its local minimum can difer from the the global optimum. But we can try to obtain some stationary points and check their optimality.

According to the Karush-Kuhn-Tucker theorem, x is a stationary point if for each user i and each link e, such that $x_{ie} > 0$, holds

$$\frac{\partial}{\partial x_{ie}}\left(SC(x) - \sum_{i=1}^{n}\gamma_i\left(\sum_{e=1}^{m}x_{ie} - w_i\right)\right) = 0$$

$$\text{or } e^{-\alpha_e\delta_e(x)}(\alpha_e\delta_e(x) - 1) = \gamma_i.$$

In equilibrium $1 - e^{-\alpha_e\delta_e(x)} = \lambda_i$ for all e, or $\alpha_e\delta_e(x) = -\ln(1-\lambda_i) = const$ by e, which satisfies the requirement to be a stationary point, but the question of its social optimality needs to be investigated more.

However, since $e^a \geq 1+a$ for $a > 0$, we can give a lower estimation $LSC(x)$ for our social costs function:

$$SC(x) \geq LSC(x) = W - \sum_{e=1}^{m}\frac{\delta_e(x)}{1+\alpha_e\delta_e(x)}.$$

The function $LSC(x)$ is convex, so it has a unique minimum, which is also global. The stationary point for $SC(x)$ is also a stationary point for its lower estimation $LSC(x)$. Thus, minimum for $LSC(x)$ and a lower estimation for

$SC(x)$ is Wardrop equilibrium profile x^{WE}, such that $\delta_e(x^{WE}) = \frac{W}{\alpha_e \sum_{e=1}^m \frac{1}{\alpha_e}}$:

$$\begin{aligned} SC(x) \geq LSC(x^{WE}) &= W - \sum_{l=1}^m \left(\frac{1}{\alpha_l} \frac{W}{\left(\sum_{e=1}^m \frac{1}{\alpha_e}\right)\left(1+\alpha_l \frac{W}{\alpha_l \sum_{e=1}^m \frac{1}{\alpha_e}}\right)} \right) \\ &= W - \sum_{e=1}^m \frac{1}{\alpha_e} \frac{W}{\left(\sum_{e=1}^m \frac{1}{\alpha_e}\right)\left(1+\frac{W}{\sum_{e=1}^m \frac{1}{\alpha_e}}\right)} \\ &= W - \frac{W}{1+\frac{W}{\sum_{e=1}^m \frac{1}{\alpha_e}}} = W\left(1 - \frac{1}{1+\frac{W}{\sum_{e=1}^m \frac{1}{\alpha_e}}}\right). \end{aligned}$$

The following example demonstrates that Wardrop equilibrium stationary point can be the worst and the best case for the different parameters of the same network. Consider a network with two paths connecting the source and distination. Total users' traffic is 1, and the path loads are $\delta(x)$ and $1 - \delta(x)$ respectively. Let first $\alpha_1 = 10$ and $\alpha_2 = 20$. In this case Wardrop equilibrium gives a maximum value of $SC(x)$ (and a minimal value of $LSC(x)$) as shown in Figure 2. Here Wardrop equilibrium is the worst case profile.

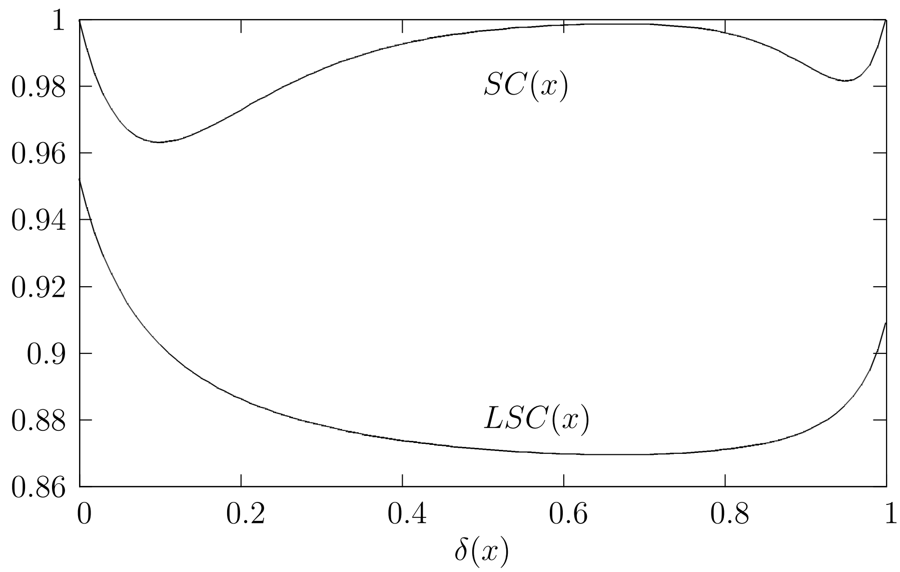

Figure 2: WE is a maximum of $SC(x)$

Now set $\alpha_1 = 1$ and $\alpha_2 = 2$. In this case Wardrop equilibrium gives a minimum value of $SC(x)$ and also a minimal value of $LSC(x)$(see Fig. 3). This Wardrop equilibrium is an optimal case.

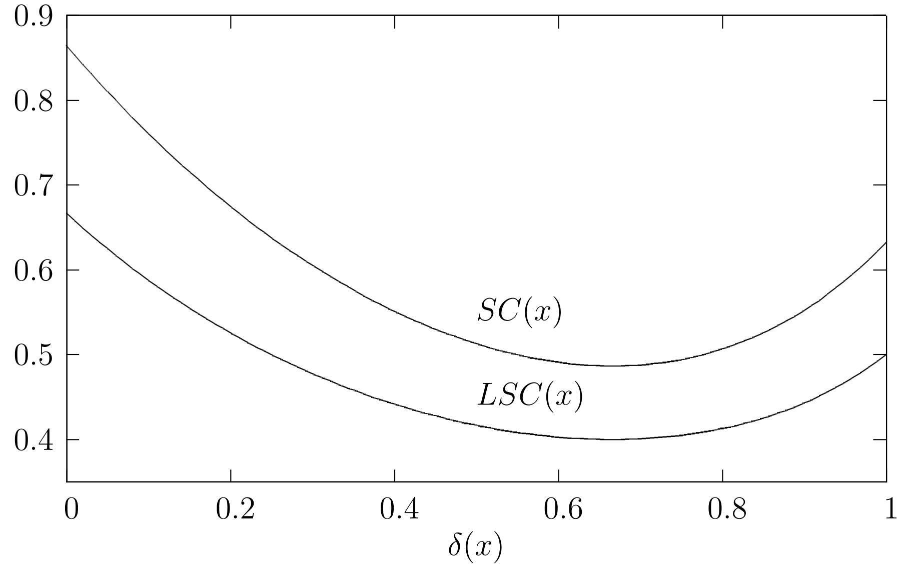

Figure 3: WE is an optimum of $SC(x)$

4.2.2 The Price of Anarchy

Now we can estimate the Price of Anarchy for the game with parallel paths, defined as a ratio of equilibrium social costs and the optimal social costs. Obviously its lower bound is 1, since Wardrop equilibrium can be optimal profile. According to the result from the previous subsection we can give an upper estimation for Price of Anarchy as follows:

$$PoA(\Gamma) = \frac{SC(x^{WE})}{SC_{opt}} \leq \left(1 - e^{-\frac{W}{\sum_{e=1}^{m}\frac{1}{\alpha_e}}}\right) / \left(1 - \frac{1}{1 + \frac{W}{\sum_{e=1}^{m}\frac{1}{\alpha_e}}}\right).$$

Denote $\frac{W}{\sum_{e=1}^{m}\frac{1}{\alpha_e}}$ as $C \geq 0$.

Then,

$$PoA \leq (1 - e^{-C})(1 + \frac{1}{C}).$$

This function has one maximum on interval $[0; +\infty)$ and its maximal value is about 1.3, leading to the total latency of each user in Wardrop equilibrium is not higher than a small constant times that of a system optimum.

5 Experimental Modeling and Simulation

Consider the multipath scheduling problem described in [11]. Traffic sent by a user is presented as a sequence of data packets each of size S located at the sender. m available paths connect the sender and the receiver, each of which could consist of a number of consecutively connected links, with the following end-to-end path characteristics: D_e - delay in the path e; B_e - bottleneck bandwidth of the path e. According to the proposed model if a packet is sent to a busy channel it will arrive to the receiver at the time $t_e^{\text{free}} + S/B_e$, where t_e^{free} indicates the time when this path becomes free after delivering previously sent packets. If N packets are sent to the same route, the next packet sent will be delayed by $N * S/B_e$. Here N is roughly the number of packets in progress, or the current load of the path δ_e.

A variation of the Fastest Path First scheduling was suggested to achieve the optimal scheduling, so that for each packet p the expected delivery time t_{pe} if sent to route e is estimated and the packet is sent to the path with minimum value of t_{pe}. In other words the optimal strategy is to distribute packets among paths according to their capacities.

Now we apply the model to our multipath multiuser routing game with the only addition that we allow more than one user to use the same network. We set parameter of the route $\alpha_e = S/B_e$. Then path load of each of our users profile in Wardrop equilibrium $\delta_e(x) = \frac{W * B_e}{\sum_{j=1}^{m} B_j}$. The loads are distributed by the routes as $\sum_{i=1}^{n} x_{ie} = \delta_e(x)$ for each path $e \in \{1, \ldots, m\}$. Equilibrium strategy for each user in the multiuser game is to distribute traffic load among the paths according to their capacities and coincides with the optimal strategy proposed for single user in [11]. The result confirms correctness of our choice of traffic delay function for approximation of TCP-controlled flows.

Now we simulate a multipath multiuser game using ns-2 network simulator [16] in order to evaluate the price of anarchy for a chosen setup. Six multipath TCP agents are attached to the source of the 3-path network connecting the source and destination nodes. The paths bandwidths were chosen as follows: 8 Mbps (megabit per second), 4 Mbps and 4 Mbps (16 Mbps network total) with the corresponding propagation delays: 60ms, 60ms and 20ms, which provide diversity in the path parameters. Each user sends 15 Mbytes of individual TCP traffic (90 Mbytes total). The resulting traffic delays for each of the six users correspond to their personal costs in equilibrium and are distributed as follows: 48.84, 47.02, 47.09, 48.23, 46.91, 45.08 s.

We compare the total equilibrium social costs $SC(eq) = 48.84$ s to the theoretical optimum, which corresponds to the minimum possible delay of 90 Mbytes traffic in such a network $SC(opt) = 45$ s. And the price of anarchy $PA = SC(eq)/SC(opt) = 1.082 < 1.3$.

6 Conclusions and Future Work

We found an upper bound for the price of anarchy in the system, where all users adhere to the equilibrium flow splitting strategies and all end-to-end subflows in the multipath network are controlled by TCP New Reno congestion control policy, providing the selfish users can successfully achieve their personal goals fairly without cooperation.

The work could be extended by modeling the dynamic multipath multiuser selfish routing network games. In dynamic routing [3], [7], network states like traffic load and congestion can vary in time. This approach could better reflect congestion situation in the real networks where performance degradation of a single data flow could be caused not only by competition between the network users but also by a natural variation in the network parameters, for example, variable quality in WLAN or HSDPA network links.

Acknowledgments

This work was supported in part by TEKES as part of the Future Internet program of the ICT cluster of the Finnish Strategic Centers for Science, Technology and Innovation.

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