Coalition Formation and Centralized Coordination

TDDE13 - Multi Agent Systems

Fredrik Präntare

LINKÖPING UNIVERSITY

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Recall

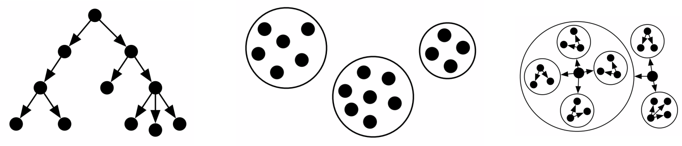
- In *cooperative games*, the focus is on the coalition.
- A common assumption in these games is that of *transferable utility*.
- Payoff distribution can be done in many ways, but *the Shapley value* is the only value that satisfies the axioms *Symmetry*, *Dummy player* and *Additivity*.
- The Shapley value is based on marginal contributions.
- *The core* is a notion of stability which describes whether coalitions have an incentive to deviate.

Coordinating and Organizing Agents

- One of the main objectives in the domain of multi-agent systems is to build agents that can take joint, coordinated actions.
- Coordinating agents can be useful in both cooperative domains, and in scenarios where they are selfish (i.e., act in their own best interests).
- The way agents are organized/coordinated can greatly influence a system (e.g., its performance).

Paradigm	Key Characteristic	Benefits	Drawbacks
Hierarchy	Decomposition	Maps to many common domains; handles scale well	Potentially brittle; can lead to bottlenecks or delays
Holarchy	Decomposition with autonomy	Exploit autonomy of functional units	Must organize holons; lack of predictable performance
Coalition	Dynamic, goal-directed	Exploit strength in numbers	Short term benefits may not outweigh organization construction costs
Team	Group level cohesion	Address larger grained problems; task-centric	Increased communication
Congregation	Long-lived, utility-directed	Facilitates agent discovery	Sets may be overly restrictive
Society	Open system	Public services; well defined conventions	Potentially complex, agents may require additional society-related capabilities
Federation	Middle-agents	Matchmaking, brokering, translation services; facilitates dynamic agent pool	Intermediaries become bottlenecks
Market	Competition through pricing	Good at allocation; increased utility through centralization; increased fairness through bidding	Potential for collusion, malicious behavior; allocation decision complexity can be high
Matrix	Multiple managers	Resource sharing; multiply-influenced agents	Potential for conflicts; need for increased agent sophistication
Compound	Concurrent organizations	Exploit benefits of several organizational styles	Increased sophistication; drawbacks of several organizational styles

From: Horling, Bryan, and Victor Lesser. "A survey of multi-agent organizational paradigms." The Knowledge engineering review 19.4 (2004): 281-316.



hierarchy coalitions holarchy

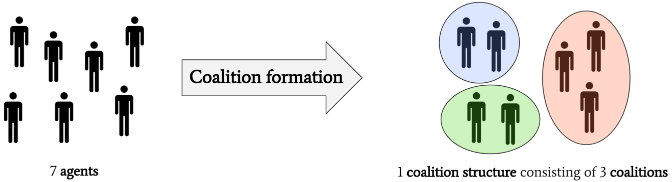
Examples of multi-agent organizational paradigms.

Coordination Paradigms

- assignment algorithms (task allocation)—both distributed and centralized;
- multi-agent reinforcement learning (e.g., policy-based);
- mechanism design ("reverse game theory"—instead of predicting outcomes, we start by defining the outcomes, and ask ourselves what mechanisms would generate those outcomes).

Coalition Formation

One of the major paradigms for organizing agents is *coalition formation*.



Coalition Formation

Applications:

- buyers can obtain lower prices through bulk purchasing;
- autonomous, heterogeneous robots can be organized in teams;
- form coalitions of delivery companies to reduce transportation costs and climate impact by sharing deliveries;
- deployment of staff/workers to locations/jobs can be automatized/analyzed;
- units in strategy games can be coordinated.

Coalition Formation

Consists of three main processes:

- **forming** a *set of coalitions*, typically via *coalition structure generation*;

  (this lecture + lab 1)
- **coordinating** within the coalitions; and

  (coming lectures ...)
- **dividing** payoff among each coalition's members.

  (previous lecture ...)

Coalition Structures

Definition 1. A *coalition structure* $CS = \{C_1, ..., C_m\}$ over the players (agents) N is a set of coalitions with:

- $C_i \cap C_j = \emptyset$ for all $i \neq j$ (disjoint); and
- $\bigcup_{i=1}^{m} C_i = N$ (exhaustive).

For example, $\{\{a_1, a_3\}, \{a_2\}\}$ and $\{\{a_1\}, \{a_2\}, \{a_3\}\}$ are two different coalition structures over $N = \{a_1, a_2, a_3\}$.

Note that we often omit the notion "over N" for brevity/clarity.

Coalition Structures

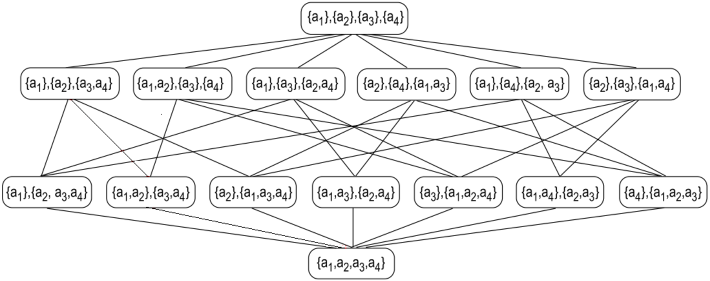
Notation:

- Π^N is the set of coalition structures over N.
- The value of a coalition structure $CS \in \Pi^N$ is denoted by $V(CS)$ and is defined as:

$$V(CS) = \sum_{C \in CS} v(C).$$

- L_k is the set of all k-sized coalitions—more formally:

$$L_k = \{C \subseteq N : |C| = k\}$$



From: Rahwan, Talal, et al. "Coalition structure generation: A survey." Artificial Intelligence 229 (2015): 139-174.

Coalition Structure Generation

Definition 2. The *coalition structure generation problem* for characteristic function games (CFGs) is the problem with the following input/output:

Input: A characteristic function game $\langle N, v \rangle$.

Output: $CS \in \arg\max_{CS \in \Pi^N} V(CS)$.

Example 1 (Coalition Structure Generation)

Suppose we have the following set of **agents** (players):

$$\{1, 2, 3\}$$

Their possible **coalitions** are:

$$\{1\} \ \{2\} \ \{3\} \ \{1,2\} \ \{1,3\} \ \{2,3\} \ \{1,2,3\}$$

The possible **coalition structures** are:

$$\{\{1\},\{2\},\{3\}\} \quad \{\{1,2\},\{3\}\} \quad \{\{2,3\},\{1\}\}$$
$$\{\{1,3\},\{2\}\} \quad \{\{1,2,3\}\}$$

Example 1 (Coalition Structure Generation)

C	$v(C)$
$\{1\}$	65
$\{2\}$	45
$\{3\}$	50
$\{1,2\}$	120
$\{1,3\}$	105
$\{2,3\}$	110
$\{1,2,3\}$	165

Example coalitional values.

CS	$V(CS)$
$\{\{1\},\{2\},\{3\}\}$	?
$\{\{1,2\},\{3\}\}$	?
$\{\{2,3\},\{1\}\}$	?
$\{\{1,3\},\{2\}\}$	?
$\{\{1,2,3\}\}$	?

Coalition structures' values.

Which coalition structure is optimal?

Example 1 (Coalition Structure Generation)

C	$v(C)$
$\{1\}$	65
$\{2\}$	55
$\{3\}$	50
$\{1,2\}$	120
$\{1,3\}$	105
$\{2,3\}$	110
$\{1,2,3\}$	165

Example coalitional values.

CS	$V(CS)$
$\{\{1\},\{2\},\{3\}\}$	170
$\{\{1,2\},\{3\}\}$	170
$\{\{2,3\},\{1\}\}$	175
$\{\{1,3\},\{2\}\}$	160
$\{\{1,2,3\}\}$	165

Coalition structures' values.

$\{\{2,3\},\{1\}\}$ is optimal!

Example 1 (Coalition Structure Generation)

... with more agents it gets more difficult.

L_1	v	L_2	v	L_3	v	L_4	v
$\{1\}$	30	$\{1,2\}$	50	$\{1,2,3\}$	90	$\{1,2,3,4\}$	140
$\{2\}$	40	$\{1,3\}$	60	$\{1,2,4\}$	120		
$\{3\}$	25	$\{1,4\}$	80	$\{1,3,4\}$	100		
$\{4\}$	45	$\{2,3\}$	55	$\{2,3,4\}$	115		
		$\{2,4\}$	70				
		$\{3,4\}$	80				

Which coalition structure is optimal now ... ?

Example 1 (Coalition Structure Generation)

... with more agents it gets more difficult.

L_1	v	L_2	v	L_3	v	L_4	v
$\{1\}$	30	$\{1,2\}$	50	$\{1,2,3\}$	90	$\{1,2,3,4\}$	140
$\{2\}$	40	$\{1,3\}$	60	$\{1,2,4\}$	120		
$\{3\}$	25	$\{1,4\}$	80	$\{1,3,4\}$	100		
$\{4\}$	45	$\{2,3\}$	55	$\{2,3,4\}$	115		
		$\{2,4\}$	70				
		$\{3,4\}$	80				

$\{\{1\},\{2\},\{3,4\}\}$ is optimal!

Coalition Structure Generation

- Combinatorial optimization problem.
- Can theoretically be solved by brute-force search.
- Brute-force typically not practicable since the number of coalition structures of n agents equals the n^{th} *Bell number* B_n, which satisfies:

$$\alpha n^{n/2} \leq B_n \leq n^n$$

  for some positive constant α.
- NP-complete—but we can do better than exhaustive search with e.g., dynamic programming and branch-and-bound for CFGs.

Embedded Coalition

Definition 3. An *embedded coalition* is a pair, $\langle C, CS \rangle$, where C is a coalition, and CS is a coalition structure over N that contains C. That is, $CS \in \Pi^N : C \in CS$.

The set of all embedded coalitions is denoted by EC.

Partition Function Games

Definition 4. A *partition function game* (PFG) is a pair $\langle N, v \rangle$ where:

- $N = \{1, ..., |N|\}$ is a finite set of players; and
- $w : EC \mapsto \mathbb{R}$ is a function called the *partition function*, that maps a value to each embedded coalition $C \in EC$.

The value of a coalition structure CS in this game type is defined as:

$$W(CS) = \sum_{C \in CS} w(C, CS).$$

Partition Function Games

- In this game type, we are interested in *externalities*: the coalitions' exerted influence over each other.
- CFGs are a special case of PFGs—in other words, CFGs form a proper subclass of PFGs.

Partition Function Games

- CSG in this setting is highly computationally challenging due to that a coalition's value may depend on the partitioning of all other agents.
- Each coalition $C \subseteq A$ can have as many different values as there are ways to partition the remaining agents $A \setminus C$.
- Thus, in general, you cannot optimally solve a CSG problem for PFGs without enumerating all possible coalition structures.
- It is possible to do better for constrained classes of externalities.

Forming and Coordinating Coalitions

(separate slides; sent on request)

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