

Large-scale Distributed Systems and Networks (TDDE35)

Slides by Niklas Carlsson (including slides based on slides by Carey Williamson)

PERFORMANCE EVALUATION

Often in Computer Science you need to:

- demonstrate that a new concept, technique, or algorithm is feasible
- demonstrate that a new method is better than an existing method

 understand the impact of various factors and parameters on the performance, scalability, or robustness of a system

PERFORMANCE EVALUATION

- There is a whole field of computer science called <u>computer systems performance evaluation</u> that is devoted to exactly this
- One classic book is Raj Jain's "The Art of Computer Systems Performance Analysis", Wiley & Sons, 1991
- Much of what is outlined in this presentation is described in more detail in [Jain 1991]
- The ACM SIG for Performance is ACM SIGMETRICS (who also have a yearly flag-ship conference)

PERF EVAL: THE BASICS

- There are three main methods used in the design of performance evaluation studies:
- Analytic approaches
 - the use of mathematics, Markov chains, queueing theory, Petri Nets, abstract models...
- Simulation approaches
 - design and use of computer simulations and simplified models to assess performance
- Experimental approaches
 - measurement and use of a real system

Analytical Example: Queueing Theory

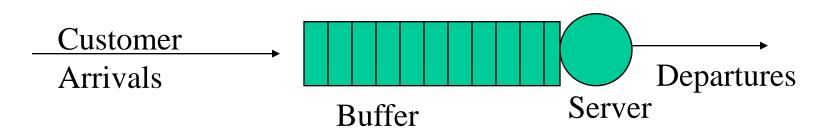
- Queueing theory is a mathematical technique that specializes in the analysis of queues; e.g.,
 - customer arrivals at a bank,
 - jobs arriving at CPU,
 - I/O requests arriving at a disk subsystem,
 - lineup at the cafeteria

• etc. ...

Queue-based Models

Queueing model represents:

- Arrival of jobs (customers) into system
- Service time requirements of jobs
- Waiting of jobs for service
- Departures of jobs from the system
- Typical diagram:



Why Queue-based Models?

- In many cases, the use of a queuing model provides a quantitative way to assess system performance
 - Throughput (e.g., job completions per second)
 - Response time (e.g., Web page download time)
 - Expected waiting time for service
 - Number of buffers required to control loss
- Reveals key system insights (properties)
- Often with efficient, closed-form calculation

Caveats and Assumptions

In many cases, using a queuing model has the following implicit underlying assumptions:

- Poisson arrival process
 - 1. Exponential interarrival times
 - 2. Independent interarrival times
- Exponential service time distribution
- Single server
- Infinite capacity queue
- First-Come-First-Serve (FCFS) discipline (also known as FIFO: First-In-First-Out)
- □ Note: important role of memoryless property!

Advanced Queueing Models

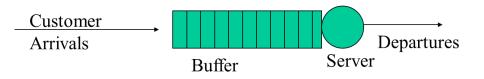
There is TONS of published work on variations of the basic model:

- Correlated arrival processes
- General (G) service time distributions
- Multiple servers
- Finite capacity systems
- Other scheduling disciplines (non-FIFO)

We will start with the basics!

Queue Notation

Queues are concisely described using the <u>Kendall notation</u>, which specifies: O Arrival process for jobs {M, D, G, ...} \bigcirc Service time distribution {M, D, G, ...} \bigcirc Number of servers {1, n} Storage capacity (buffers) {B, infinite} • Service discipline {FIFO, PS, SRPT, ...} Examples: M/M/1, M/G/1, M/M/c/c



The M/M/1 Queue

Assumes:

 Poisson arrival process, exponential service times, single server, FCFS service discipline, infinite capacity for storage, with no loss

□Notation: M/M/1

Markovian arrival process (Poisson)
 Markovian service times (exponential)

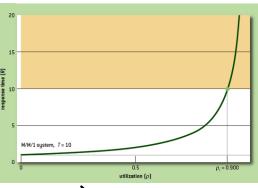
Single server (FCFS, infinite capacity)

The M/M/1 Queue (cont'd)

- □ Arrival rate: λ (e.g., customers/sec)
 - \circ Inter-arrival times are exponentially distributed (and independent) with mean 1 / λ
- □ Service rate: µ (e.g., customers/sec)
 - $\circ\,$ Service times are exponentially distributed (and independent) with mean 1 / $\mu\,$
- □ System load: $\rho = \lambda / \mu$

 $0 \le \rho \le 1$ (also known as utilization factor)

□ Stability criterion: $\rho < 1$ (single server systems)

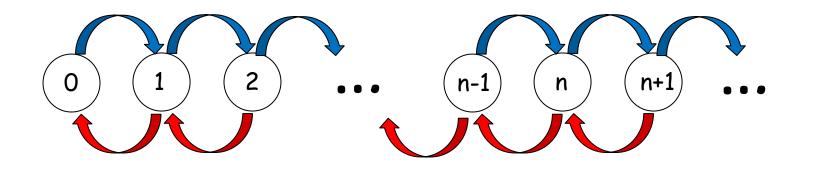


Queue Performance Metrics

- N: Avg number of customers in system as a whole, including any in service
- Q: Avg number of customers in the queue (only), excluding any in service
- □W: Avg waiting time in queue (only)
- T: Avg time spent in system as a whole, including wait time plus service time
- **Note:** Little's Law: $N = \lambda T$ (on average)

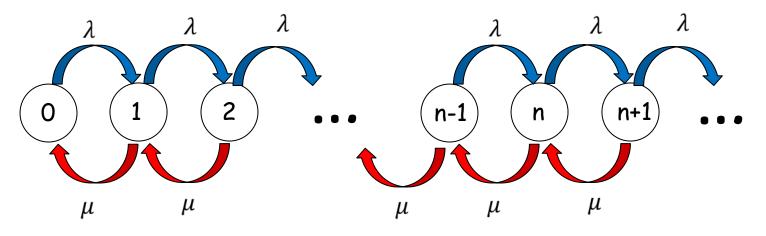
Arrival rate λ

$$N = \lambda T$$

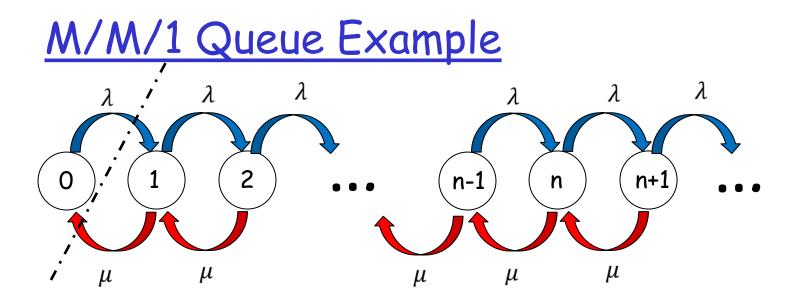


Consider system state (# of customers in system)

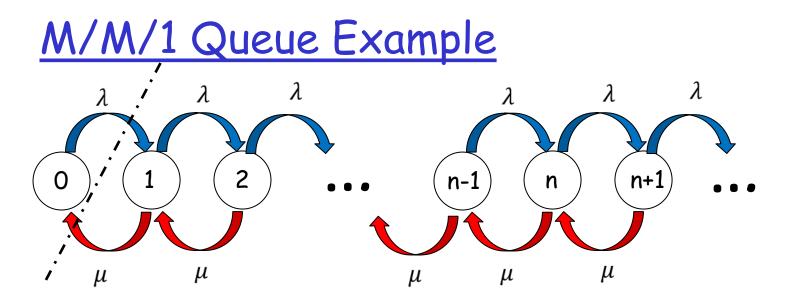
- If arrival, then move up one state ...
- If departure, then move down one state ...



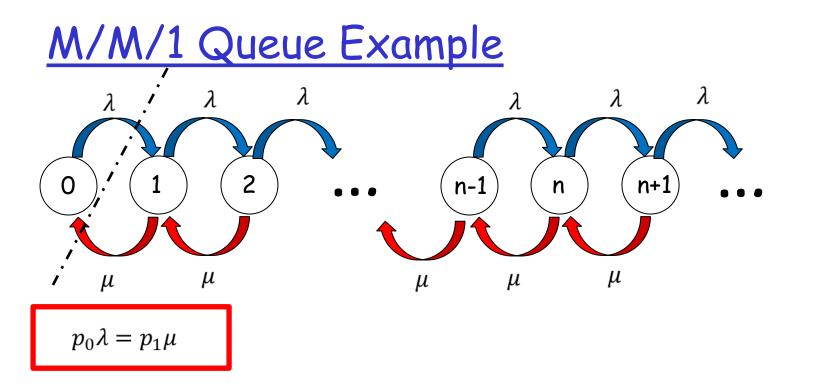
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



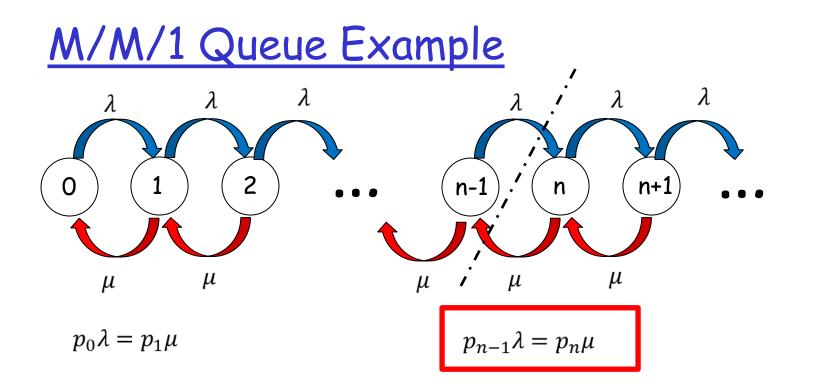
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



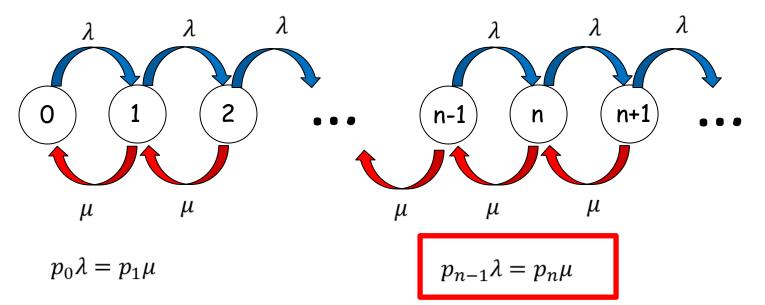
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



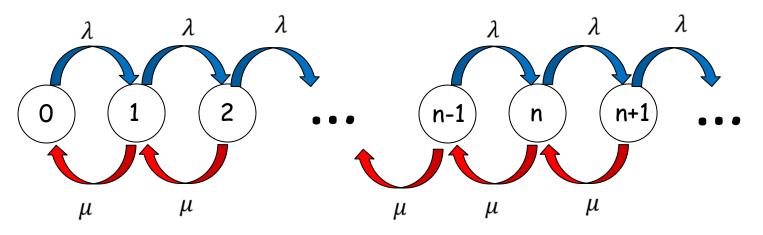
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



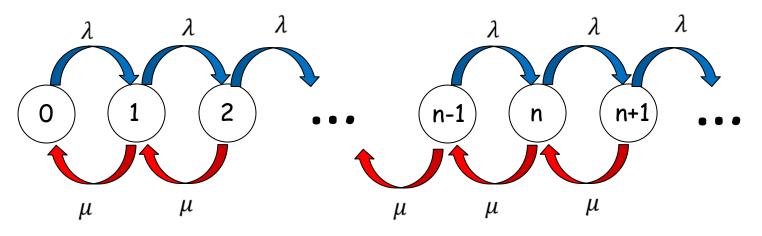
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1}$$

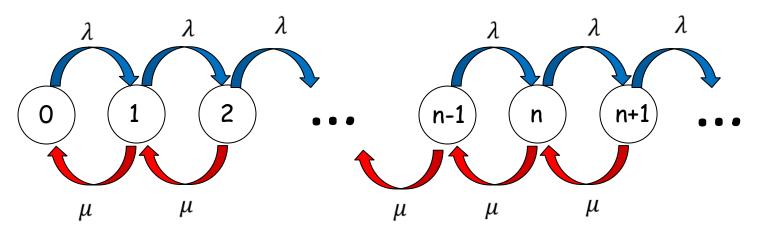
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2}$$

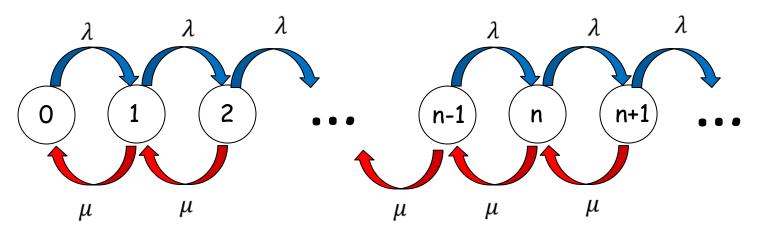
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0$$

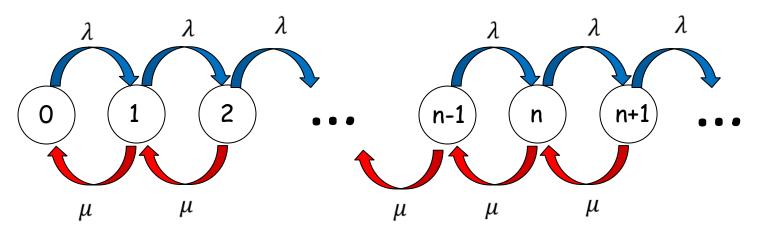
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$

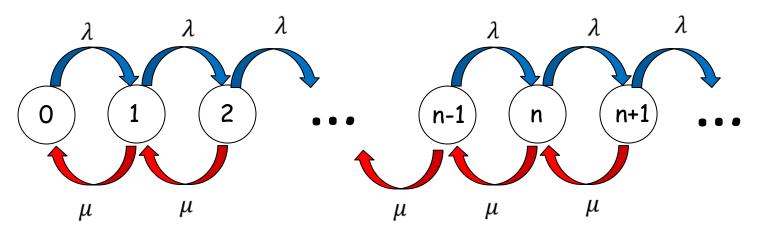


$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$1 = \sum_{n=0}^{\infty} p_n$$

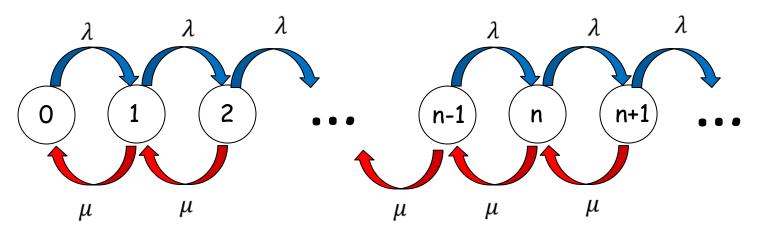
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$
$$1 = \sum_{n=0}^{\infty} p_n$$

$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$

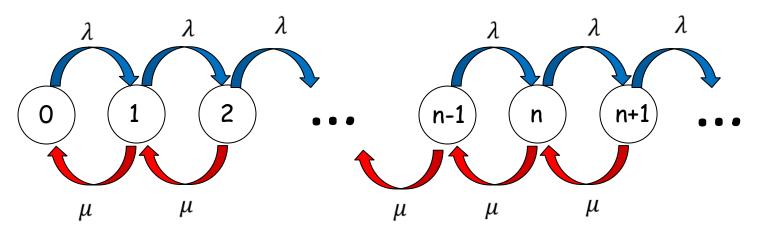


$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n$$

$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$

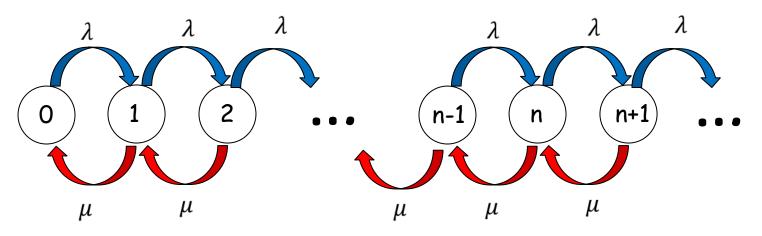


$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho}$$

$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$

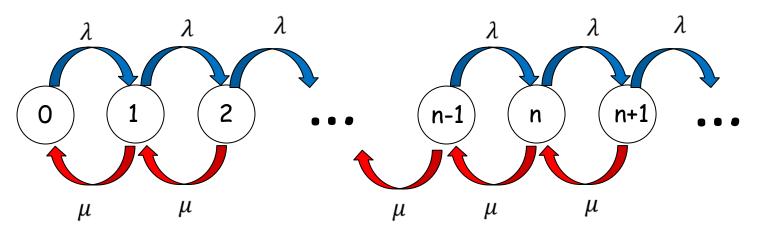


$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho} \qquad \qquad p_0 = 1-\rho$$

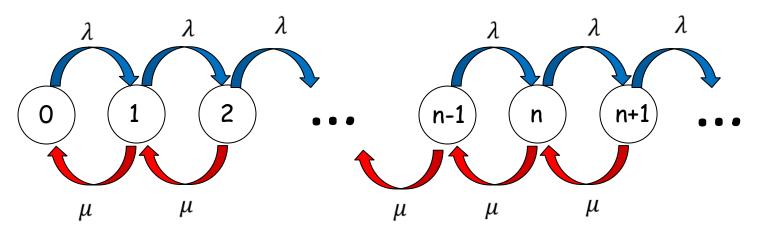
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



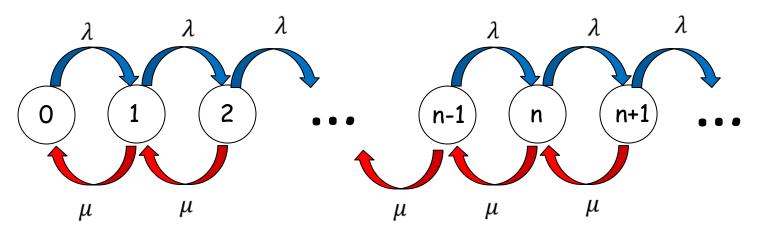
$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho}$$

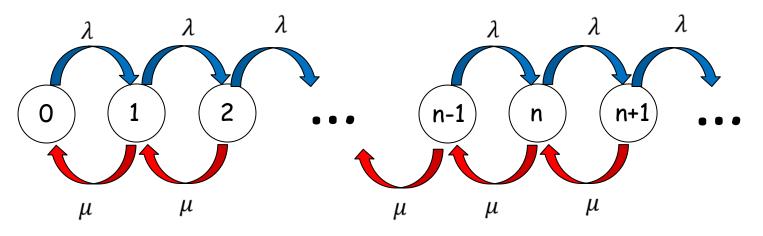
$$p_0 = 1-\rho$$

$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

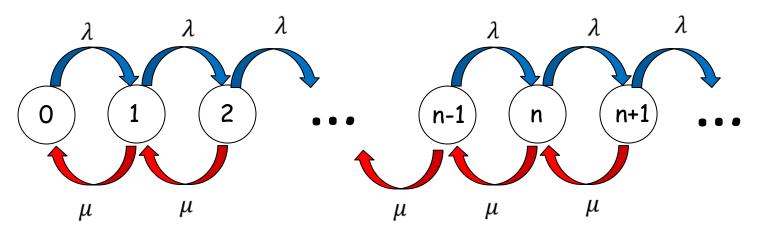
$$p_{n} = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^{2} p_{n-2} = \left(\frac{\lambda}{\mu}\right)^{n} p_{0} = \rho^{n} p_{0} = (1-\rho)\rho^{n}$$
$$1 = \sum_{n=0}^{\infty} p_{n} = p_{0} \sum_{n=0}^{\infty} \rho^{n} = p_{0} \frac{1}{1-\rho}$$
$$p_{0} = 1-\rho$$
$$N = E[n] = \sum_{n=0}^{\infty} n p_{n}$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0 = (1-\rho)\rho^n$$

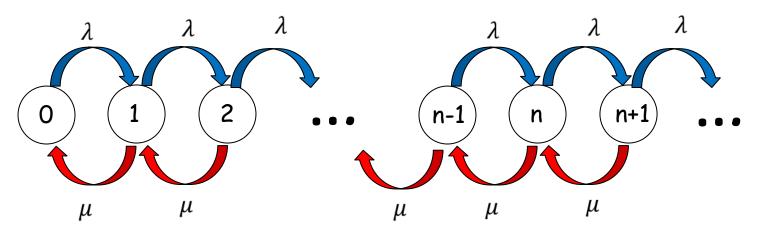
$$N = E[n] = \sum_{n=0}^{\infty} n p_n$$



$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0 = (1-\rho)\rho^n$$

$$N = E[n] = \sum_{n=0}^{\infty} n \, p_n = (1 - \rho) \sum_{n=0}^{\infty} n \, \rho^n$$



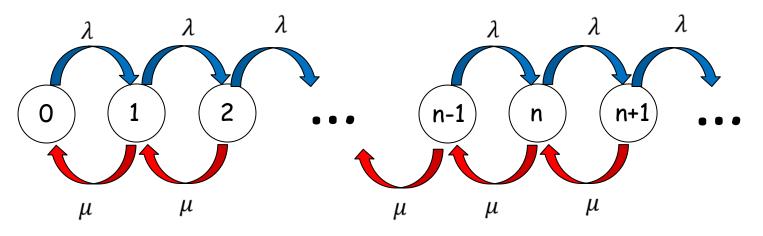
$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0 = (1-\rho)\rho^n$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho} \qquad \qquad p_0 = 1-\rho$$

$$N = E[n] = \sum_{n=0}^{\infty} n \, p_n = (1-\rho) \sum_{n=0}^{\infty} n \, \rho^n = (1-\rho) \rho \sum_{n=0}^{\infty} n \, \rho^{n-1}$$

M/M/1 Queue Example



 $p_0\lambda = p_1\mu$

$$p_{n-1}\lambda = p_n\mu$$

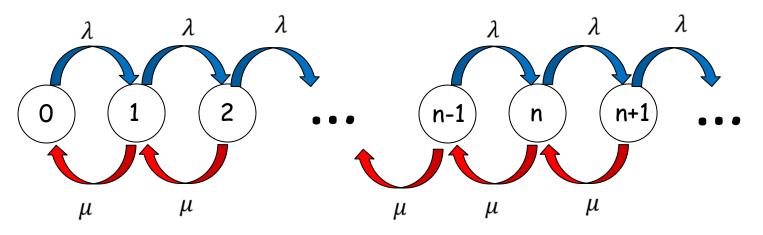
$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0 = (1-\rho)\rho^n$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho} \qquad \qquad p_0 = 1-\rho$$

$$\begin{split} N &= E[n] = \sum_{n=0}^{\infty} n \, p_n = (1-\rho) \sum_{n=0}^{\infty} n \, \rho^n = (1-\rho) \rho \sum_{n=0}^{\infty} n \, \rho^{n-1} \\ &= (1-\rho) \rho \frac{d}{d\rho} (\sum_{n=0}^{\infty} \rho^n) \end{split}$$

37

M/M/1 Queue Example



 $p_0\lambda = p_1\mu$

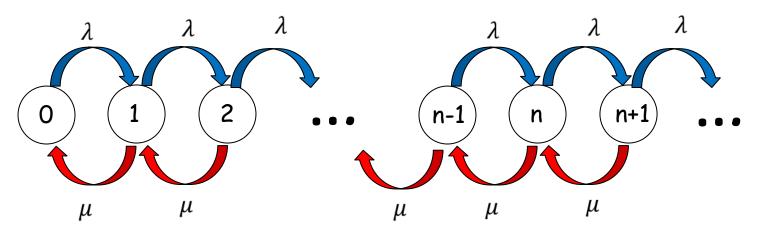
$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0 = (1-\rho)\rho^n$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho} \qquad \qquad p_0 = 1-\rho$$

$$N = E[n] = \sum_{n=0}^{\infty} n \, p_n = (1-\rho) \sum_{n=0}^{\infty} n \, \rho^n = (1-\rho)\rho \sum_{n=0}^{\infty} n \, \rho^{n-1}$$
$$= (1-\rho)\rho \frac{d}{d\rho} (\sum_{n=0}^{\infty} \rho^n) = (1-\rho)\rho \frac{d}{d\rho} \left(\frac{1}{1-\rho}\right)$$

M/M/1 Queue Example



 $p_0\lambda = p_1\mu$

$$p_{n-1}\lambda = p_n\mu$$

$$p_n = \frac{\lambda}{\mu} p_{n-1} = \left(\frac{\lambda}{\mu}\right)^2 p_{n-2} = \left(\frac{\lambda}{\mu}\right)^n p_0 = \rho^n p_0 = (1-\rho)\rho^n$$

$$1 = \sum_{n=0}^{\infty} p_n = p_0 \sum_{n=0}^{\infty} \rho^n = p_0 \frac{1}{1-\rho} \qquad \qquad p_0 = 1-\rho$$

$$N = E[n] = \sum_{n=0}^{\infty} n \, p_n = (1-\rho) \sum_{n=0}^{\infty} n \, \rho^n = (1-\rho) \rho \sum_{n=0}^{\infty} n \, \rho^{n-1}$$
$$= (1-\rho) \rho \frac{d}{d\rho} \left(\sum_{n=0}^{\infty} \rho^n \right) = (1-\rho) \rho \frac{d}{d\rho} \left(\frac{1}{1-\rho} \right) = \frac{\rho}{1-\rho}$$

M/M/1 Queue Results

Average number of customers in the system: N = ρ / (1 - ρ)
 Variance: Var(N) = ρ / (1 - ρ)²

 \bigcirc Note: Little's Law: N = \land T

The M/D/1 Queue

Assumes:

Poisson arrival process, deterministic
 (constant) service times, single server,
 FCFS service discipline, infinite capacity
 for storage, no loss

□Notation: M/D/1

• Markovian arrival process (Poisson)

- Deterministic service times (constant)
- Single server (FCFS, infinite capacity)

M/D/1 Queue Results

□ Average number of customers: $Q = \rho/(1 - \rho) - \rho^2 / (2 (1 - \rho))$

Waiting time: W = x p / (2 (1 - p)) where x is the mean service time

Note that lower variance in service time means less queueing occurs [©]

$$\circ$$
 E.g., M/M/1 has W = (1/µ) ρ / (1 - ρ) 42

Queueing Theory (cont'd)

- These simple models can be cascaded in series and in parallel to create arbitrarily large complicated queueing network models
- **Two main types:**
 - closed queueing network model (finite pop.)
 - open queueing network model (infinite pop.)
- Software packages exist for solving these types of models to determine steady-state performance (e.g., delay, throughput, util.)

Simulation Example: TCP Throughput

- Can use an existing simulation tool, or design and build your own custom simulator
- Example: ns-3 network simulator
 - A discrete-event simulator with detailed TCP protocol models
 - Configure network topology and workload
 - Run simulation using pseudo-random numbers and produce statistical output

OTHER ISSUES

Simulation <u>run length</u>

- choosing a long enough run time to get statistically meaningful results (equilibrium)
- Simulation <u>start-up effects</u> and <u>end effects</u>
 - deciding how much to "chop off" at the start and end of simulations to get proper results

Replications

 ensure repeatability of results, and gain greater statistical confidence in the results given

Experimental Example: Benchmarking

- The design of a performance study requires great care in experimental design and methodology
- Need to identify
 - experimental <u>factors</u> to be tested
 - <u>levels</u> (settings) for these factors
 - performance <u>metrics</u> to be used
 - <u>experimental design</u> to be used



- Factors are the main "components" that are varied in an experiment, in order to understand their impact on performance
 - Examples: request rate, request size, read/write ratio, num concurrent clients
- Need to choose factors properly, since the number of factors affects size of study

LEVELS

Levels are the precise settings of the factors that are to be used in an experiment

• Examples: req size S = 1 KB, 10 KB, 1 MB

- Example: num clients C = 10, 20, 30, 40, 50
- Need to choose levels realistically
- Need to cover useful portion of the design space

PERFORMANCE METRICS

- Performance <u>metrics</u> specify what you want to measure in your performance study
 - Examples: response time, throughput, pkt loss
- Must choose your metrics properly and instrument your experiment accordingly



EXPERIMENTAL DESIGN

- Experimental design refers to the organizational structure of your experiment
- Need to methodically go through factors and levels to get the full range of experimental results desired
- There are several "classical" approaches to experimental design





One factor at a time

- vary only one factor through its levels to see what the impact is on performance
- Two factors at a time
 - vary two factors to see not only their individual effects, but also their interaction effects, if any

Full factorial

 try every possible combination of factors and levels to see full range of performance results

<u>SUMMARY</u>

- Computer systems performance evaluation defines standard methods for designing and conducting performance studies
- Great care must be taken in experimental design and methodology if the experiment is to achieve its goal, and if results are to be fully understood
- Very many examples of these important methodologies and their applications ...

Scalability example: Broadcast protocol

Streaming Popular Content

Consider a popular media file

- O Playback rate: 1 Mbps
- Ouration: 90 minutes
- Request rate: once every minute

How can a video server handle such high loads?

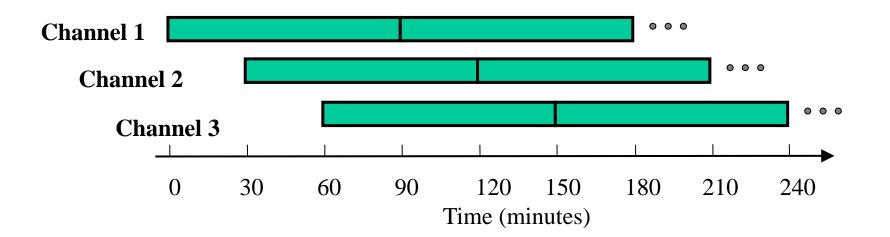
- Approach 1: Start a new "stream" for each request
- Allocate server and disk I/O bandwidth for each request
- Sandwidth required at server= 1 Mbps x 90

Streaming Popular Content using Batching

- Approach 2: Leverage the multipoint delivery (e.g., multicast/broadcast) capability of modern networks
- Playback rate = 1 Mbps, duration = 90 minutes

Streaming Popular Content using Batching

- Approach 2: Leverage the multipoint delivery (e.g., multicast/broadcast) capability of modern networks
- Playback rate = 1 Mbps, duration = 90 minutes
- □ Consider case of high request rate and D=30min...
 - Max. start-up delay = 30 minutes
 - Group requests in non-overlapping intervals of 30 min
 - Bandwidth required = 3 channels = 3 Mbps



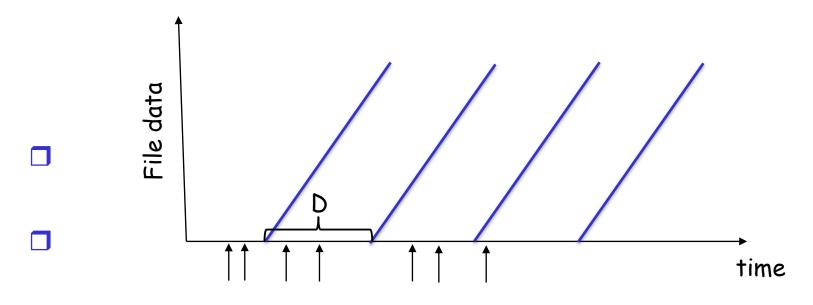
Streaming Popular Content using Batching

- Approach 2: Leverage the multipoint delivery (e.g., multicast/broadcast) capability of modern networks
- Playback rate = 1 Mbps, duration = 90 minutes

An optimal batching protocol (and analysis)???
 Define protocol

0

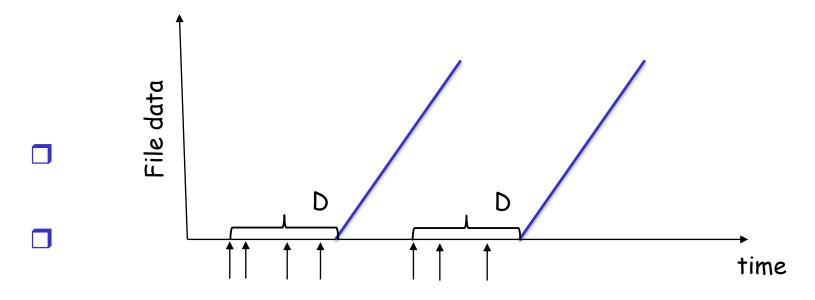
N Carlsson, D. Eager, and M. K. Vernon, Multicast Protocols for Scalable On-demand Download, Performance Evaluation, 2006.



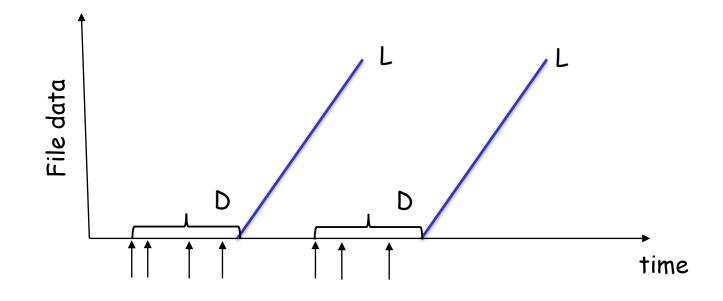
An optimal batching protocol (and analysis)???
 Define protocol

0

N Carlsson, D. Eager, and M. K. Vernon, Multicast Protocols for Scalable On-demand Download, Performance Evaluation, 2006.



- An optimal batching protocol (and analysis)???
 - Define protocol
 - How to evaluate?
 - Analytically?
 - Simulations?
 - Experiments?



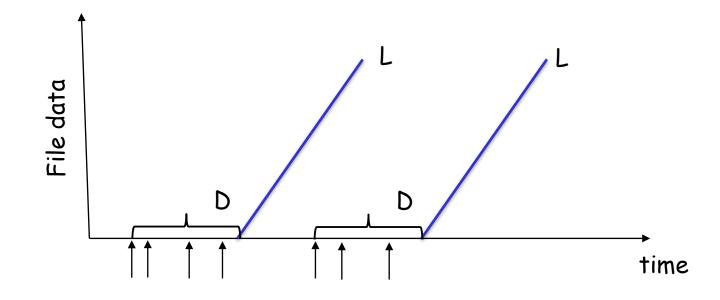
Optimal batching protocol

• Max delay = D

Poisson process

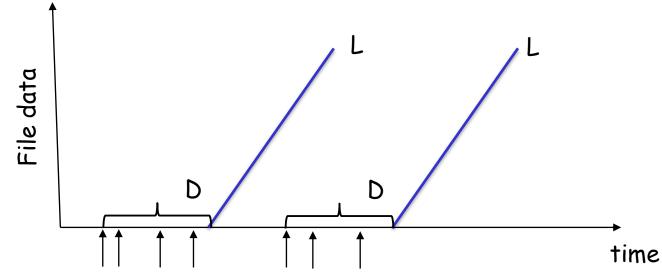
- Inter-arrival times (i) exponentially distributed and (ii) independent
- Memory less arrival process

N Carlsson, D. Eager, and M. K. Vernon, Multicast Protocols for Scalable On-demand Download, Performance Evaluation, 2006.



Renewal process

- Identify and analyze "renewal periods" (statistically the same) • $B = L / (D+1/\lambda)$
- □ Poisson Arrivals See Time Average (PASTA) property • $A = [D(1+\lambda D/2)]/[1+\lambda D]$



Little's law

• # in system = (arrival rate into system) x (average time in system)

Systems considered where

- System = "waiting queue" (for first bit)
 - Average time in system = A; Arrival rate = λ
 - E[# in system] = $\lambda [D(1+\lambda D/2)]/[1+\lambda D]$
- System = "queue or being served" (to get all bits)
 - Average time in system A+L/r; Arrival rate = λ
 - E[#in system] = $\lambda [D(1+\lambda D/2)]/[1+\lambda D] + \lambda L/r$



Batching Issues

Bandwidth increases linearly with decrease in start-up delays

Can we reduce or eliminate "start-up" delays?
 Periodic Broadcast Protocols

Stream Merging Protocols

Periodic Broadcast Example

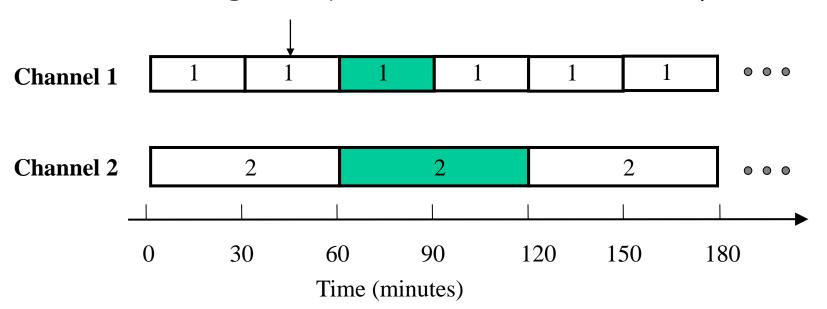
Partition the media file into 2 segments with relative sizes {1, 2}. For a 90 min. movie:

• Segment 1 = 30 minutes, Segment 2 = 60 minutes

□ Advantage:

- Max. start-up delay = 30 minutes
- Bandwidth required = 2 channels = 2 Mbps

Disadvantage: Requires increased client capabilities

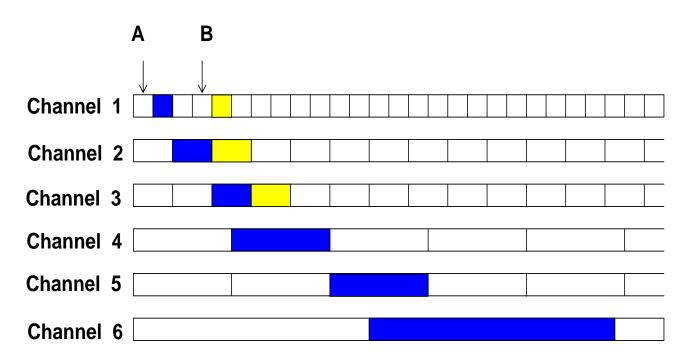


70

Skyscraper Broadcasts (SB)

Divide the file into K segments of increasing size

- Segment size progression: 1, 2, 2, 5, 5, 12, 12, 25, ...
- Multicast each segment on a separate channel at the playback rate
- Aggregate rate to clients: 2 x playback rate



Comparing Batching and SB

Server	Start-up Delay	
Bandwidth	Batching	SB
1 Mbps	90 minutes	90 minutes
2 Mbps	45 minutes	30 minutes
6 Mbps	15 minutes	3 minutes
10 Mbps	9 minutes	30 seconds

Playback rate = 1 Mbps, duration = 90 minutes

- □ Limitations of Skyscraper:
 - Ad hoc segment size progress
 - Does not work for low client data rates

Reliable Periodic Broadcasts (RPB)

[Mahanti et al. 2001, 2003, 2004]

Optimized PB protocols (no packet loss recovery)

- o client fully downloads each segment before playing
- required server bandwidth near minimal
- Segment size progression is not ad hoc
- Works for client data rates < 2 × playback rate</p>
- extend for packet loss recovery
- extend for "bursty" packet loss
- extend for client heterogeneity

Reliable Periodic Broadcasts (RPB)

[Mahanti et al. 2001, 2003, 2004]

Optimized PB protocols (no packet loss recovery)

- client fully downloads each segment before playing
- required server bandwidth near minimal
- Segment size progression is not ad hoc
- Works for client data rates < 2 × playback rate</p>
- extend for packet loss recovery
- extend for "bursty" packet loss
- extend for client heterogeneity

Optimized Periodic Broadcasts

- Playback rate assumed equal to 1
- r = segment streaming rate
- □ s = maximum # streams client listens to concurrently
- b = client data rate = s x r

Optimized Periodic Broadcasts



- Playback rate assumed equal to 1
- r = segment streaming rate
- □ s = maximum # streams client listens to concurrently
- b = client data rate = s x r

Optimized Periodic Broadcasts



- Playback rate assumed equal to 1
- r = segment streaming rate = 1
- s = maximum # streams client listens to concurrently = 2
- b = client data rate = s x r = 2
- I length of first s segments: $\frac{1}{r}l_k = \frac{1}{r}l_1 + \sum_{i=1}^{k-1}l_i$

I length of segment k > s: $\frac{1}{r}l_k = \sum_{j=1}^{k-1} l_j$

Comparison with Skyscraper

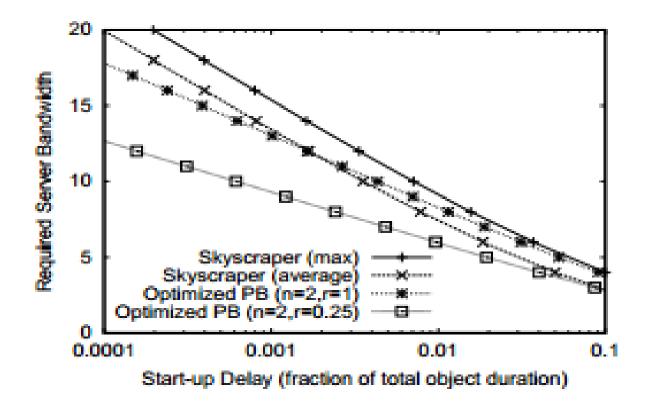


Figure 5: Performance Comparison of Optimized PB and Skyscraper Broadcasts

<u>Immediate service:</u> <u>Hierarchical Stream Merging</u>

D. Eager, M. Vernon, and J. Zahorjan, "Minimizing Bandwidth Requirements for On-Demand Data Delivery", IEEE TKDE, 2001.

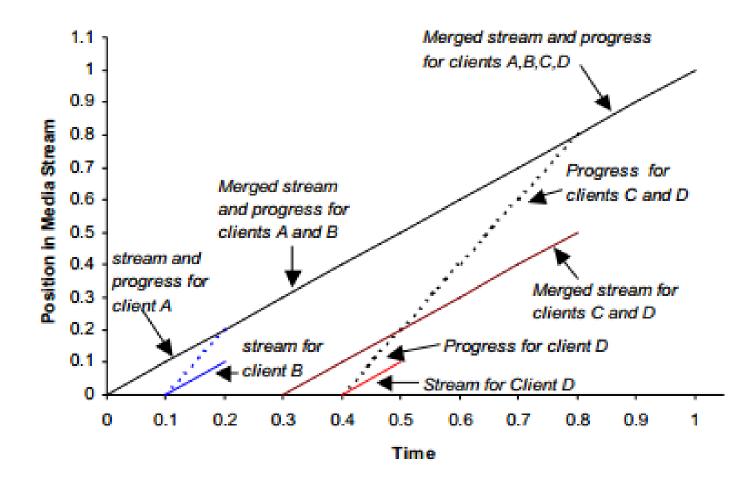


Figure 5: Example of Hierarchical Multicast Stream Merging