



Syntax Analysis, Parsing

Peter Fritzon, Christoph Kessler,
IDA, Linköpings universitet, 2010.

Parser



■ A parser for a CFG (*Context-Free Grammar*) is a program which determines whether a string w is part of the language $L(G)$.

Function

- Produces a parse tree if $w \in L(G)$.
- Calls semantic routines.
- Manages syntax errors, generates error messages.

Input:

- String (finite sequence of tokens)
- Input is read from left to right.

Output:

- Parse tree / error messages

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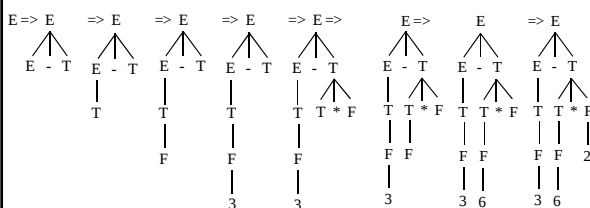
5.2

Top-Down Parsing



■ Example: **Top-down** parsing with input: $3 - 6 * 2$

$E \rightarrow E - T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow \text{Integer} \mid (E)$



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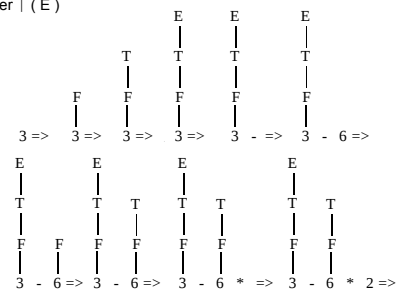
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Bottom-up Parsing



■ Example: **Bottom-up** parsing with input: $3 - 6 * 2$

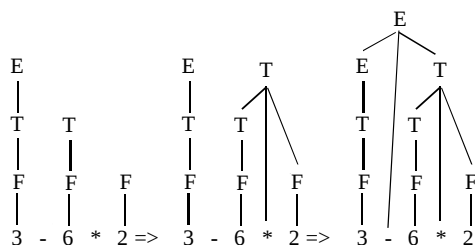
(same CFG as in previous example)
 $E \rightarrow E - T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow \text{Integer} \mid (E)$



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Bottom-up Parsing cont.



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Top-Down Analysis



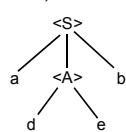
■ How do we know in which order the string is to be derived?

- Use one or more tokens lookahead.

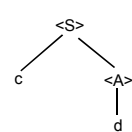
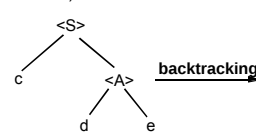
■ Example: **Top-down analysis with backtracking**

$\langle S \rangle \rightarrow a \langle A \rangle b$ 1 token lookahead works well
 $\quad \quad \quad c \langle A \rangle$ 1 token lookahead works well
 $\langle A \rangle \rightarrow d e$ test right side until something fits
 $\quad \quad \quad d$ fits

■ a) adeb



b) cd



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Top-down Analys with Backtracking, cont.

- Top-down analys with backtracking is implemented by writing a **procedure or a function for each nonterminal** whose task is to find one of its right sides:

```
bool A() { /* A → d e | d */
    char* savep;
    savep = inpptr;

    if (*inpptr == 'd') {
        scan(); /* Get next token, move inpptr a step */
        if (*inpptr == 'e') {
            scan();
            return true; /* 'de' found */
        }
    }
    inpptr = savep;
    /* 'de' not found, backtrack and try 'd' */
    if (*inpptr == 'd') {
        scan(); return true; /* 'd' found, OK */
    }
    return false;
}
```

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Top-down Analys with Backtracking, cont.

```
bool S() { /* S → a A b | c A */
    if (*inpptr == 'a') {
        scan();
        if A() {
            if (*inpptr == 'b') {
                scan();
                return true;
            } else return false;
        } else return false;
    }
    else if (*inpptr == 'c') {
        scan();
        if A() return true; else return false;
    }
    else return false;
}
```

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Construction of a top-down parser

- Write a procedure for each nonterminal.
- Call scan directly *after each token is consumed*.
 - Reason: The look-ahead token should be available
- Start by calling the procedure for the start symbol.

At each step check the leftmost non-treated vocabulary symbol.

- If it is a *terminal symbol*
 - Match it with the current token, and read the next token.
- If it is a *nonterminal symbol*
 - Call the routine for this nonterminal.
- In case of error call the error management routine.

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Example: An LL(1) grammar which describes binary numbers

```
S → BinaryDigit BinaryNumber
BinaryNumber → BinaryDigit BinaryNumber
               | ε
BinaryDigit → 0 | 1
```

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Sketch of a Top-Down Parser (recursive descent)

```
void TopDown(input,output) {
    /* main program */
    scan();
    S();
    if not eof then error(...);
}
```

Grammar:

```
S → BinaryDigit BinaryNumber
BinaryNumber → BinaryDigit
               BinaryNumber
               | ε
BinaryDigit → 0 | 1
```

```
void BinaryDigit()
{
    if (token==0 || token==1) scan();
    else error(...);
} /* BinaryDigit */

void BinaryNumber()
{
    if (token==0 || token==1)
    {
        BinaryDigit();
        BinaryNumber();
    } /* OK for the case with ε */
} /* B' */

void S()
{
    BinaryDigit();
    BinaryNumber();
} /* S' */
```

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A Top-Down Parser that does not Work, Infinite Recursion:

```
void TopDown(input,output)
{
    /* main program */
    scan();
    S();
    if not eof then error(...);
}
```

Grammar:

```
S → BinaryDigit BinaryNumber
BinaryNumber → BinaryNumber
               BinaryDigit
               | ε
BinaryDigit → 0 | 1
```

```
void BinaryDigit()
{
    if (token==0 || token==1) scan();
    else error(...);
} /* BinaryDigit */

void BinaryNumber()
{
    if (token==0 || token==1)
    {
        BinaryNumber(); /* Infinite Recursion here */
        BinaryDigit();
    } /* OK for the case with ε */
} /* B' */

void S()
{
    BinaryDigit();
    BinaryNumber();
} /* S' */
```

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Non-LL(1) Structures in a Grammar:

- Left recursion, example:

$$E \rightarrow E - T$$

$$| T$$

- Productions for a nonterminal with the same prefix in two or more right-hand sides, example:

$$\text{arglist} \rightarrow ()$$

$$| (\text{args})$$

or

$$A \rightarrow a b$$

$$| a c$$

- The problem can be solved in most cases by rewriting the grammar to an LL(1) grammar

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Convert a grammar for top-down parsing?

1. Eliminate left recursion

a) Transform the grammar to iterative form

EBNF (Extended BNF) Notation:

- $\{\beta\}$ same as the regular expression: β^*
- $[\beta]$ same as the regular expression: $\beta | \epsilon$
- $()$ left factoring, e.g. $A \rightarrow ab | ac$ in EBNF is rewritten:
 $A \rightarrow a (b | c)$

Transform the grammar to be iterative using EBNF

- $A \rightarrow A \alpha | \beta$ (where β may not be preceded by A) in EBNF is rewritten:
 $A \rightarrow \beta \{\alpha\}$

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1b) Transform the Grammar to Right Recursive Form Using a Rewrite Rule:

- $A \rightarrow A \alpha | \beta$ (where β may not be preceded by A)

is rewritten to

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' | \epsilon$$

Generally:

- $A \rightarrow A \alpha_1 | A \alpha_2 | \dots | A \alpha_m | \beta_1 | \beta_2 | \dots | \beta_n$
(where $\beta_1, \beta_2, \dots$ may not be preceded by A)

is rewritten to:

$$A \rightarrow \beta_1 A' | \beta_2 A' | \dots | \beta_n A'$$

$$A' \rightarrow \alpha_1 A' | \alpha_2 A' | \dots | \alpha_m A' | \epsilon$$

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2. Left Factoring Using $()$ or $[]$

Original Grammar:

$$\langle \text{stmt} \rangle \rightarrow \text{if} \langle \text{expr} \rangle \text{ then} \langle \text{stmt} \rangle$$

$$| \text{if} \langle \text{expr} \rangle \text{ then} \langle \text{stmt} \rangle \text{ else} \langle \text{stmt} \rangle$$

Original Grammar

- $A \rightarrow ab | ac$

Solution using EBNF:

- $A \rightarrow a (b | c)$

Solution using EBNF:

$$\langle \text{stmt} \rangle \rightarrow \text{if} \langle \text{expr} \rangle \text{ then} \langle \text{stmt} \rangle$$

$$[\text{else} \langle \text{stmt} \rangle]$$

Solution using rewriting:

$$\langle \text{stmt} \rangle \rightarrow \text{if} \langle \text{expr} \rangle \text{ then} \langle \text{stmt} \rangle \langle \text{rest-if} \rangle$$

$$\langle \text{rest-if} \rangle \rightarrow \text{else} \langle \text{stmt} \rangle | \epsilon$$

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Summary LL(1) and Recursive Descent

Summary of the LL(1) grammar:

- Many CFGs are not LL(1)
- Some can be rewritten to LL(1)
- The underlying structure is lost (because of rewriting).

Two main methods for writing a top-down parser

- Table-driven, LL(1)
- Recursive descent

LL(1)	Recursive Descent
Table-driven	Hand-written
+ fast	- Much coding, + fast
+ Good error management and restart	+ Easy to include semantic actions; good error mgmt

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Small Rewriting Grammar Exercise

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Example: A recursive Descent Parser for Pascal Declarations, Orig. Grammar



```
<declarations> → <constdecl> <vardecl>
<constdecl> → CONST <consdeflist>
                | ε
<consdeflist> → <consdeflist> <consdef>
                | <consdef>
<consdef> → id = number ;
<vardecl> → VAR <vardeflist>
                | ε
<vardeflist> → <vardeflist> <idlist> : <type> ;
                | <idlist> : <type> ;
<idlist> → <idlist> , id
                | id
<type> → integer
                | real
```

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Rewrite in EBNF so that a Recursive Descent Parser can be Written



```
<declarations> → <constdecl> <vardecl>
<constdecl> → CONST <consdef> { <consdef> }
                | ε
<consdef> → id = number ;
<vardecl> → VAR <vardef> { <vardef> }
                | ε
<vardef> → id { , id } : ( integer | real ) ;
```

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A Recursive Descent Parser for the New Pascal Declarations Grammar in EBNF



- We have one character lookahead.
- scan should be called when we have consumed a character.

```
void declarations()
/* <declarations> → <constdecl> <vardecl> */
{
    constdecl();
    vardecl();
} /* declarations */

void constdecl()
/* <constdecl> → CONST <consdef>
   { <consdef> }
   | ε */
{
    if (token == CONST) {
        scan();
        if (token == id)
            constdef();
        else
            error("Missing id after CONST");
        while (token == id) constdef();
    }
} /* constdecl */
```

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Pascal Declarations Parser cont 1



```
void constdef()
/* <constdef> → id = number ; */
{
    scan(); /* consume ID, get next token */
    if (token == '=')
        scan();
    else
        error("Missing '=' after id");
    if (token == NUMBER) then
        scan();
    else
        error("Missing number");
}

void vardecl()
/* <vardecl> → VAR <vardef> { <vardef> }
   | ε */
{
    if (token == VAR) {
        scan();
        if (token == ID)
            vardef();
        else
            error("Missing id after VAR");
        while (token == ID) {
            vardef();
        }
    }
    if (token == ';')
        scan(); /* consume ';', get next token */
    else
        error("Missing ';' after const decl");
} /* vardecl */
```

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Pascal Declarations Parser cont 2



```
void vardef() /* <vardef> → id { , id } : ( integer | real ) ; */
{
    scan();
    while (token == ',') {
        scan();
        if (token == ID)
            scan();
        else
            error("id expected after ','");
    } /* while */

    if (token == ':') {
        scan();
        if ((token == INTEGER) || (token == REAL))
            if (token != eof_token) then error(...);
        else
            error("Incorrect type of variable");
        if (token == ';')
            scan();
        else
            error("Missing ';' in variable decl.");
    } else
        error("Missing ':' in var. decl.");
} /* vardef */

/* main */
scan(); /* lookahead token */
declarations();
if (token != eof_token) then error(...);
} /* main */
```

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TDDD16 Compilers and interpreters
TDD44 Compiler Construction



LL Parsing Issues Beyond Recursive Descent

LL(k)
LL items
Finite pushdown automaton
FIRST and FOLLOW
Table-driven Predictive Parser

Peter Fritzon, Christoph Kessler,
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LL(k)

- Given:
 - Context-free grammar $G = (N, \Sigma, P, S)$
 - Integer $k > 0$

- G is (in) **LL(k)** if:

for any two leftmost derivations

- $S \Rightarrow_{lm}^* uY\alpha \Rightarrow^* u\beta\alpha \Rightarrow^* ux$ and
- $S \Rightarrow_{lm}^* uY\alpha \Rightarrow^* u\gamma\alpha \Rightarrow^* uy$

with $x[1:k] = y[1:k]$

the k first tokens of x and y are equal

it holds $\beta = \gamma$.

- That is, for fixed left context u , the choice for the "right" production to apply to Y is uniquely determined by the next k input tokens.

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Example

- The following grammar is LL(1)
(terminals are bold-face):

```

S -> if ident then S else S fi
      | while ident do S od
      | begin S end
      | ident := ident
    
```

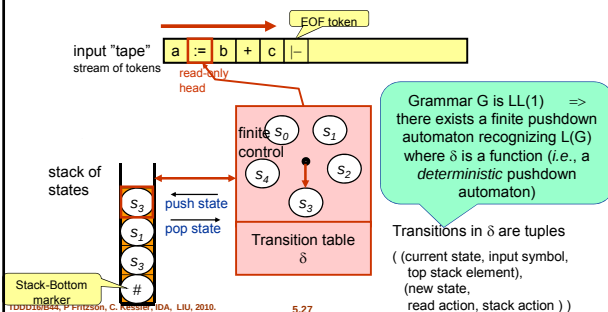
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Automaton Model for Parsing Context-Free Languages

Finite pushdown automaton (FPA)

- a finite automaton with a stack of states



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Context-Free Items

Given CFG G , construct states of the finite pushdown automaton:

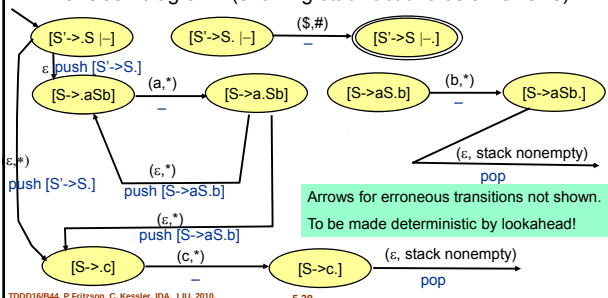
- Add new start symbol S' with $S' \rightarrow S$ (| means End-of-Input)
- For each production $A \rightarrow \alpha_1 \dots \alpha_k$ e.g. $A \rightarrow aBc$ create $k+1$ **context-free items** (= states)
 - e.g., $[A \rightarrow aBc]$, $[A \rightarrow aBc]$, $[A \rightarrow aBc]$, $[A \rightarrow aBc]$
- Construct a **predictive parser** as finite pushdown automaton:
 - start in state $[S' \rightarrow S]$ with empty stack (#)
 - halt and accept in state $[S' \rightarrow S]$ with empty stack (#)
 - at $[A \rightarrow \alpha.b\gamma]$: read input symbol, i.e., $[A \rightarrow \alpha.b\gamma] \rightarrow [A \rightarrow \alpha.b\gamma]$
 - at $[A \rightarrow \alpha.B\gamma]$: push $[A \rightarrow \alpha.B\gamma]$, determine new production $B \rightarrow \beta$ and start from $[B \rightarrow \beta]$ **Prediction!**
 - at $[B \rightarrow \beta.]$: pop state $[A \rightarrow \alpha.B\gamma]$ to restore context (if #, error)

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Example

- Grammar with productions $\{ S \rightarrow aSb \mid c \}$
- Add new start symbol S' : $\{ S' \rightarrow S; S \rightarrow aSb; S \rightarrow c \}$
- Transition diagram (showing **stack actions** below arrows):

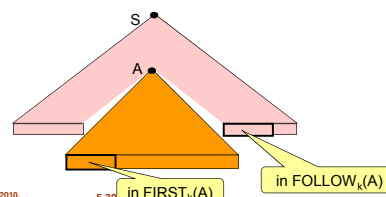


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FIRST and FOLLOW

- For a sentential form α in $(N \cup \Sigma)^+$, $FIRST(\alpha)$ denotes the set of all terminals with can be **first** in a string derived from α .
- For a nonterminal A in N , $FOLLOW(A)$ denotes the set of all terminals (e.g. a) that could appear immediately **after** A in a sentential form i.e., there exists $S \Rightarrow^* \alpha A \beta$ for arbitrary α, β



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Small FIRST and FOLLOW Exercise

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Computing FIRST = FIRST₁

For all grammar symbols X :

- If X is a terminal, then $\text{FIRST}(X) = \{X\}$.
- If $X \rightarrow \epsilon$ is a production, then add ϵ to $\text{FIRST}(X)$.
- If X is a nonterminal and $X \rightarrow Y_1 Y_2 \dots Y_q$ is a production,
 - then place all those a of Σ in $\text{FIRST}(X)$ where for some i , a is in $\text{FIRST}(Y_i)$ and ϵ is in all of $\text{FIRST}(Y_1), \dots, \text{FIRST}(Y_{i-1})$ (that is, $Y_1, \dots, Y_{i-1}$ all may derive ϵ).
 - If ϵ is in $\text{FIRST}(Y_j)$ for all $j=1,2,\dots,q$ then add ϵ to $\text{FIRST}(X)$.

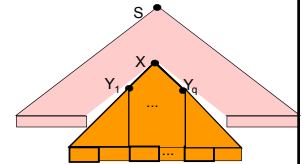
Apply these rules until no more terminals or ϵ can be added to any FIRST set.

For the example grammar
 $S' \rightarrow S$; $S \rightarrow aSb$; $S \rightarrow c$

$\text{FIRST}(a) = \{a\}$, $\text{FIRST}(b) = \{b\}$,
 $\text{FIRST}(c) = \{c\}$

$\text{FIRST}(S') = \text{FIRST}(S)$

TD: $\text{FIRST}(S) = \{a, c\}$



Computing FIRST (cont.)

For any string $X_1 X_2 \dots X_n$ of grammar symbols:

- Add to $\text{FIRST}(X_1 X_2 \dots X_n)$ all non- ϵ symbols of $\text{FIRST}(X_1)$.
- If ϵ in $\text{FIRST}(X_1)$, add also all non- ϵ symbols of $\text{FIRST}(X_2)$, otherwise done.
- If ϵ also in $\text{FIRST}(X_2)$, add also all non- ϵ symbols of $\text{FIRST}(X_3)$, otherwise done.
- ...
- If ϵ also in $\text{FIRST}(X_n)$, add ϵ to $\text{FIRST}(X_1 X_2 \dots X_n)$

For the example grammar
 $S' \rightarrow S$; $S \rightarrow aSb$; $S \rightarrow c$

$\text{FIRST}(abc) = \{a\}$

$\text{FIRST}(Sb) = \text{FIRST}(S) = \{a, c\}$

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Computing FOLLOW

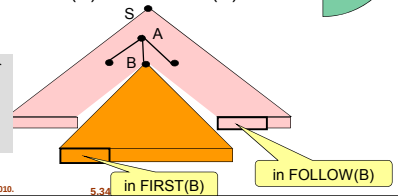
Compute $\text{FOLLOW}(B)$ for each nonterminal B :

- Add $|$ to $\text{FOLLOW}(S)$
- If there is a production $A \rightarrow^* \alpha B \beta$ for arbitrary α, β then add all of $\text{FIRST}(\beta)$ except ϵ to $\text{FOLLOW}(B)$
- If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where ϵ in $\text{FIRST}(\beta)$, i.e. $\beta \Rightarrow^* \epsilon$, then add all of $\text{FOLLOW}(A)$ to $\text{FOLLOW}(B)$.

Apply these rules until no more terminals or ϵ can be added to any FOLLOW set.

For the example grammar
 $S \rightarrow aSb$; $S \rightarrow c$

$\text{FOLLOW}(S) = \{ |, b \}$

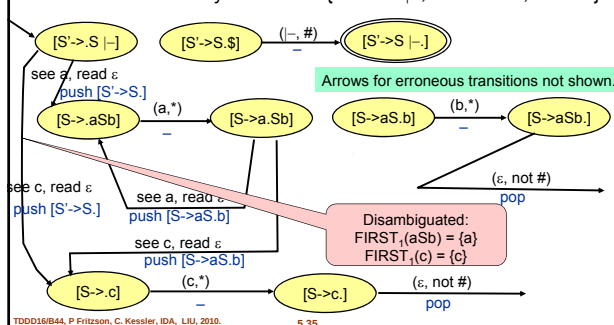


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Example Cont.: Finite Pushdown Automaton (FPA) Made Deterministic

- Grammar with productions $\{ S \rightarrow aSb \mid c \}$
- Added new start symbol S' : $\{ S' \rightarrow S \mid ; \}; S \rightarrow aSb; S \rightarrow c$



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Example (cont.): Transition table ($k=1$)

state	final ?	lookahead a	lookahead b	lookahead c	lookahead
$[S' \rightarrow S -]$	no	push $[S' \rightarrow S.]$; $[S \rightarrow aSb]$	[Error]	push $[S' \rightarrow S.]$; $[S \rightarrow c.]$	[Error]
$[S' \rightarrow S. -]$	no	[Error]	[Error]	[Error]	read ; $[S' \rightarrow S -]$
$[S' \rightarrow S -]$	yes				
$[S \rightarrow aSb]$	no	read a; $[S \rightarrow aSb.]$	[Error]	[Error]	[Error]
$[S \rightarrow aSb.]$	no	push $[S \rightarrow aSb.]$; $[S \rightarrow aSb.]$	[Error]	push $[S \rightarrow aSb.]$; $[S \rightarrow c.]$	[Error]
$[S \rightarrow aSb.]$	no	[Error]	read b; $[S \rightarrow aSb.]$	[Error]	[Error]
$[S \rightarrow aSb.]$	no	[Error]	pop state	[Error]	pop state
$[S \rightarrow c.]$	no	[Error]	[Error]	read c; $[S \rightarrow c.]$	[Error]
$[S \rightarrow c.]$	no	[Error]	pop state	[Error]	pop state

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General Approach: Predictive Parsing



At any production $A \rightarrow \alpha$

- If ε is not in $\text{FIRST}(\alpha)$:
 - Parser expands by production $A \rightarrow \alpha$ if current lookahead input symbol is in $\text{FIRST}(\alpha)$.
- otherwise (i.e., ε in $\text{FIRST}(\alpha)$):
 - Expand by production $A \rightarrow \alpha$ if current lookahead symbol is in $\text{FOLLOW}(A)$ or if it is $|-$ and $|-$ is in $\text{FOLLOW}(A)$.

Use these rules to fill the transition table.
(pseudocode: see [ASU86] p. 190, [ALSU06] p. 224)

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Summary: Parsing LL(k) Languages



- **Predictive LL parser**
 - iterative, based on finite pushdown automaton
 - transition-table-driven
 - can be generated automatically
- **Recursive-descent parser**
 - recursive
 - manually coded
 - easier to fix intermediate code generation, error handling
- **Both require lookahead** (or backtracking) to predict the next production to apply
 - Removes nondeterminism
 - Necessary checks derived from FIRST and FOLLOW sets
 - FIRST and FOLLOW are also useful for syntax error recovery

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Homework



- Now, read again the part on recursive descent parsers and find the equivalent of
 - Context-free items (Pushdown automaton (PDA) states)
 - The stack of states
 - Pushing a state to stack
 - Popping a state from stack
 - Start state, final statein a recursive descent parser.

TDD016/B44, P Fritzon, C. Kessler, IDA, LIU, 2010.

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