# TDDD95 Algorithmic Problem Solving 6hp, vt2022 

## Fredrik Heintz

Dept of Computer and Information Science Linköping University

- What is algorithmic problem solving?
- Why is algorithmic problem solving important?
- What will be studied in this course?
- A method for algorithmic problem solving
- Common algorithmic problem solving approaches
- Common data structures and algorithms
- Pragmatic algorithmic problem solving using Kattis


## What is Algorithmic Problem Solving?

- Algorithmic problem solving is about developing correct and working algorithms to solve classes of problems.
- The problems are normally very well defined and you know there is a solution, but they can still be very hard.



## Swedsoft

Mjukvaranär själen i svensk industri

## Those that really understand

 and take advantage of software technology owns the future!facebook


## Background

Problems in Computer Science are often classified as belonging to a certain class of problems (e $\mathrm{g}, \mathrm{NP}$, Unsolvable, Recursive). In this problem you will be analyzing a property of an algorithm whose classification is not known for all possible inputs

## The Problem

Consider the following algorithm:

| 1. | Anpat a |
| :---: | :---: |
| 2. | prist a |
| 3. | 15n-1 $=$ man stop |
| 6. | If $a 12$ ofa then |

8. 

Goro 2

## Given the irput 22, the following sequence of numbers will be printed 221134175226134020105168421

It is conjectured that the algorithm above will terminate (when a 1 is printed) for any istegral input value. Despite the simplicity of the algorithm, it is unknown whether this conjecture is true. It has been verified, however, for all integers $n$ such that $0<n<1,000,000$ (and, in fact, for many more numbers than this.)
Given an input $n$, it is possible to deternine the number of numbers printed (including the 1). For a given $n$ this is called the cycle-length of $n$ In the example above, the cycle length of 22 is 16 .
For any two mumbers $i$ and $j$ you are to determine the maxinum cycle length over all numbers between $i$ and $j$.

## The Input

The input will consist of a series of pairs of integers $i$ and $j$, one pair of integers per line All integers will be less than $1,000,000$ and greater than 0 .
You should process all pairs of integers and for each pair determine the maximum cycle length over all integers between and including $i$ and $j$.
You can assume that no operation overflows a 32 -bit integer

## The Output

For each pair of input integers $i$ and $j$ you should output $i$, $j$, and the maximum cycle length for integers between and including $i$ and $j$. These three numbers should be separated by at least one space with all three numbers on one line and with one line of output for each line of input. The integers $i$ and $j$ must appear in the output in the same order in which they appeared in the input and should be followed by the maximum cycle length (on the same line)

## Sample Input

110
100200
201210
2001000
Sample Output
12020
100200129
$\begin{array}{lll}201 & 210 & 82 \\ 200 & 1000 & 174\end{array}$

## Example: The $3 n+1$ problem

## 100 The $3 n+1$ problem

## Background

Problems in Computer Science are often classified as belonging to a certain class of problems (e.g., NP, Unsolvable, Recursive). In this problem you will be analyzing a property of an algorithm whose classification is not known for all possible inputs.

## The Problem

Consider the following algorithm:

1. imput $n$
2. print $n$
3. if $n=1$ then STOP
4. if $n$ is odd then $n \leftarrow 3 n+1$
5. else $n \leftarrow n / 2$
6. GOTO 2

Given the input 22, the following sequence of numbers will be printed

## 221134175226134020105168421

It is conjectured that the algorithm above will terminate (when a 1 is printed) for any integral input value. Despite the simplicity of the algorithm, it is unknown whether this conjecture is true. It has been verified, however, for all integers $n$ such that $0<n<1,000,000$ (and, in fact, for many more numbers than this.)

Given an input $n$, it is possible to determine the number of numbers printed before and including the 1 is printed. For a given $n$ this is called the cycle-length of $n$. In the example above, the cycle length of 22 is 16 .

For any two numbers $i$ and $j$ you are to determine the maximum cycle length over all numbers between and including both $i$ and $j$.

## Example: The $3 n+1$ problem

## The Input

The input will consist of a series of pairs of integers $i$ and $j$, one pair of integers per line. All integers will be less than 10,000 and greater than 0 .

You should process all pairs of integers and for each pair determine the maximum cycle length over all integers between and including $i$ and $j$.

## The Output

For each pair of input integers $i$ and $j$ you should output $i, j$, and the maximum cycle length for integers between and including $i$ and $j$. These three numbers should be separated by at least one space with all three numbers on one line and with one line of output for each line of input. The integers $i$ and $j$ must appear in the out put in the same order in which they appeared in the input and should be followed by the maximum cycle length (on the same line).

## Sample Input

```
110
100200
201210
9001000
```

Sample Output
11020
100200125
20121089
$900 \quad 1000 \quad 174$

## Example: The $3 n+1$ problem

- Follow the instructions in the problem!
- Memoization to speed it up.
- Table lookup to solve it in constant time.
- Gotchas:
- $j$ can be smaller than $i$.
- $j$ can equal $i$.
- The order of $i$ and $j$ in output must be the same as the input, even when $j$ is smaller than $i$.


## Course Goals

The goals of the course are you should be able to:

- analyze the efficiency of different approaches to solving a problem to determine which approaches will be reasonably efficient in a given situation,
- compare different problems in terms of their difficulty,
- use algorithm design techniques such as greedy algorithms, dynamic programming, divide and conquer, and combinatorial search to construct algorithms to solve given problems,
- strategies for testing and debugging algorithms and data structures,
- quickly and correctly implement a given specification of an algorithm or data structure,
- communicate and cooperate with other students during problem solving in groups.


## Examination

- LABı 4hp
- individually solving the 4 lab assignments and
- actively participating in at least 3 problem solving sessions.
- UPPG1 2hp,
- individually solving the 13 weekly homework exercises, e.g.:
- Data structures
- Greedy Problems and Dynamic Programming
- Graph Algorithms
- Search
- Math-related Problems
- Computational Geometry.


## Exercises

## UPG1

To pass the UPG1 part of the course, you must fulfill the following requirements:

1. Solve at least one problem from each of the 13 exercise sets, before or after the corresponding deadline.
2. Accumulate a sufficient number of UPG1 points. This also determines your UPG1 grade.

The problems come in three difficulty classes: A, B, and C. You are awarded 1 point in the corresponding difficulty class for each problem you solve before its deadline, and 0.5 points if you solve it after the deadline. The table below specifies how many points you must accumulate for the different UPG1 grades.

| A | B | C | Grade UPG1 |
| :---: | :---: | :---: | :---: |
| 13 | 0 | 0 | 3 |
| 13 | 7 | 0 | 4 |
| 13 | 7 | 6 | 5 |

You may of course replace easier problems with harder problems. For example, if you have 6 A points and 7 B points, you have 13 points at difficulty A or above, but not an additional 7 points at difficulty B or above. You would therefore get grade 3 .

## The Schedule - VT 1

- 19/1 How to solve algorithmic problems, intro seminar
- 20/1 Practice problem solving session: 13.30-17.00 with discussion
- 28/1 Deadline Ex 1 (Greedy and DP 1) - Seminar Exı and Data structures
- 4/2 Deadline Ex2 (Data structures) - Seminar Ex2 and Arithmetic
- 11/2 Deadline Ex3 (Arithmetic) - Seminar Ex3 and Problem solving approaches
- 17/2 Deadline Lab Assignment 1 (Data structures, Greedy/Dynamic, Arithmetic)
- 17/2 Problem solving session (individual based on Lab 1)
- 18/2 Deadline Ex4 (Greedy and DP II) - Seminar Ex 4 and Graphs I
- 25/2 Deadline Ex5 (Graphs I) - Seminar Ex5 and Graphs II
- 4/3 Deadline Ex6 (Graphs II) - Seminar Ex6 and Graphs III
- 10/3 Deadline Lab Assignment 2 (Graphs)
- 10/3 Problem solving session (individual based on Lab 2)


## The Schedule - VT 2

- 28/3 Problem solving session (groups based on Lab 1-2)
- 1/4 Deadline Ex 7 (Graphs III) - Seminar Ex7 and Strings I
- 8/4 Deadline Ex8 (Strings I) - Seminar Ex8 and Strings II
- 13/4 Deadline Ex9 (Strings II) - Seminar Ex9 and Number Theory
- 25/4 Deadline Lab Assignment 3 (Strings, String Matching and Number Theory)
- 25/4 Problem solving session (individual based on Lab 3)
- 22/4 Deadline Exio (Number Theory) - Seminar Ex 11 and Combinatorial Search
- 29/4 Deadline Exı1 (Combinatorial Search) - Seminar Exı2 and Computational Geometry
- 6/5 Deadline Exı2 (Computational Geometry)
- 9/5 Deadline Lab Assignment 4 (Computational Geometry)
- 9/5 Problem solving session (individual based on Lab 4)
- 13/5 Deadline Exı3 (Mixed)
- 16/5 Problem solving session (groups based on Lab 3-4)


## Steps in solving algorithmic problems

- Estimate the difficulty
- Theory (size of inputs, known algorithms, known theorems, ...)
- Coding (size of program, many cases, complicated data structures, ...)
- Have you seen this problem before? Have you solved it before? Do you have useful code in your code library?
- Understand the problem!
- What is being asked for? What is given? How large can instances be?
- Can you draw a diagram to help you understand the problem?
- Can you explain the problem in your own words?
- Can you come up with good examples to test your understanding?


## Steps in solving algorithmic problems

- Determine the right algorithm or algorithmic approach
- Can you solve the problem using brute force?
- Can you solve the problem using a greedy approach?
- Can you solve the problem using dynamic programming?
- Can you solve the problem using search?
- Can you solve the problem using a known algorithm in your code library?
- Can you modify an existing algorithm? Can you modify the problem to suite an existing algorithm?
- Do you have to come up with your own algorithm?
- Solve the problem! ©


## Time Limits and Computational Complexity

- The normal time limit for a program is a few seconds.
- You may assume that your program can do about 100 M operations within this time limit.

| n | Worst AC Complexity | Comments |
| :--- | :--- | :--- |
| $\leq[10 . .11]$ | $O(n!), O\left(n^{6}\right)$ | Enumerating permutations |
| $\leq[15 . .18]$ | $O\left(2^{n} \times n^{2}\right)$ | DP TSP |
| $\leq[18 . .22]$ | $O\left(2^{n} \times n\right)$ | DP with bitmask technique |
| $\leq 100$ | $O\left(n^{4}\right)$ | DP with 3 dimensions and $\mathrm{O}(\mathrm{n})$ loop |
| $\leq 450$ | $O\left(n^{3}\right)$ | Floyd Warshall's (APSP) |
| $\leq 2 \mathrm{~K}$ | $O\left(n^{2} \log _{2} n\right)$ | 2-nested loops + tree search |
| $\leq 10 \mathrm{~K}$ | $O\left(n^{2}\right)$ | Bubble/Selection/Insertion sort |
| $\leq 1 \mathrm{M}$ | $O\left(n \log _{2} n\right)$ | Merge Sort, Binary search |
| $\leq 100 \mathrm{M}$ | $O(n), O\left(\log _{2}\right), O(1)$ | Simulation, find average |

## Important Problem Solving Approaches

- Simulation/Ad hoc
- Do what is stated in the problem
- Example: Simulate a robot
- Greedy approaches
- Find the optimal solution by extending a partial solution by making locally optimal decisions
- Example: Minimal spanning trees, coin change in certain currencies
- Divide and conquer
- Take a large problem and split it up in smaller parts that are solved individually
- Example: Merge sort and Quick sort
- Dynamic programming
- Find a recursive solution and compute it "backwards" or use memoization
- Example: Finding the shortest path in a graph and coin change in all currencies
- Search
- Create a search space and use a search algorithm to find a solution
- Example: Exhaustive search (breadth or depth first search), binary search, heuristic search ( $\mathrm{A}^{*}$, best first, branch and bound)


## Complete Search a.k.a. Brute Fo

- When a problem is small or (almost) all possibilities have to be tried complete search is a candidate approach.
- To determine the feasibility of complete search estimate the number of calculations that have to be made in the worst case.
- Iterative complete search uses nested loops to generate every possible complete solution and filter out the valid ones.
- Iterating over all permutations using next_permutation
- Iterating over all subsets using bit set technique
- Recursive complete search extends a partial solution with one element until a complete and valid solution is found.
- This approach is often called recursive backtracking.
- Pruning is used to significantly improve the efficiency by removing partial solutions that can not lead to a solution as soon as possible. In the best case only valid solutions are generated.


## Complete Search a.k.a. Brute Fore

We have three different integers, $x, y$ and $z$, which satisfy the following three relations:

- $x+y+z=A$
- $x y z=B$
- $x^{2}+y^{2}+z^{2}=C$

You are asked to write a program that solves for $x, y$ and $z$ for given values of $A, B$ and $C$.

## Divide and Conquer

- Divide and conquer is very common and powerful technique which divides a problem into smaller parts, solves each part recursively and then puts together the answer from the pieces.
- Many well known algorithms are based on divide and conquer such as quick sort, merge sort and binary search.
- Binary search is a very versatile and useful technique which can be used to
- find a particular value in a sorted range,
- find the parameters of a (convex) function that gives a particular value,
- find the minimum/maximum value of a function.
- Binary search can be implemented either using built in functions (lower_bound/upper_bound), iterating until the difference between the end points is small enough or iterate a constant but sufficiently large number of times.


## Quick-Sort

- Quick-sort is a randomized sorting algorithm based on the divide-and-conquer paradigm:
- Divide: pick a random element $\boldsymbol{x}$ (called pivot) and partition $S$ into
- L elements less than $\boldsymbol{x}$
- $\boldsymbol{E}$ elements equal $\boldsymbol{x}$
- $\boldsymbol{G}$ elements greater than $\boldsymbol{x}$
- Recur: sort $\boldsymbol{L}$ and $\boldsymbol{G}$
- Conquer: join $\boldsymbol{L}, \boldsymbol{E}$ and $\boldsymbol{G}$



## Quick-Sort Example

- Pivot selection



## Quick-Sort Example

- Partition, recursive call, pivot selection



## Quick-Sort Example

- Partition, recursive call, base case



## Quick-Sort Example

- Recursive call, ..., base case, join



## Quick-Sort Example

- Recursive call, pivot selection



## Quick-Sort Example

- Partition, ..., recursive call, base case



## Quick-Sort Example

Join, join


## Greedy

- An algorithm is said to be greedy if it makes a locally optimal choice in each step towards the globally optimal solution.
- For a greedy algorithm to give a globally optimal result a problem must have two properties:
- It has optimal sub-structures, i.e. an optimal solution contains the optimal solutions to sub problems.
- It has the greedy choice property, i.e. if we extend a partial solution by making a locally optimal choice we will get the optimal complete solution without reconsidering previous choices.
- Classical examples: Coin change in some currencies, interval coverage and load balancing.
- Greedy algorithms can be very useful as heuristics for example in branch-and-bound search algorithms.
- In combinatorics matroids and the generalization greedoids characterize classes of problems with greedy solutions.


## Kruskal's MST Algorithm

Consider an undirected, weight graph

## Kruskal's MST Algorithm

Sort the edges by increasing edge weight


| edge | $\boldsymbol{d}_{\boldsymbol{v}}$ |  |
| :---: | :---: | :--- |
| $(\mathrm{D}, \mathrm{E})$ | 1 |  |
| $(\mathrm{D}, \mathrm{G})$ | 2 |  |
| $(\mathrm{E}, \mathrm{G})$ | 3 |  |
| $(\mathrm{C}, \mathrm{D})$ | 3 |  |
| $(\mathrm{G}, \mathrm{H})$ | 3 |  |
| $(\mathrm{C}, \mathrm{F})$ | 3 |  |
| $(\mathrm{~B}, \mathrm{C})$ | 4 |  |


| edge | $\boldsymbol{d}_{\boldsymbol{v}}$ |  |
| :---: | :---: | :--- |
| $(\mathrm{B}, \mathrm{E})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{~F})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{H})$ | 4 |  |
| $(\mathrm{~A}, \mathrm{H})$ | 5 |  |
| $(\mathrm{D}, \mathrm{F})$ | 6 |  |
| $(\mathrm{~A}, \mathrm{~B})$ | 8 |  |
| $(\mathrm{~A}, \mathrm{~F})$ | 10 |  |

## Kruskal's MST Algorithm

Select first |V|-1 edges which do not generate a cycle


| edge | $\boldsymbol{d}_{\boldsymbol{v}}$ |  |
| :---: | :---: | :---: |
| $(\mathrm{D}, \mathrm{E})$ | 1 | $\sqrt{ }$ |
| $(\mathrm{D}, \mathrm{G})$ | 2 |  |
| $(\mathrm{E}, \mathrm{G})$ | 3 |  |
| $(\mathrm{C}, \mathrm{D})$ | 3 |  |
| $(\mathrm{G}, \mathrm{H})$ | 3 |  |
| $(\mathrm{C}, \mathrm{F})$ | 3 |  |
| $(\mathrm{~B}, \mathrm{C})$ | 4 |  |


| edge | $\boldsymbol{d}_{\boldsymbol{v}}$ |  |
| :---: | :---: | :--- |
| $(\mathrm{B}, \mathrm{E})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{~F})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{H})$ | 4 |  |
| $(\mathrm{~A}, \mathrm{H})$ | 5 |  |
| $(\mathrm{D}, \mathrm{F})$ | 6 |  |
| $(\mathrm{~A}, \mathrm{~B})$ | 8 |  |
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| $(\mathrm{D}, \mathrm{G})$ | 2 | $\sqrt{ }$ |
| $(\mathrm{E}, \mathrm{G})$ | 3 |  |
| $(\mathrm{C}, \mathrm{D})$ | 3 |  |
| $(\mathrm{G}, \mathrm{H})$ | 3 |  |
| $(\mathrm{C}, \mathrm{F})$ | 3 |  |
| $(\mathrm{~B}, \mathrm{C})$ | 4 |  |


| edge | $\boldsymbol{d}_{\boldsymbol{v}}$ |  |
| :---: | :---: | :--- |
| $(\mathrm{B}, \mathrm{E})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{~F})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{H})$ | 4 |  |
| $(\mathrm{~A}, \mathrm{H})$ | 5 |  |
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| $(\mathrm{D}, \mathrm{E})$ | 1 | $\sqrt{ }$ |
| $(\mathrm{D}, \mathrm{G})$ | 2 | $\sqrt{ }$ |
| $(\mathrm{E}, \mathrm{G})$ | 3 | $\chi$ |
| $(\mathrm{C}, \mathrm{D})$ | 3 |  |
| $(\mathrm{G}, \mathrm{H})$ | 3 |  |
| $(\mathrm{C}, \mathrm{F})$ | 3 |  |
| $(\mathrm{~B}, \mathrm{C})$ | 4 |  |


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| :---: | :---: | :--- |
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| $(\mathrm{~B}, \mathrm{~F})$ | 4 |  |
| $(\mathrm{~B}, \mathrm{H})$ | 4 |  |
| $(\mathrm{~A}, \mathrm{H})$ | 5 |  |
| $(\mathrm{D}, \mathrm{F})$ | 6 |  |
| $(\mathrm{~A}, \mathrm{~B})$ | 8 |  |
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Accepting edge (E,G) would create a cycle

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| $(\mathrm{C}, \mathrm{D})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{G}, \mathrm{H})$ | 3 |  |
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| $(\mathrm{C}, \mathrm{F})$ | 3 | $\sqrt{ }$ |
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| :---: | :---: | :---: |
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| $(\mathrm{~B}, \mathrm{H})$ | 4 |  |
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| $(\mathrm{G}, \mathrm{H})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{C}, \mathrm{F})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{~B}, \mathrm{C})$ | 4 | $\sqrt{ }$ |


| edge | $\boldsymbol{d}_{v}$ |  |
| :---: | :---: | :---: |
| $(\mathrm{~B}, \mathrm{E})$ | 4 | $\chi$ |
| $(\mathrm{~B}, \mathrm{~F})$ | 4 | $\chi$ |
| $(\mathrm{~B}, \mathrm{H})$ | 4 | $\chi$ |
| $(\mathrm{~A}, \mathrm{H})$ | 5 |  |
| $(\mathrm{D}, \mathrm{F})$ | 6 |  |
| $(\mathrm{~A}, \mathrm{~B})$ | 8 |  |
| $(\mathrm{~A}, \mathrm{~F})$ | 10 |  |

## Kruskal's MST Algorithm

Select first |V|-1 edges which do not generate a cycle


| edge | $\boldsymbol{d}_{v}$ |  |
| :---: | :---: | :---: |
| $(\mathrm{D}, \mathrm{E})$ | 1 | $\sqrt{ }$ |
| $(\mathrm{D}, \mathrm{G})$ | 2 | $\sqrt{ }$ |
| $(\mathrm{E}, \mathrm{G})$ | 3 | $\chi$ |
| $(\mathrm{C}, \mathrm{D})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{G}, \mathrm{H})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{C}, \mathrm{F})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{~B}, \mathrm{C})$ | 4 | $\sqrt{ }$ |


| edge | $\boldsymbol{d}_{v}$ |  |
| :---: | :---: | :---: |
| $(\mathrm{~B}, \mathrm{E})$ | 4 | $\chi$ |
| $(\mathrm{~B}, \mathrm{~F})$ | 4 | $\chi$ |
| $(\mathrm{~B}, \mathrm{H})$ | 4 | $\chi$ |
| $(\mathrm{~A}, \mathrm{H})$ | 5 | $\sqrt{ }$ |
| $(\mathrm{D}, \mathrm{F})$ | 6 |  |
| $(\mathrm{~A}, \mathrm{~B})$ | 8 |  |
| $(\mathrm{~A}, \mathrm{~F})$ | 10 |  |

## Kruskal's MST Algorithm

Select first |V|-1 edges which do not generate a cycle


| edge | $\boldsymbol{d}_{\boldsymbol{v}}$ |  |
| :---: | :---: | :---: |
| $(\mathrm{D}, \mathrm{E})$ | 1 | $\sqrt{ }$ |
| (D,G) | 2 | $\sqrt{ }$ |
| (E,G) | 3 | $\chi$ |
| $(\mathrm{C}, \mathrm{D})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{G}, \mathrm{H})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{C}, \mathrm{F})$ | 3 | $\sqrt{ }$ |
| $(\mathrm{~B}, \mathrm{C})$ | 4 | $\sqrt{ }$ |

\(\left.\begin{array}{|c|c|c|}\hline edge \& \boldsymbol{d}_{\boldsymbol{v}} \& <br>
\hline(\mathrm{B}, \mathrm{E}) \& 4 \& \chi <br>
\hline(\mathrm{~B}, \mathrm{~F}) \& 4 \& \chi <br>
\hline(\mathrm{~B}, \mathrm{H}) \& 4 \& \chi <br>
\hline(\mathrm{~A}, \mathrm{H}) \& 5 \& \sqrt{ } <br>
\hline(\mathrm{D}, \mathrm{F}) \& 6 \& <br>
\hline(\mathrm{~A}, \mathrm{~B}) \& 8 \& <br>
\hline(\mathrm{~A}, \mathrm{~F}) \& 10 \& <br>

\hline\end{array}\right\}\)| not |
| :--- |
| considere |

## Done

Total Cost $=\Sigma d_{v}=21$

## Dynamic Programming

- Dynamic Programming is a problem solving approach which computes the answer for every possible state exactly once.
- For DP to be suitable a problem must have two properties:
- It has optimal sub-structures, i.e. an optimal solution contains the optimal solutions to sub problems.
- Overlapping sub-problems, i.e. the same subproblem occurs many times.
- Top-down (memoization) vs Bottom-up
- Top-down: no need to consider the order of computations, only compute states actually used, natural transition from complete search,
- Bottom-up: no recursion, computes every state, table size can be reduced if only the previous row of states is used then only two rows are required.
- Displaying the optimal solution
- Store the previous state for each solution
- Use the DP table and the optimal sub-structures property to compute the path.


## Classical DP Problems

- Max ${ }^{2}$ D sum
- Max 2D sum
- Longest increasing subsequence (LIS)
" Longest decreasing subsequence (LDS)
- o-1 Knapsack (subset sum)
- Coin Change (general version)
- Travelling Salesman Problem (TSP)

|  | 1D RSQ | 2D RSQ | LIS | Knapsack | CoinChange | TSP |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| State | $(\mathrm{i})$ | $(\mathrm{i}, \mathrm{j})$ | $(\mathrm{i})$ | $($ id,remW) | $(\mathrm{v})$ | (pos,mask) |
| Space | $O(n)$ | $O\left(n^{2}\right)$ | $O(n)$ | $O(n S)$ | $O(V)$ | $O\left(n 2^{n}\right)$ |
| Transition | subarray | submatrix | all $\mathfrak{j}<\mathrm{i}$ | take/ignore | all $n$ coins | all $n$ cities |
| Time | $O(1)$ | $O(1)$ | $O\left(n^{2}\right)$ | $O(n S)$ | $O(n V)$ | $O\left(2^{n} n^{2}\right)$ |

## Common Data Structures

| (5) 2 [ 7 [-5 16 12 ? ? ? |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |



## capacity $=12$


hash
keys




## Important Data Structures and Algorithms

- Data structures
- Standard library data structures
- Vector, stack, queue, heap, priority queue, sets, maps
- Other data structures
- Graph (adjacency list and adjacency matrix), Union/find, Segment tree, Fenwick tree, Trie
- Sorting
- Quick sort, Merge sort, Radix sort, Bucket sort
- Strings
- String matching (Knuth Morris Pratt, Aho-Corasick), pattern matching, trie, suffix trees, suffix arrays, recursive decent parsing


## Important Data Structures and Algorithms

- Dynamic programming
- Longest common subsequence, Longest increasing subsequence, o/1 Knapsack, Coin Change, Matrix Chain Multiplication, Subset sum, Partitioning
- Graphs
- Traversal (pre-, in- and post-order), finding cycles, finding connected components, finding articulation points, topological sort, flood fill, Euler cycle/Euler path, SSSP - Single source shortest path (Dijkstra, BellmanFord), APSP - All pairs shortest path (Floyd Warshall), transitive closure (Floyd Warshall), MST - Minimum spanning tree (Prim, Kruskal (using Union/find)), Maximal Bipartite Matching, Maximum flow, Maximum flow minimal cost, Minimal cut
- Search
- Exhaustive search (depth-first, breadth-first search, backtracking), binary search (divide and conquer), greedy search (hill climbing), heuristic search ( $\mathrm{A}^{*}$, branch and bound), search trees


## Important Data Structures and Algorithms

- Mathematics
- Number theory (prime numbers, greatest common divisor (GCD), modulus), big integers, combinatorics (count permutations), number series (Fibonacci numbers, Catalan numbers, binomial coefficients), probabilities, linear algebra (matrix inversion, linear equations systems), finding roots to polynomial equations, diofantic equations, optimization (simplex)
- Computational geometry
- Representations of points, lines, line segments, polygons, finding intersections, point localization, triangulation, Voronoi diagrams, area and volume calculations, convex hull (Graham scan), sweep line algorithms


## Kattis（https：／／liu．kattis．com）

（4）AAPS／AAPS16－Kattis，Linköping...$\times+$

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## Advanced Algorithmic Problem Solving－AAPS／AAPS16

This course offering has no end date
I am a student taking this course and I want to register for it on Kattis．
－Course website
－Course offering website
－Problem list
－Students
－Export course data

## Teachers

－Henrik Adolfsson
－Tommy Färnquist

## Problem groups associated with this course：

－AAPS16 Exercise 1 （student results）
－AAPS16 Exercise 2 （student results）
－AAPS16 Exercise 3 （student results）
－AAPS16 Exercise 4 （student results）
－AAPS16 Exercise 5 （student results）
－AAPS16 Exercise 6 （student results）
－AAPS16 Exercise 7 （student results）
－AAPS16 Lab 1 （student results）
－AAPS16 Lab 2 （student results）

## How Kattis checks a program

## Compiles?

Compilation Error

For each test case


Runtime Error

Time Limit
Exceeded

Wrong
Answer

Accepted

## UVA Online Judge

## UVa

## http://uva.onlinejudge.org/

## Online Judge

## Home

```
Google" Custom Search
```


## Search

## Main Menu

## Home

My Account
Contact Us
TOOLS on the Old UVa OJ Site
ACM-ICPC Live Archive
Logout

## Online Judge

Quick Submit
Migrate submissions
My Submissions
My Statistics
My uHunt with Virtual Contest Service
Browse Problems

## UVa OJ fundraising campaing

As you may already discovered by the widget shown on the left, we have started a fundraising campaing to create a whole new UVa Online Judge. Please, take a couple of minutes to read the reasons for this on the campaing website, by clicking on the widget.

## Welcome to the UVa Online Judge

Categorized set of problems
Here you will find hundreds of problems. They are like the ones used during programming contests, and are available in HTML and PDF formats. You can submit your sources in a variety of languages, trying to solve any of the problems available in our database.

See the new Contest Rankings section at the Live Rankings link.

Now you can use the new Quick access, info and search option on the left menu for and easier navigation. (The tool will be updated next days

This book contains a collection of relevant data structures, algorithms, and programming tips written for University students who want to be more competitive in the

ACM International Collegiate Programming

Contest (ICPC), high school students who are aspiring to be competitive
in the International Olympiad in Informatics

## Programming languages

- Allowed languages are C, C++, Java, and Python.
- C++ or Java is strongly recommended, use the language that you are most familiar with and want to learn more about.
- Get to know their standard libraries.
- Get to know input and output. Remember that I/O in Java is very slow, use Kattio. Remember that cout/cerr also is relatively slow, learn how to use scanf/printf if you use C++.
- Learn to use an appropriate IDE such as eclipse, emacs, or vim
- Create a problem template to speed up problem solving and to create a common format for your problems.


## Pragmatic Algorithmic Problem Solving

```
/* -*- Mode+ [++ -*- */
/**
* XXX NAME C++ "Approach"
*
* Started*
* Finished*
* Total time*
*
* Submission 1+
*
* Comments;
*
* Lessons learned+
*/
#include <algorithm>
#include <cassert>
#include <cmath>
#include <cstdio>
#include <cstdlib>
#include <cstring>
#include <functional>
#include <iomanip>
#include <iostream>
#include <sstream>
#include <map>
#include <set>
#include <queue>
#include <stack>
#include <string>
#include <utility>
#include <vector>
using namespace std;
typedef vector<int> vi;
int
main(int argc, char* argu[])
{
\ return 0;
```


## Testing and debugging

- Always create an example input (.in) and example output (.out) file with verbatim copies of the example input and output from the problem statement!
- For most problems it is enough to diff your output with the example output: ./prog < prog.in | diff - prog.out
- Create additional tests, such as:
" Extreme inputs, i.e. smallest and largest values ( 0,1, "", empty line, $2^{\wedge} 31-1$ )
- Small inputs that you can compute by hand
- Potentially tricky cases such as when all inputs are equal, in the case of floating points numbers when you have to round both up and down
- Very large cases, randomly generated to test that your program computes an answer fast enough (even though you might not know the correct answer).
- Use a correct but slow algorithm to compute answers.
- Print intermediate information, such as values of relevant variables.
cout $\ll$ " $\mathrm{a}=$ " $\ll$ a $\ll$ "; $\mathrm{b}=$ " $\ll$ b $\ll$ endl; Remember to remove the debug output before submitting! (or use cerr)


## Summary

- What is algorithmic problem solving?
- Why is algorithmic problem solving important?
- What will be studied in this course?
- A method for algorithmic problem solving
- Common algorithmic problem solving approaches
- Common data structures and algorithms
- Pragmatic algorithmic problem solving using Kattis

