Amortized complexity: Insertion in a dynamic array

n	cap	allocate	copy	write	free	$2^{\lfloor \log(n-1) \rfloor}$
0	0					
		1	0	1	0	
1	1					–
		2	1	1	1	
2	2					1
		4	2	1	2	
3	4					2
		0	0	1	0	
4	4					2
		8	4	1	4	
5	8					4
		0	0	1	0	
6	8					4
		0	0	1	0	
7	8					4
		0	0	1	0	
8	8					4
		16	8	1	8	
9	16					8
		0	0	1	0	
10	16					8

$$\text{sum}(n) = 2\cdot al\cdot c(n) + cp\cdot c(n) + wr\cdot n + fr\cdot c(n)$$

$$= (2\cdot al + cp + fr)\cdot c(n) + wr\cdot n = a\cdot c(n) + b\cdot n$$

where $c(n) = 1 + 2 + 4 + \_\_\_\_ + 2^{\lfloor \log(n-1) \rfloor}$

$$= 2^{\lfloor \log(n-1) \rfloor + 1} - 1$$

you can show: $\alpha\cdot n \leq c(n) \leq \beta\cdot n$

with $0 < \alpha \leq \beta$

so there is a $k \in \mathbb{N}$ such that, for any $n > 0$:

$$\frac{\text{sum}(n)}{n} \leq k$$

and hence, in average, insertion is in $O(1)$