TIMED & HYBRID AUTOMATA

1. Restrictions of the synchronous FSM model

2. Timed automata

3. Timed automata as a particular case of hybrid automata

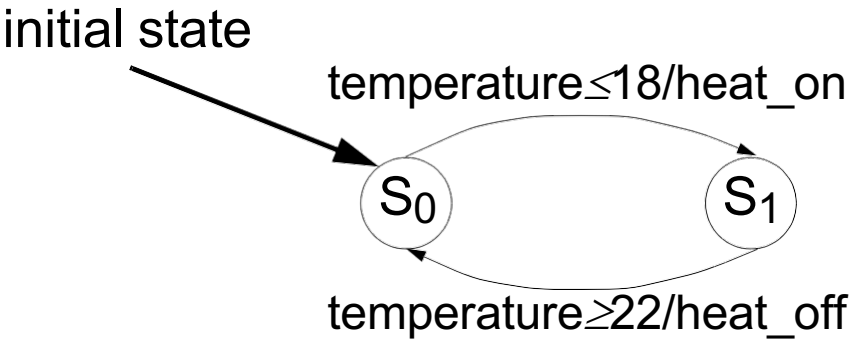
4. Hybrid automata

Restrictions/Assumptions with synchronous FSM

- FSMs react to inputs and generate, as response, outputs.
- The inputs are either present or absent; when inputs are present, the FSM reacts and generates outputs.
- Between the time instants when inputs are present, nothing interesting occurs; at each instant when inputs occur, a reaction (outputs) is computed *instantly* by the FSM.
- FSMs operate in a sequence of *discrete* reactions.
- The clock can be explicitly modelled as a FSM delivering ticks for the whole system; all transitions in the system are synchronised on this clock tick. Time (non-negative integer) is captured counting these clock ticks $\Rightarrow$ *discrete time model*.

Example FSM: Thermostat

- Input event: {temperature}
  - Signal *temperature* is received at certain instants of time from a sensor;
  - When input *temperature* is received and the guard on the transition is true, the system generates an output.
- Outputs: {heat_on, heat_off}
- States: {S_0, S_1}
  - S_0: system cools (heating off)
  - S_1: system is heating



- We want to keep the temperature close to 20°;
- To avoid *chattering* (turning on and off rapidly, all the time), we allow temperature to be inside a band (technique called *hysteresis*).

Timed Automata

- For modeling real-time asynchronous systems, continuous time models are the natural representation.
- Real-time systems require measuring the passage of (continuous) time and performing actions at specific times.

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- Real-time systems require measuring the passage of (continuous) time and performing actions at specific times.
- In timed automata time is considered a continuous quantity. No global synchronisation, in the sense of a unique clock, is assumed.
- Timed automata are an extension of the FSM model which allows modelling of certain real-time systems and formal reasoning about time.
  - A timed automaton is a finite automaton (similar to a FSM) *augmented with a finite set of real-valued clocks*.

Example: Thermostat with Timed Automata

- **Clock: x**
- **Input event: {temperature}**
- **Outputs: {heat_on, heat_off}**
- **States: {S_0, S_1}**
  - **S_0: system cools (heating off)**
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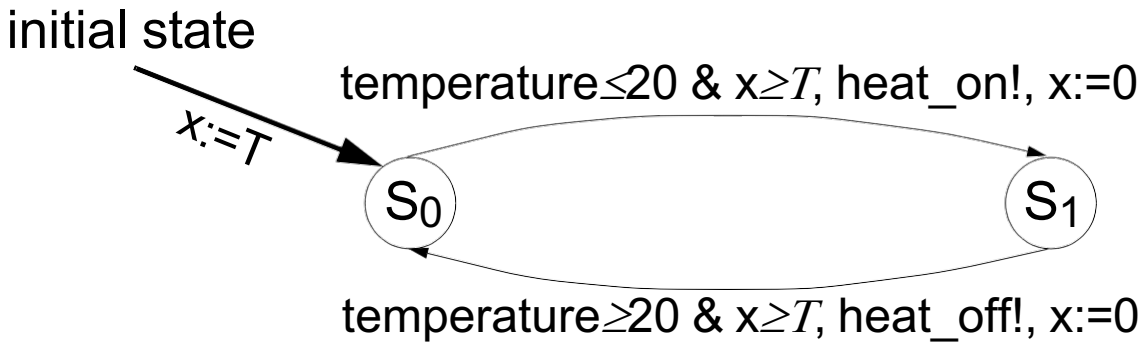

- **We use a single temperature threshold - the desired level of 20°;**
  To avoid chattering, the heater remains on/off for a minimum required time *T*.

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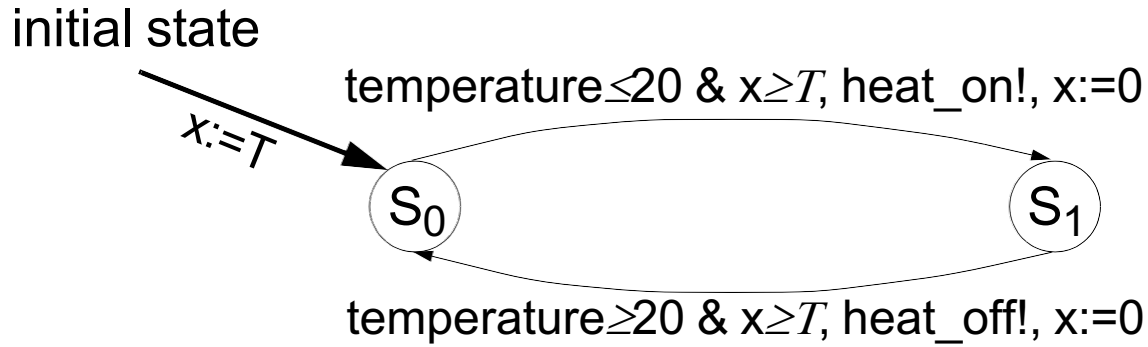
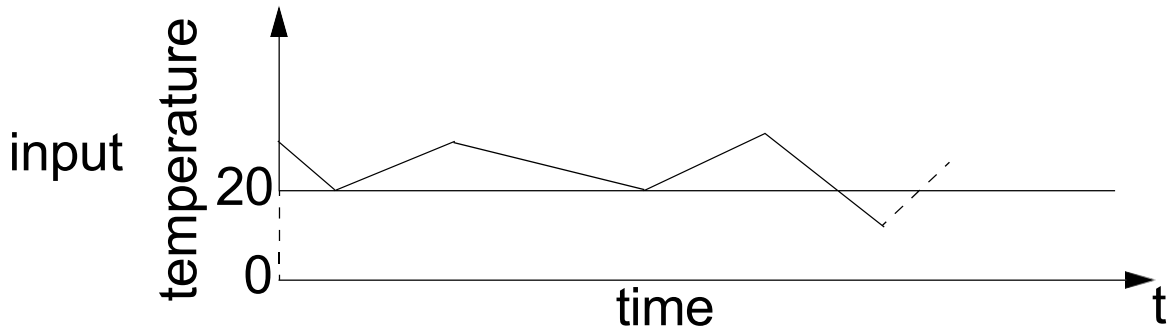
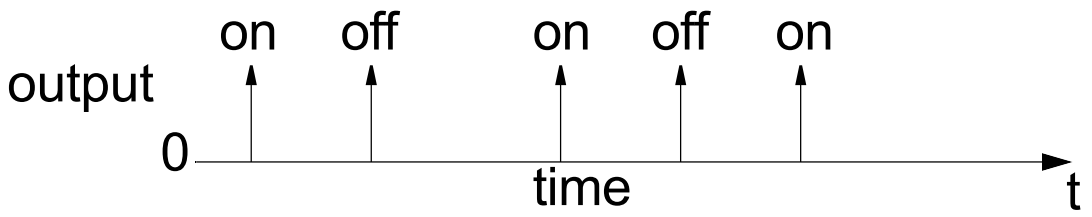
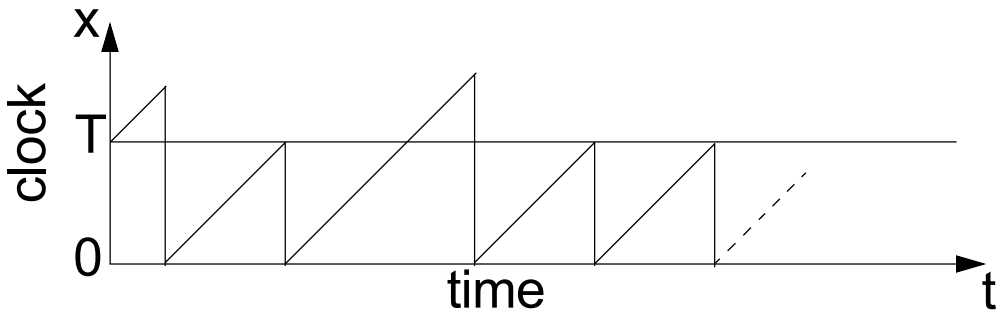








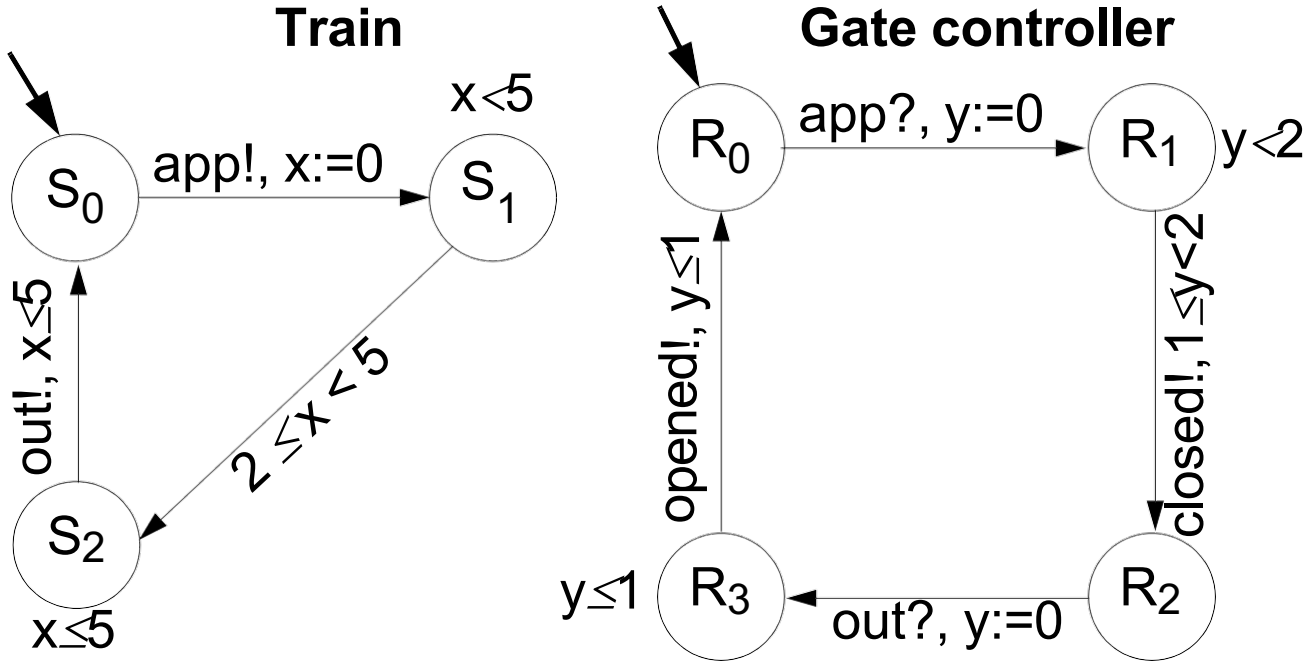
Example: Gate Control System

Specification:

- When the train approaches, it sends signal *app* at least 2 minutes before it enters the crossing; after leaving the crossing it sends signal *out*; it leaves the crossing maximum 5 minutes after signalling *app*.

- When the controller gets signal *app* it closes the gate, which takes at least 1 minute, but less than 2; then it waits for signal *out*; when *out* arrives it opens the gate within maximum 1 minute.

Example: Gate Control System



x, y: clocks

S_0: train far away
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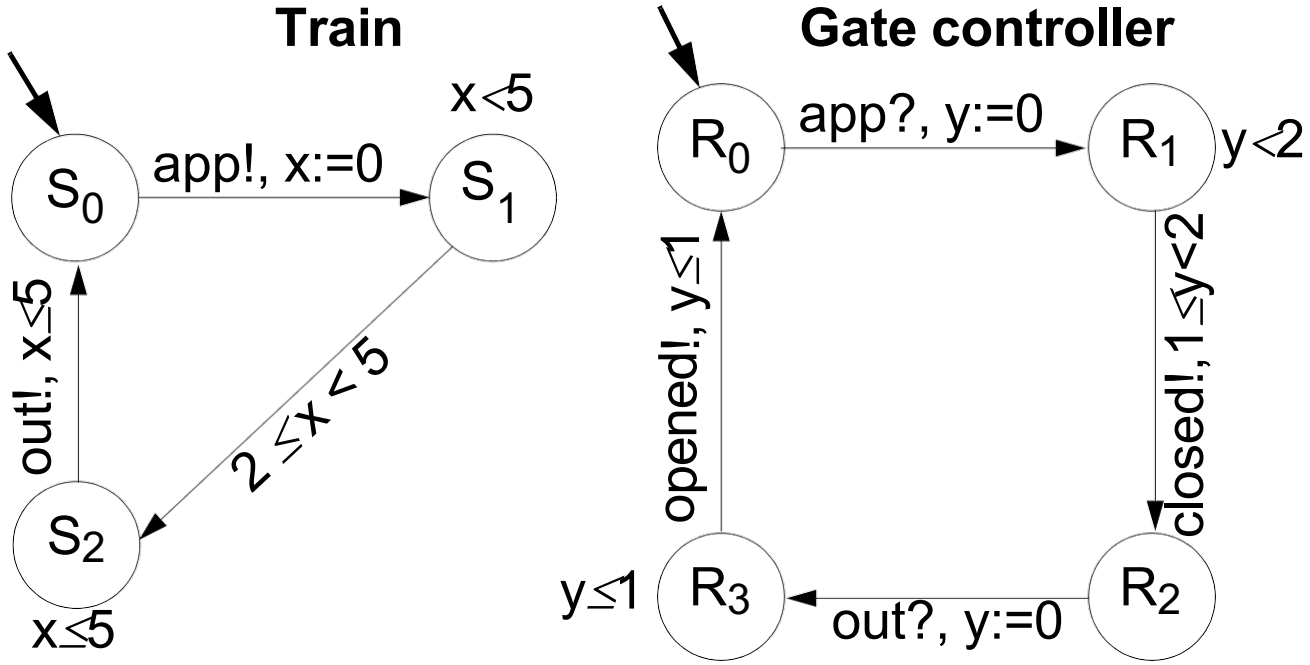

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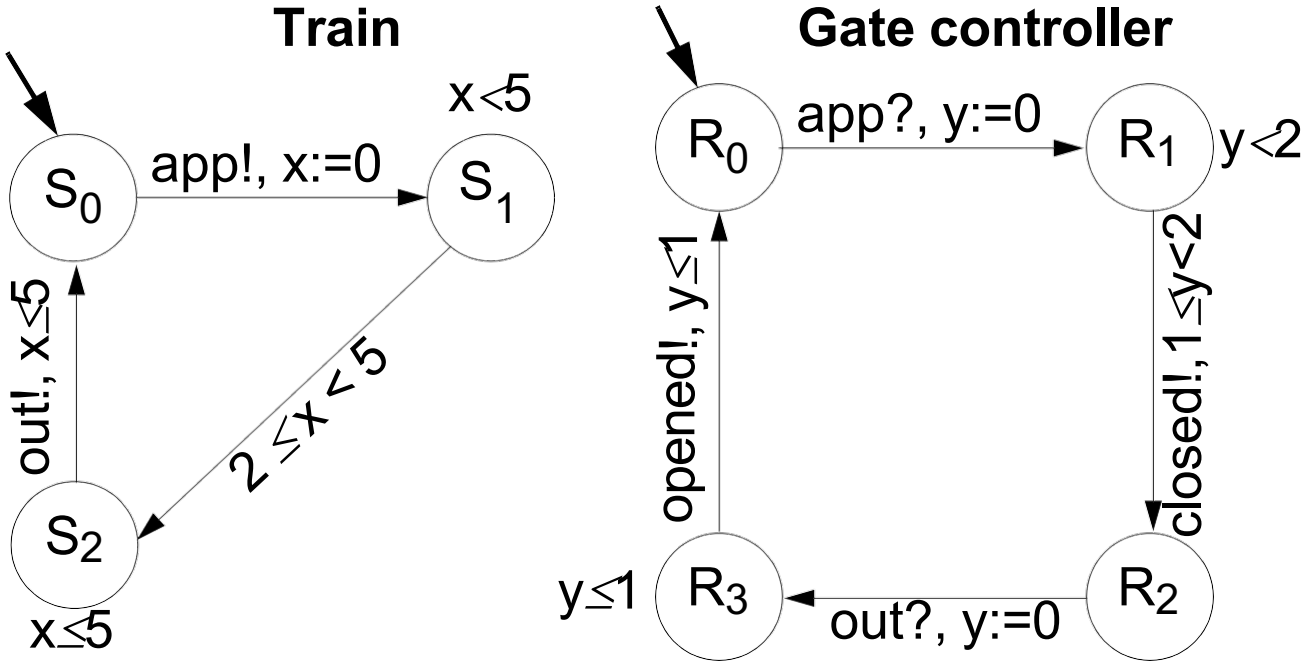
- Once the above model is realised, one can formally verify (e.g. using model checking tools) properties such as: the train will only be in state S_2 (crossing) when, simultaneously, the gate is in R_2 (down).

Timed Automata: Rules/Properties



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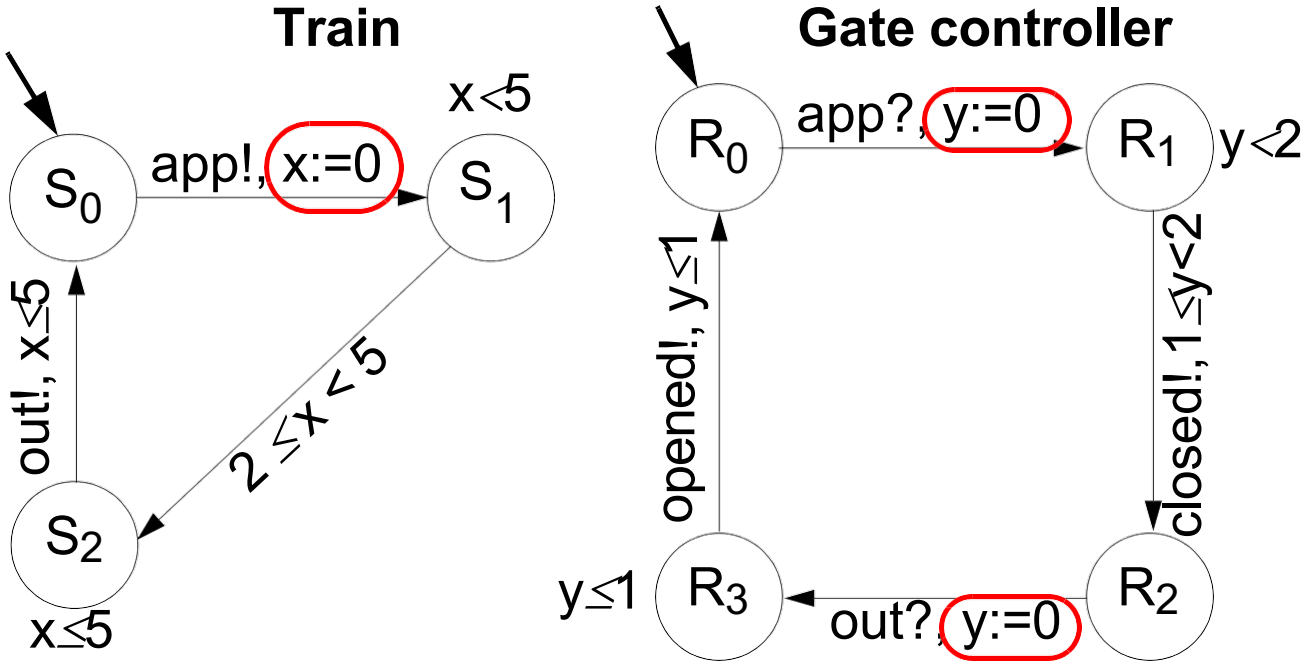

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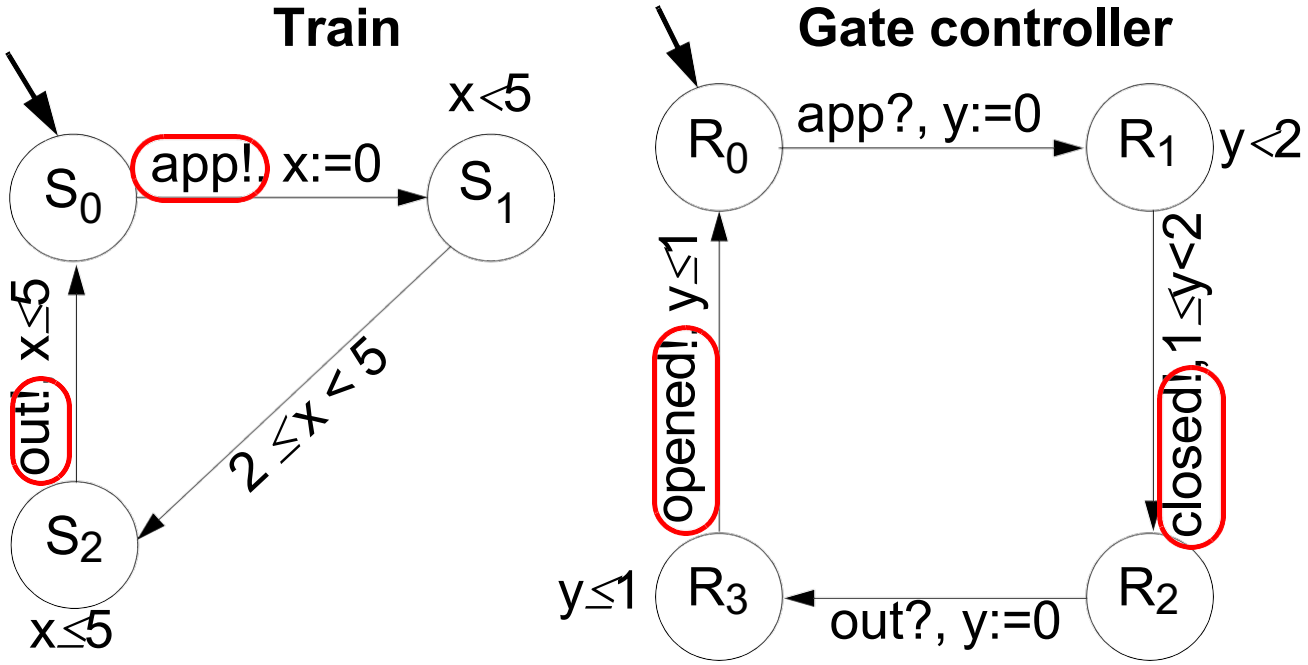

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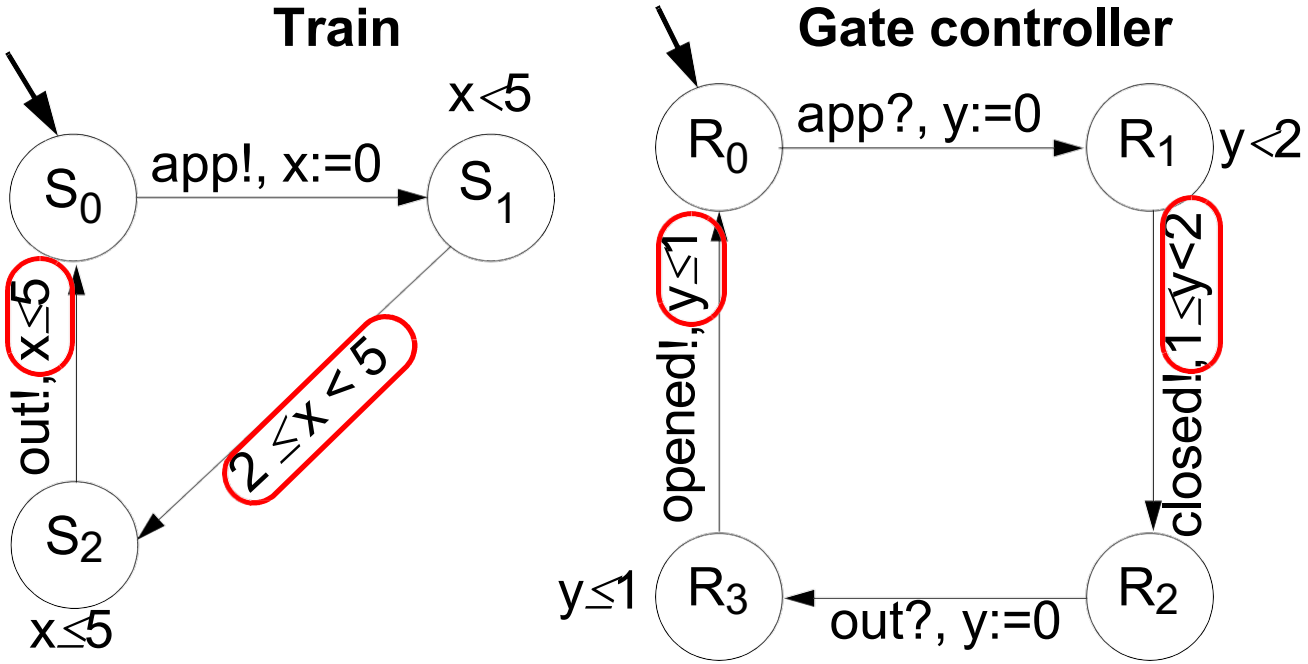
- Transitions are instantaneous; time elapses when the automaton is in a state.
- When a transition occurs clocks can be reset.
- Time passes at the same rate for all clocks.
- When a transition occurs, signals (events) can be generated.

Timed Automata: Rules/Properties



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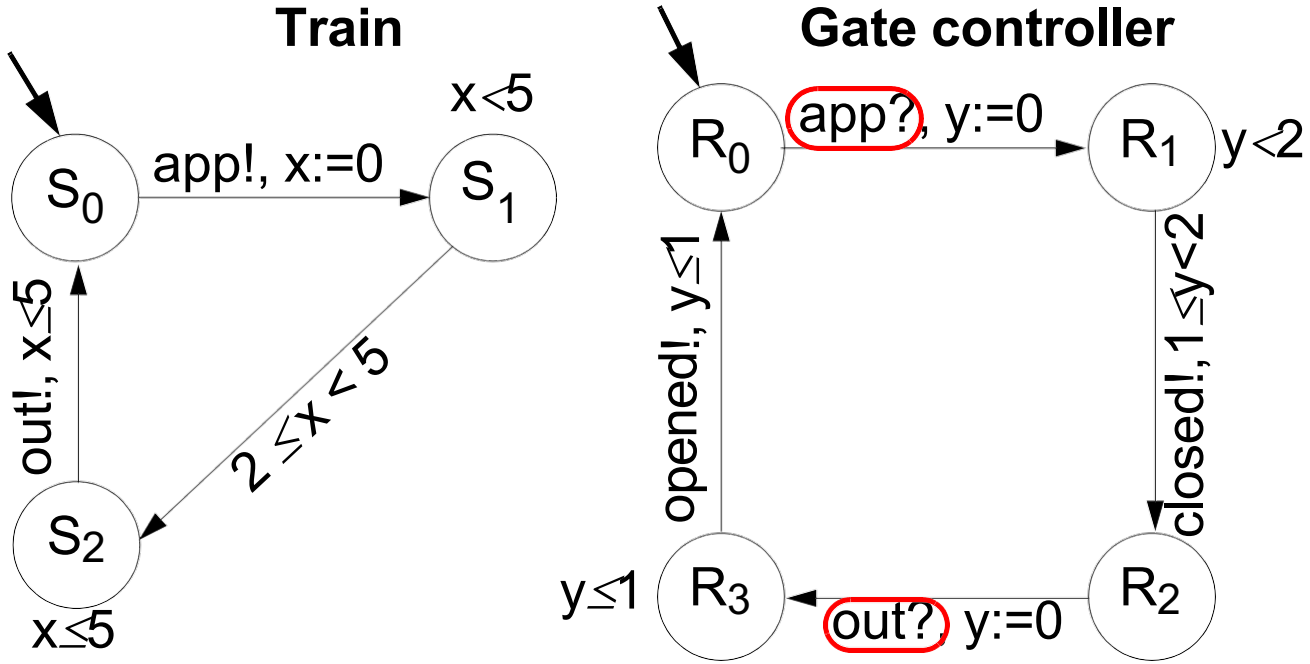

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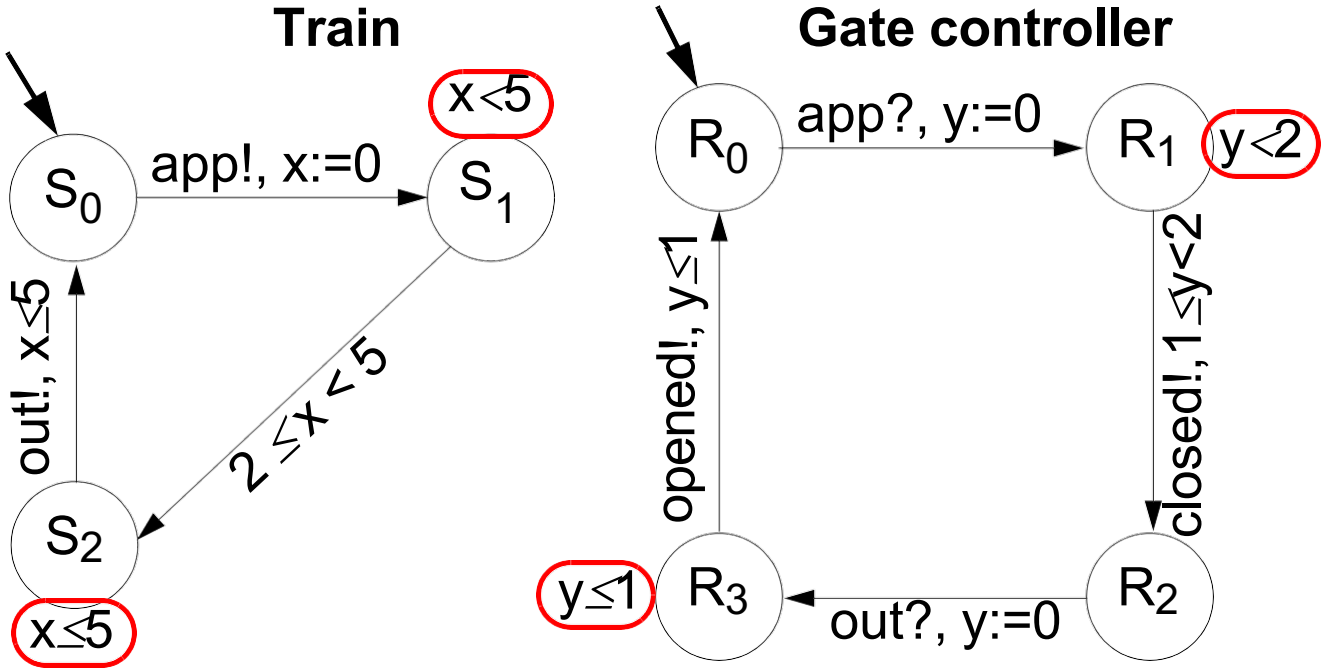

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- Transitions can have input signals (events) associated; when the signal arrives and the associated guard is satisfied, the transition will be taken.
- States can have associated invariants, expressed as conditions on the clocks; the automaton can stay in that state as long as the invariant is true.

Timed Automata: Rules/Properties

- Like FSMs, timed automata can be extended with variables.
  - Actions on variables can be associated to transitions.
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  - Actions on variables can be associated to transitions.
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- Timed automata are, by definition, *infinite state models*: At any time moment, the state of the system is defined not only by the actual state in the state machine (e.g. S_0, S_1, etc.), but also by the current values of the clocks!

  However, for verification, timed automata admit *finite state representations* (by exploiting equivalence relations on certain portions of the state space)!
  - Model checking techniques can be used to prove properties of timed automata.
  - The state explosion problem is more severe than for synchronous concurrent FSMs!

Hybrid Automata

- Timed automata are FSMs with addition of clocks.

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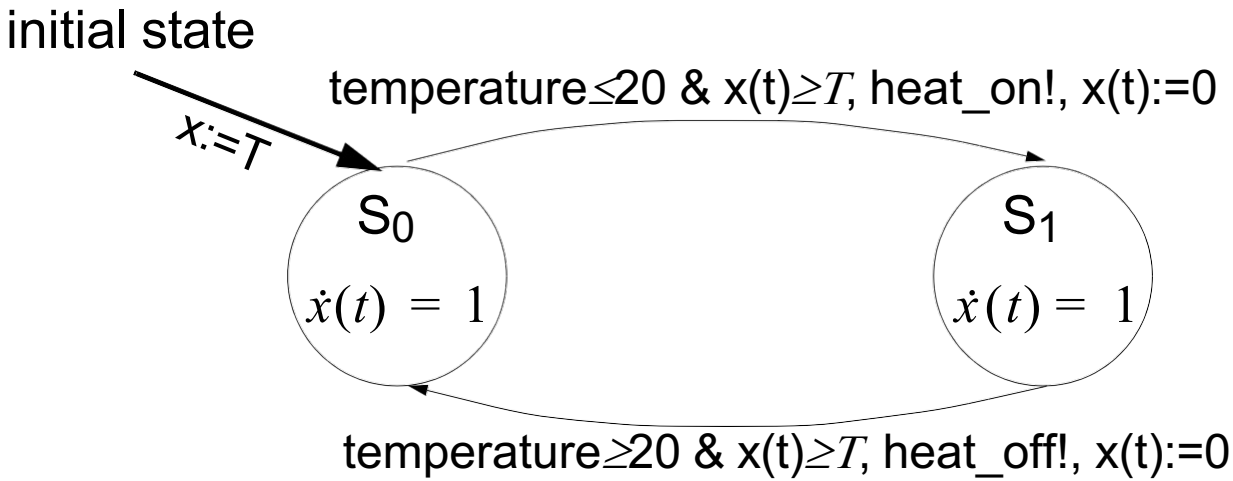
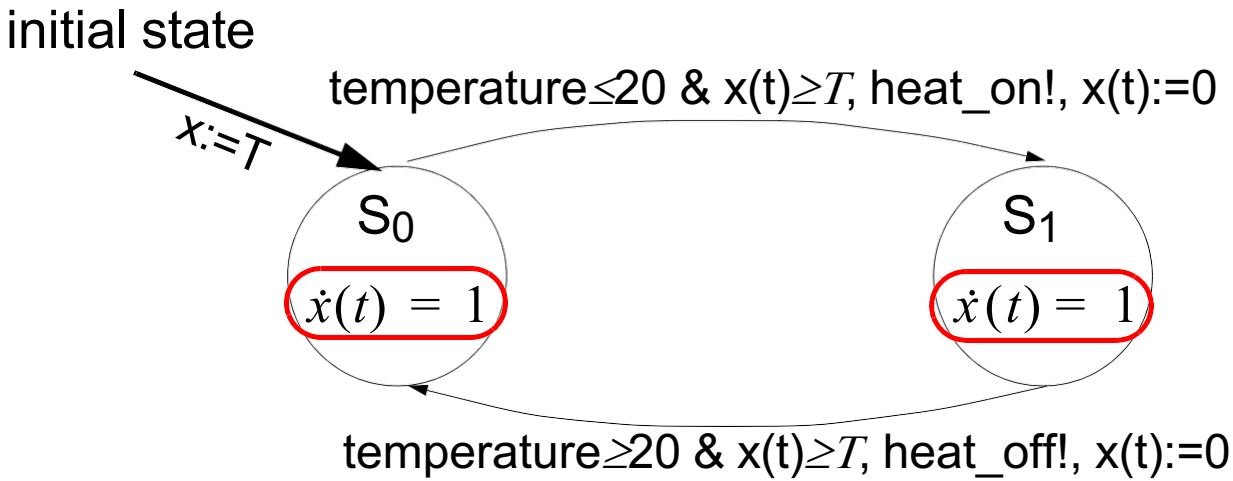
- A clock is a continuous variable whose value can be described by the differential equation: $\dot{x}(t) = 1$.
- Timed automata are, in fact, the simplest form of *hybrid automata*.
- Hybrid automata are FSMs combined with a finite set of continuous variables whose values are described by a set of ordinary differential equations.

The Thermostat as a Hybrid Automaton



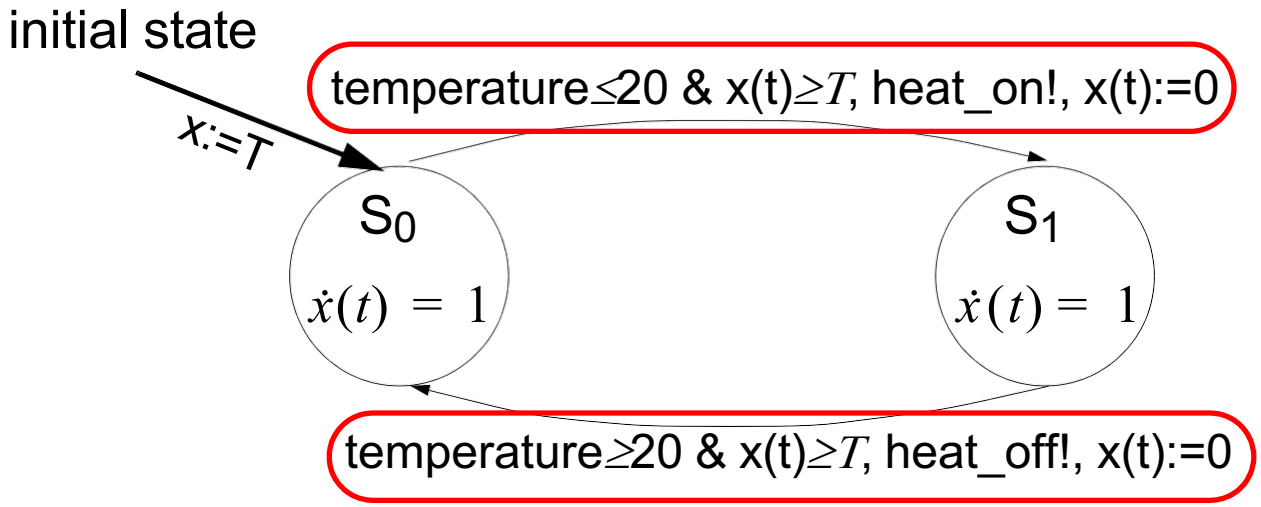
- **In the above model, the equation describing the continuous variable representing clock x is made explicit. This model is completely equivalent with the one based on Timed Automata.**

Hybrid Automata: Rules/Properties



- **Hybrid Automata associate with each state of an FSM a dynamic behavior.**
- **The dynamic behavior in each state is specified by a *state refinement*; a state refinement describes the dynamics of the outputs as a function of the inputs.**
- **State refinements are specified as ordinary *differential equations*.**

Hybrid Automata: Rules/Properties



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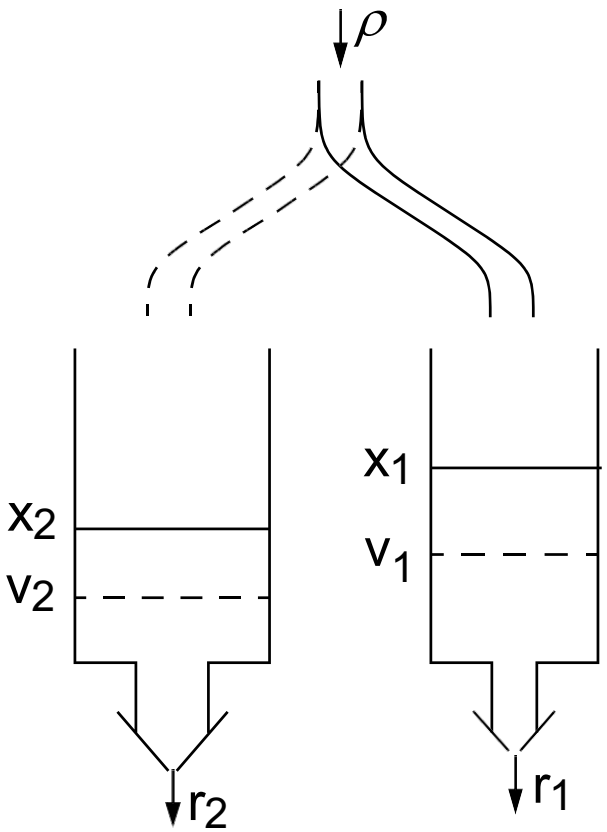
- **The dynamic behavior in each state is specified by a *state refinement*; a state refinement describes the dynamics of the outputs as a function of the inputs.**
- **State refinements are specified as ordinary *differential equations*.**
- **Transitions can have associated guards, assignments to variables, outputs...**
- **Hybrid automata are extremely strong in their expressive power; they combine discrete and continuous behavior in one single model.**

Example: Water Tank



- Each tank is leaking at constant rate (r_1, r_2).
- Water is added at a constant rate ρ.
- One tank is filled at a time; filling switches from one tank to the other in zero time.
- The goal is to keep the water *volume* above v_1 and v_2 respectively.
- The current water volme is x_1, x_2.

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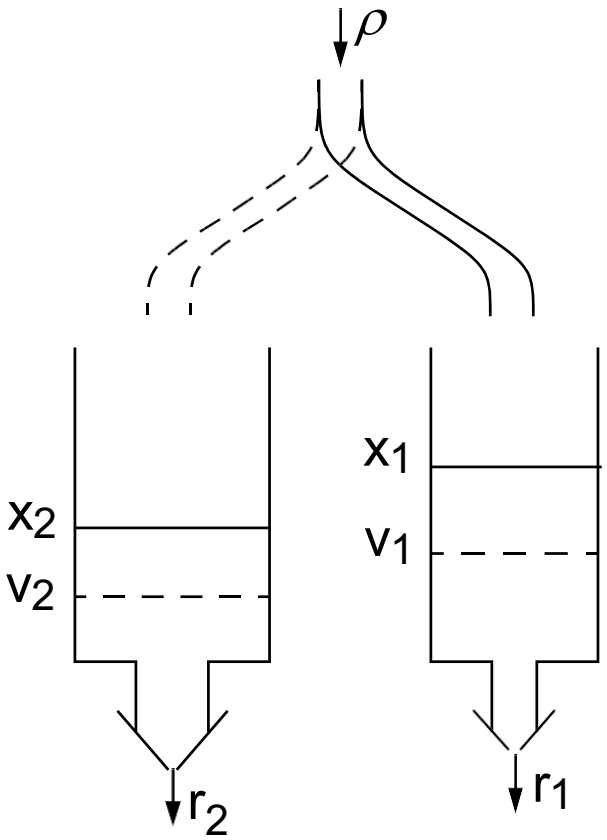

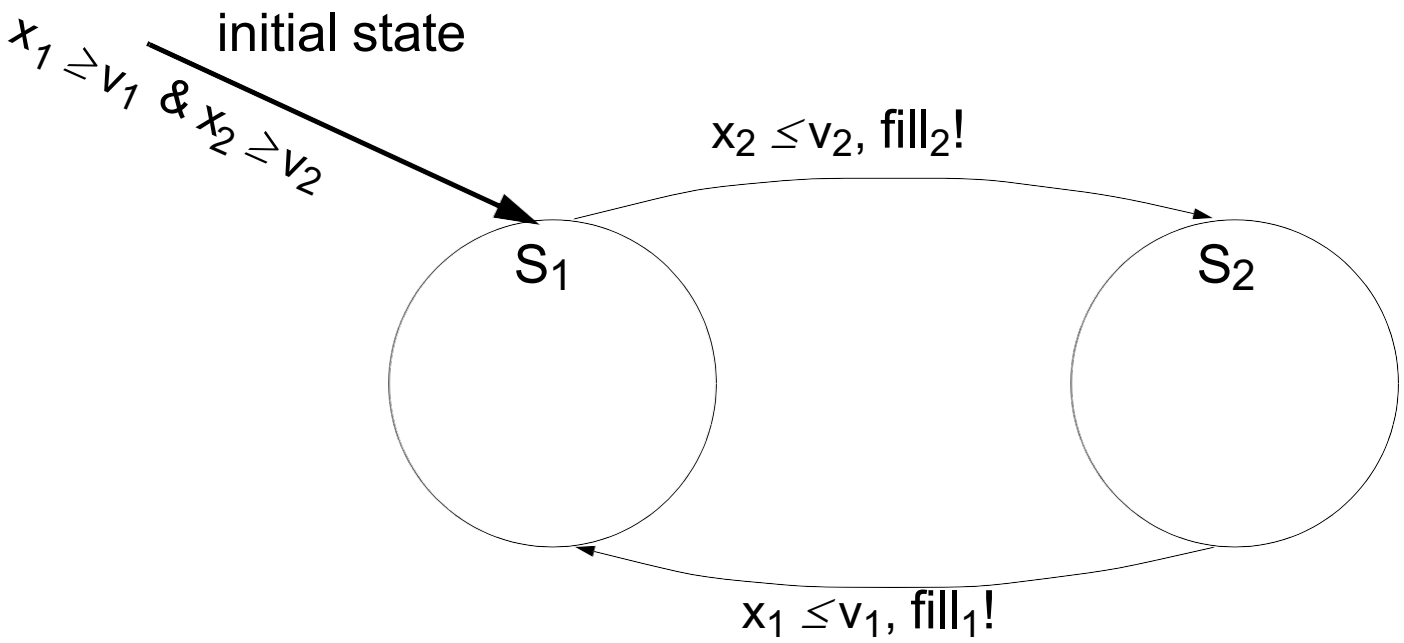

- **Inputs: {x_1, x_2}**
- **Outputs: {$fill_1$, $fill_2$}**
- **States: {S_1, S_2}**
  - **S_1: tank 1 is filled**
  - **S_2: tank 2 is filled**

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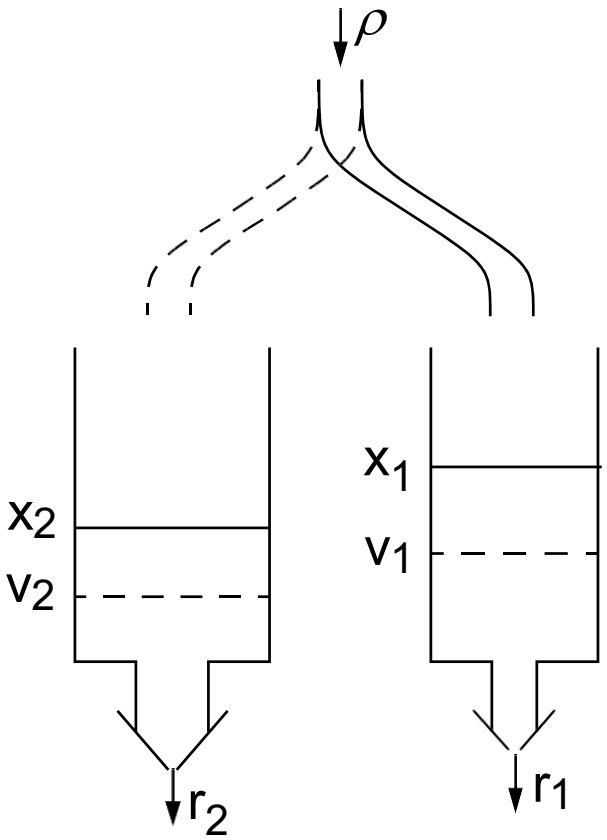

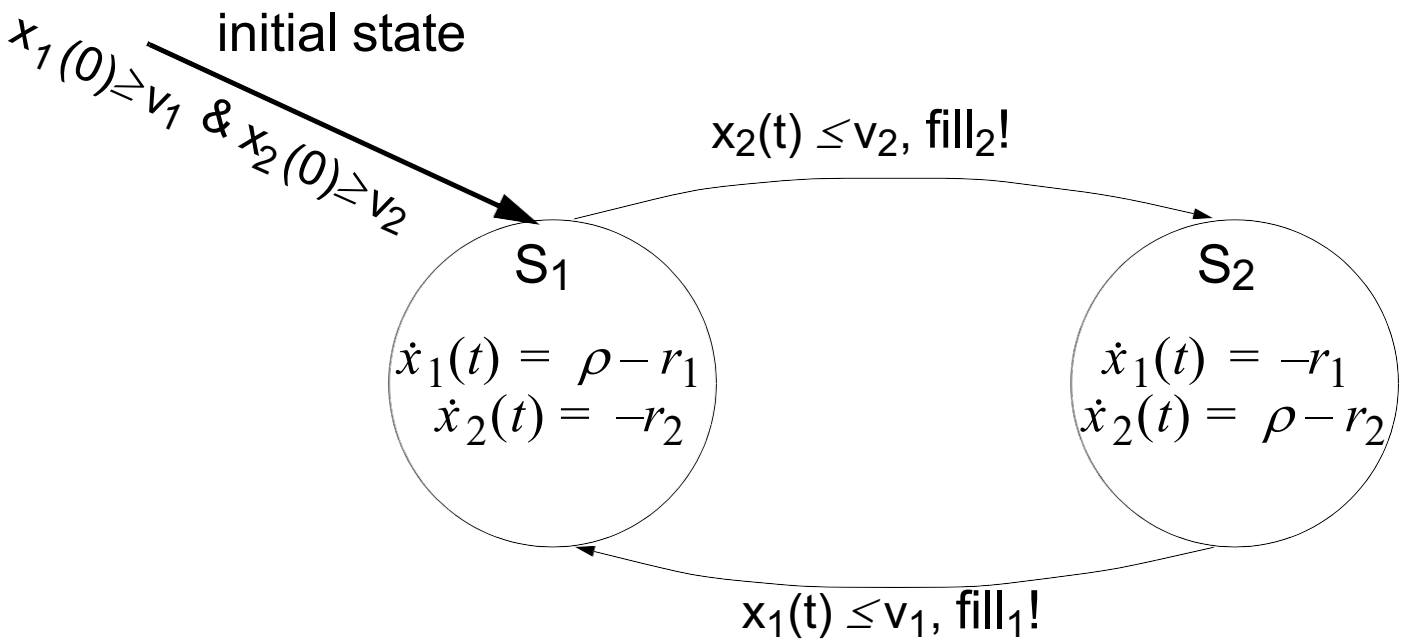

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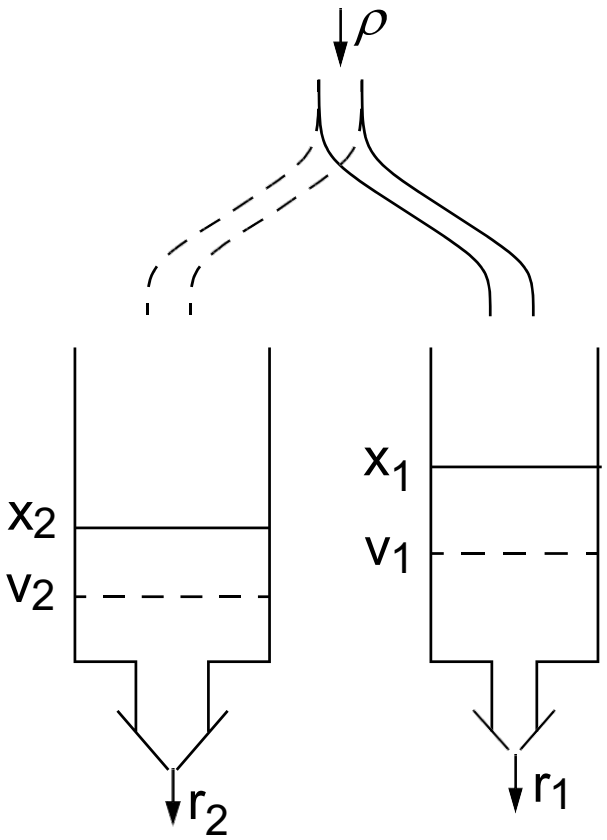

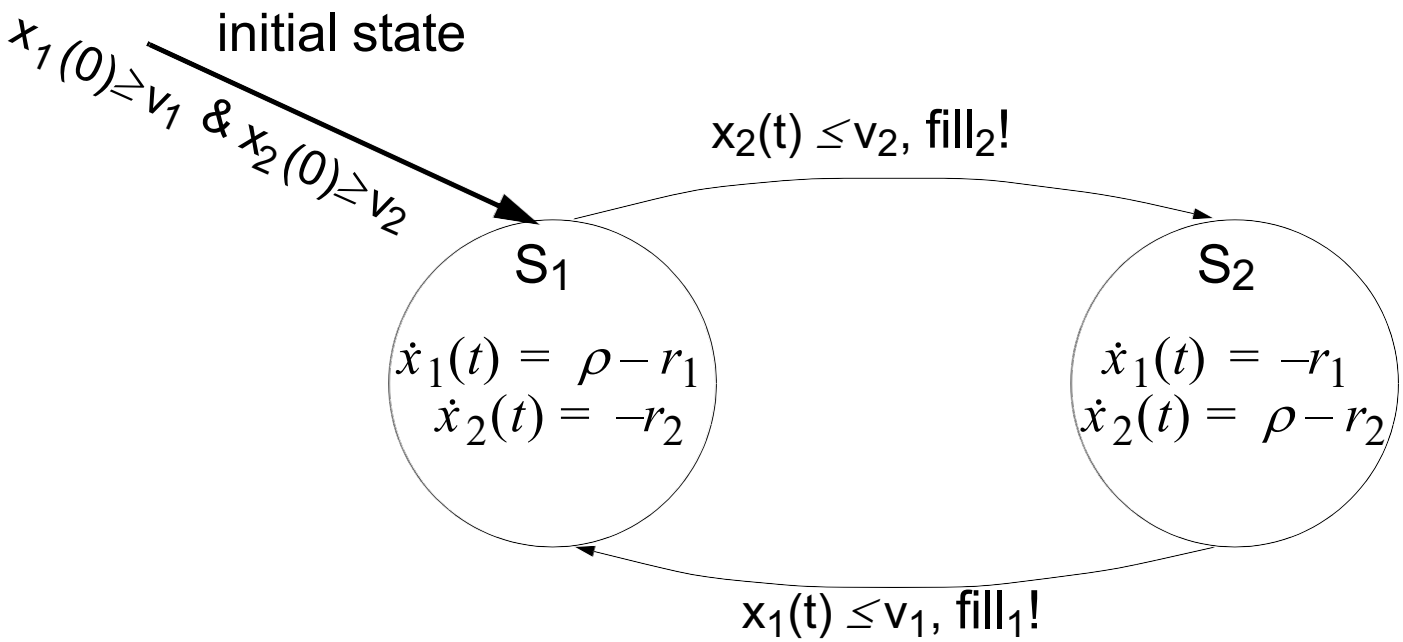

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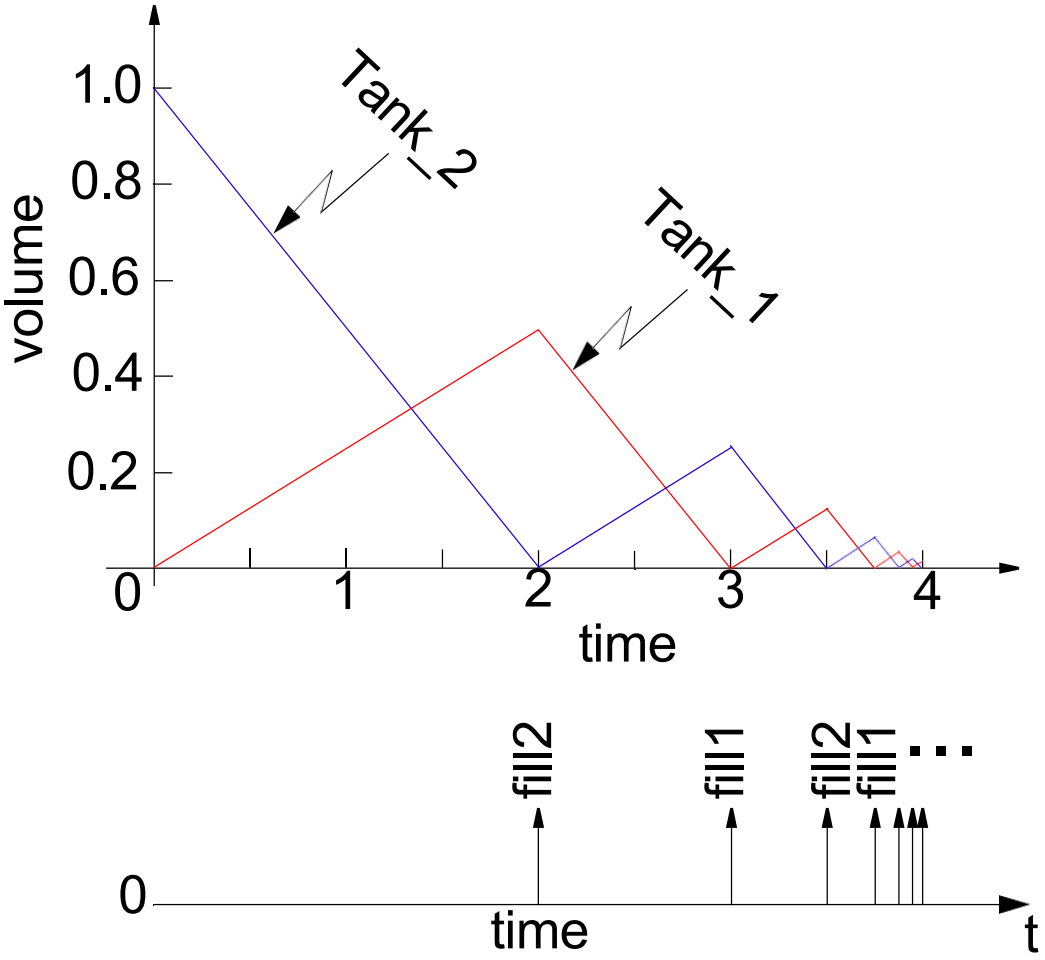
- **The system might reach a situation (when water level is very close to the target) in which the number of switches per time unit is continuously increasing (Zeno system).**

Example: Water Tank



This is a simulation considering:

- Leaking rates: $r_1=r_2= 0.5$
- Water inflow rate: $\rho= 0.75$
- Keep the water above $v_1=v_2=0$
- Initial level: $x_1(0)=0$, $x_2(0)=1$

- Since ρ is too small, both tanks will, eventually, become empty.
- As the tanks come close to the 0 level, the number of switches per time unit increases.

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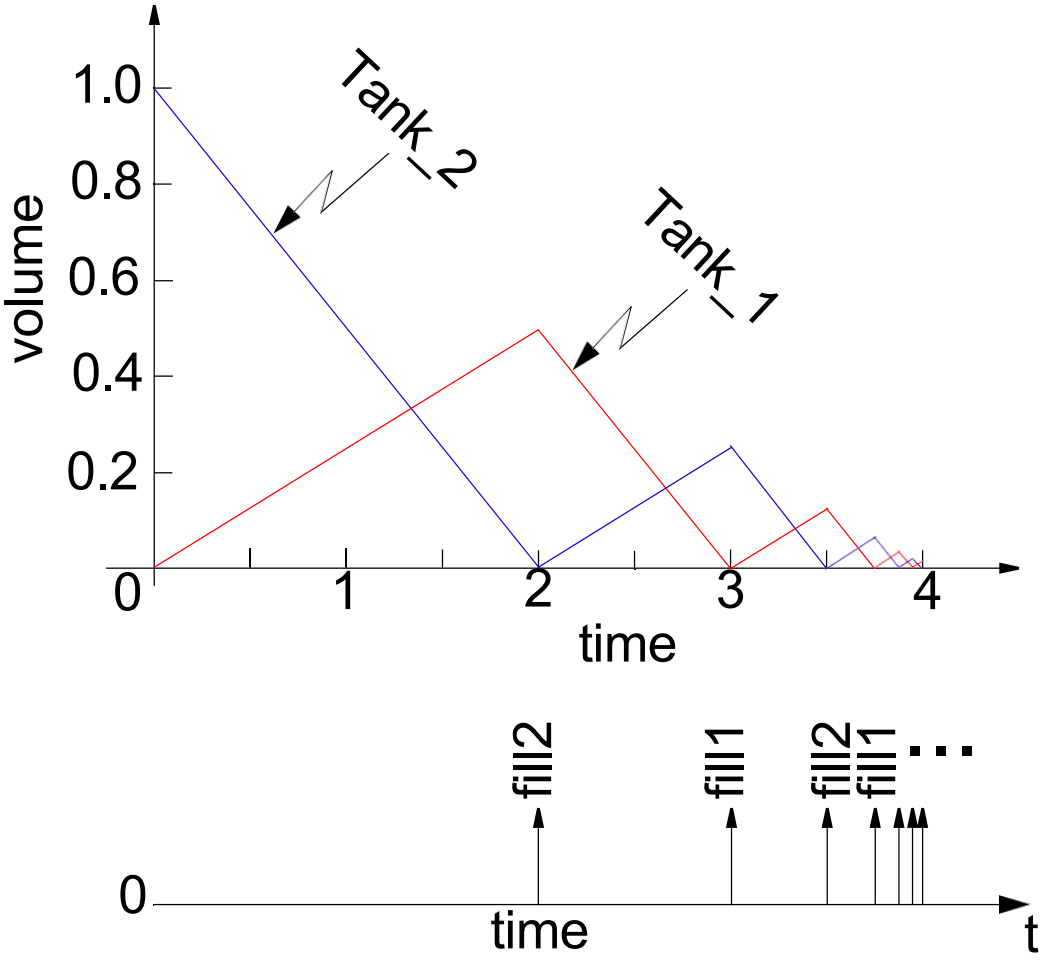

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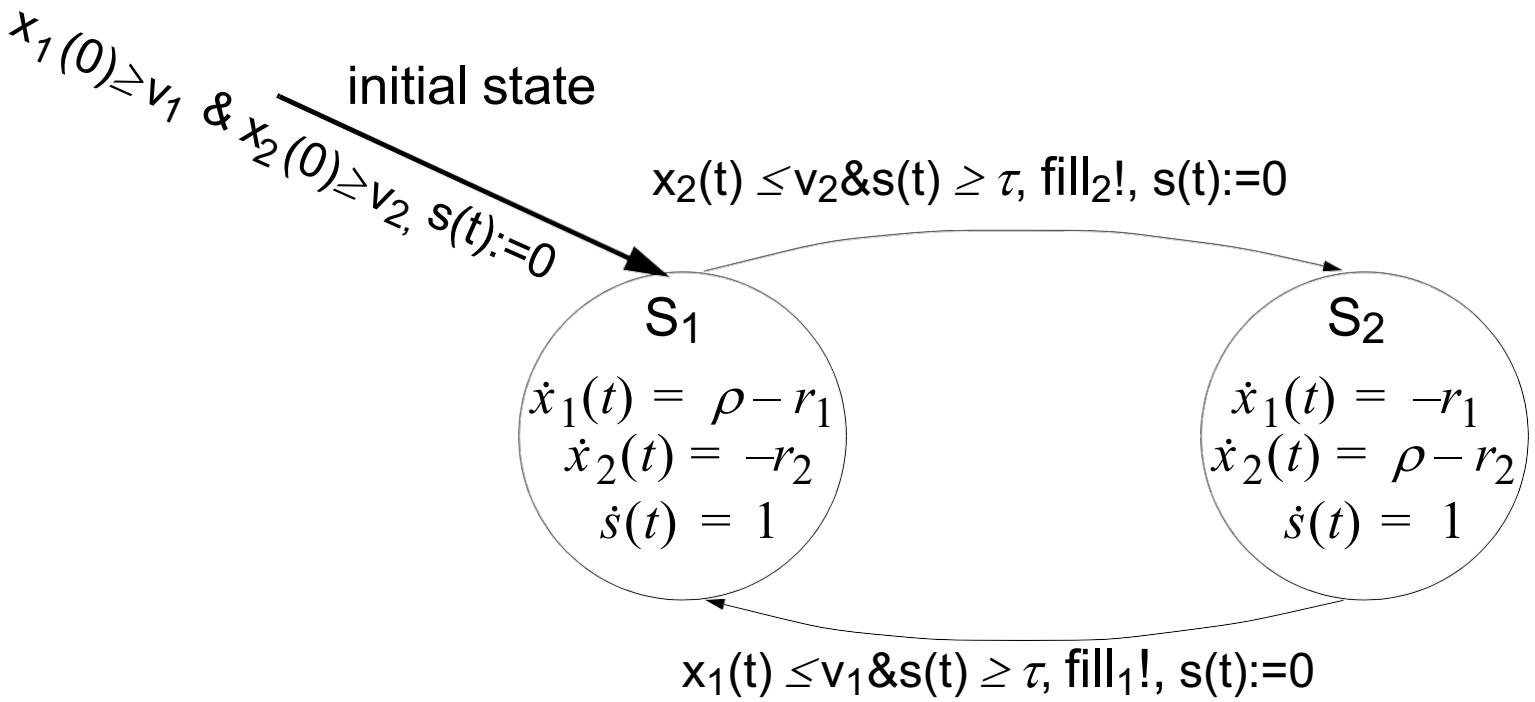
- With such a Hybrid Automata model, one can use formal verification to answer questions like: *will tank_1 become empty before time 7?* You cannot ask this with a simple timed automata model (since the flow equations are not part of the model).

Example: Water Tank

- **In order to avoid Zeno behaviour one possible solution is to allow switches only after the system spent a minimum amount of time τ in a state.**



Hybrid Automata: Final Comments

- Hybrid automata models can be used for simulation and formal verification.

- Hybrid automata, like timed automata, are, by definition, *infinite state models*. However, they admit a *finite state representation* (by exploiting equivalence relations on certain portions of the state space)!
  - Model checking techniques can be used to prove properties of hybrid automata.
  - The state explosion problem is more severe than for timed automata!

- Available frameworks/tools: Ptolemy II, HyTech.

What Modeling Approach to Choose?

- It depends on the characteristics of the system:
  - control or data flow dominated;
  - synchronous or asynchronous; centralised or distributed;
  - how large?
  - what aspects related to timing are we interested in?
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  - formal verification
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- It depends on what tools you have available and which approach you (or your company or your boss!) prefer.

What Modeling Approach to Choose?

- Don’t use the “strongest”! Go for exactly that expressive power you need; not more!
  - Large expressive power:
    - Can specify “anything”.
    - No formal reasoning possible (or extremely complex).
  - Limited expressive power, based on well chosen computation model:
    - Only particular systems can be specified.
    - Formal reasoning is possible.
    - Efficient implementation

Modeling Languages

- The choice of a modeling language is, to a large extent, connected to the choice of the modeling approach.

  This, because certain modeling languages are strongly connected to a particular model of computation:

  - Communicating asynchronous state machines: SDL, Lotos
  - Synchronous FSM systems: Esterel, StateCharts;
  - Dataflow computation: Matlab, Lustre, Silage
  - Discrete event: SystemC, VHDL, Verilog