



Automated Planning

Pattern Databases:
"Who cares about that?"

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- **First** main idea behind pattern databases:
 - Let's care about a few facts and ignore all others – everywhere
 - Goals, preconditions, effects, states

A form of relaxation...

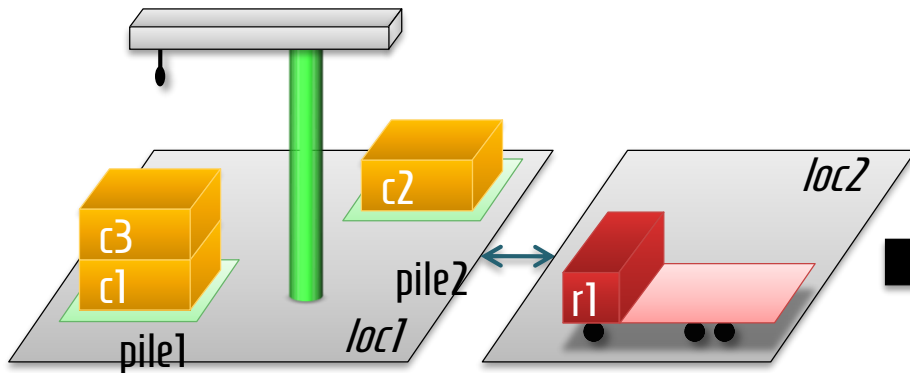
With some special features
(later!)



PDB 2: Dock Worker Robots

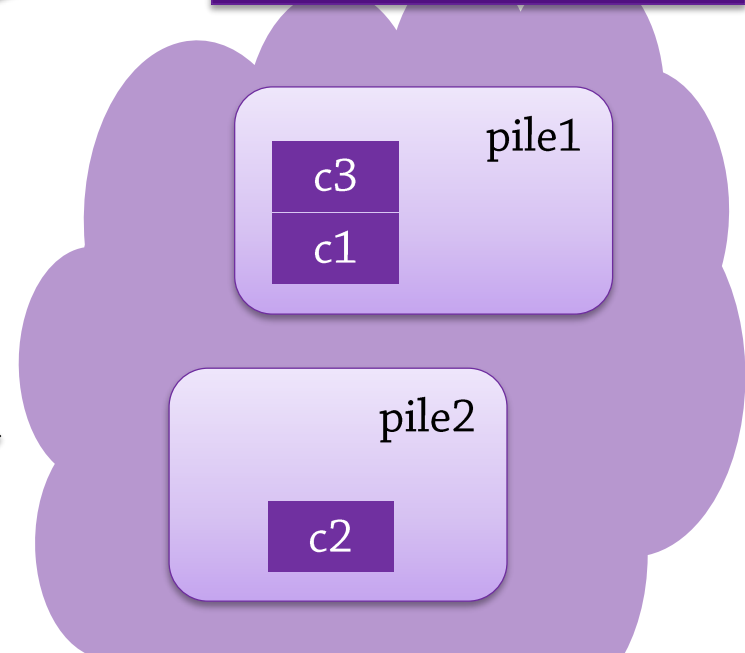
- Example from Dock Worker Robots
 - Could choose to care about container locations
 - *in(container, pile), top(container, pile), on(c1, c2), ...*
 - **Ignore** robot locations, crane locations, location adjacency, ...

Ordinary state in problem P,
all facts included



States are
"grouped together"!

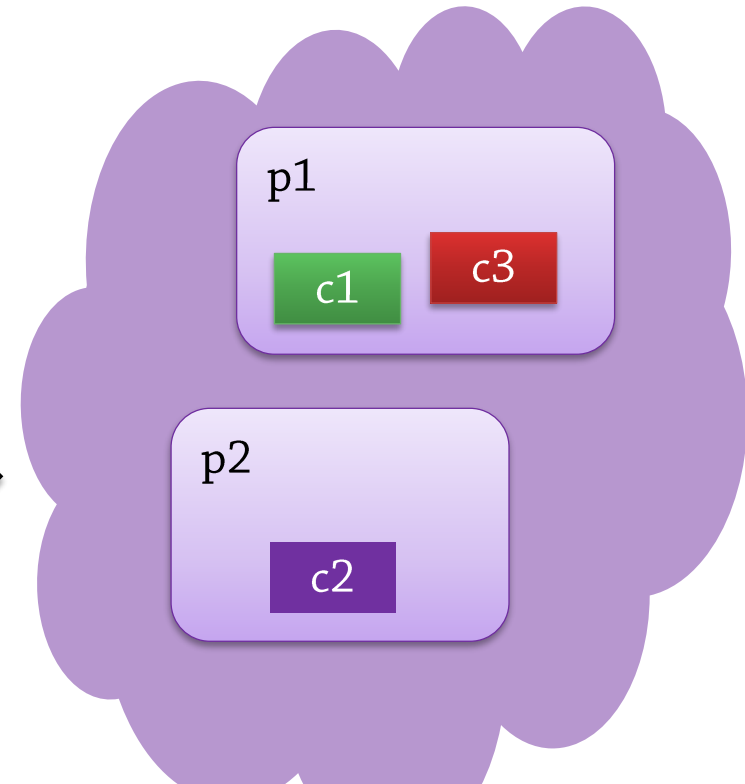
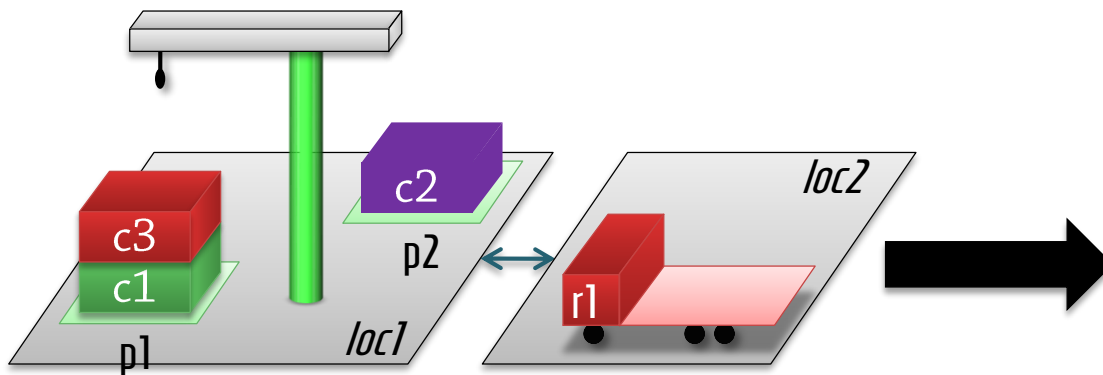
Abstract state in P',
representing
many states in P
with different
robot locations, ...



PDB 3: Planning in Patterns

- In P' we (pretend that we) can use the crane at p1 to:
 - **pick up** c3 (should be possible)
 - **place** something on r1 (too far away, but we don't care)
 - **place** five containers on a single truck
- But we can't:
 - pick up c1 (we do care about pile ordering)
 - immediately place c1 below c2, ...

New paths
to the goal!



Let's formalize!

-

- All ground atoms (facts) in this problem instance:

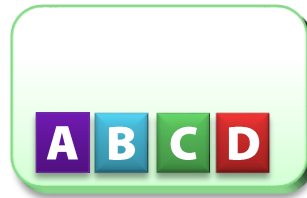
- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| (on A A) | (on A B) | (on A C) | (on A D) |
| (on B A) | (on B B) | (on B C) | (on B D) |
| (on C A) | (on C B) | (on C C) | (on C D) |
| (on D A) | (on D B) | (on D C) | (on D D) |

(ontable A) (ontable B) (ontable C) (ontable D)
(clear A) (clear B) (clear C) (clear D)
(holding A) (holding B) (holding C) (holding D)

(handempty)

- Example: only consider 5 ground facts related to block A
 - "Pattern": $p = \{(\text{on A B}), (\text{on A C}), (\text{on A D}), (\text{clear A}), (\text{ontable A})\}$

- **Initial state:**



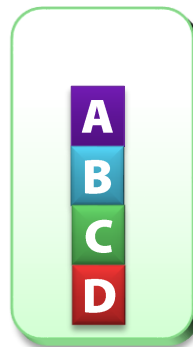
ontable(A)
ontable(B)
ontable(C)
ontable(D)
clear(A)
clear(B)
clear(C)
clear(D)
handempty



ontable(A)
ontable(B)
ontable(C)
ontable(D)
clear(A)
clear(B)
clear(C)
clear(D)
handempty

An "abstract state"

- **Goal:**



clear(A)
on(A,B)
on(B,C)
on(C,D)
ontable(D)
handempty



clear(A)
on(A,B)
on(B,C)
on(C,D)
ontable(D)
handempty

An "abstract goal"

BW4: Transformed Actions

- Pattern $p = \{(\text{on A B}), (\text{on A C}), (\text{on A D}), (\text{clear A}), (\text{ontable A})\}$

- **Example action:** (unstack A B)

- **Before transformation:**

:precondition (and (handempty) (**clear A**) (**on A B**))

:effect (and (not (handempty)) (holding A) (**not (clear A)**) (clear B)
(**not (on A B)**))

Loses **some** preconditions and effects

- **After transformation:**

:precondition (and (clear A) (on A B))

:effect (and (not (clear A)) (not (on A B)))

Let's call this action

$a \cap p$

(not a set, but "restricted to p ")

- **Example action:** (unstack C D)

- **Before transformation:**

:precondition (and (handempty) (clear C) (on C D))

:effect (and (not (handempty)) (holding C) (not (clear C)) (clear D)
(not (on C D)))

- **After transformation:**

:precondition (and)

:effect (and)

Loses **all** preconditions and effects → never used!

PDB Heuristics: Patterns, Abstract States

- Given a **pattern** (set of ground facts) p
 - A state s is represented by the **abstract state** $s \cap p$

(clear A)
(on A B)
(on B C)
(on C D)
(ontable D)
(handempty)

$\approx$

(clear A)
(on A B)
(ontable B)
(clear C)
(on C D)
(ontable D)
(handempty)

$\approx$

(clear A)
(on A B)
(on B D)
(on D C)
(ontable C)
(handempty)

represented
by a single
abstract
state

(clear A)
(on A B)

(clear A)
(ontable A)
(clear B)
(on B C)
(on C D)
(ontable D)
(handempty)

$\approx$

(clear A)
(ontable A)
(holding B)
(clear C)
(on C D)
(ontable D)

$\approx$

(clear A)
(ontable A)
(clear B)
(on B D)
(on D C)
(ontable C)
(handempty)

represented
by a single
abstract
state

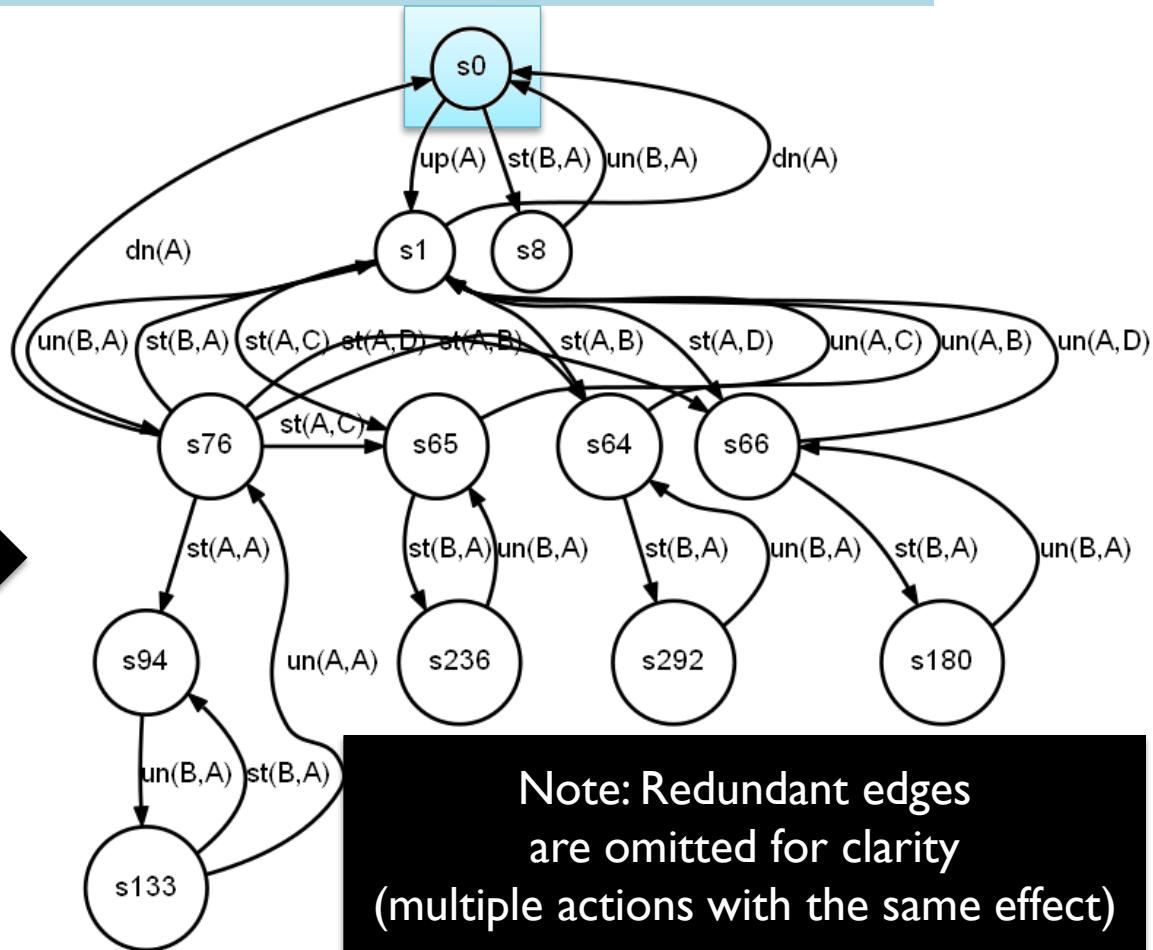
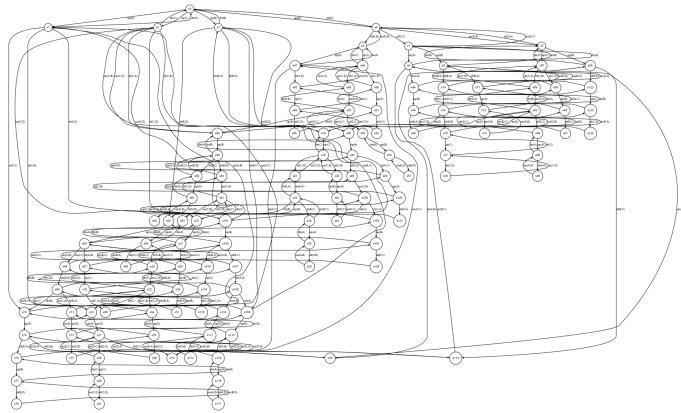
(clear A)
(ontable A)

BW4: Smaller State Transition Graph

- **Reachable state transition graph** for the given pattern:

- **Current state**: Everything on the table, hand empty, all blocks clear

- Abstract state: $s_0 = \{ (\text{ontable } A), (\text{clear } A) \}$

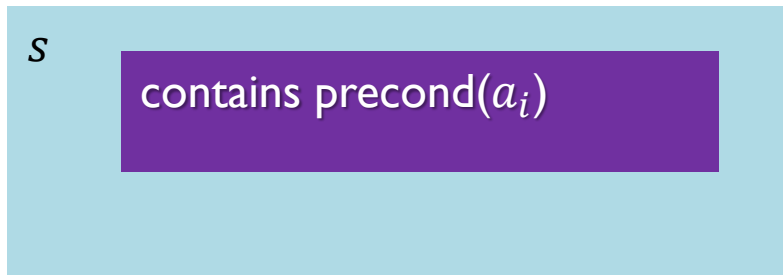


Relaxation!

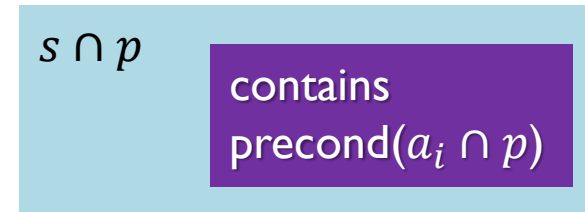
Relaxation! (1)

- Why is this a relaxation?

- Step 1: In s , we can execute a_i $\rightarrow$ In $s \cap p$, we can execute $a_i \cap p$



ontable(A)	clear(B)
ontable(B)	clear(C)
ontable(C)	clear(D)
ontable(D)	handempty
clear(A)	



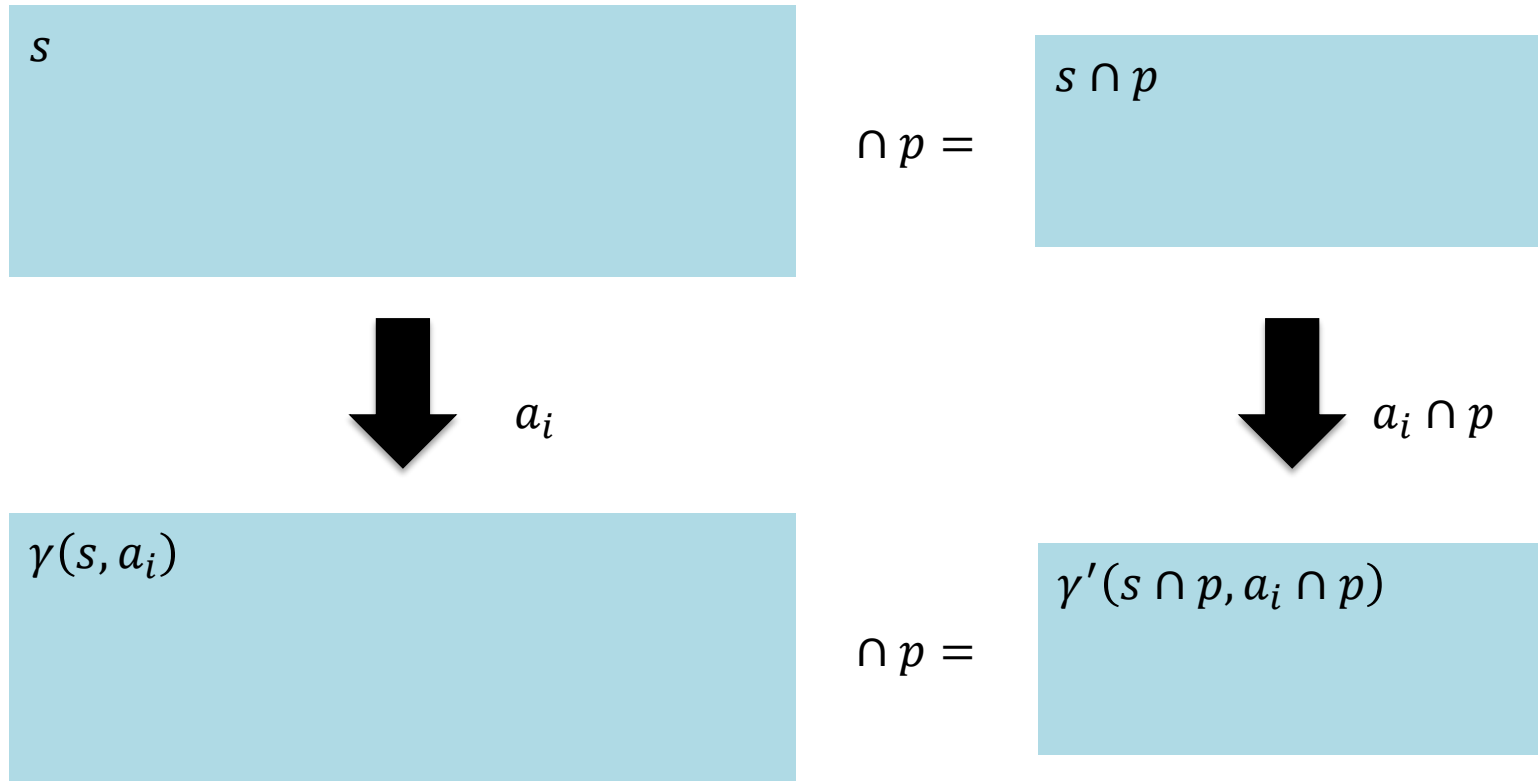
ontable(A)	clear(B)
ontable(B)	clear(C)
ontable(C)	clear(D)
ontable(D)	handempty
clear(A)	

Relaxation! (2)

■ Relaxation 2:

- If γ' is the state transition function for transformed actions/states, then:

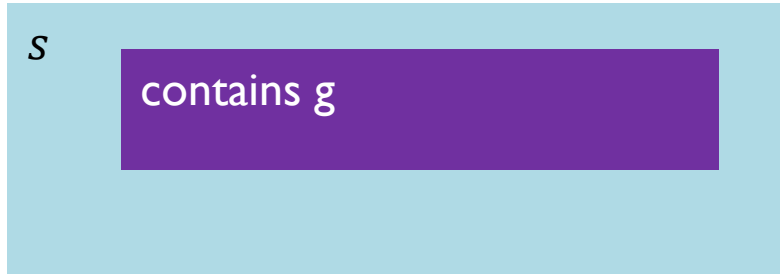
$$\gamma'(s \cap p, a_i \cap p) = \gamma(s, a_i) \cap p$$



So: Executable action sequences are preserved

Relaxation! (3)

- Relaxation 3:
 - If $g \subseteq s$, then $g \cap p \subseteq s \cap p$



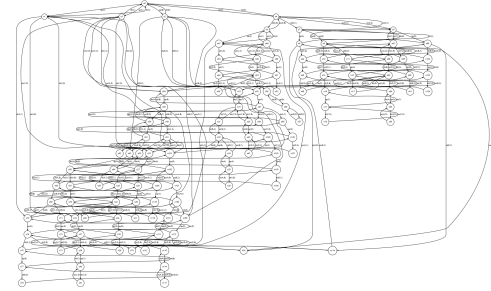
- So: **Solutions are preserved** (but new solutions may arise)

State Representation for PDBs

- For PDB heuristics, a state variable representation is useful
 - Typically:
 - Reduces the number of facts
 - Provides more information about which states are actually reachable!
 - Model problems using the state variable representation, or let planners convert automatically from predicate representation

PDB Heuristics: State Variables (2)

- Repetition: Blocks world with 4 blocks
 - **536,870,912 states** (reachable and unreachable) in the standard predicate representation
- But in **all 125 states reachable** from "all-on-table" (all "normal" states):
 - Block A satisfies **exactly one** of the following:
 - **(holding A)** – Held in the gripper
 - **(clear A)** – At the top of a tower
 - **(on B A)** – Below B
 - **(on C A)** – Below C
 - **(on D A)** – Below D
 - ➔ Remove those facts, introduce state variable **aboveA** $\in \{ \text{gripper, nothing, B, C, D} \}$



■ Example, continued

- 536,870,912 states (reachable and unreachable) in predicate representation
- 20,000 states (reachable and unreachable) in state variable representation:
 - **aboveA** $\in \{ \text{nothing, B, C, D, gripper} \}$
 - **aboveB** $\in \{ \text{nothing, A, C, D, gripper} \}$
 - **aboveC** $\in \{ \text{nothing, A, B, D, gripper} \}$
 - **aboveD** $\in \{ \text{nothing, A, B, C, gripper} \}$
 - **posA** $\in \{ \text{on-table, other} \}$
 - **posB** $\in \{ \text{on-table, other} \}$
 - **posC** $\in \{ \text{on-table, other} \}$
 - **posD** $\in \{ \text{on-table, other} \}$
 - **hand** $\in \{ \text{empty, full} \}$

The state variable *translation* is not part of the PDB heuristic!

Using state variables is *useful* because PDBs work better with fewer "irrelevant states" in the state space...

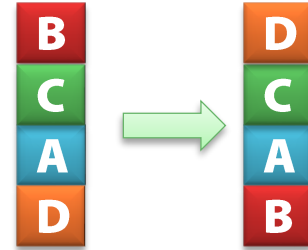
Allows structure to remain in the abstract search space:
Preserves the fact that A can't be under B and under C

Also useful when choosing facts: Ignore where A is, care about where B is

PDB Heuristics: Rewriting the Problem

- **Rewriting** works as before

- Suppose the pattern is { **aboveB**, **aboveD**, **posB**, **posD** }



- **Rewrite** the goal

- **Original:** { aboveB=A, aboveA=C, aboveC=D, aboveD=nothing, hand=empty }
- **Abstract:** { aboveB=A, aboveD=nothing }

- **Rewrite** actions, removing some preconds / effects

- (unstack A D) no longer requires aboveA = nothing
- (unstack B C) still requires aboveB = nothing

- ...

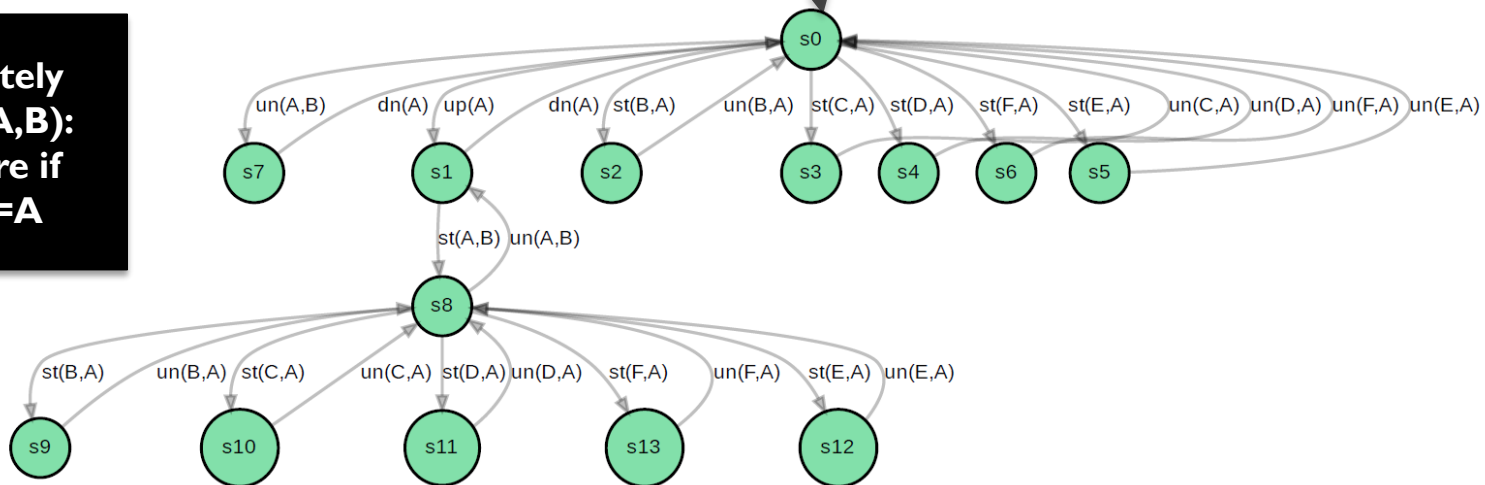
aboveA	∈ { nothing, B, C, D, gripper }
aboveB	∈ { nothing, A, C, D, gripper }
aboveC	∈ { nothing, A, B, D, gripper }
aboveD	∈ { nothing, A, B, C, gripper }
posA	∈ { on-table, other }
posB	∈ { on-table, other }
posC	∈ { on-table, other }
posD	∈ { on-table, other }
hand	∈ { empty, full }

PDB Heuristics: State Space Size

- **Abstract** states reachable from "all on table", **by pattern p...**

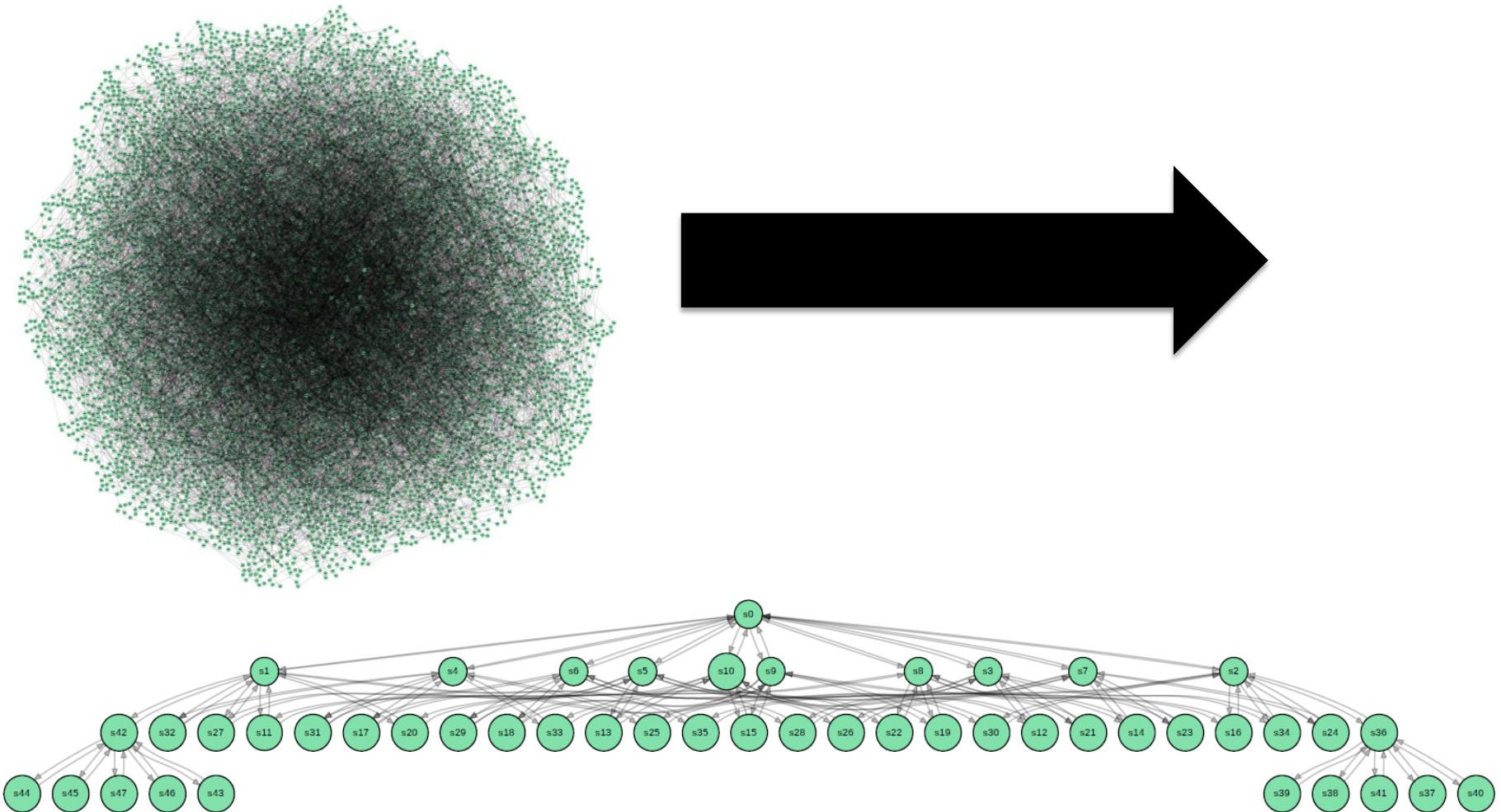
Blocks	All vars	$p=\{\text{aboveA}\}$	$p=\{\text{aboveA}, \text{posA}\}$	$P=\{\text{aboveA}, \text{aboveB}\}$
4	125	5	10	24
5	866	6	12	35
6	7057	7	14	48
7	65990	8	16	63
8	695417	9	18	80
9	8145730	10	20	99

Immediately
unstack(A,B):
Don't care if
aboveB=A



PDB Heuristics: State Space Size (2)

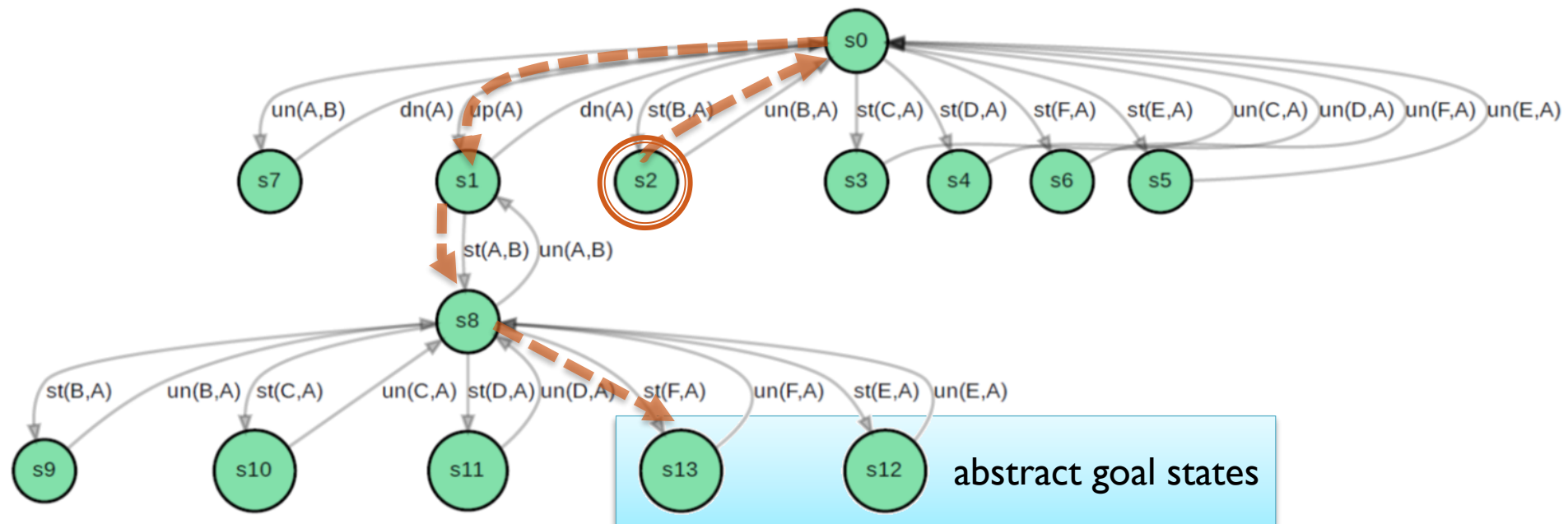
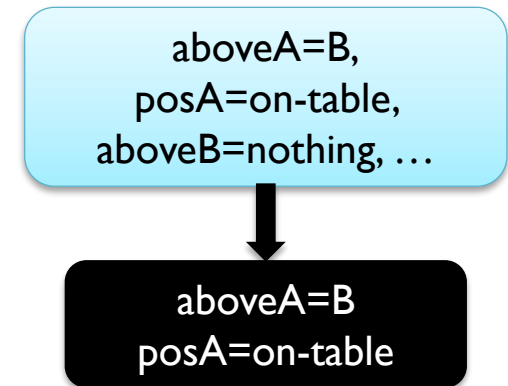
- For 6 blocks, originally 7057 reachable states
 - Pattern **{aboveA,aboveB}** → 48 reachable states



Computing PDB Heuristics

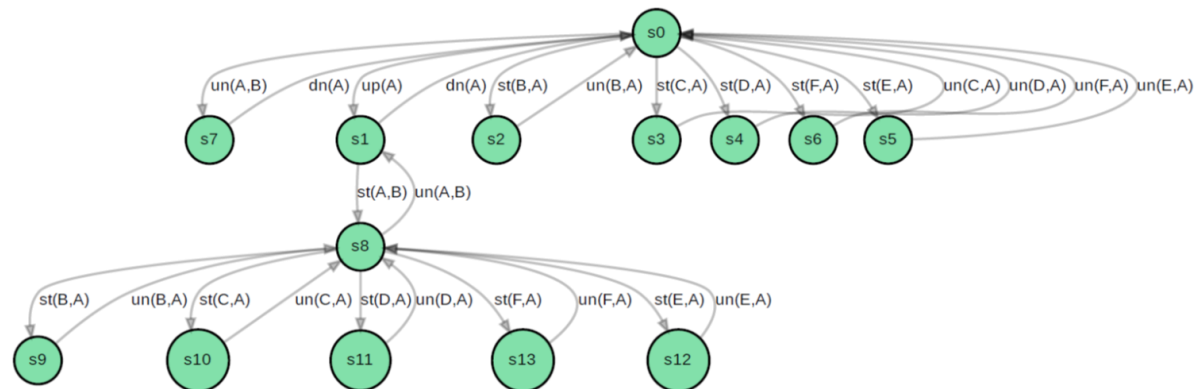
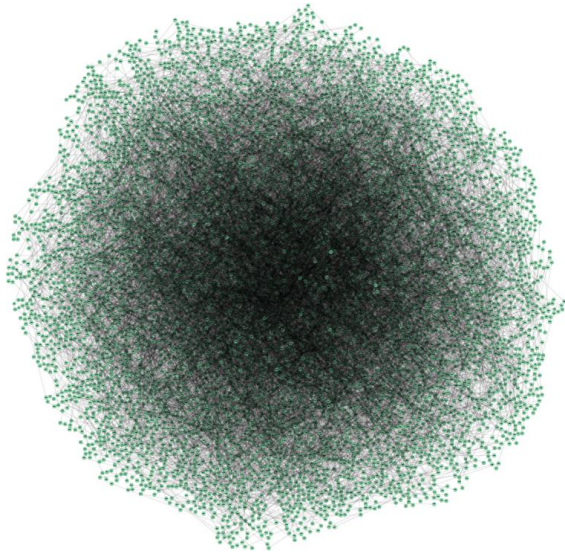
PDB Computation: Main Idea

- To calculate $h(s)$ for a new state s :
 - Example: 6 blocks, pattern **{aboveA, posA}**
 - Convert s to *abstract* state:
 - Find **optimal** path to abstract goal state – in a much smaller search space
 - Current abstract state is s_2 :



Tweak 1: Caching

- Because we keep *few* state variables:
 - Many real states map to the *same* abstract state
 - → An abstract state may be encountered **many** times during search
 - → **Cache** calculated costs



Tweak 2: Databases!

- Dijkstra efficiently finds optimal paths from all abstract states
 - → Precalculate all heuristic values for each pattern
 - Store in a look-up table – a database

Second main idea!

Database Creation

- Preprocessing step 1 (6 blocks, pattern {**aboveA**, **posA**})
 - Create the initial *abstract* state...

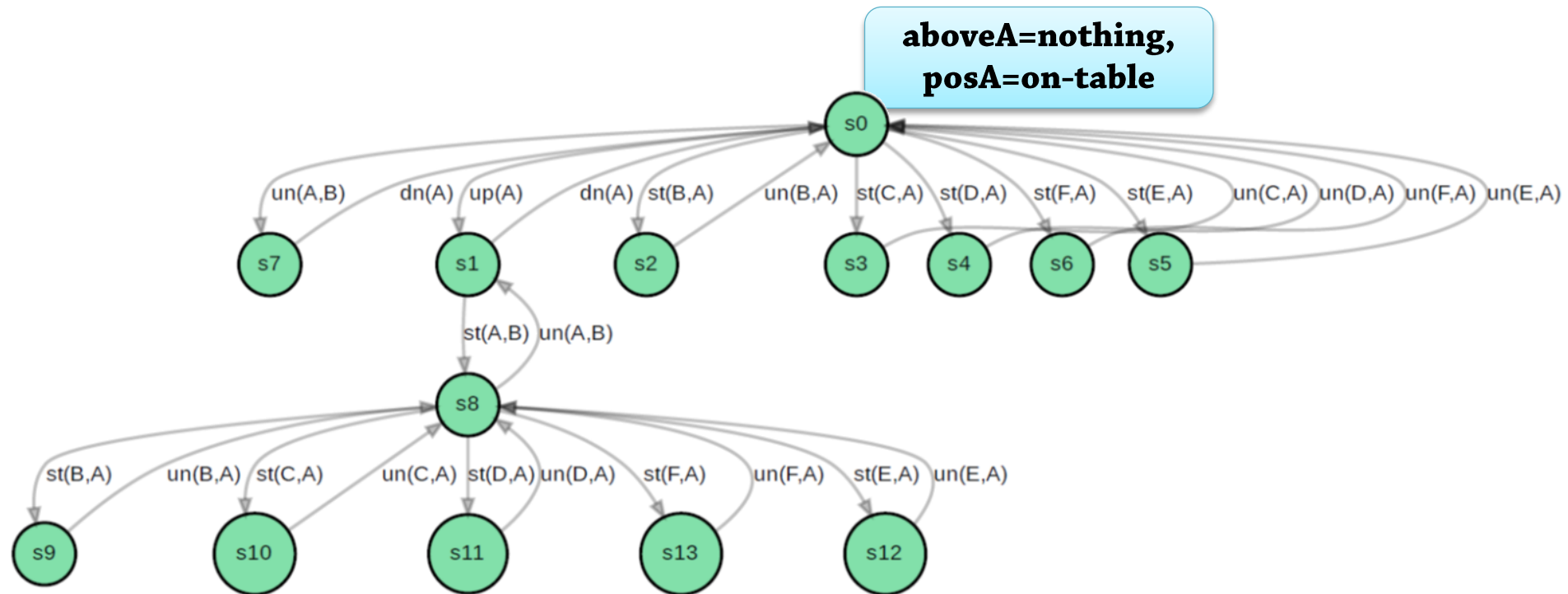
aboveA=nothing,
aboveB=nothing,
aboveC=nothing,
aboveD=nothing,
aboveE=nothing,
aboveF=nothing,
posA=on-table,
posB=on-table,
posC=on-table,
posD=on-table,
posE=on-table,
posF=on-table,
hand=empty



aboveA=nothing,
aboveB=nothing,
aboveC=nothing,
aboveD=nothing,
aboveE=nothing,
aboveF=nothing,
posA=on-table,
posB=on-table,
posC=on-table,
posD=on-table,
posE=on-table,
posF=on-table,
hand=empty

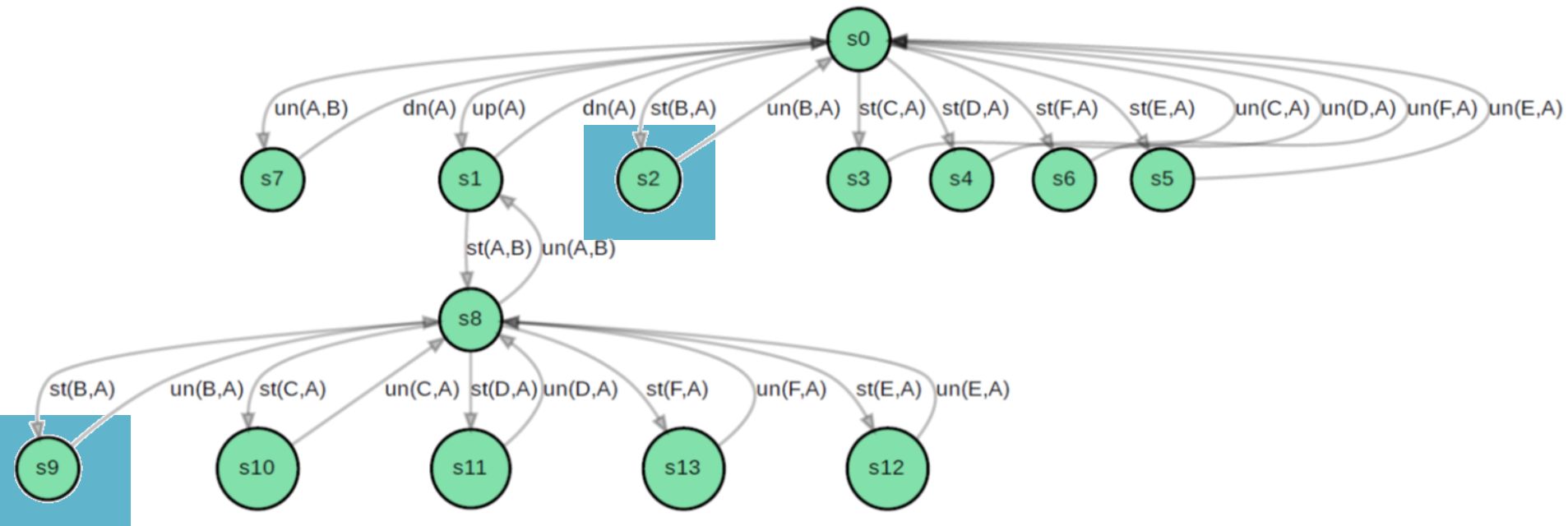
■ Preprocessing step 2

- Find all reachable abstract states
 - Start in the abstract initial state (s0 below)
 - Apply all applicable actions to all states you find (DFS, BFS, ...)
 - Small, therefore *fast*



Preprocessing (3)

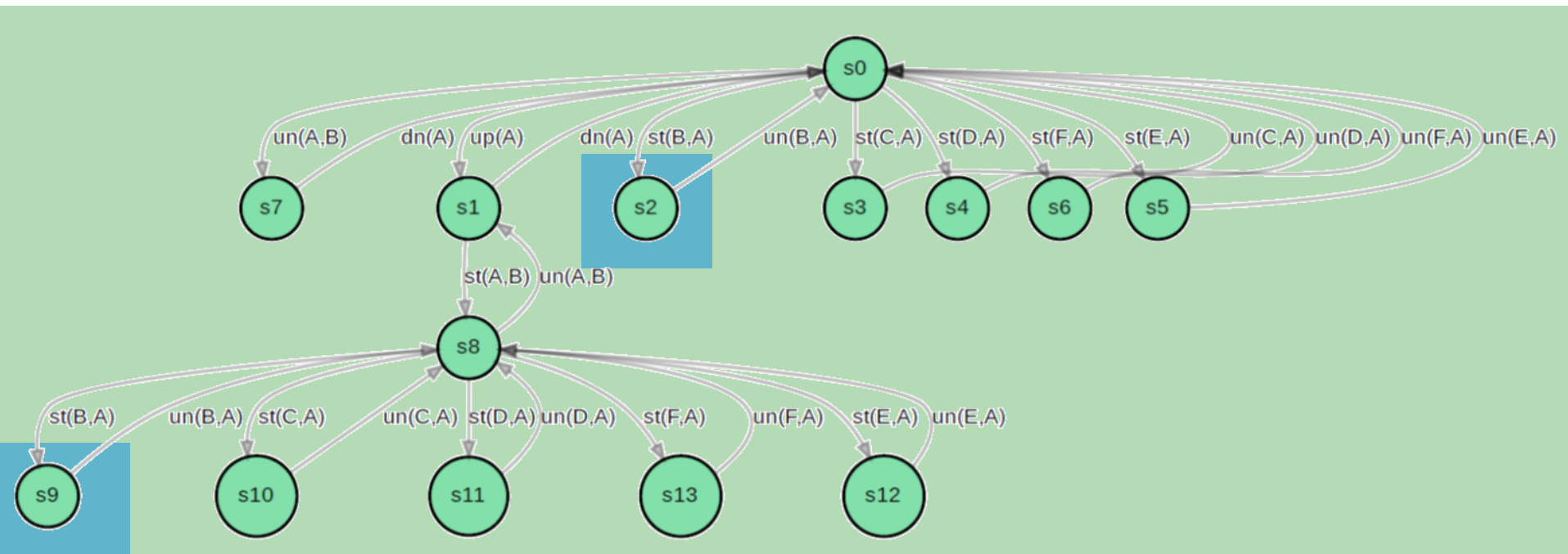
- Step 3: Which abstract states satisfy the **abstract goal**?
 - Real goal = { aboveA=B, aboveB=C, aboveC=D, aboveD=E, aboveE=F }
 - Abstract goal = { aboveA=B }
 - Satisfied in **two** of the reachable states



Preprocessing (4)

- Preprocessing step 4: Compute the database

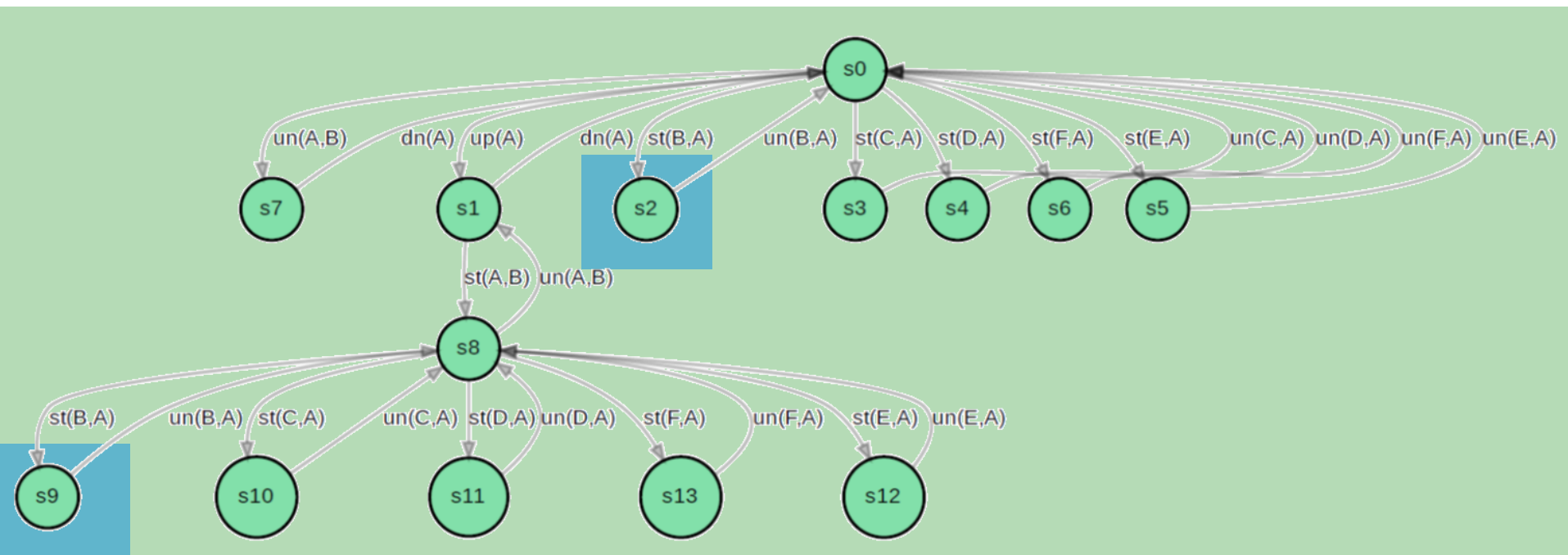
- For every abstract state reachable from the abstract initial state,
- find a cheapest path to any abstract goal state



Preprocessing (5)

- More efficient computation:

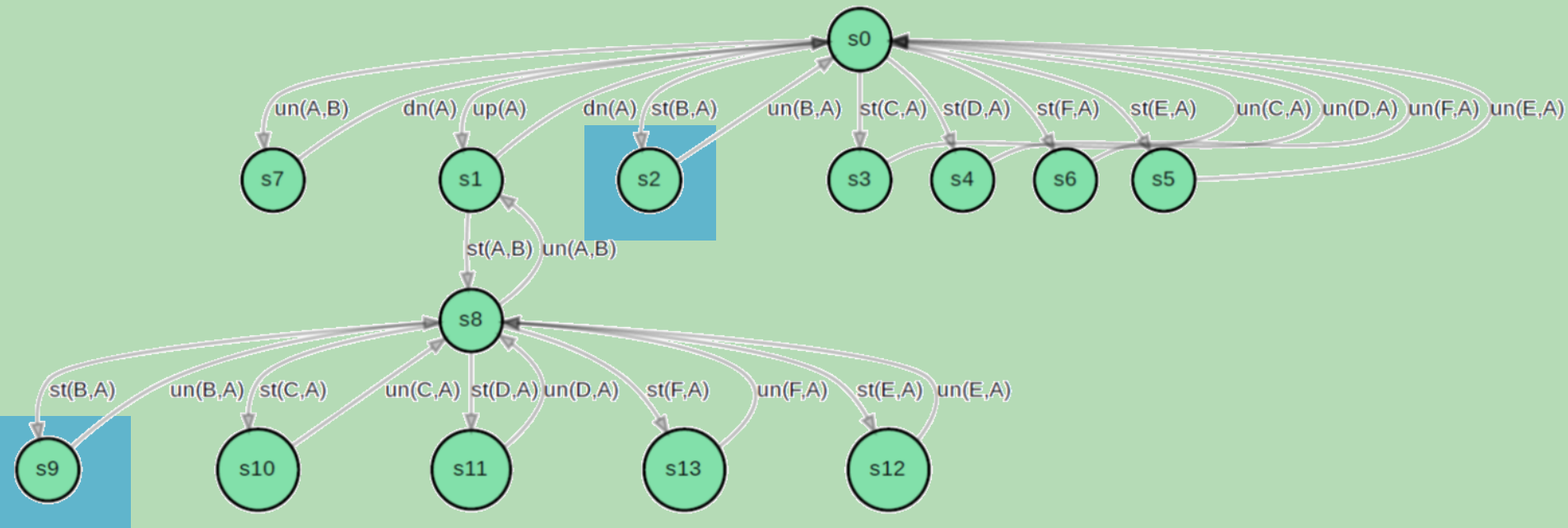
- Start with cost=0 for all abstract goal states
- Search backward from these states – single call to Dijkstra
 - Queue initialized with multiple nodes/states of cost 0 (s2, s9)
 - Edges followed backward (in explicitly computed graph, no γ^{-1} necessary)



Preprocessing (6)

- Example: Priority queue contains $\langle s2/\text{cost}=0, s9/\text{cost}=0 \rangle$
 - Pick $s2$
 - A "successor" is $s0$, reached backwards by $\text{stack}(B,A)$ of cost 2;
 - $s0$ inserted in queue with cost 2
 - Priority queue contains $\langle s9/c=0, s0/\text{cost}=2 \rangle$, pick $s9$
 - A "successor" is $s8$, ...

Assuming $\text{cost}(\text{stack}/\text{unstack})=2$,
 $\text{cost}(\text{pickup}/\text{putdown})=1$



-

[illegible]

Using PDB Heuristics during Search

PDB Heuristics in Forward Search (1)



- Step 1: **Automatically** generate a pattern
 - A selection of state variables to consider
 - Choosing a **good** pattern is a **difficult problem!**
 - Different approaches exist...

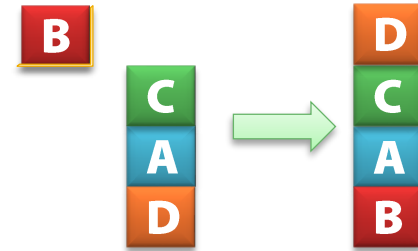
- Step 2: Calculate the pattern database
 - As already discussed

PDB Heuristics in Forward Search (2)

■ Step 3: Forward search in the original problem

- For each new successor state s_1 , calculate heuristic value $h_{pdb}(s_1)$

- Example: $s_1 = \{$ aboveD = A, aboveA = C, aboveC = nothing, aboveB = gripper,
 posA = other, posB = other,
 posC = other, posD = on-table,
 hand = full $\}$



- Convert this to an *abstract* state

- Example: $s'_1 = \{$ aboveB = gripper, aboveD = A, posB = other, posD = on-table $\}$

- Use the database to quickly look up $h_{pdb}(s_1) =$
the cost of reaching the nearest abstract goal from s'_1

aboveB = gripper, aboveD = A, posB = other, posD = on-table $\rightarrow$ cost $n1$
aboveB = gripper, aboveD = A, posB = other, posD = other $\rightarrow$ cost $n2$
...

How informative can PDBs be?

- **How close** to $h^*(n)$ can an admissible PDB-based heuristic be?
 - Wrong question!
 - A pattern can include **all** state variables → **equal** to $h^*(n)$

Then computing the database takes too much time...

HOW much time?

Dijkstra's algorithm:
 $O(|E| + |V| \log |V|)$

$|V|$ = number of
reachable abstract states

of reachable abstract:
Exponential in # of
selected state vars

Exponential in the number of state variables in the pattern...

Possible state variables: Linear in problem size (length of PDDL file, ...)
Select **all** → PDB computation exponential in problem size

Computation Time (2)



We want: Polynomial in the problem size (length of PDDL file, ...)

Counteract this:

Exponential in the number of state variables in the pattern...

Using this:

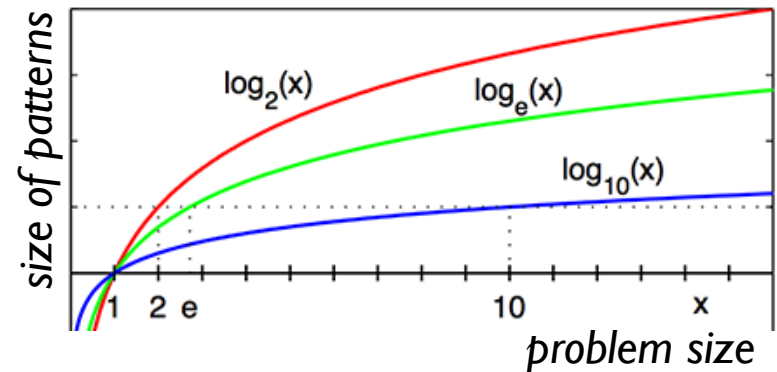
Select a logarithmic number of state variables – growing as log(problem size)

Computation time: $O(2^{\log \text{problem size}})$, which is polynomial in problem size

Asymptotic Accuracy: Single PDB Heuristic

- **Assume** (as before) an *infinite* set of problems $\{P_i | i \geq 0\}$

- Assume a **strategy** for selecting a single **pattern** for each problem
 - So that pattern size grows at most as $O(\log k)$ for problem size k



- What is the best **accuracy guarantee** any strategy can give?
 - $h(n) \leq \text{cost of reaching the **most expensive subgoal** of size } O(\log k)$

Subgoal size is not constant but grows with problem size k – good!

But still, $\log(k)$ grows much slower than k ...

- ➔ For any given single pattern, **asymptotic** accuracy is 0 in many domains
 - As before, *practical results* are usually better!

PDB Heuristics: Gripper Example

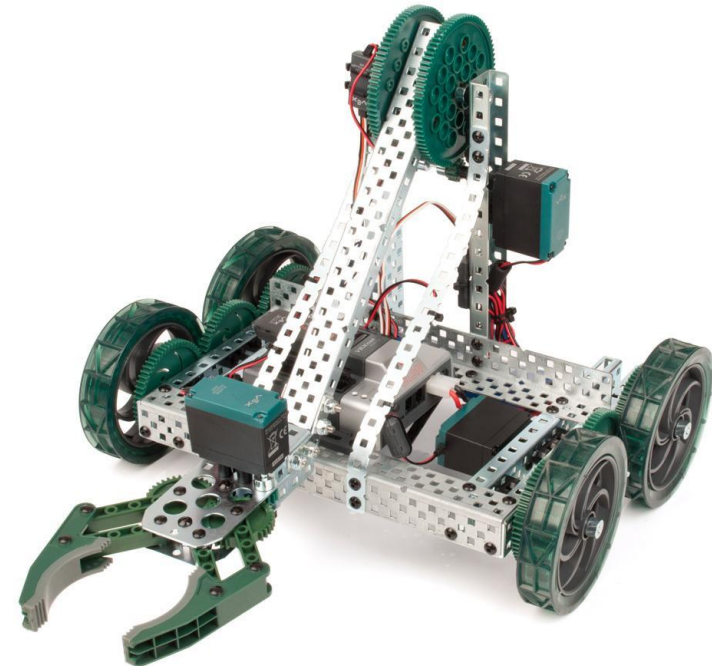
- A common *restricted* gripper domain:
 - One robot with two grippers
 - Two rooms
 - All n balls originally in the first room
 - Objective: All balls in the second room

Compact state variable representation:
 $\text{loc}(\text{ball}_k) \in \{ \text{room1}, \text{room2}, \text{gripper1}, \text{gripper2} \}$
 $\text{loc-robot} \in \{ \text{room1}, \text{room2} \}$

$2 * 4^n$ states, some unreachable – which ones?

Some possible patterns for $k \geq 1$ balls:

$\{ \text{loc}(\text{ball}_1) \}$	→ 4 abstract states
$\{ \text{loc}(\text{ball}_1), \text{loc-robot} \}$	→ 8 abstract states
$\{ \text{loc}(\text{ball}_i) \mid i \leq k \}$	→ 4^k abstract states
$\{ \text{loc}(\text{ball}_i) \mid i \leq \log(k) \}$	→ $4^{\log(k)}$ abstract states



Multiple Patterns:

**More information,
Same pattern size**

■ Subproblem 2: Some facts related to **B**

- **Current state:** Everything on the table, hand empty, all blocks clear

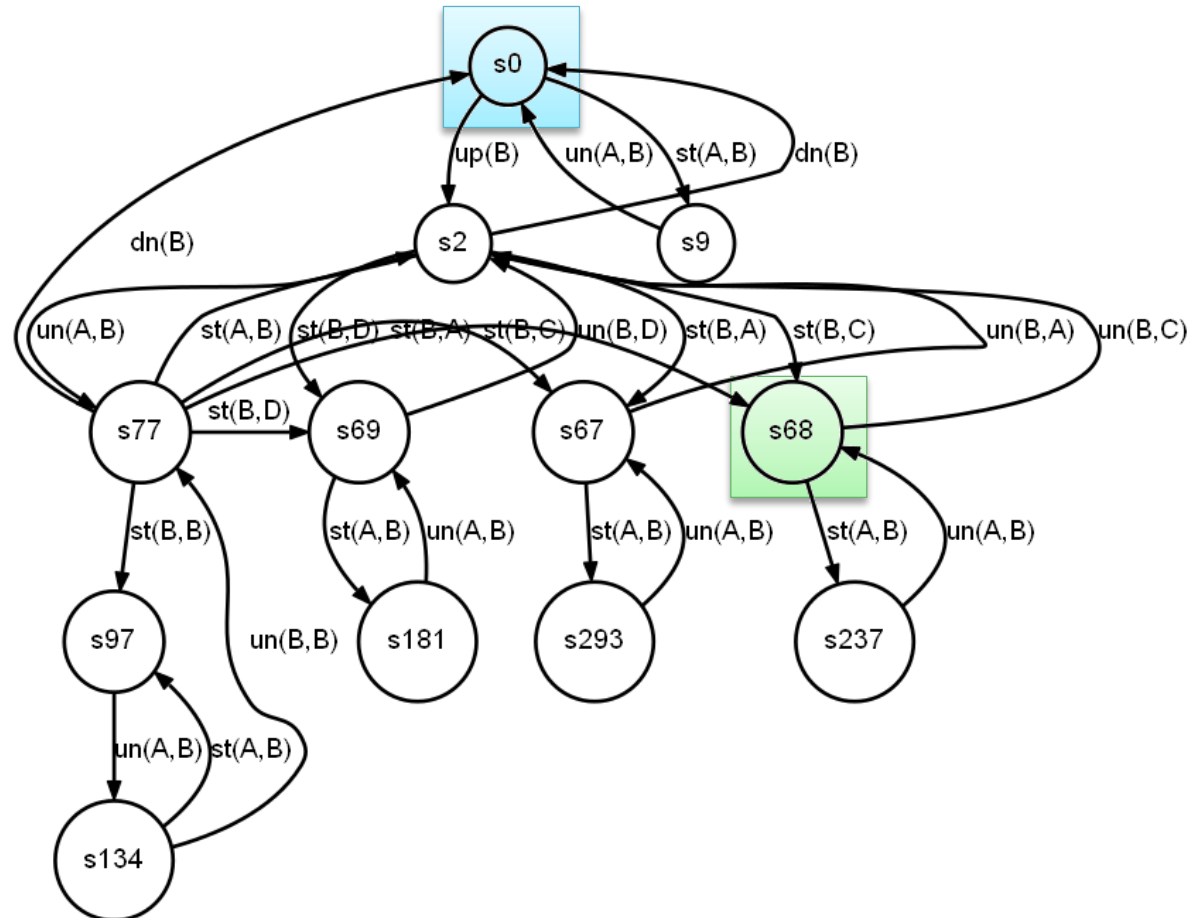
- Abstract state: { (**ontable B**), (**clear B**) }

- **Goal state:**

A on B on C on D

- Abstract goal:
{ (**on B C**) }

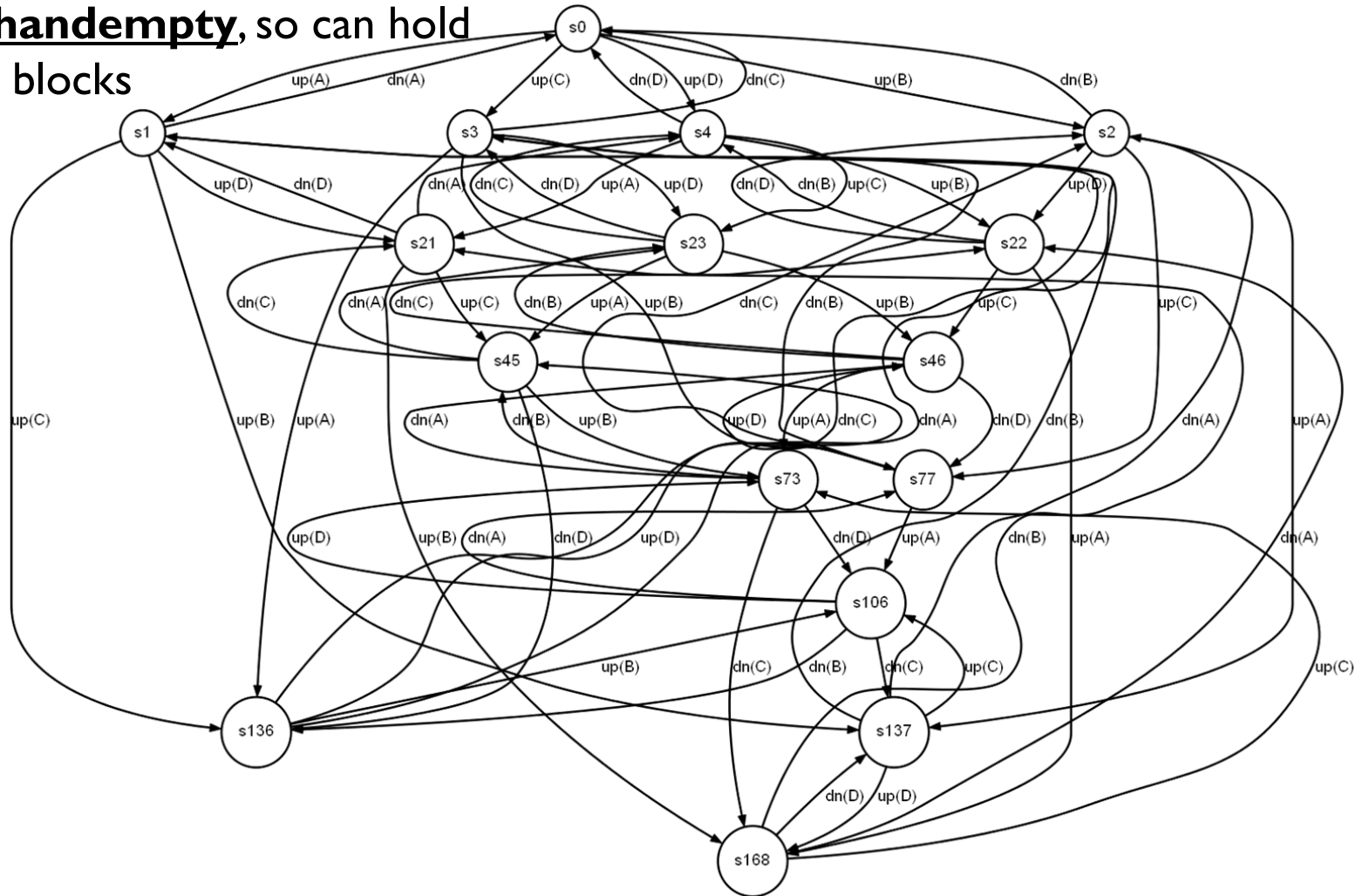
- Find a path,
compute its cost



BW4: Subproblem 3

- Subproblem 3: Only consider (holding ?x) facts...

- Not handempty, so can hold many blocks



- For each pattern, compute a separate pattern database
 - Each such cost is an admissible heuristic

What then? Do we sum the costs?

Possibly in satisficing planning – but would generally not be admissible...

Pattern 1, aboveB:
I need stack(A,B), cost 2

Pattern 2, aboveC:
I need stack(B,C), cost 2

Sum:
2+2=4

Could have used buildTower(A,B,C), cost 3
Only detectable if we consider the whole problem

But the maximum of two admissible heuristics is also admissible!
And polynomial time: k patterns require k computations

- What is the new level of accuracy?
 - Still 0... *asymptotically* / *worst case*
 - But this can still work well *in practice*!

Additive PDB Heuristics

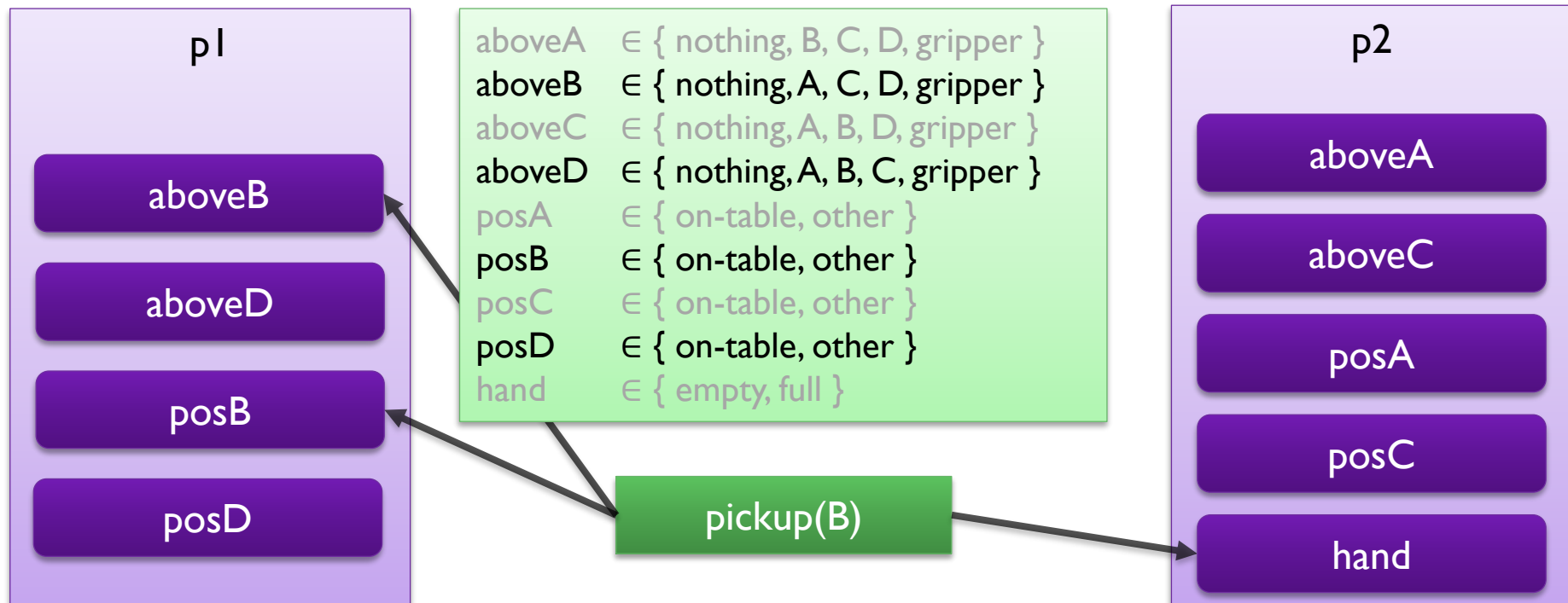
Additive PDB Heuristics (1)



- To create admissible heuristics by summing PDB heuristics:
 - Each fact should be in at most one pattern
 - Each action should affect facts in at most one pattern
- → Additive pattern database heuristics

Additive PDB Heuristics (2)

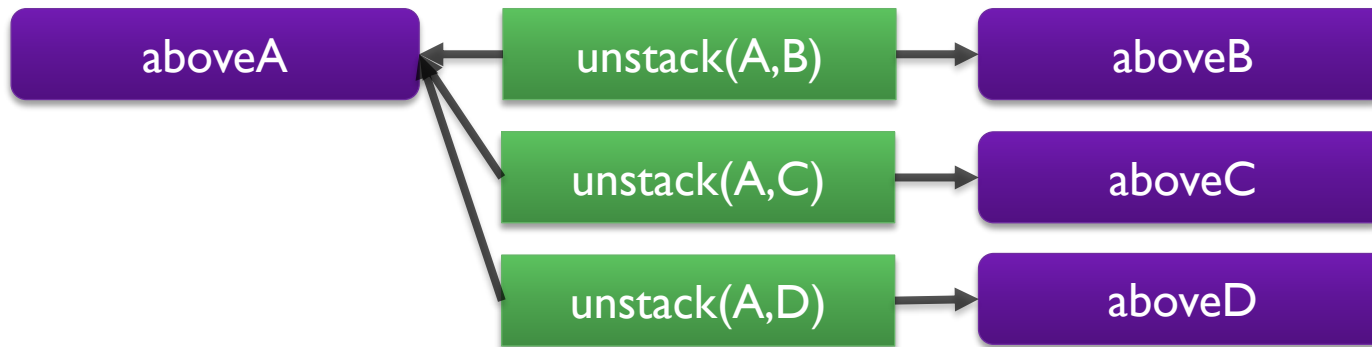
- BW: Is $p1 = \{\text{facts in even rows}\}$, $p2 = \{\text{facts in odd rows}\}$ *additive*?
 - No: pickup(B) affects {aboveB, posB} in $p1$, {hand} in $p2$



One potential problem:
Both patterns could use pickup(B) in their optimal solutions
→ sum counts this twice! This is what we're trying to avoid...

Additive PDB Heuristics (3)

- BW: Is $p1=\{\text{aboveA}\}$, $p2=\{\text{aboveB}\}$ additive?
 - No: $\text{unstack}(A,B)$ affects $\{\text{aboveB}\}$ in $p1$, $\{\text{aboveA}\}$ in $p2$
 - True for *all* combinations of aboveX

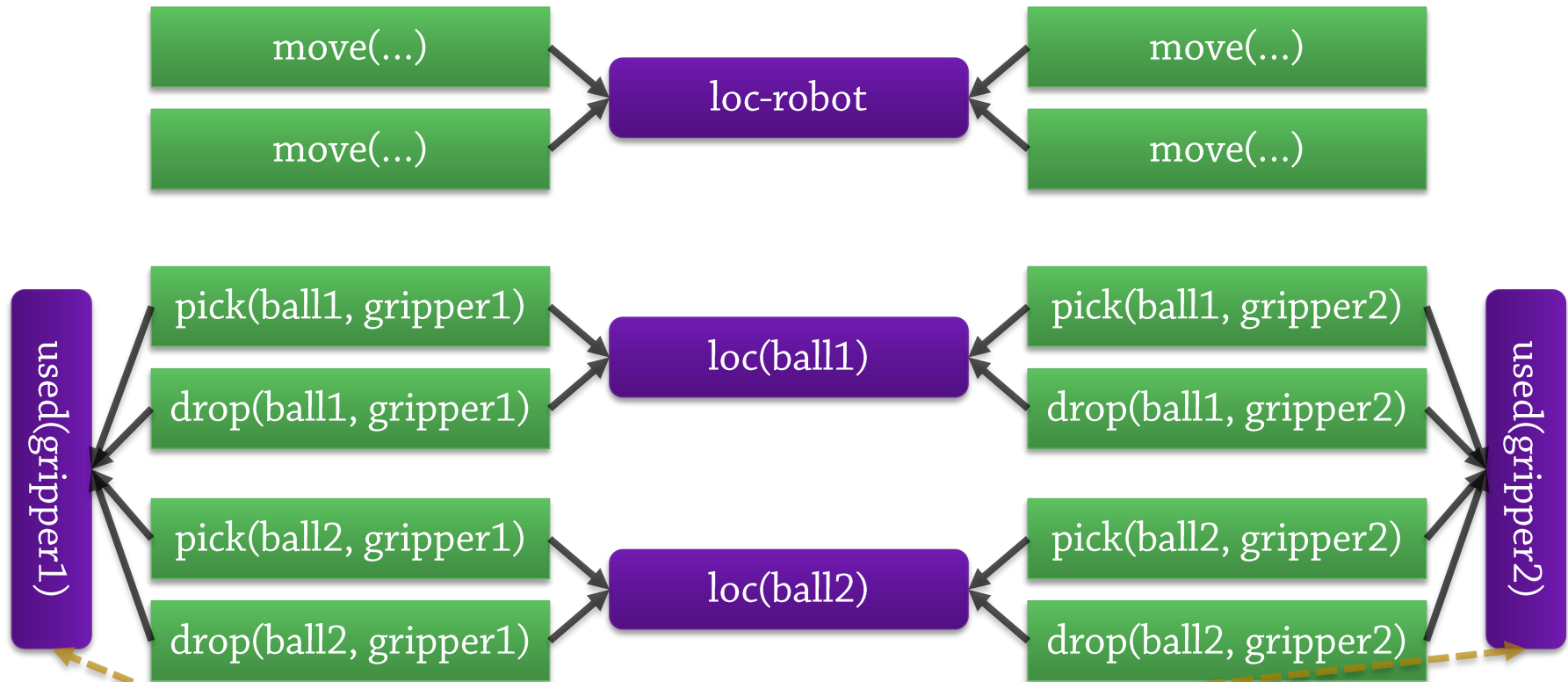


- An additive PDB heur. could use one of these:
 - $p1 = \{\text{aboveA}\}$
 - $p1 = \{\text{aboveA}, \text{aboveC}, \text{aboveD}\}$
 - ...
- Can't have two separate patterns $p1, p2$ both of which include an aboveX
 - Those aboveX will be directly connected by some unstack action

**This formulation of the
Blocks World is
"connected in the wrong way"
for this approach
to work well**

Additive PDB Heuristics (4)

- "Separating" patterns in the Gripper domain:

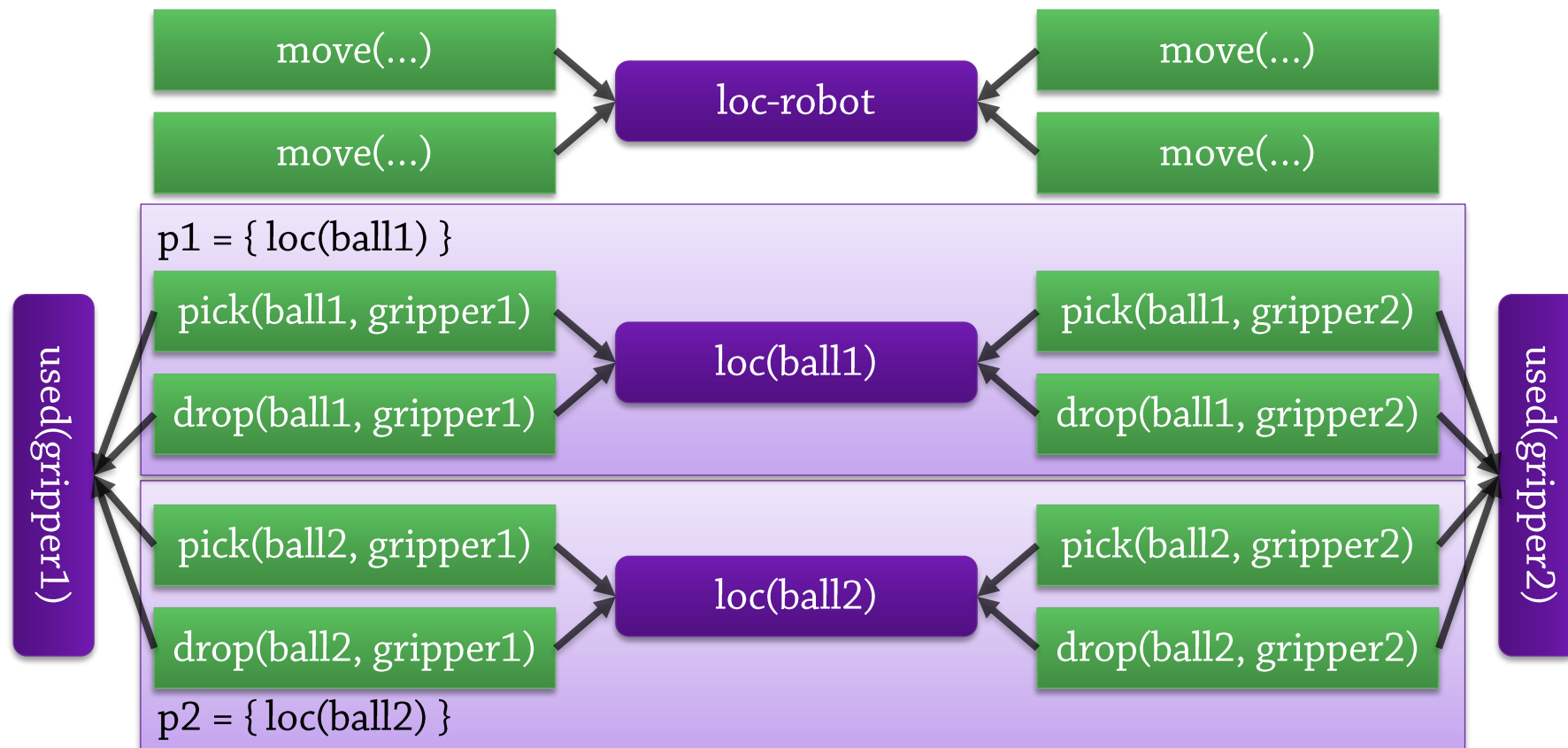


Are these a problem?

$\text{loc}(\text{ball}_k) \in \{ \text{room1}, \text{room2}, \text{gripper1}, \text{gripper2} \}$
 $\text{loc-robot} \in \{ \text{room1}, \text{room2} \}$
 $\text{used}(\text{gripper}_k) \in \{ \text{true}, \text{false} \}$

Additive PDB Heuristics (5)

- No problem: We don't have to use all variables in patterns!



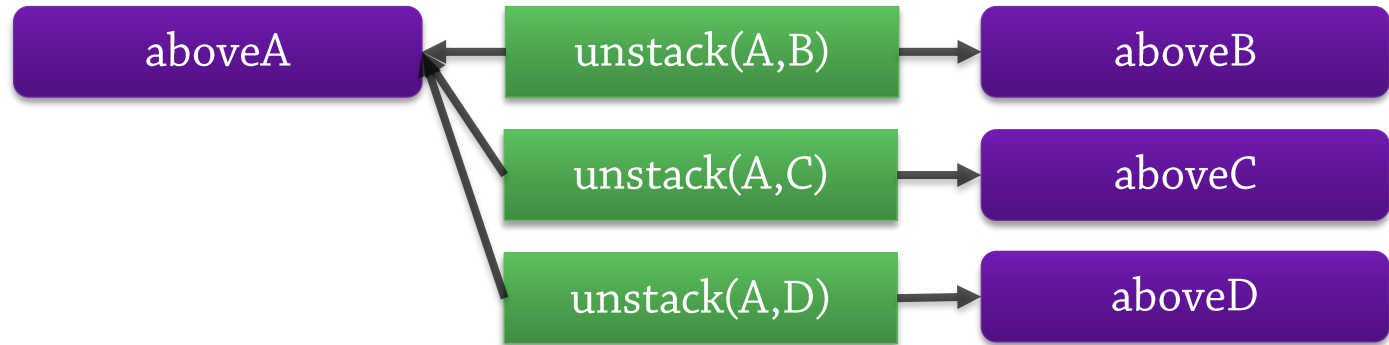
For each pattern we chose one variable

Then we have to include all actions affecting it

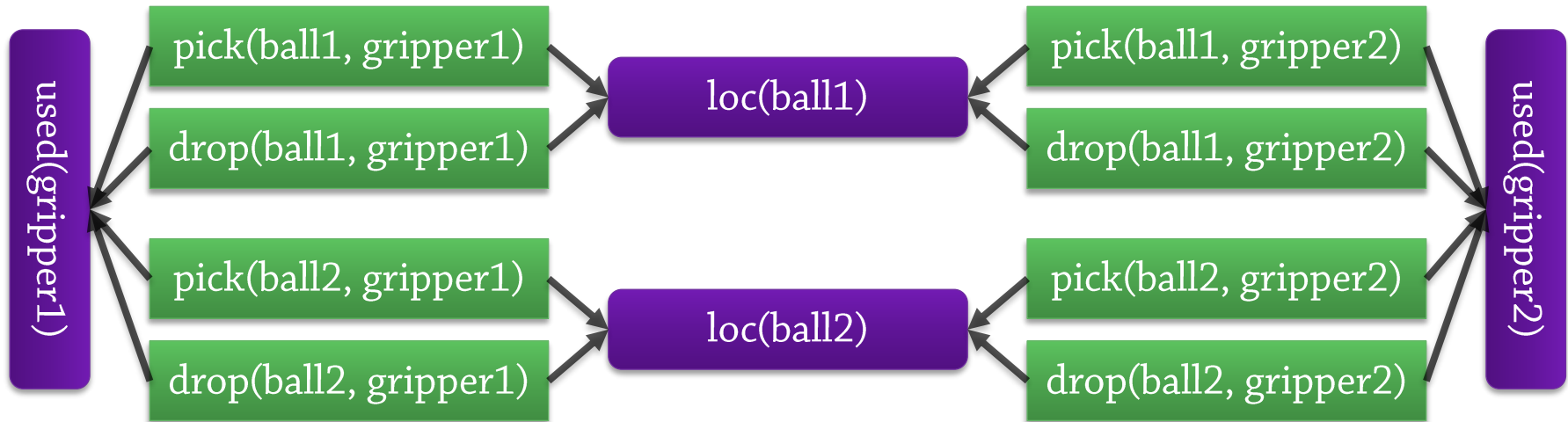
The other variables those actions affect [`used()`] don't have to be part of *any* pattern!

Additive PDB Heuristics (6)

- Notice the difference in structure!



BW: Every pair of aboveX facts has a *direct connection* through an action

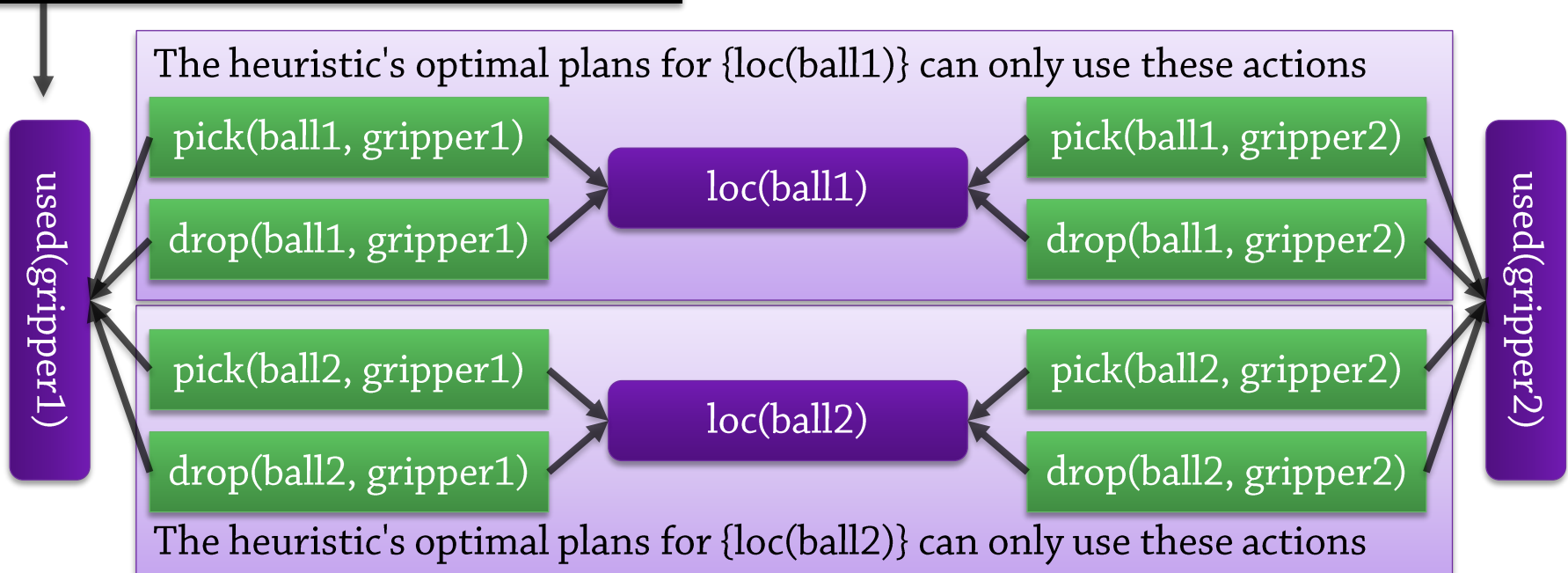


Gripper: No pair of loc() facts has a *direct connection* through an action

Additive PDB Heuristics (7)

- When every action affects facts in at most *one* pattern:
 - The subproblems we generated are completely *disjoint*
 - They achieve **different aspects** of the *goal*
 - Optimal solutions **must** use **different actions**

The heuristic never tries to generate optimal plans for `used(gripper1)` – we have not included it in any pattern



Additive PDB Heuristics (8)

- Avoids the overestimation problem

Problem:

Goal: p and q

A1: effect p

A2: effect q

A3: effect p and q // BuildTower

To achieve p: Heuristic uses A1

To achieve q: Heuristic uses A2

Sum of costs is 4 – optimal cost is 3, using A3

This cannot happen
when every action affects facts
in at most one pattern

- The costs are additive
for multiple patterns
- Adding costs
from multiple heuristics
yields an
admissible heuristic!

Additive PDB Heuristics (9)

- Can be taken one step further...
 - Suppose we have several sets of additive patterns:
 - Can calculate an admissible heuristic from **each** additive set, then take the **maximum** of the results as a **stronger** admissible heuristic

$$\text{Max} \rightarrow \text{admissible heuristic } h_{pdb}^3(s) = \max(h_{pdb}^1(s), h_{pdb}^2(s))$$

$$\text{Sum} \rightarrow \text{admissible heuristic } h_{pdb}^1(s)$$

p1

p2

p3

p4

4 patterns satisfying
additive constraints

$$\text{Sum} \rightarrow \text{admissible heuristic } h_{pdb}^2(s)$$

p5

p6

p7

p8

p9

5 patterns satisfying
additive constraints

Additive PDB Heuristics: Accuracy

Additive PDB Heuristics (10)



- **How close** to $h^*(n)$ can an **additive** PDB-based heuristic be?
 - For additive PDB heuristics with a single sum,
asymptotic accuracy as problem size approaches infinity...

■ In Gripper:

- In state s_n there are n balls in **room1**, and no balls are carried
- Additive PDB heuristic $h_{add}^{PDB}(s_n)$:
 - Use a separate pattern for each ball location variable **loc(ball_k)**
 - For **each** pattern/ball, the optimal PDB cost is 2
 - **pick(ball,room1,gripper1): loc(ball)=room1 → loc(ball)=gripper1**
 - **drop(ball,room2,gripper1): loc(ball)=gripper1 → loc(ball)=room2**
 - $h_{add}^{PDB}(s_n) = \text{sum for } n \text{ balls} = 2n$
- Real cost:
 - Using both grippers:
 - *pick, pick, move(room1,room2),
drop, drop, move(room2,room1)*
 - Repeat $n/2$ times, total cost $\approx 6n/2 = 3n$
- → Asymptotic accuracy $2n/3n = 2/3$

Additive PDB Heuristics (12)

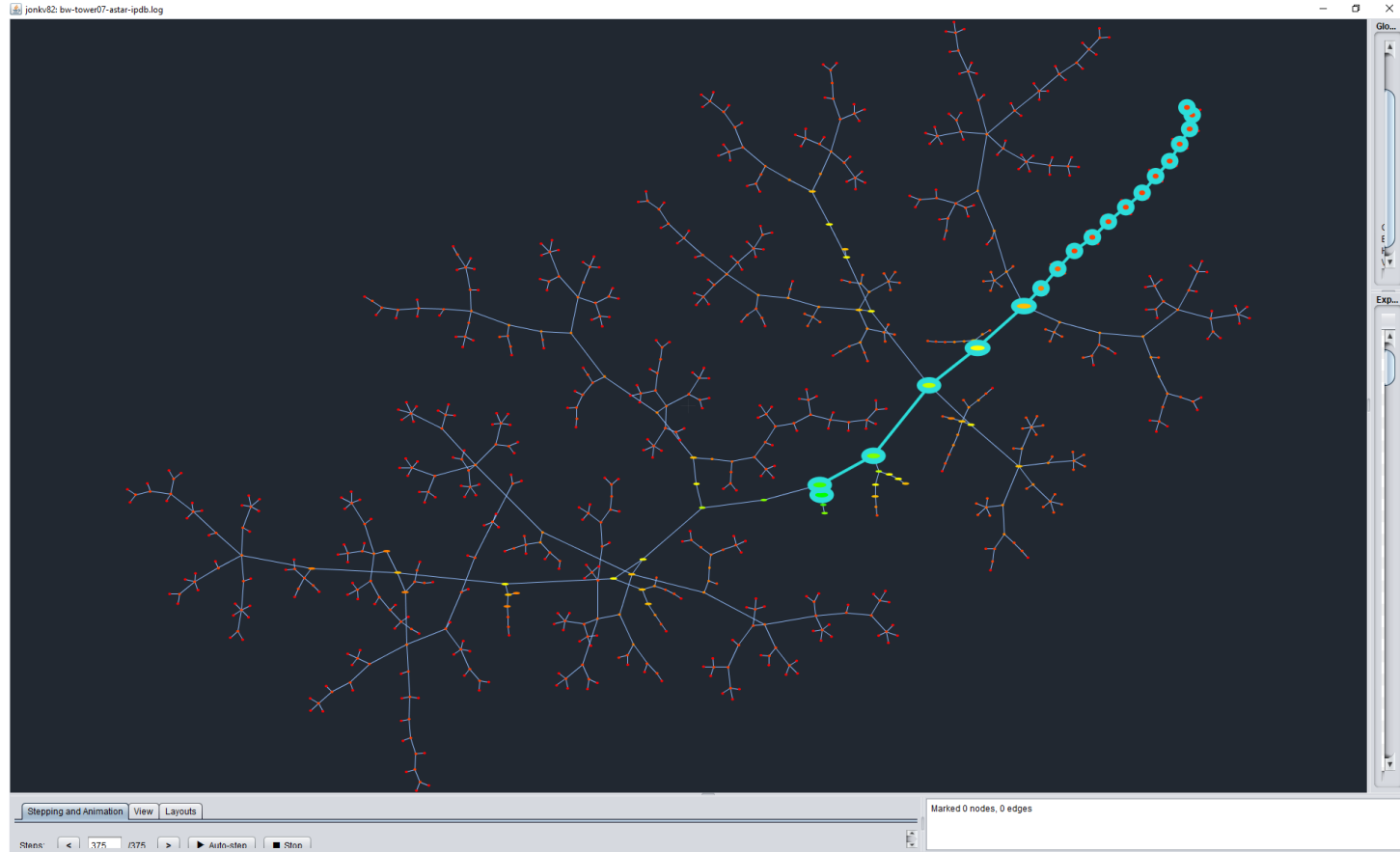
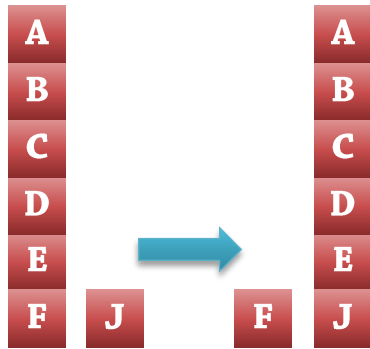


- What quality **guarantees** can an **additive** PDB heuristic give?
 - For additive PDB heuristics with a single sum,
asymptotic accuracy (fraction of h^*) as problem size approaches infinity:

	h^+ (too slow!)	Additive PDB
Gripper	2/3	2/3
Logistics	3/4	1/2
Blocks world	1/4	0
Miconic-STRIPS	6/7	1/2
Miconic-Simple-ADL	3/4	0
Schedule	1/4	1/2
Satellite	1/2	1/6

- **Only guaranteed** if the planner **finds** the best combination of patterns!
 - This is a very difficult problem in itself!
- But as usual, this is a worst-case analysis...

bw-tower07-astar-ipdb: Only 7 blocks, A* search, based on PDB variation



- Blind A*: 43150 states calculated, 33436 visited
- A* + goal count: 6463 states calculated, 3222 visited
- A* + iPDB: 1321 states calculated, 375 visited

No heuristic is perfect – visiting some additional states is fine!