

## TDDD55, Exercises Lesson 2, Example Solutions

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a.

$xz=213213$ ,  $zyx=323132$ ,  $z^2=323323$ ,  $x^7 = 2222222$

b.

$A^1=\{1,2,3\}$ ,  $A^2=\{11,12,13,21,22,23,31,32,33\}$ ,  $A^0 = \epsilon$

c.

$A^* = A^0 \cup A^1 \cup A^2 \cup \dots$

$A^+ = A^1 \cup A^2 \cup A^3 \cup \dots$

## 2.

1.  $\langle \text{exp} \rangle ::= \langle \text{term} \rangle$
2.  $\quad | \langle \text{exp} \rangle + \langle \text{term} \rangle$
3.  $\quad | \langle \text{exp} \rangle - \langle \text{term} \rangle$
4.  $\langle \text{term} \rangle ::= \langle \text{factor} \rangle$
5.  $\quad | \langle \text{term} \rangle * \langle \text{factor} \rangle$
6.  $\quad | \langle \text{term} \rangle / \langle \text{factor} \rangle$
7.  $\langle \text{factor} \rangle ::= ( \langle \text{exp} \rangle )$
8.  $\quad | \langle \text{ident} \rangle$
9.  $\langle \text{ident} \rangle ::= A \mid B \mid C \dots \mid Z$

a.

Example derivations:  $A^*B-C$

Starting with  $\langle \text{exp} \rangle$

$\langle \text{exp} \rangle \xrightarrow{3} \langle \text{exp} \rangle - \langle \text{term} \rangle$

$\langle \text{exp} \rangle - \langle \text{term} \rangle \xrightarrow{4} \langle \text{exp} \rangle - \langle \text{factor} \rangle$

$\langle \text{exp} \rangle - \langle \text{factor} \rangle \xrightarrow{8} \langle \text{exp} \rangle - \langle \text{ident} \rangle$

$\langle \text{exp} \rangle - \langle \text{ident} \rangle \xrightarrow{9} \langle \text{exp} \rangle - C$

$\langle \text{exp} \rangle - C \xrightarrow{1} \langle \text{term} \rangle - C$

$\langle \text{term} \rangle - C \xrightarrow{5} \langle \text{term} \rangle * \langle \text{factor} \rangle - C$

$\langle \text{term} \rangle * \langle \text{factor} \rangle - C \xrightarrow{8} \langle \text{term} \rangle * \langle \text{ident} \rangle - C$

$\langle \text{term} \rangle * \langle \text{ident} \rangle - C \xrightarrow{9} \langle \text{term} \rangle * B - C$

$\langle \text{term} \rangle * B - C \xrightarrow{4} \langle \text{factor} \rangle * B - C$

$\langle \text{factor} \rangle * B - C \xrightarrow{8} \langle \text{ident} \rangle * B - C$

$\langle \text{ident} \rangle * B - C \xrightarrow{9} A * B - C$

A\*(B-C)

Starting with <exp>

<exp> -> **1** -> <term>

<term> -> **5** -> <term> \* <factor>

<term> \* <factor> -> **7** -> <term> \* ( <exp> )

<term> \* ( <exp> ) -> **3** -> <term> \* ( <exp> - <term> )

<term> \* ( <exp> - <term> ) -> **4** -> <term> \* ( <exp> - <factor> )

<term> \* ( <exp> - <factor> ) -> **8** -> <term> \* ( <exp> - <ident> )

<term> \* ( <exp> - <factor> ) -> **9** -> <term> \* ( <exp> - C )

<term> \* ( <exp> - C ) -> **1** -> <term> \* ( <term> - C )

<term> \* ( <term> - C ) -> **4** -> <term> \* ( <factor> - C )

<term> \* ( <factor> - C ) -> **8** -> <term> \* ( <ident> - C )

<term> \* ( <factor> - C ) -> **9** -> <term> \* ( B - C )

<term> \* ( B - C ) -> **4** -> <factor> \* ( B - C )

<factor> \* ( B - C ) -> **8** -> <ident> \* ( B - C )

<ident> \* ( B - C ) -> **9** -> A \* ( B - C )

A/B/C

Starting with <exp>

<exp> -> **1** -> <term>

<term> -> **6** -> <term> / <factor>

<term> / <factor> -> **8** -> <term> / <ident>

<term> / <ident> -> **9** -> <term> / C

<term> / C -> **6** -> <term> / <factor> / C

<term> / <factor> / C -> **8** -> <term> / <ident> / C

<term> / <ident> / C -> **9** -> <term> / B / C

<term> / B / C -> **4** -> <factor> / B / C

<factor> / B / C -> **8** -> <ident> / B / C

<ident> / B / C -> **9** -> A / B / C

-A\*B can not be derived since we do not have unary minus in the grammar

Parse trees: (Available on the following pages)

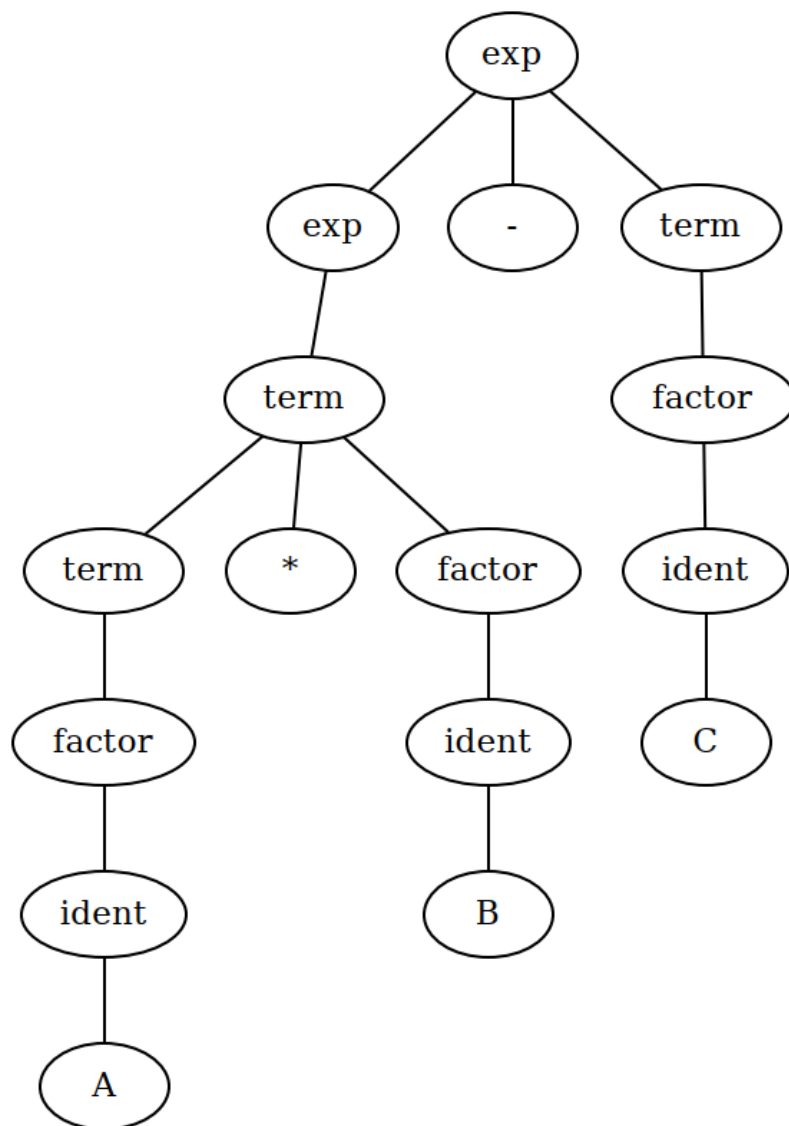


Figure 1 Parse Tree for  $A * B - C$

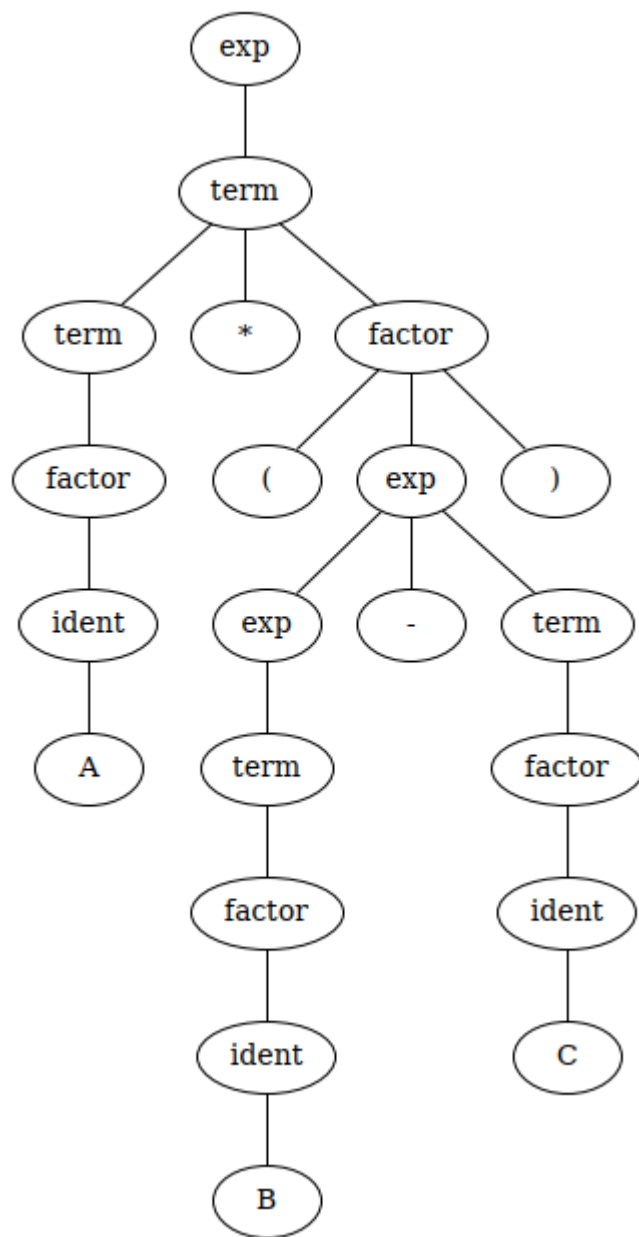


Figure 2 Parse Tree for  $A * (B - C)$

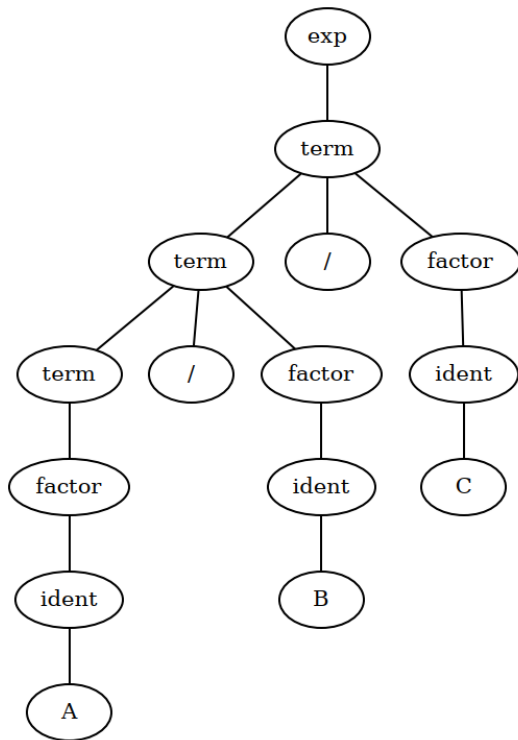


Figure 3 Parse Tree for  $A / B / C$

**b.** `<term> ::= <term>*<factor>`, Position 0

(See lecture 3 for a detailed explanation of handle and position)

c.

If every derivation step is rightmost, then this is a canonical derivation.

d. It cannot be derived

e.

$$V = N \cup \Sigma$$

$$\Sigma = \{A, B, \dots, Z, (, ), +, -, /*\}$$

$$N = \{\langle \text{exp} \rangle, \langle \text{term} \rangle, \langle \text{factor} \rangle, \langle \text{ident} \rangle\}$$

f.

$$\Sigma^+$$

Strings of terminal symbols only, containing at least one terminal symbol.

**(This is in this set but not in the language  $L(G)$ ).**

**$L(G)$**

The language generated by grammar G

4.

a) The grammar is left-recursive. Either we use a parser technique capable to deal with left recursive grammar or we need to transform the grammar into a grammar without left-recursion.

b) Demonstrated on the lesson

c) Demonstrated on the lesson

5.

$$\langle \text{even} \rangle ::= \langle \text{start} \rangle \langle \text{middle} \rangle \langle \text{end} \rangle \mid \langle \text{evennr} \rangle$$

$$\langle \text{oddnr} \rangle ::= 1 \mid 3 \mid 5 \mid 7$$

$$\langle \text{evennr} \rangle ::= 2 \mid 4 \mid 6 \mid 8$$

$$\langle \text{start} \rangle ::= \langle \text{oddnr} \rangle \mid \langle \text{evennr} \rangle$$

$$\langle \text{middle} \rangle ::= \langle \text{oddnr} \rangle \langle \text{middle} \rangle$$

$$\mid \langle \text{evennr} \rangle \langle \text{middle} \rangle$$

$$\mid 0 \langle \text{middle} \rangle$$

$$\mid \text{epsilon}$$

$$\langle \text{end} \rangle ::= \langle \text{evennr} \rangle \mid 0$$

6.

a.

Exactly one vowel

$$\langle \text{start} \rangle ::= \langle A \rangle \langle \text{vowel} \rangle \langle A \rangle$$

$$\langle A \rangle ::= \langle \text{not\_vowel} \rangle \langle A \rangle \mid \text{epsilon}$$

$$\langle \text{not\_vowel} \rangle ::= b \mid c \mid d \mid f \mid g \mid h$$

$$\langle \text{vowel} \rangle ::= a \mid e \mid i$$

b.

At least one vowel

$\langle \text{start} \rangle ::= \langle A \rangle \langle \text{vowel} \rangle \langle A \rangle$

$\langle A \rangle ::= \langle \text{not\_vowel} \rangle \langle A \rangle \mid \langle \text{vowel} \rangle \langle A \rangle \mid \text{epsilon}$

$\langle \text{not\_vowel} \rangle ::= b \mid c \mid d \mid f \mid g \mid h$

$\langle \text{vowel} \rangle ::= a \mid e \mid i$

## 7.

a.  $a^* (b+c)^* a^*$

b.  $a^n b^n c^n, n \geq 0$

No, regular expressions "can't count".

See for instance ***Automata and Computability***, Dexter C. Kozen, Springer Verlag.

## 8.

a.

$1^n 0^n 1^m 0^m \mid n > 0, m > 0$

b.

$1^n 0^m 1^m 0^n \mid n \geq 0, m \geq 0$

c.

$1^n 1^m 0^m \mid n > 0, m \geq 0$

OR

$1^m 0^m 0^n \mid n > 0, m \geq 0$