Homework 1

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Submit your solutions in the form of two files: a text document (possibly with pictures for the computation trees and automata) and a SPIN file for problem 4. The SPIN file should be well documented and clear. Make sure you separate the transitions of the frogs (a process for each frog) from the property whose negation captures success.

Problem 1

Assume CTL^*. Express each of the following using (boolean combinations) of $true$, the path formulas `f` and `g`, the temporal binary operator **U** and the path quantifer **E**:

1. $(\mathbf{F}\ \texttt{f}) = \ldots$
2. $(\mathbf{G}\ \texttt{f}) = \ldots$
3. $(\mathbf{A}\ \texttt{f}) = \ldots$
4. $(\texttt{f}\ \mathbf{R}\ \texttt{g}) = \ldots$

Problem 2

Assume the atomic propositions `blk` (for block), `esc` (for escape), `tai` (for taint), `ted` (for tainted) `run` (for run) and `res` (for reset). Consider the sequence of CTL formulas:

1. $\mathbf{AX}(\texttt{blk})$
2. $\mathbf{EG}(\texttt{esc})$
3. $\mathbf{AG}(\texttt{tai} \implies \mathbf{AF}(\texttt{ted}))$
4. $\mathbf{AG}(\texttt{run} \implies \mathbf{EF}(\texttt{res}))$

For each furmula, give the first levels of a computational tree (similar to those describing formluas $M, s_0 \models \mathbf{EF}\ g$ and $M, s_0 \models \mathbf{AF}\ g$ in the slides) where the corresponding atomic propositions appear at least once. For instance, the computational tree associated to formula (1) should have at least one node satisfying `blk`.

Problem 3

Assume atomic propositions `En` (for Enabled) and `Ex` (for Executed). Consider the following two LTL formulas:

1. $\mathbf{GF}(\neg\texttt{En} \vee \texttt{Ex})$

2. $\mathbf{GF}(\texttt{En}) \implies \mathbf{GF}(\texttt{Ex})$

For each formula:

1. Give an infinite word on $\{\texttt{En}, \texttt{Ex}\}$ that violates the formula
2. Give a Büchi automaton that accepts all negations on $\{\texttt{En}, \texttt{Ex}\}$, of the formula.

Problem 4

Model the farmer, wolf, goat and cabbage puzzle in Spin[1] where a farmer aims to move a wolf, a goat and a cabbage from one side of the river to the other. The farmer has a small boat where he can only take one of the cabbage, the goat, the wolf or nothing. The goat will eat the cabbage if left unattended with it, and the wolf would eat the goat if left unattended with it.
The objective is to formalize a model and a property whose counter-example, found by SPIN, is a sequence of moves that allows all frogs to switch side. Your model should not encode the solution. Instead, it should model the possible transitions and it is up to SPIN, given a property you formalize, to find the sequence of actions that will allow the farmer to move the wolf, the goat and the cabbage to the other side of the river. Send your mordel.

[1] https://illuminations.nctm.org/BrainTeasers.aspx?id=4992