Software Verification

Introduction
Model Checking and Temporal Logic

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Outline

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This course

- Introduces principals behind software verification approaches including model checking, Hoare-style reasoning, satisfiability modulo theory and abstract interpretation
- Uses **assignments** to allow for experimenting with the introduced notions. The assignments will have a "theoretical" part and a "practical" part. The practical part involves hands-on assignments on representative tools of the different verification techniques.
- Concludes with an **exam**

Plan

- Explicit model checking
  - 2 lectures + Assignment.
- Bounded/symbolic verification
  - 2 lectures + Assignment.
- Axiomatic reasoning
  - 2 lectures + Assignment.
- Scalable over-approximation
  - 2 lectures + Assignment.
- Exam June 2025.

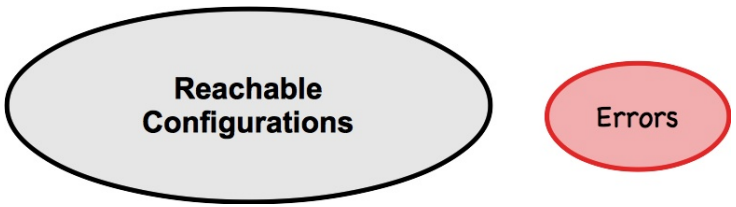
Assignments

- Jonathan Hjort is the new course assistant
- There are four assignments
- Expected to work in pairs
- Register to webreg before April 3rd
- Demonstrate your solution in a scheduled lab session
- Deadline for submissions: last lab session

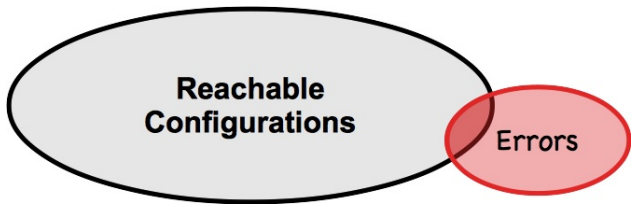
Outline

Verification

- We want to answer whether some program behaves correctly. We define "correctness" soon.
- For now, assume that means some erroneous configurations are not reachable
- We say the program is **safe**



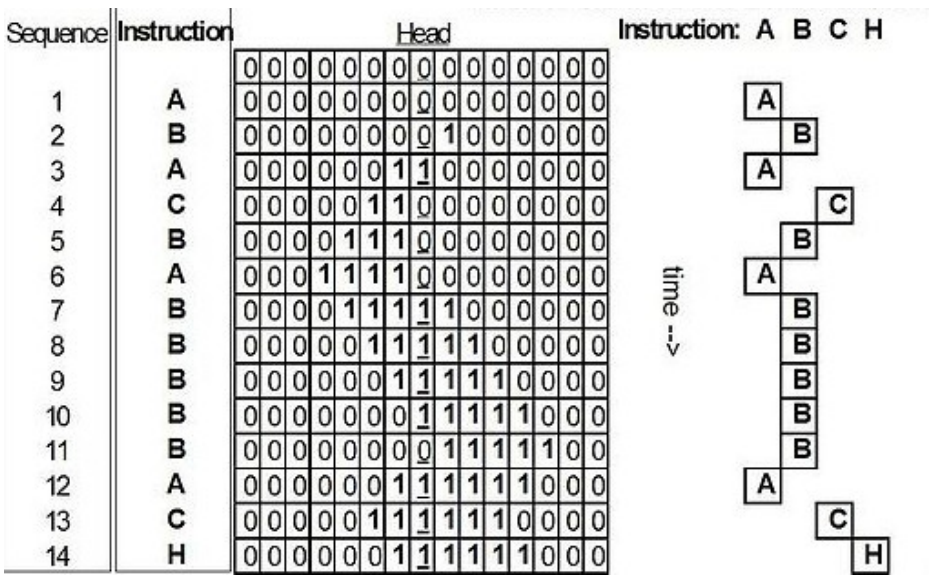
Safe Program

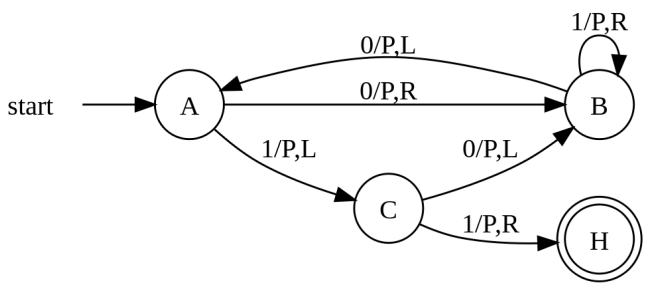


Unsafe Program

The general verification problem is “very difficult”

- Deciding whether all possible executions of a program are error-free is hard. If we could write a program that does it for arbitrary programs to be analyzed then we would always be able to answer whether a Turing machine halts.
- This problem is proven to be undecidable.





Problem is “very difficult”: what to do?

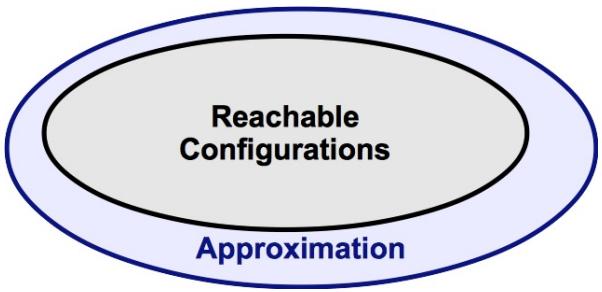
- Identify sub-problems on which one can decide: e.g. finite state machines, push-down automata, timed automata, Petri nets, well-structured transition systems.
- Proceed with approximations that will hopefully give some guarantees.

Verification problem and approximations

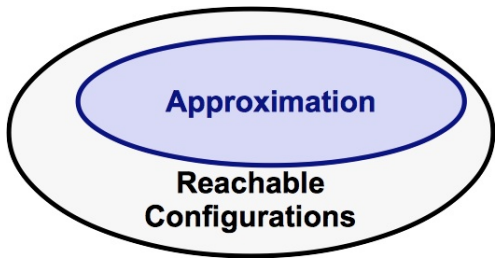
- An analysis procedure takes as input a program to be checked against a property. The procedure is an analysis algorithm if it is guaranteed to terminate.
- An analysis algorithm is **sound** in the case where each time it reports the program is safe wrt. some errors, then the original program is indeed safe wrt. those errors (pessimistic analysis)
- An algorithm is **complete** in the case where each time it is given a program that is safe wrt. some errors, then it does report it to be safe wrt. those errors (optimistic analysis)
- In general, you have to give up on one of the three: termination, soundness or completeness.

Verification problem and approximations

- The idea is then to come up with efficient approximations to give correct answers in as many cases as possible.



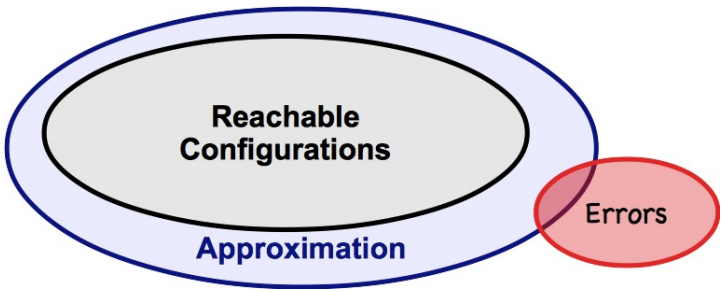
Over-approximation



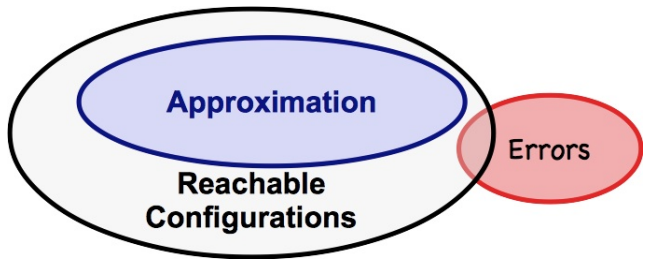
Under-approximation

Program verification and the price of approximations

- A sound analysis cannot give **false negatives**
- A complete analysis cannot give **false positives**



False Positive



False Negative

We will introduce the following techniques

- Explicit model checking
  - represents “behaviors” explicitely. Aims for exactness on sub-classes.
- Symbolic based techniques
  - represents “behaviors” symbolically. Can be used for sound or complete approaches.
- Axiomatic reasoning
  - Uses predicates and can prove anything provable by a human, but with human intervention.
- Scalable over-approximation
  - Uses sound approximations that may lead to false positives.

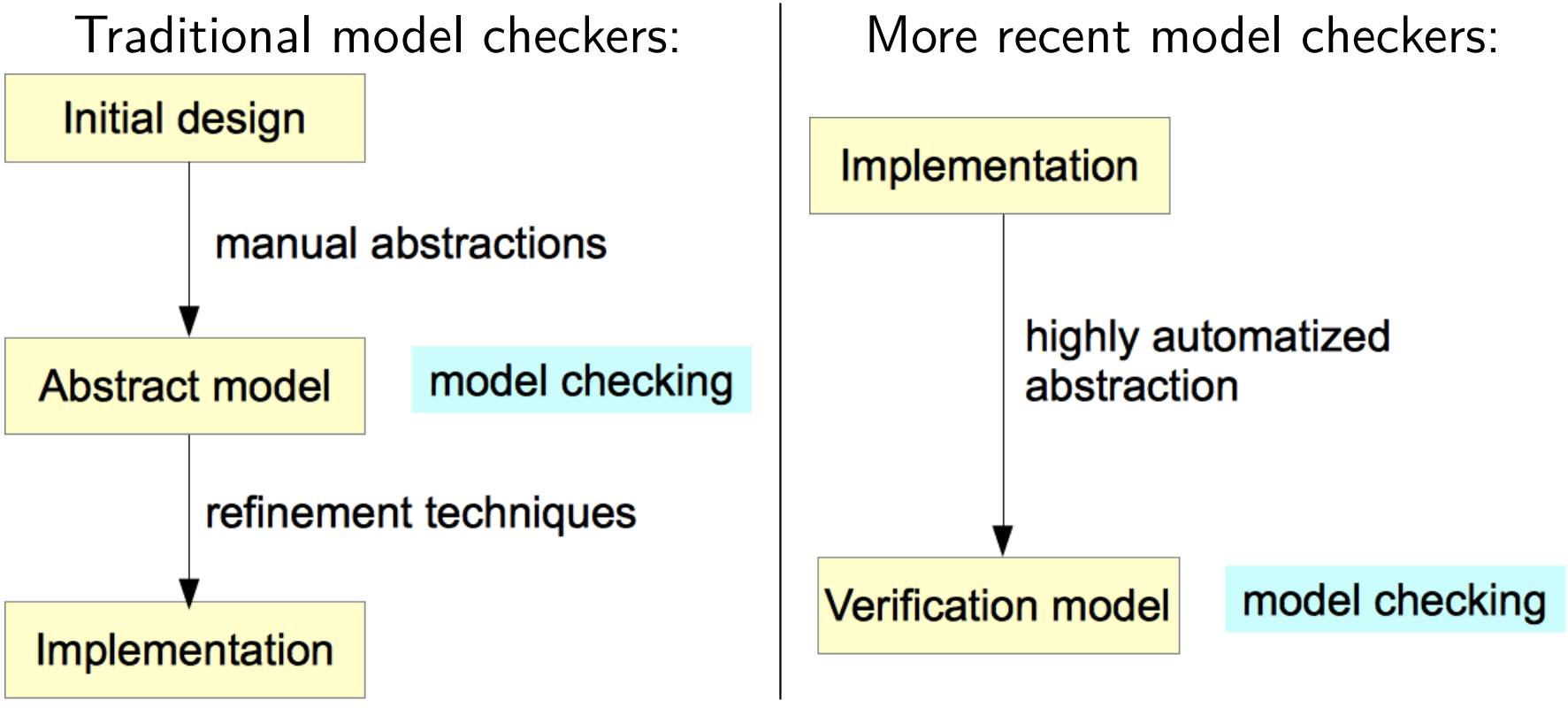
Outline

Model checking

$$M \stackrel{?}{\models} \Phi$$

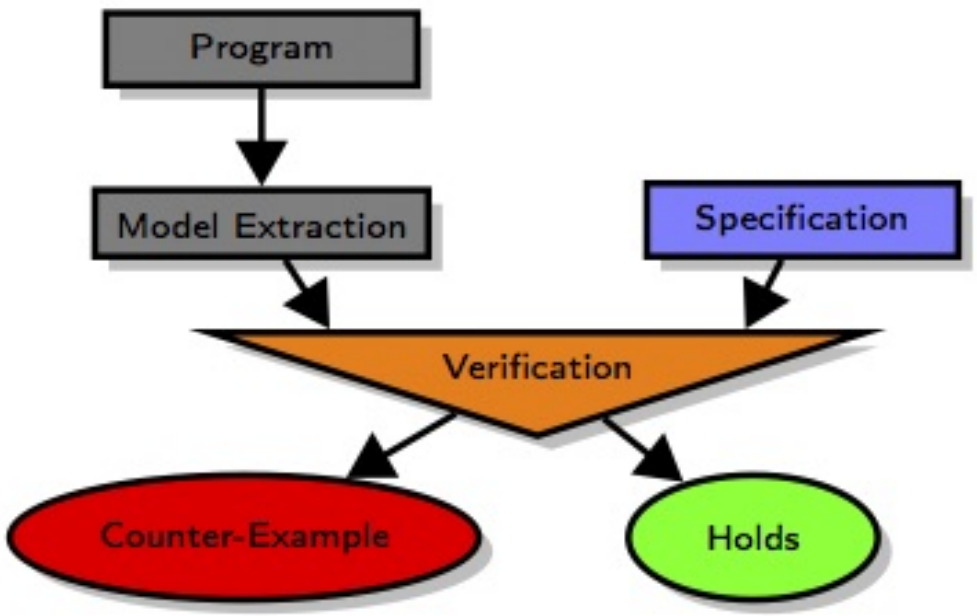
- Model checking is a push button verification approach
- Given:
  - a model M of the system to be verified, and
  - a correctness property Φ to be checked: absence of deadlocks, livelocks, starvation, violations of constraints/assertions, etc
- The model checking tool returns:
  - a counter example in case M does not model Φ, or
  - a "proof" that M does model Φ

Model Checking in Practice



Model Checking: Verification vs debugging

- Model checking tools are used both:
  - To establish correctness of a model M with respect to a correctness property Φ
  - More importantly, to find bugs and errors in M early during the design



M as a Kripke structure

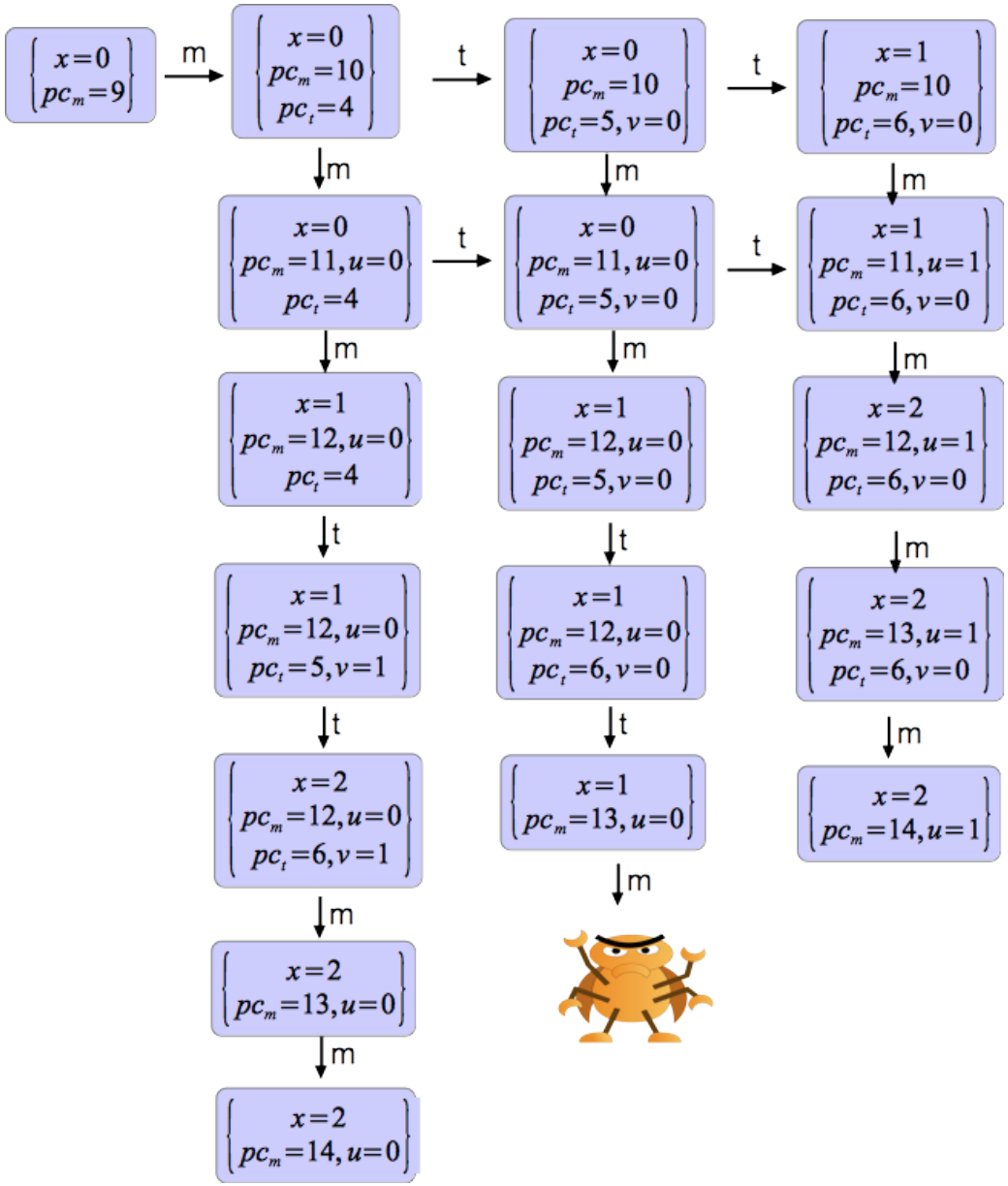
Assume a set of atomic propositions AP. A *Kripke structure* M is a tuple (S, S_0, R, L) where:

1. S is a finite set of states
2. $S_0 \subseteq S$ is the set of initial states
3. $R \subseteq S \times S$ is the transition relation s.t. for any $s \in S$, $R(s, s')$ holds for some $s' \in S$
4. $L : S \to 2^{AP}$ labels each state with the atomic propositions that hold on it.

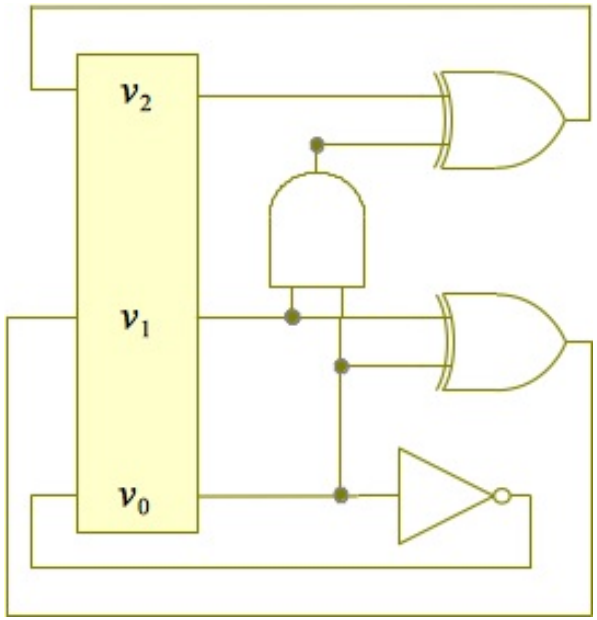
Intuitively, AP are properties whose evolution (when moving from one state to the other) we want to track. Kripke structures can be used to capture the behavior of very different systems.

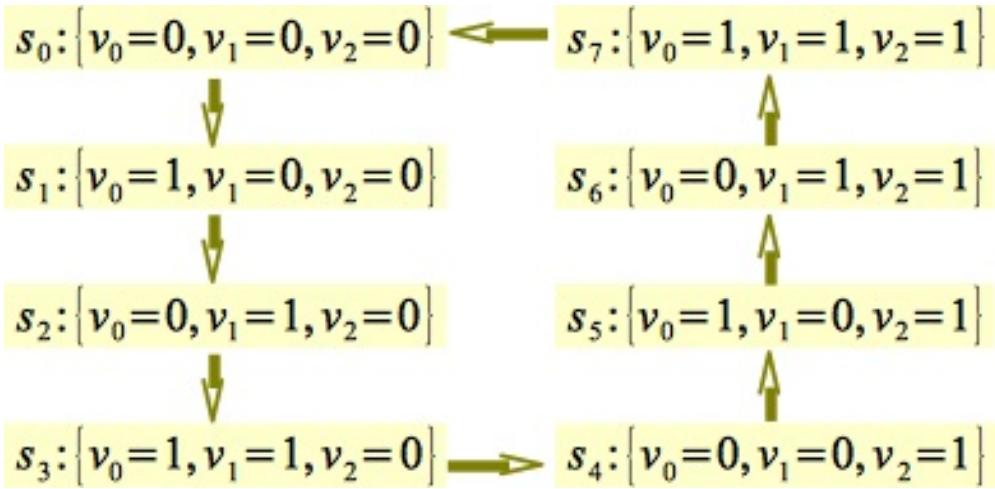
Programs as Kripke structures

```
1  int x = 0;
2
3  void thread(){
4    int v = x;
5    x = v + 1;
6  }
7
8  void main(){
9    fork(thread);
10   int u = x;
11   x = u + 1;
12   join(thread);
13   assert(x == 2);
14 }
```



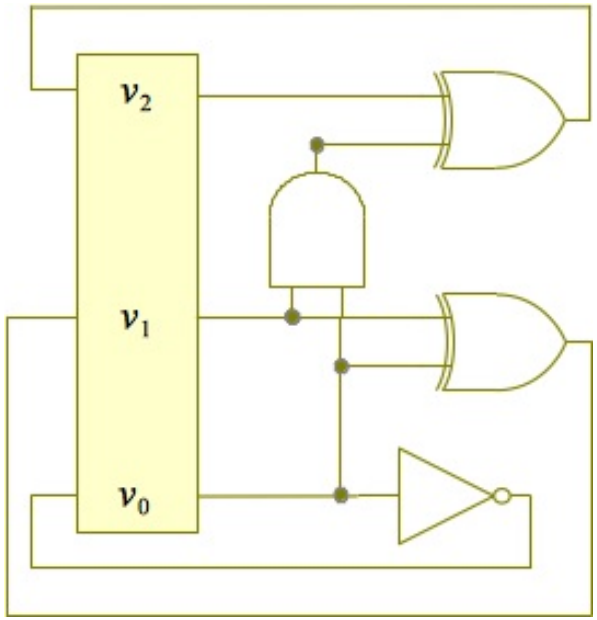
Synchronous circuits as Kripke structures

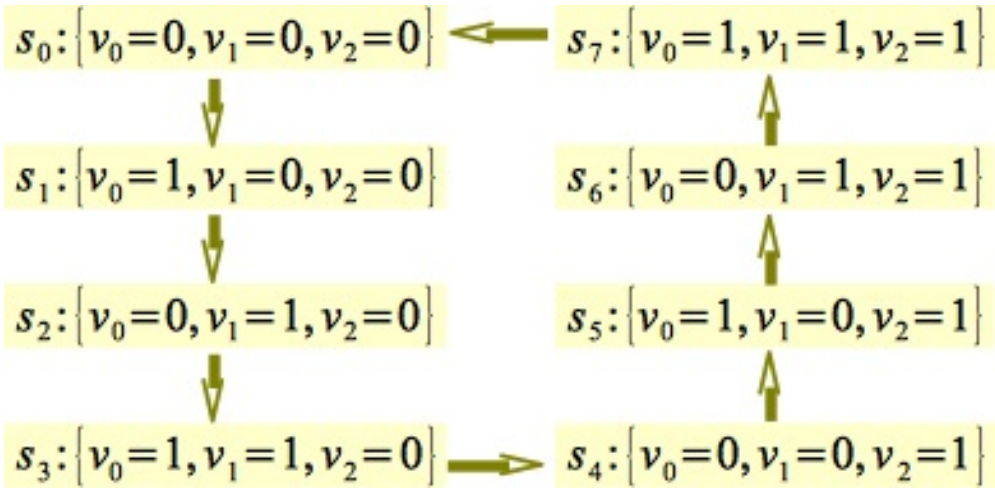




$$v_0' = \neg v_0 \quad (1)$$
$$v_1' = v_0 \oplus v_1 \quad (2)$$
$$v_2' = (v_0 \wedge v_1) \oplus v_2 \quad (3)$$

Synchronous circuits as Kripke structures





$$v_0' = \neg v_0 \quad (1)$$
$$v_1' = v_0 \oplus v_1 \quad (2)$$
$$v_2' = (v_0 \wedge v_1) \oplus v_2 \quad (3)$$

Asynchronous circuits handled using a disjunctive R instead of a conjunctive one like for synchronous circuits.

Temporal Logics

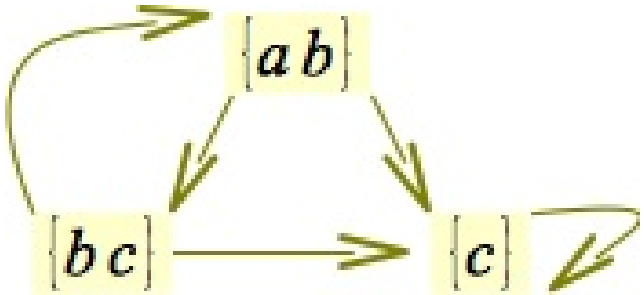
- We are intereseted in describing sequences of transitions of Kripke structures
- Many *Reactive Systems* are designed to continously react to their environement
- An input/output description is not suitable
- Describing sequences makes more sense

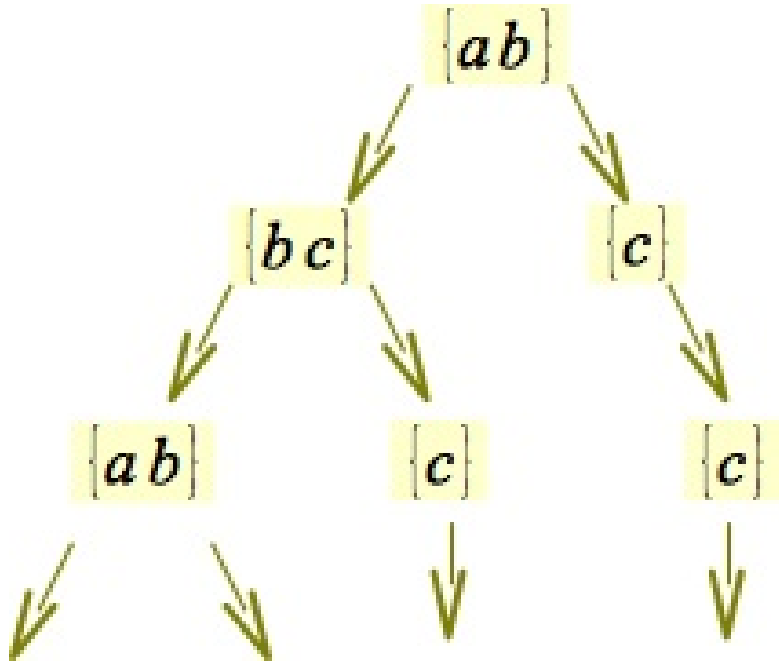
ϕ as a formula in some temporal logics

- Temporal logics are formalisms to describe sequences (hence the notion of time) of transitions
- Temporal operators are used to express that certain properties in AP are:
  - never reached
  - eventually reached
  - more complex combinations of those

The *CTL** Temporal Logic

Computation trees are obtained by unwinding the Kripke structure





The *CTL** Temporal Logic (cont.)

- A *CTL** formua is composed of *path quantifiers* and *temporal operators*
- Path quantifiers (**A**, **E**) describe the branching of the tree. Given a state, **A** (resp. **E**) specify that all (resp. some) path starting at the state have some property
- Temporatl operators (**X**, **F**, **G**, **U**, **R**) describe properties of a given path in the computations tree

The CTL^* Temporal Logic: syntax

The following are *state fromulas*

- p if $p \in AP$
- $\neg f$, $f \wedge g$ and $f \vee g$ if f, g are state formulas
- $\mathbf{A} f$, $\mathbf{E} f$ if f is a path formula

The following are *path fromulas*

- f if it is also a state formula
- $\neg f$, $f \wedge g$, $f \vee g$, $\mathbf{X}\ f$, $\mathbf{F}\ f$, $\mathbf{G}\ f$, $f\,\mathbf{U}\ G$ and $f\,\mathbf{R}\ g$ if f, g are path formulas

CTL^* is the set of state formulas generated by the above rules.

The CTL^* Temporal Logic: notation

- A path $\pi = s_0 s_1 \ldots$ in a computation tree (obtained from a Kripke structure) is any infinite sequence of states with $R(s_i, s_{i+1})$ for each $i \in \mathbb{N}$
- Write π^i to mean the path starting from s_i in $\pi = s_0 s_1 \ldots$
- Write $M, s \models f$ to mean that state formula f holds at state s in the Kripke structure M
- Write $M, \pi \models f$ to mean that path formula f holds along path π in the Kripke structure M

The CTL^* Temporal Logic: semantics

f_1 and f_2 are state formulas, g_1 and g_2 are path formulas.

$$
\begin{array}{lcl}
M, s \models p & \Leftrightarrow & p \in L(s) \\
M, s \models \neg f_1 & \Leftrightarrow & M, s \not\models f_1 \\
M, s \models f_1 \vee f_2 & \Leftrightarrow & M, s \models f_1 \text{ or } M, s \models f_2 \\
M, s \models f_1 \wedge f_2 & \Leftrightarrow & M, s \models f_1 \text{ and } M, s \models f_2 \\
M, s \models \mathbf{E}\ g_1 & \Leftrightarrow & \text{there is a path } \pi \text{ from } s \text{ s.t. } M, \pi \models g_1 \\
M, s \models \mathbf{A}\ g_1 & \Leftrightarrow & \text{for every path } \pi \text{ starting from } s,\ M, \pi \models g_1
\end{array}
$$

The CTL^* Temporal Logic: semantics (cont.)

f_1 and f_2 are state formulas, g_1 and g_2 are path formulas.

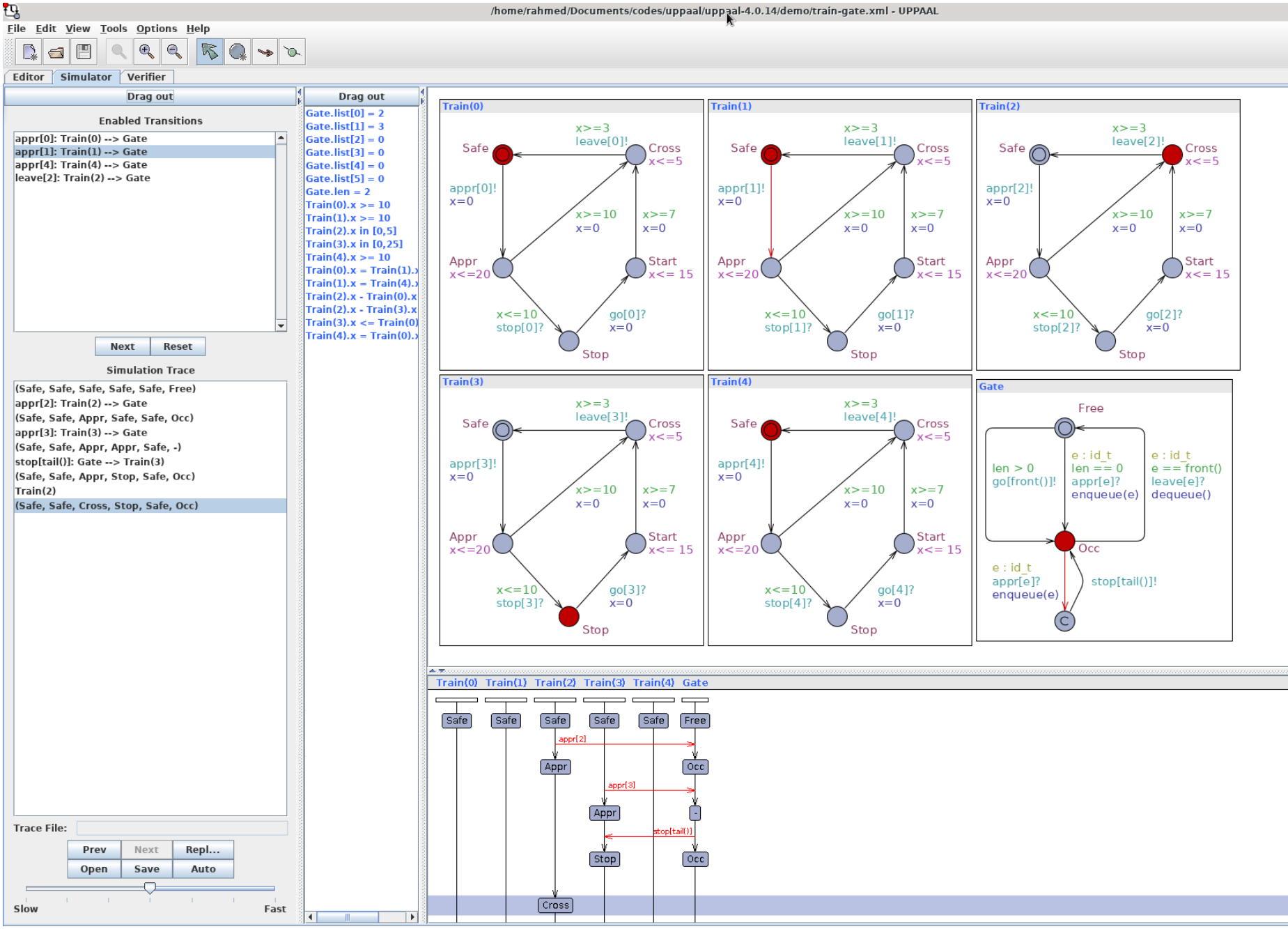
$$
\begin{array}{lcl}
M, \pi \models f_1 & \Leftrightarrow & \text{if } \pi = s_0 s_1 \ldots \text{ then } M, s_0 \models f_1 \\
M, \pi \models \neg g_1 & \Leftrightarrow & M, \pi \not\models g_1 \\
M, \pi \models g_1 \vee g_2 & \Leftrightarrow & M, \pi \models g_1 \text{ or } M, \pi \models g_2 \\
M, \pi \models g_1 \wedge g_2 & \Leftrightarrow & M, \pi \models g_1 \text{ and } M, \pi \models g_2 \\
M, \pi \models \mathbf{X}\ g_1 & \Leftrightarrow & M, \pi^1 \models g_1 \\
M, \pi \models \mathbf{F}\ g_1 & \Leftrightarrow & \text{there exists a } k \geq 0 \text{ s.t. } M, \pi^k \models g_1 \\
M, \pi \models \mathbf{G}\ g_1 & \Leftrightarrow & \text{for all } k \geq 0 \text{ s.t. } M, \pi^k \models g_1 \\
M, \pi \models g_1 \mathbf{U}\ g_2 & \Leftrightarrow & \text{there exists a } k \geq 0 \text{ s.t. } M, \pi^k \models g_2 \\
 & & \text{and for all } 0 \leq j < k, M, \pi^j \models g_1 \\
M, \pi \models g_1 \mathbf{R}\ g_2 & \Leftrightarrow & \text{for all } j \geq 0 \text{ if for every } i < j, M, \pi^i \not\models g_1 \\
 & & \text{then } M, \pi^j \models g_2
\end{array}
$$

The CTL^* Temporal Logic (cont.)

Assignment: Express each of the following using $f, g, \neg, \mathbf{U}, \mathbf{E}$:

- $(\mathbf{F}\ f)\ = ?$
- $(\mathbf{G}\ f)\ = ?$
- $(\mathbf{A}\ f)\ = ?$
- $(f \mathbf{R}\ g)\ = ?$

The UPPAAL model checker



Outline

Further readings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Further readings


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