Exam Software Verification (TDDE34)

May 31, 2022

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- Time kl 14.00 - 18.00
- Submit a "main" pdf, word or text file. If you join pictures, reference them from the main file.
- This is an open book exam. You can access internet.
- It is however strictly forbidden to contact and discuss the exam, during the exam period, with any person other than the examiner, whether the person is related to the course or not.

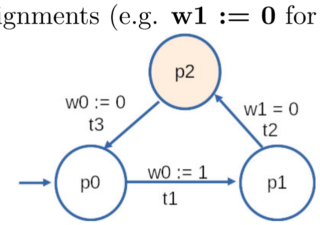
1 Branching time (6p)

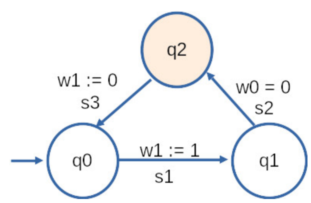
Assume `Req`, `crit@p`, `crit@q` and `Ack` are atomic propositions. Express the following CTL properties using (boolean combinations of) **EG**, **EU** and the atomic propositions above:

- **EF**(`Req`) (1p)
- **AG**(`!crit@p` $\wedge$ `!crit@q`) (1p)
- **EF**(**AG**(`!Req`)) (2p)
- **AG** (((**AG**(`!Req`)) $\Rightarrow$ (**AG**(`!Ack`))) (2p)

2 Mutual exclusion (16p)

Assume the following description of a simple mutual exclusion algorithm for two processes **p** and **q**. State **p0** (resp. **q0**) is the initial state of process **p** (resp. process **q**). State **p2** (resp. **q2**) is the critical section of process **p** (resp. process **q**). Variable **w0** is only written by process **p**. It is 1 when **p** wants to access its critical section (**p2**). Similarly, variable **w1** is only written by process **q**. It is 1 when **q** wants to access its critical section (**q2**). Variables **w0, w1** take their values in **{0,1}**. Transitions are either tests (e.g. **w1=0** for transition **t2**) or assignments (e.g. **w1 := 0** for transition **s3**).





2.1 Part A: (10p)

In the following, we use **@pi** to mean the proposition stating process **p** is at state **pi**. We do the same for process **q**. For instance, the proposition **@q2** is true in a configuration when process **q** is at its critical section. We use the following set of atomic propositions:

- Location propositions: $\{@\mathbf{pi} \mid 0 \leq \mathbf{i} \leq 2\} \cup \{@\mathbf{qi} \mid 0 \leq \mathbf{i} \leq 2\}$
- Values' propositions: $\{\mathbf{x} = \mathbf{v} \mid \mathbf{x} \text{ in } \{\mathbf{w0}, \mathbf{w1}\} \text{ and } \mathbf{v} \text{ in } \{\mathbf{0}, \mathbf{1}\}\}$

Answer the following questions:

- The LTL formula **G**(!@**p2** $\vee$!@**q2**) states that mutual exclusion is always respected. Does it hold? argue or give a run violating it (2p)
- Write an LTL formula corresponding to the starvation freedom of **p**, i.e., each time **p** wants to access its critical section it eventually succeeds. (2p)

- Write an LTL formula $\varphi_{eat-alone}$ that states that it is always the case that: if process **q** stabilizes (i.e., stays forever, possibly after a preliminary phase) in its initial state, then each time **p** wants to access its critical section it eventually succeeds. (3p)
- Give a Büchi automaton for the formula $\varphi_{eat-alone}$. Explain it. (3p)

2.2 Part B: (6p)

We assume the transitions are atomic. Transitions from different processes can be interleaved (a scheduler schedules one process at a time to execute a number of transitions). Transitions corresponding to assignements (e.g., **t1** or **s3**) are enabled if the corresponding process is at the start of the transition (e.g., **@q2** holds for **s3**). Transitions corresponding to tests (e.g., **t2** or **s2**) are enabled if the corrsponding process is at the start of the transition and the test is true (e.g., **@q1** and **w0=0** for **s2**). We write **En(t)** to mean transition **t** is enabled. We write **Ex(t)** to mean transition **t** is indeed executed. For instance **Ex(s2)** is true if **En(s2)** and process **q** moves from **q1** to **q2**. To simplify the discussion, we will hereafter discuss LTL formulas over {**En**(**t**) | **t** is a transition} and {**Ex**(**t**) | **t** is a transition}. You should not use the atomic propositions from part A. It is common to assume schedulers behave "reasonably". A way to account for this assumption is to restrict runs to those satisfying "reasonable" constraints. Consider the following constraint:

Φ: for all transition **u** of processes **p** and **q**. GF(!**En(u)** or **Ex(u)**)

- Is restricting scheduler's behavior to Φ enough to ensure starvation freedom of process **p**? argue or give a counter-example. (3p)
- Is restricting scheduler's behavior to Φ enough to ensure $\varphi_{eat-alone}$? argue or give a counter-example. (3p)

3 Symbolic representation (8p)

1. Assume integer variables x_1, x_2 and x_3. A pure function f that associates integers to integers (assume the domain of f conatins all integers). An integer-indexed array Arr containing integer values (assume the size of Arr is infinite and all integers are valid indices). The following formula involves expressions in Linear Integer Arithmetic (LIA), Equality over Uninterpreted Functions (EUF) and Arrays (A) fragments.

$$\left(\begin{array}{c}(0 < x_1 \wedge x_1 < 3 \wedge 1 < x_2 \wedge x_2 < 3 \wedge x_1 + 1 = x_2) \\ \wedge \\ ((f(x_1) = f(1)) \Rightarrow (rd(wr(Arr, x_2, x_3), x_1 + 1) = (x_1 + x_3)))\end{array}\right)$$

   Give a model (i.e., values for the variables) for the formula if it is satisfiable, otherwise argue why it is not satisfiable. (4p)

2. Consider the formula $f(v_0, v_1, v_2, v_3)$ defined as $(v_0 \wedge v_1) = (v_2 \vee v_3)$ where v_0, v_1, v_2 and v_3 are boolean variables. Give a BDD for f assuming the order $v_0 < v_1 < v_2 < v_3$ (i.e., starting from the root, variable v_0 appears first, then variable v_1, ... etc). If it makes the submission simpler, you can draw the BDD on paper, take a picture and join it to your submission. (4p).

4 Partial and total correctness (10p)

Consider the following simple program:

$$
\begin{array}{l}
\{Q : x = 0 \wedge y = 0 \wedge i = 0 \wedge 0 \leq n\} \\
\ \textbf{do} \quad i < n \quad \rightarrow \quad i := i + 1; x := x + 3; y := y - 2 \\
\ \textbf{od} \\
\{R : x - y = 5 * n\}
\end{array}
$$

- Find a suitable invariant and use it to show that if the loop terminates after starting from a state satisfying Q then it terminates in a state satisfying R (6p)
- Find a suitable variant function and use it to show the loop terminates. (4p)