TDDD55, Exercises Lesson 2, Example Solutions

Author: Jonas Wallgren, updated by John Tinnerholm 2021 and 2022

a.

xz=213213, zyx=323132, z^2=323323, x^7 = 2222222

b.

A^1={1,2,3}, A^2={11,12,13,21,22,23,31,32,33}, A^0 = ε

c.

$A^* = A^0 \cup A^1 \cup A^2 \cup \ldots$

$A^+ = A^1 \cup A^2 \cup A^3 \cup \ldots$

2.

1. <exp> ::= <term>
2. | <exp> + <term>
3. | <exp> - <term>
4. <term> ::= <factor>
5. | <term> * <factor>
6. | <term> / <factor>
7. <factor> ::= (<exp>)

8 | <ident>

9. <ident> ::= A | B | C ... | Z

a.

Example derivations: A*B-C

Starting with <exp>

<exp> **-> 3 ->** <exp> - <term>

<exp> - <term> **-> 4 ->** <exp> - <factor>

<exp> - <factor> **-> 8 ->** <exp> - <ident>

<exp> - <ident> **-> 9 ->** <exp> - C

<exp> - C **-> 1 ->** <term> - C

<term> - C **-> 5 ->** <term> * <factor> - C

<term> * <factor> - C **-> 8 ->** <term> * <ident> - C

<term> * <ident> - C **-> 9 ->** <term> * B - C

<term> * B - C **-> 4 ->** <factor> * B - C

<factor> * B - C **-> 8 ->** <ident> * B - C

<ident> * B - C **-> 9 ->** A * B - C

A*(B-C)
Starting with <exp>
<exp> -**> 1 ->** <term>
<term> -**> 5 ->** <term> * <factor>
<term> * <factor> -**> 7 ->** <term> * (<exp>)
<term> * (<exp>) -**> 3 ->** <term> * (<exp> - <term>)
<term> * (<exp> - <term>) -**> 4 ->** <term> * (<exp> - <factor>)
<term> * (<exp> - <factor>) -**> 8 ->** <term> * (<exp> - <ident>)
<term> * (<exp> - <factor>) -**> 9 ->** <term> * (<exp> - C)
<term> * (<exp> - C) -**> 1 ->** <term> * (<term> - C)
<term> * (<term> - C) -**> 4 ->** <term> * (<factor> - C)
<term> * (<factor> - C) -**> 8 ->** <term> * (<ident> - C)
<term> * (<factor> - C) -**> 9 ->** <term> * (B - C)
<term> * (B - C) **-> 4 ->** <factor> * (B - C)
<factor> * (B - C) **-> 8 ->** <ident> * (B - C)
<ident> * (B - C) **-> 9 ->** A * (B - C)

A/B/C
Starting with <exp>
<exp> -**> 1 ->** <term>
<term> -**> 6 ->** <term> / <factor>
<term> / <factor> -**> 8 ->** <term> / <ident>
<term> / <ident>-**> 9 ->** <term> / C
<term> / C -**> 6 ->** <term> / <factor> / C
<term> / <factor> / C -**> 8 ->** <term> / <ident> / C
<term> / <ident> / C -**> 9 ->** <term> / B / C
<term> / B / C -**> 4 ->** <factor> / B / C
<factor> / B / C -**> 8 ->** <ident> / B / C
<ident> / B / C -**> 9 ->** A / B / C

-A*B can not be derived since we do not have unary minus in the grammar

<u>Parse trees:</u> (Available on the following pages)

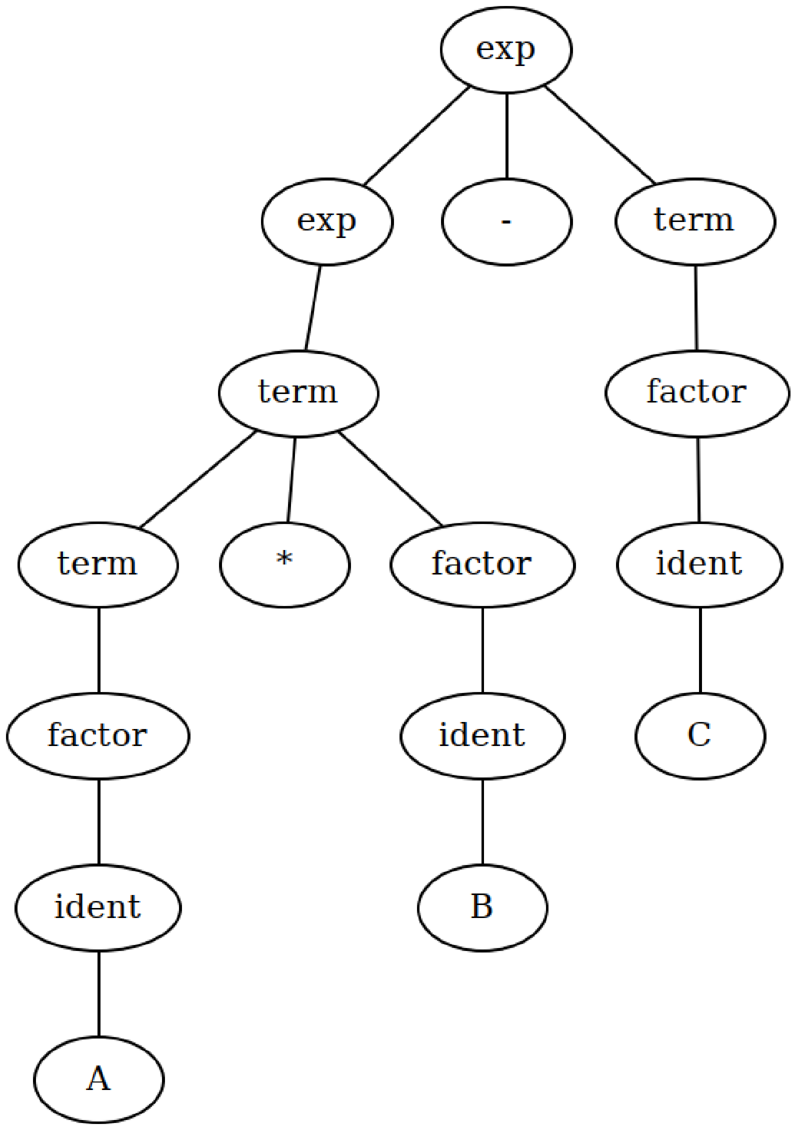

*Figure 1 Parse Tree for A*B-C*

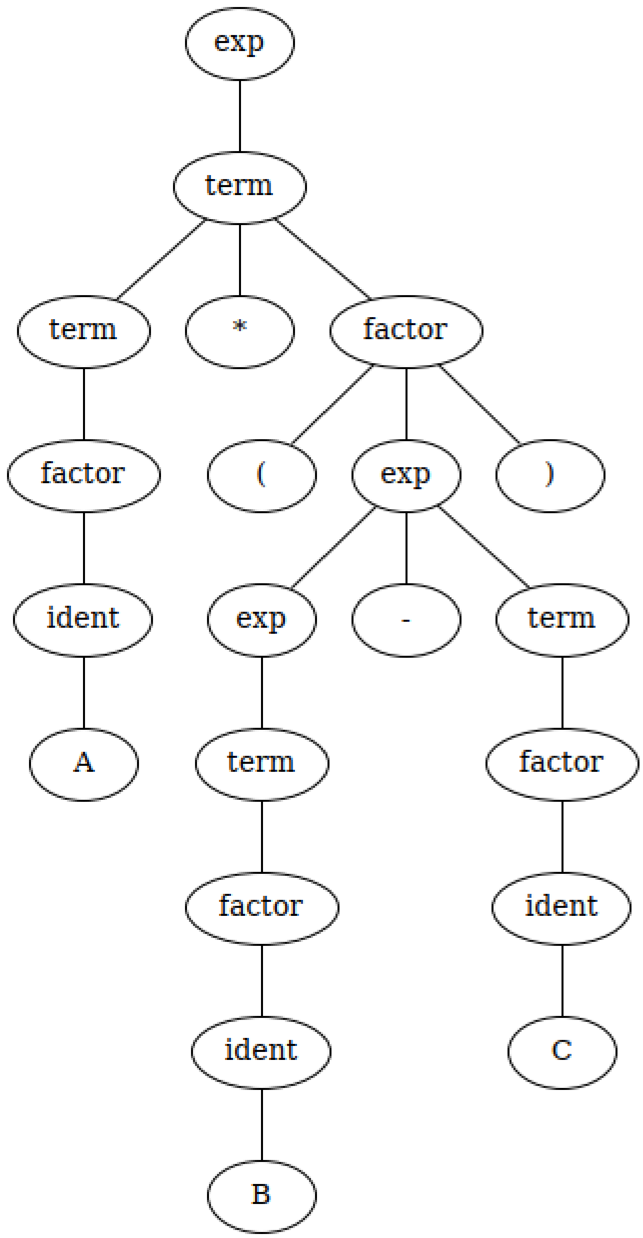

*Figure 2 Parse Tree for A * (B - C)*

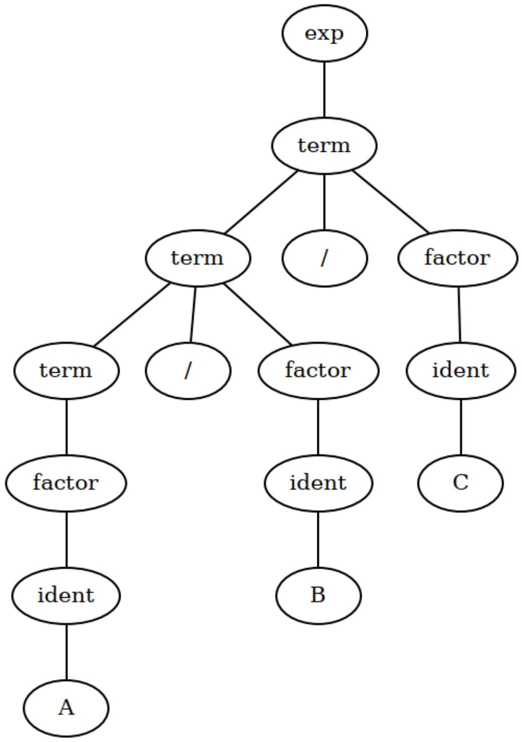

Figure 3 Parse Tree for A / B / C

b. <term> ::= <term>*<factor>, Position 0

(See lecture 3 for a detailed explanation of handle and position)

c.
If every derivation step is rightmost, then this is a canonical derivation.

d. It cannot be derived

e.

$V = N \cup \Sigma$

$\Sigma\ = \{A, B, \dots, Z, (,), +, -, /*\}$

N={<exp>,<term>,<factor>,<ident>}

f.

Σ^+

Strings of terminal symbols only, containing at least one terminal symbol.
(This is in this set but not in the language L(G)).
L(G)
The language generated by grammar G

4.
a) The grammar is left-recursive. Either we use a parser technique capable to deal with left recursive grammar or we need to transform the grammar into a grammar without left-recursion.
b) Demonstrated on the lesson
c) Demonstrated on the lesson

<u>5.</u>
<even> ::= <start> <middle> <end> | <evennr>
<oddnr> ::= 1 | 3 | 5 | 7
<evennr> ::= 2 | 4 | 6 | 8
<start> ::= <oddnr> | <evennr>
<middle> ::= <oddnr><middle>
| <evennr><middle>
| 0 <middle>
| epsilon
<end> ::= <evennr> | 0

<u>6.</u>
a.
Exactly one vowel
<start> ::= <A> <vowel> <A>
<A> ::= <not_vowel> <A> | epsilon
<not_vowel> ::= b | c | d | f | g | h
<vowel> ::= a | e | i

b.

At least one vowel
<start> ::= <A> <vowel> <A>
<A> ::= <not_vowel> <A> | <vowel> <A> | epsilon
<not_vowel> ::= b | c | d | f | g | h
<vowel> ::= a | e | i

7.

a. $a^* (b+c)^* a^*$

b. $a^n b^n c^n$, n=>0
No, regular expressions "can't count".
See for instance ***Automata and Computability***, *Dexter C. Kozen, Springer Verlag.*

8.

a.
$1^n 0^n 1^m 0^m$ | n>0, m>0
b.
$1^n 0^m$ 1^m 0^n | n=>0, m=>0
c.
$1^n 1^m 0^m$ | n>0, m=>0
OR
$1^m 0^m 0^n$ | n>0, m=>0