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// Size of the problem is  $n = hi - lo$ .
// You can assume  $n = 2^p$  for some integer  $p$ .
int f5(int a[], int lo, int hi, int x){
    if(lo >= hi){
        return 0;
    }
    int m = lo + (hi - lo)/2;
    int a = f5(a, lo, m, x);
    int b = f5(a, m+1, hi, x);
    if(a[m] == 0){
        return a + b + 1;
    }
    else{
        return a + b;
    }
}

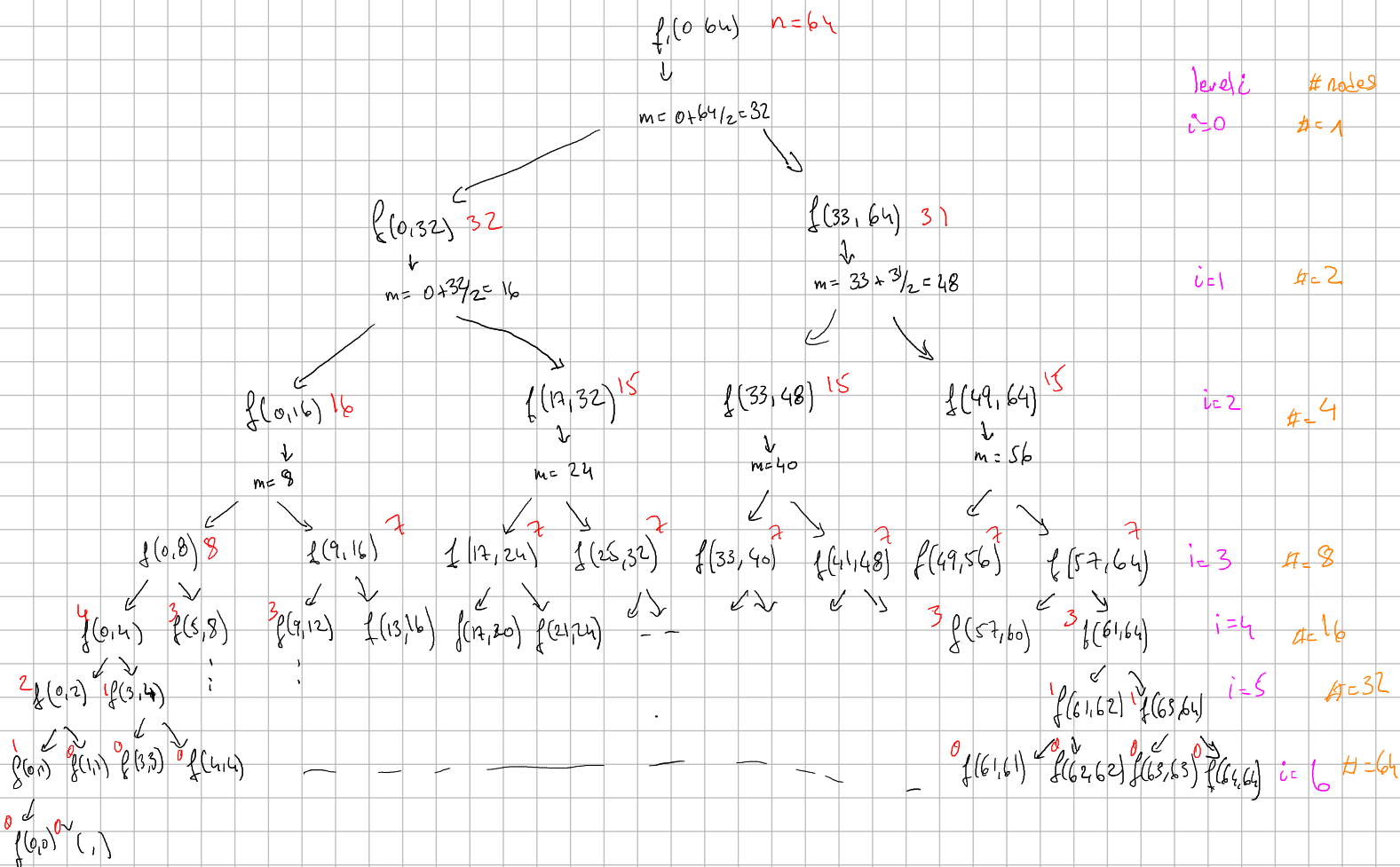
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Just to illustrate, suppose:

$lo = 0$ and $hi = 64$.

we write $f(0, 64)$ to mean a call to

$f_5(a, 0, 64, x)$ for some a and x .



- There are $(\log_2 n) + 2$ levels. The last level (i , $i = \log_2 n + 1$) contains 2 nodes. All other levels satisfy #nodes at level i is 2^i (2^0 at first level, 2^1 at second, ...).
- Each node corresponds to an instance of f_5 . Each node will have an amount of work between c_1 (just a test and a return) and c_2 (a pair of tests and calls, a return, and a bounded number of arithmetic operations and assignments).
- The amount of work $w(i)$ at level i is therefore bounded by $2^i c_1$ and $2^i c_2$.
- Observe that $\sum_{i=0}^{\log_2 n} 2^i = 2^{(\log_2 n) + 1} - 1 = 2n - 1$.
- The total amount of work T satisfies: $c_1(2n - 1) + k \leq T \leq c_2(2n - 1) + k$ where k is the constant amount of work at level $\log_2(n) + 1$ which has 2 nodes.
- The function is therefore in $\Theta(n)$ (both for the best and the worst cases).