
TDDD16 Lesson 2, November 5 2010, Example Solutions

1.

a.

$xz=2323$, $zyx=323132$, $z^2=323323$, $x^7 = 2222222$

b.

$A^1=\{1,2,3\}$, $A^2=\{11,12,13,21,22,23,31,32,33\}$, $A^0=\{\epsilon\}$

c.

$A^* = A^0 \cup A^1 \cup A^2 \cup \dots$

$A^+ = A^1 \cup A^2 \cup A^3 \cup \dots$

2.

1. $\langle \text{exp} \rangle ::= \langle \text{term} \rangle$

2. $\quad \quad \quad | \langle \text{exp} \rangle + \langle \text{term} \rangle$

3. $\quad \quad \quad | \langle \text{exp} \rangle - \langle \text{term} \rangle$

4. $\langle \text{term} \rangle ::= \langle \text{factor} \rangle$

5. $\quad \quad \quad | \langle \text{term} \rangle * \langle \text{factor} \rangle$

6. $\quad \quad \quad | \langle \text{term} \rangle / \langle \text{factor} \rangle$

7. $\langle \text{factor} \rangle ::= (\langle \text{exp} \rangle)$

8. $\quad \quad \quad | \langle \text{ident} \rangle$

9. $\langle \text{ident} \rangle ::= A | B | C \dots | Z$

a.

Example derivations:

A^*B-C

Starting with $\langle \text{exp} \rangle$

$\langle \text{exp} \rangle \rightarrow \underline{3} \rightarrow \langle \text{exp} \rangle - \langle \text{term} \rangle$

$\langle \text{exp} \rangle - \langle \text{term} \rangle \rightarrow \underline{4} \rightarrow \langle \text{exp} \rangle - \langle \text{factor} \rangle$

$\langle \text{exp} \rangle - \langle \text{factor} \rangle \rightarrow \underline{8} \rightarrow \langle \text{exp} \rangle - \langle \text{ident} \rangle$

$\langle \text{exp} \rangle - \langle \text{ident} \rangle \rightarrow \underline{9} \rightarrow \langle \text{exp} \rangle - C$

$\langle \text{exp} \rangle - C \rightarrow \underline{1} \rightarrow \langle \text{term} \rangle - C$

$\langle \text{term} \rangle - C \rightarrow \underline{5} \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle - C$

$\langle \text{term} \rangle * \langle \text{factor} \rangle - C \rightarrow \underline{8} \rightarrow \langle \text{term} \rangle * \langle \text{ident} \rangle - C$

$\langle \text{term} \rangle * \langle \text{ident} \rangle - C \rightarrow \underline{9} \rightarrow \langle \text{term} \rangle * B - C$

$\langle \text{term} \rangle * B - C \rightarrow \underline{4} \rightarrow \langle \text{factor} \rangle * B - C$

$\langle \text{factor} \rangle * B - C \rightarrow \underline{8} \rightarrow \langle \text{ident} \rangle * B - C$

$\langle \text{ident} \rangle * B - C \rightarrow \underline{9} \rightarrow A * B - C$

$A^*(B-C)$

Starting with $\langle \text{exp} \rangle$

$\langle \text{exp} \rangle \rightarrow \underline{1} \rightarrow \langle \text{term} \rangle$

$\langle \text{term} \rangle \rightarrow \underline{5} \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle$

$\langle \text{term} \rangle * \langle \text{factor} \rangle \rightarrow \underline{7} \rightarrow \langle \text{term} \rangle * (\langle \text{exp} \rangle)$

$\langle \text{term} \rangle * (\langle \text{exp} \rangle) \rightarrow \underline{3} \rightarrow \langle \text{term} \rangle * (\langle \text{exp} \rangle - \langle \text{term} \rangle)$

$\langle \text{term} \rangle * (\langle \text{exp} \rangle - \langle \text{term} \rangle) \rightarrow \underline{4} \rightarrow \langle \text{term} \rangle * (\langle \text{exp} \rangle - \langle \text{factor} \rangle)$

$\langle \text{term} \rangle * (\langle \text{exp} \rangle - \langle \text{factor} \rangle) \rightarrow \underline{8} \rightarrow \langle \text{term} \rangle * (\langle \text{exp} \rangle - \langle \text{ident} \rangle)$

$\langle \text{term} \rangle * (\langle \text{exp} \rangle - \langle \text{factor} \rangle) \rightarrow \underline{9} \rightarrow \langle \text{term} \rangle * (\langle \text{exp} \rangle - C)$

$\langle \text{term} \rangle * (\langle \text{exp} \rangle - C) \rightarrow \underline{1} \rightarrow \langle \text{term} \rangle * (\langle \text{term} \rangle - C)$

$\langle \text{term} \rangle * (\langle \text{term} \rangle - C) \rightarrow \underline{4} \rightarrow \langle \text{term} \rangle * (\langle \text{factor} \rangle - C)$

$\langle \text{term} \rangle * (\langle \text{factor} \rangle - C) \rightarrow \underline{8} \rightarrow \langle \text{term} \rangle * (\langle \text{ident} \rangle - C)$

$\langle \text{term} \rangle * (\langle \text{factor} \rangle - C) \rightarrow \underline{9} \rightarrow \langle \text{term} \rangle * (B - C)$

$\langle \text{term} \rangle * (B - C) \rightarrow 4 \rightarrow \langle \text{factor} \rangle * (B - C)$
 $\langle \text{factor} \rangle * (B - C) \rightarrow 8 \rightarrow \langle \text{ident} \rangle * (B - C)$
 $\langle \text{ident} \rangle * (B - C) \rightarrow 9 \rightarrow A * (B - C)$

A/B/C

Starting with $\langle \text{exp} \rangle$

$\langle \text{exp} \rangle \rightarrow 1 \rightarrow \langle \text{term} \rangle$
 $\langle \text{term} \rangle \rightarrow 6 \rightarrow \langle \text{term} \rangle / \langle \text{factor} \rangle$
 $\langle \text{term} \rangle / \langle \text{factor} \rangle \rightarrow 8 \rightarrow \langle \text{term} \rangle / \langle \text{ident} \rangle$
 $\langle \text{term} \rangle / \langle \text{ident} \rangle \rightarrow 9 \rightarrow \langle \text{term} \rangle / C$
 $\langle \text{term} \rangle / C \rightarrow 6 \rightarrow \langle \text{term} \rangle / \langle \text{factor} \rangle / C$
 $\langle \text{term} \rangle / \langle \text{factor} \rangle / C \rightarrow 8 \rightarrow \langle \text{term} \rangle / \langle \text{ident} \rangle / C$
 $\langle \text{term} \rangle / \langle \text{ident} \rangle / C \rightarrow 9 \rightarrow \langle \text{term} \rangle / B / C$
 $\langle \text{term} \rangle / B / C \rightarrow 4 \rightarrow \langle \text{factor} \rangle / B / C$
 $\langle \text{factor} \rangle / B / C \rightarrow 8 \rightarrow \langle \text{ident} \rangle / B / C$
 $\langle \text{ident} \rangle / B / C \rightarrow 9 \rightarrow A / B / C$

$-A*B$ can not be derived since we do not have unary minus in the grammar

Parse trees: (These were drawn during the lesson)

b. $\langle \text{term} \rangle ::= \langle \text{term} \rangle * \langle \text{factor} \rangle$, Position 0

c.

If every derivation step is rightmost, then this is a canonical derivation.

d. It cannot be derived

e.

$$V = N \cup \Sigma$$

$$\Sigma = \{A, B, \dots, Z, (,), +, -, /, *\}$$

$$N = \{\langle \text{exp} \rangle, \langle \text{term} \rangle, \langle \text{factor} \rangle, \langle \text{ident} \rangle\}$$

f.

$$\sum^+$$

Strings of terminal symbols only, containing at least one terminal symbol. (is in this set but not in the language $L(G)$).

$L(G)$

Language generated by grammar G

4.

$\langle \text{even} \rangle ::= \langle \text{start} \rangle \langle \text{middle} \rangle \langle \text{end} \rangle \mid \langle \text{evennr} \rangle$
 $\langle \text{oddnr} \rangle ::= 1 \mid 3 \mid 5 \mid 7$
 $\langle \text{evennr} \rangle ::= 2 \mid 4 \mid 6 \mid 8$
 $\langle \text{start} \rangle ::= \langle \text{oddnr} \rangle \mid \langle \text{evennr} \rangle$
 $\langle \text{middle} \rangle ::= \langle \text{oddnr} \rangle \langle \text{middle} \rangle$
 $\quad \mid \langle \text{evennr} \rangle \langle \text{middle} \rangle$
 $\quad \mid 0 \langle \text{middle} \rangle$
 $\quad \mid \text{epsilon}$
 $\langle \text{end} \rangle ::= \langle \text{evennr} \rangle \mid 0$

5.

$(0 + 1)^+ ((- + +) (0 + 1)^+)^*$

6.

a.

Exactly one vowel

$\langle \text{start} \rangle ::= \langle A \rangle \langle \text{vowel} \rangle \langle A \rangle$
 $\langle A \rangle ::= \langle \text{not_vowel} \rangle \langle A \rangle \mid \text{epsilon}$
 $\langle \text{not_vowel} \rangle ::= b \mid c \mid d \mid f \mid g \mid h$
 $\langle \text{vowel} \rangle ::= a \mid e \mid i$

b.

At least one vowel

$\langle \text{start} \rangle ::= \langle A \rangle \langle \text{vowel} \rangle \langle A \rangle$
 $\langle A \rangle ::= \langle \text{not_vowel} \rangle \langle A \rangle \mid \langle \text{vowel} \rangle \langle A \rangle \mid \text{epsilon}$
 $\langle \text{not_vowel} \rangle ::= b \mid c \mid d \mid f \mid g \mid h$
 $\langle \text{vowel} \rangle ::= a \mid e \mid i$

7.

a. $a^* (b+c)^* a^*$

b. $a^n b^n c^n$, $n \geq 0$

No, regular expressions "can't count".

See for instance *Automata and Computability*, Dexter C. Kozen, Springer Verlag.

8.

a.

$1^n 0^n 1^m 0^m \mid n > 0, m > 0$

b.

$1^n 0^m 1^m 0^n \mid n \geq 0, m \geq 0$

c.

$1^n 1^m 0^m \mid n > 0, m \geq 0$

OR

$1^m 0^m 0^n \mid n > 0, m \geq 0$