Assignment Set 1

Multi Agent Systems (TDDE13), Linköping University
By Fredrik Präntare, Autumn Semester 2024

Directions: *Individually* (not in groups or pairs) solve the assignments below and send your solutions (with a clear and precise line of reasoning!) to your TA's mail address (Daniel de Leng, **daniel.de.leng@liu.se**) before the deadline. It is important that you

1. use the course's *LaTeX*/Word template for the answers (only submit the compiled .pdf; bad formats and file types will be rejected);
2. use "**TDDE13: Assignment Set 1**" as the header in your mail; and
3. send the answers from your *LiU student account*.

Deadline: See the course's webpage. After the deadline, you receive only half the points for correct answers.

Prerequisites: Course lectures + the following chapters in the course's textbook (*Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*):

- Chapter 3: Introduction to Noncooperative Game Theory: Games in Normal Form;
- Chapter 4: Computing Solution Concepts of Normal-Form Games;
- Chapter 5: Games with Sequential Actions;
- Chapter 12: Teams of Selfish Agents: An Introduction to Coalitional Game Theory.

Exercises:

1 ***(0.5p)*** What is a von Neumann and Morgenstern *utility function* and what does it represent? Define such a utility function for a situation/context relevant to your own daily life, and describe what it represents.

2 ***(0.5p)*** If any, what are the key differences between *common-payoff games*, *zero-sum games* and *constant-sum games*? Describe and define an example you know of for two of them. Your examples must be distinct (i.e., they cannot be the same example) and *not* taken from the course's book!

3 ***(1.5p)*** Find all existing *pure* **and** *mixed Nash equilibria* (NE) and the players' expected payouts (in the NEs) in the following normal-form games:

	L	R
U	1, 1	1, 0
D	−1, 1	0, 1

	X	Y
A	1, 0	0, 1
B	0, 1	1, 0

	C1	C2
R1	1, 3	2, 1
R2	2, 1	1, 4

$\boxed{4}$ ***(0.5p)*** Find the row player's *pure maxmin solution* (both value and strategy) in the following (simultaneous move) normal-form game:

	A	B
X	4, −3	−2, 1
Y	0, 0	5, −6

$\boxed{5}$ ***(0.5p)*** In game theory, what are the key differences between *cooperative* and *non-cooperative* settings? Give an example you know of for each. Your examples must be distinct and *not* taken from the course's textbook.

$\boxed{6}$ Design and define your own *non-convex 3-player (characteristic function) coalitional game* (with transferable utility), for which each coalition (except the empty coalition) must have a distinct (= unique) non-zero value. The coalitional game that you define needs to represent a (potentially fictional) scenario that you can think of.

(a) ***(0.5p)*** Describe what the coalitional game that you defined represents.

(b) ***(0.5p)*** Compute one player's *marginal contributions* and her *Shapley value*. Also, clearly describe (with your scenario as basis) what her Shapley value represents.

(c) ***(0.5p)*** Prove that your game is **not** *convex*.

(d) ***(0.5p)*** Show that the game that you defined is (or that it is not, depending on the game that you defined) *additive* and/or *superadditive*.

$\boxed{7}$ ***(1.0p)*** Recall that a coalitional *partition function game* is defined as a tuple $\langle N, v\rangle$ where:

- $N = \{1, ..., n\}$ is a set of agents;
- $v((C, CS)) \mapsto \mathbb{R}$ is a function that maps a value (e.g., potential utility) to every *embedded coalition* (C, CS) over N; see Definition 1.

Definition 1. An *embedded coalition* (over the agents N) is a pair (C, CS), where CS is a coalition structure (see Definition 2) over N, and C is a coalition with $C \in CS$.

Definition 2. A *coalition structure* $CS = \{C_1, ..., C_{|CS|}\}$ over the set of agents N is a set of coalitions with $C_i \subseteq N$ for $i = 1, ..., |CS|$, $C_i \neq \emptyset$ for $i = 1, ..., |CS|$, $C_i \cap C_j = \emptyset$ for all $i \neq j$, and $\bigcup_{i=1}^{|CS|} C_i = N$.

Now, suppose we have a partition function game $\langle\{1, 2, 3\}, v\rangle$ with:

$$v((C, CS)) = \begin{cases} 2 & \text{if } 1 \in C \text{ and } 2 \in C; \\ 1 & \text{else if } |CS| = 2 \text{ and } |C| = 1; \\ 3 & \text{else if } |CS| = 2,\ C \neq \emptyset \text{ and } C \neq \{2, 3\}; \\ 1.5 & \text{else if } C = \{3\}; \\ 0 & \text{otherwise.} \end{cases}$$

Your task is to compute $\max_{CS \in \Pi} \left\{ \sum_{C \in CS} v((C, CS)) \right\}$ and $\arg\max_{CS \in \Pi} \left\{ \sum_{C \in CS} v((C, CS)) \right\}$, where Π denotes the set of all coalition structures over N.

$\boxed{8}$ ***(0.5p)*** Discuss and critique using the Shapley value for dividing payoff in **partition** function games.