



Linköping University



# Automated Planning

## The Partial Order Causal Link Search Space

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Partly adapted from slides by Dana Nau

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# Motivations

# Motivating Problem

- Simple planning problem:

- Two crates

- At A
    - Should be at B



- One robot

- Can carry up to two crates
    - Can move between locations, which requires one unit of fuel
    - Has only two units of fuel



Let's see what a forward-chaining planner *might* do (depending on heuristics)...

# Motivating Problem 2: Forward Search

## We have:

robotat(A)  
at(c1,A)  
at(c2,A)  
has-fuel(2)

pickup(c1,A)

**effects:**  
holding(c1),  
¬at(c1,A)

robotat(A)  
**holding(c1)**  
at(c2,A)  
has-fuel(2)

drive(A,B)

**effects:**  
¬robotat(A),  
robotat(B)

**robotat(B)**  
holding(c1)  
at(c2,A)  
**has-fuel(1)**

put(c1, B)

**effects:**  
at(c1,B),  
¬holding(c1)

robotat(B)  
**at(c1,B)**  
at(c2,A)  
has-fuel(1)

pickup(c1, B)

**effects:**  
¬at(c1,B),  
holding(c1)

robotat(B)  
**holding(c1)**  
at(c2,A)  
has-fuel(1)

Cycle, backtrack

drive(B,A)

**effects:**  
¬robotat(B),  
robotat(A)

**robotat(A)**  
at(c1,B)  
at(c2,A)  
**has-fuel(0)**

pickup(c2,A)

**effects:**  
holding(c2),  
¬at(c2,A)

robotat(A)  
at(c1,B)  
**holding(c2)**  
has-fuel(0)

Dead end,  
backtrack

drive(B,A)

**effects:**  
robotat(A),  
¬robotat(B)

**robotat(A)**  
holding(c1)  
at(c2,A)  
**has-fuel(0)**

Dead end, backtrack

Why is this  
not a cycle?

## We want:

at(c1,B)  
at(c2,B)



# Motivating Problem 3

## We have:

robotat(A)  
at(c1,A)  
at(c2,A)  
has-fuel(2)

pickup(c1,A)

**effects:**  
holding(c1),  
¬at(c1,A)

robotat(A)  
**holding(c1)**  
at(c2,A)  
has-fuel(2)

drive(A,B)

**effects:**  
¬robotat(A),  
robotat(B)

robotat(B)  
holding(c1)  
at(c2,A)  
has-fuel(1)

put(c1, B)

**effects:**  
at(c1,B),  
¬holding(c1)

robotat(B)  
at(c1,B)  
at(c2,A)  
has-fuel(1)

Keep  
backtracking...

pickup(c2,A)

**effects:**  
holding(c2),  
¬at(c2,A)

robotat(A)  
holding(c1)  
**holding(c2)**  
has-fuel(2)

drive(A,B)

**effects:**  
¬robotat(A),  
robotat(B)

**robotat(B)**  
holding(c1)  
holding(c2)  
**has-fuel(1)**

put(c1, B)

**effects:**  
at(c1,B),  
¬holding(c1)

robotat(B)  
**at(c1,B)**  
holding(c2)  
has-fuel(1)

put(c2, B)

**effects:**  
at(c2,B),  
¬holding(c2)

robotat(B)  
at(c1,B)  
**at(c2,B)**  
has-fuel(1)

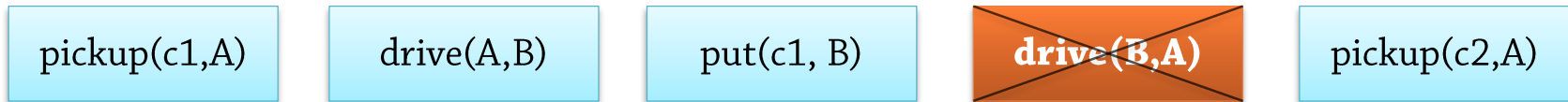
## We want:

at(c1,B)  
at(c2,B)

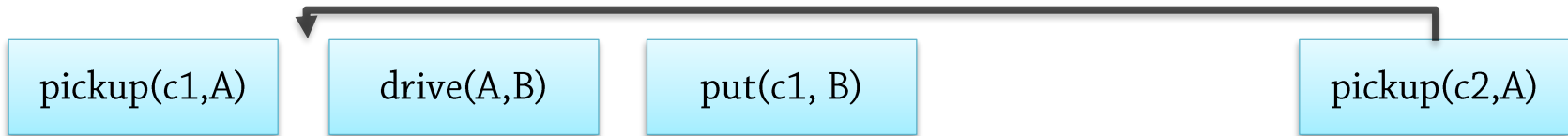
# Motivating Problem 4

## ■ Observations:

- **Most** actions we added before backtracking were **useful** and **necessary**!



- At first, we added them in the **wrong order**



- **Forward** and **backward** planning **commits** immediately to action order
  - Puts each action in its **final place** in the plan
- State space **heuristics** must be smart enough to tell us:
  - Which actions are **useful**
  - ***When to add them to the plan***

What if we could **rearrange** actions?

# Partial Order Causal Link: Plan Structure

# First Step: Insertion

- **Sequences** with arbitrary insertion: Useful?

- Add actions in sequence, as in state space planning...

pickup(c1,A)

drive(A,B)

put(c1, B)

- Realize you need another one...

pickup(c2,A)

How to decide which action to *insert*, if not at the end

- Make a space...

pickup(c1,A)

drive(A,B)

put(c1, B)

How to decide where to insert it?

- ...and place the action there

pickup(c1,A)

pickup(c2,A)

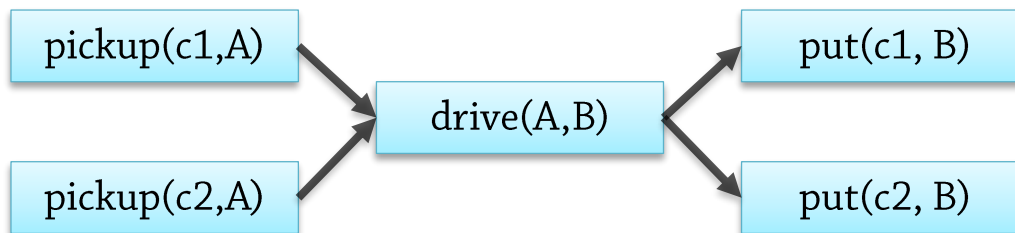
drive(A,B)

put(c1, B)

How to check that "old" preconditions remain satisfied?

# Second Step: Partial Order

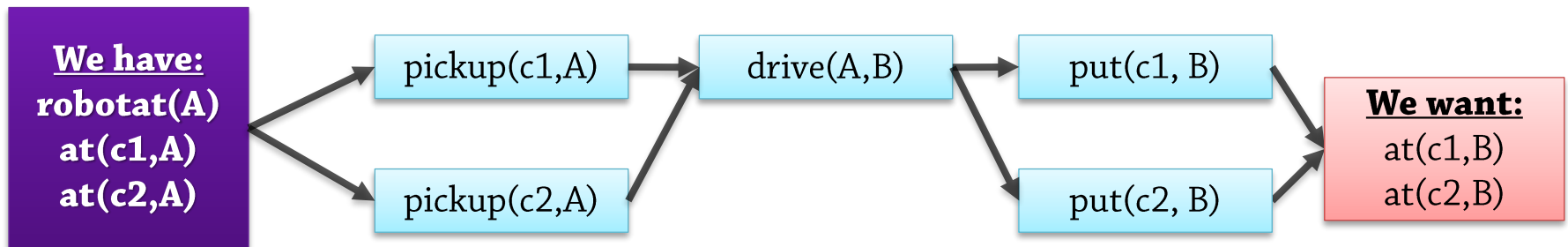
- If we must deal with this complexity:
  - We can "get more for the same price"
- Let's skip sequences completely – a plan could be *partially ordered*:



- A set of **actions**  $A = \{ a_1, a_2, a_3, \dots \}$
- A set of **precedence constraints**  $\{ a_1 < a_2, \quad a_1 < a_3, \quad \dots \}$ 
  - $a_1$  must finish before  $a_2$  starts, ...
  - Here: **solid arrows**

How do we *generate* these plans?

- **Partial Order Causal Link** (POCL) planning
  - Use a partial order, as described
    - Not when executing the plan
    - Only to **delay commitment** to ordering
  - As in backward search:
    - Add **useful** actions to achieve necessary conditions
    - Keep track of what **remains** to be achieved
    - But: Insert actions "**at any point**" in a plan

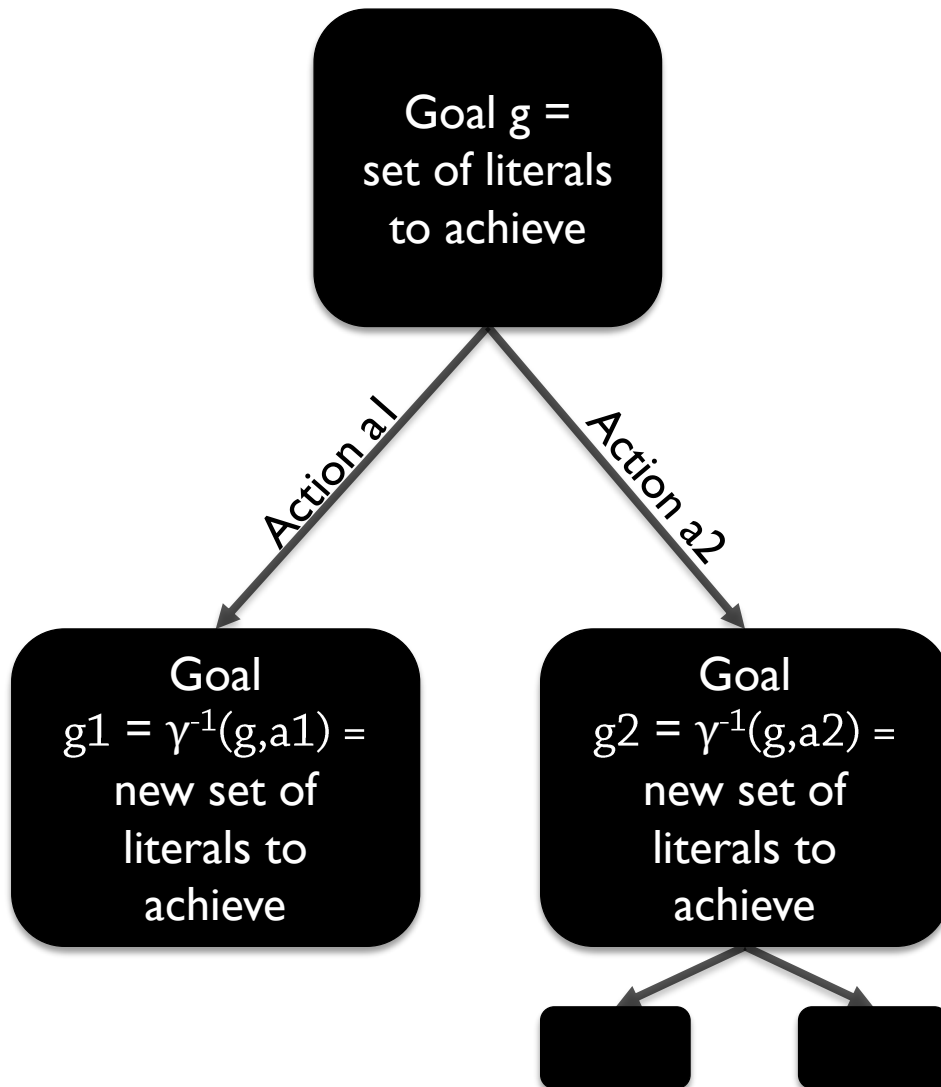


More sophisticated "bookkeeping" required!

# POCL 2: Comparison to Backward Search



- Search tree for backward search, earlier:



The goal is a set of literals – simple!

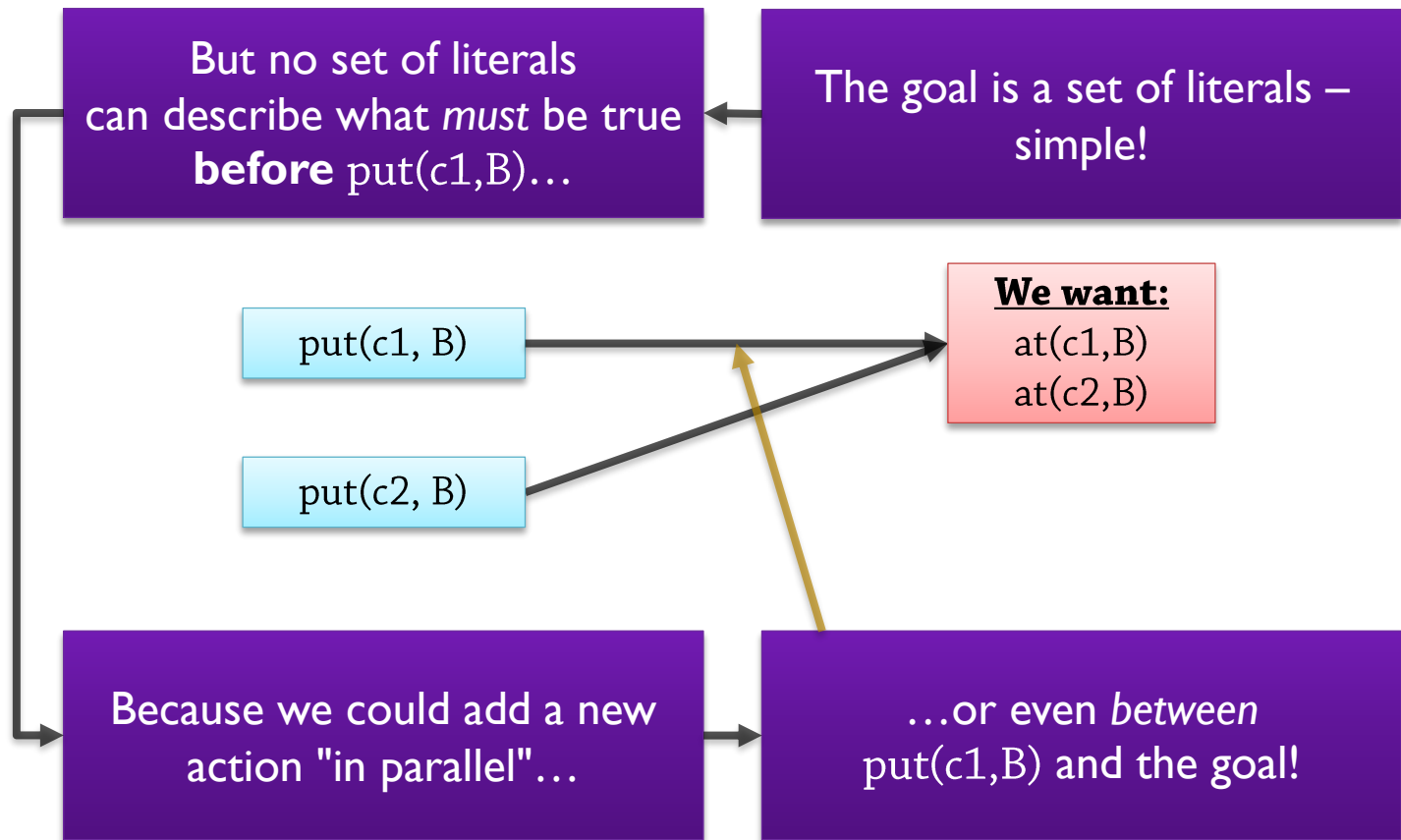
Every step takes you to a new set of literals to achieve

From a search node, you know how to reach the goal using a sequence of actions

A search node [2] can simply be a goal set

# POCL 3: Comparison to Backward Search

- In POCL planning:
  - There is no sequence – and no clear "before" relation!



Has consequences for the POCL plan structure and the node structure...



# POCL 4: Conditions; Goal Action

- Must keep track of individual propositions to be achieved

- Throughout the plan – not a single state  $g1 = \gamma^{-1}(g, a1)$

- May come from preconditions of every action in the plan

Notation chosen for this presentation: Preconditions on the left/top side

robotat(B)

holding(c1)

**put(c1, B)**

- May come from the problem goal as in backward search

- Trick: Use a uniform representation

- Add a "fake" goal action to every plan, with the goals as preconditions!

at(c1, B)

at(c2, B)

**goalaction**

preconds: the goal  
effects: none

# POCL 5: Effects; Initial Action

- Must keep track of individual propositions that are achieved
  - Throughout the plan – not from a single *relevant* action
  - May come from effects of every action in the plan

Notation chosen for this presentation: Effects on the right/bottom side

robotat(B)

holding(c1)

**put(c1, B)**

at(c1,B)

-holding(c1)

- May come from the initial state
  - Trick: Add a "fake" initial action with the initial state as effects!
- Effects are sometimes omitted from the slides, due to lack of space...

**initaction**

preconds: none  
effects:  
the initial state

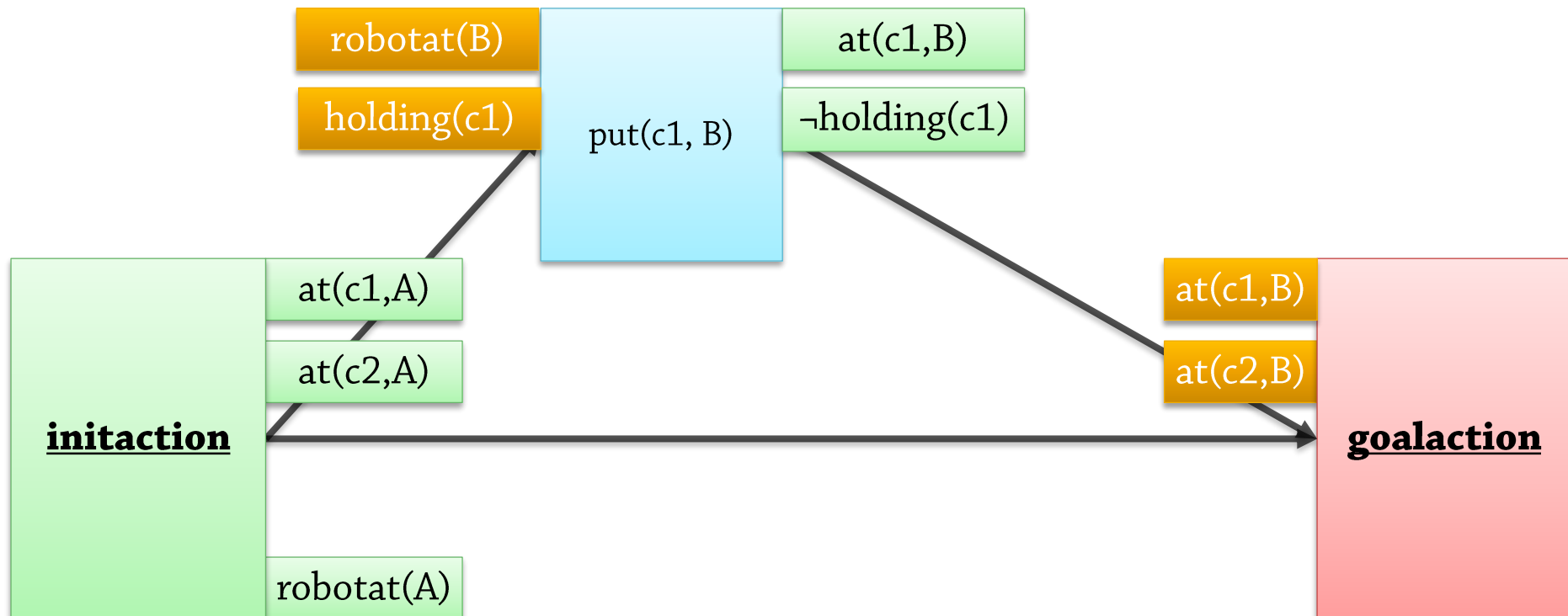
at(c1,A)

at(c2,A)

robotat(A)

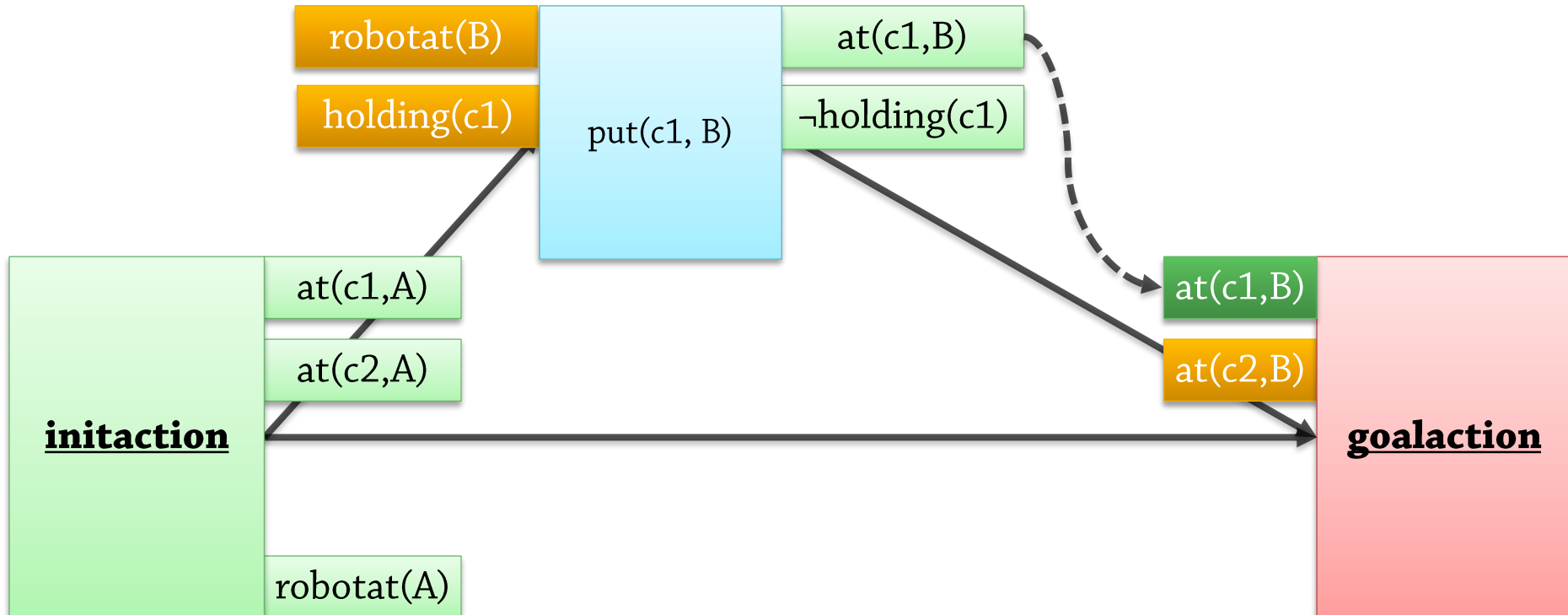
# POCL 6: Precedence Constraints

- Plan structure so far:



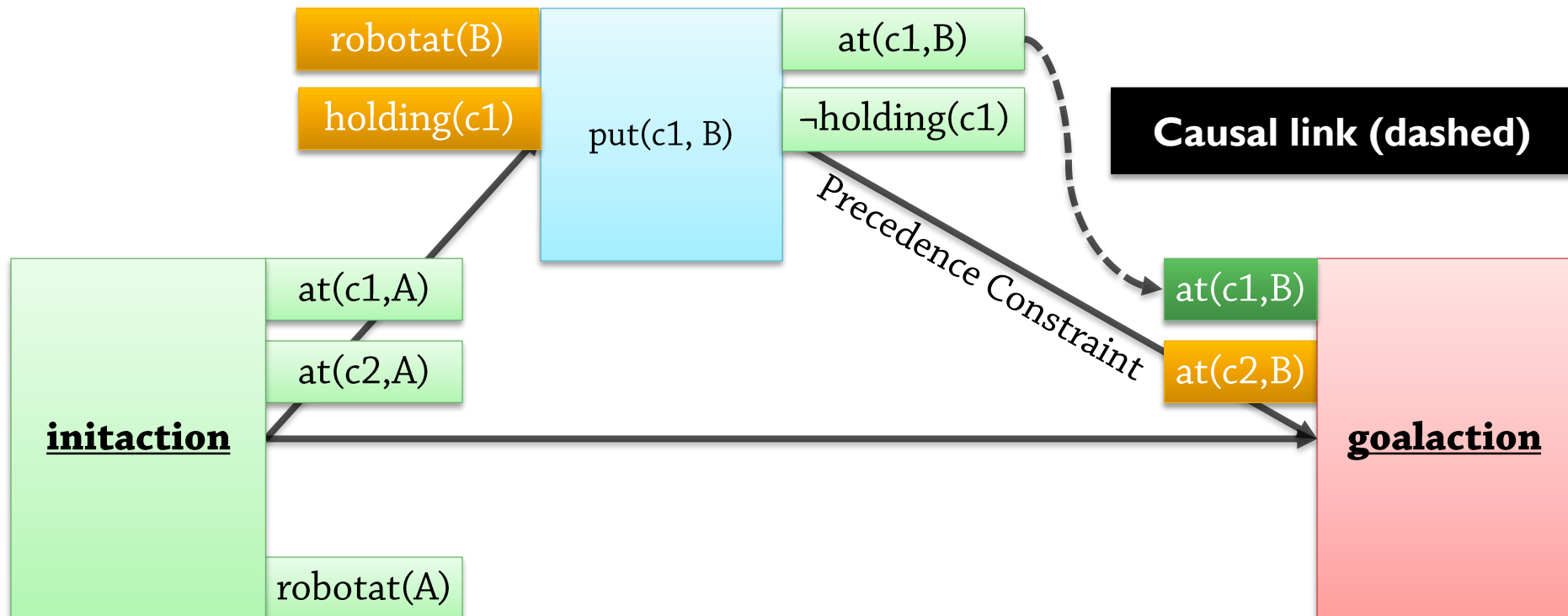
- Let's keep track of which action achieves which precondition
  - Causal links

Causal link (dashed):  
at(c1,B) must **remain true**  
between the end of *put* and the beginning of *goalaction*.  
No one must delete it!  
Important for *threat management* (later)



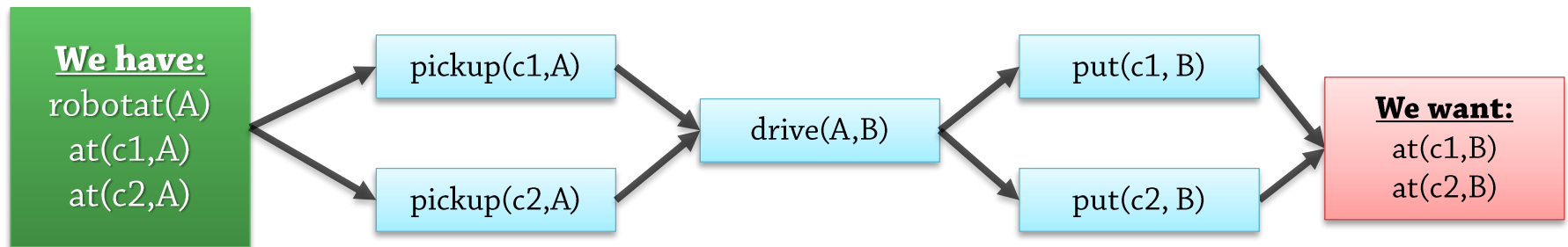
# POCL 8: Partial-Order Plans

- To summarize, a ground **partial-order plan** consists of:
  - A set of **actions**
  - A set of **precedence constraints**  $a \rightarrow b$ 
    - Action  $a$  must precede  $b$
  - A set of **causal links**  $a \xrightarrow{p} b$ 
    - Action  $a$  establishes the precondition  $p$  needed by  $b$



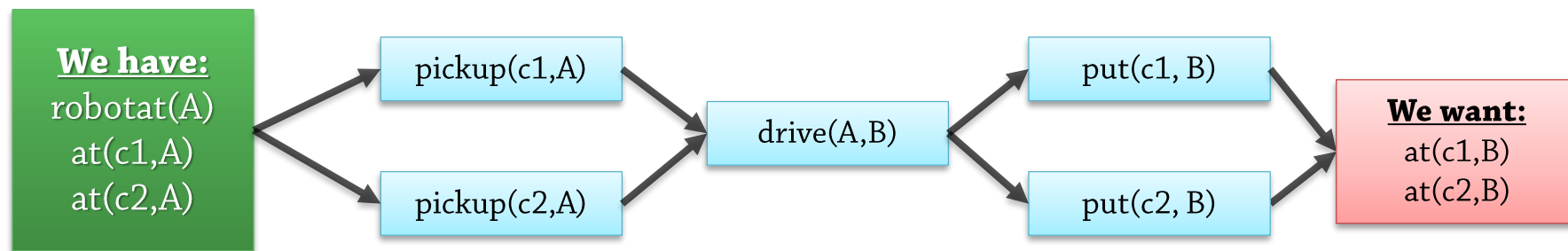
## ■ Original motivation: performance

- Therefore, a partial-order plan is a solution iff all sequential plans satisfying the ordering are solutions
  - Similarly, executable iff corresponding sequential plans are executable
  - $\langle \text{pickup}(c1,A), \text{pickup}(c2,A), \text{drive}(A,B), \text{put}(c1,B), \text{put}(c2,B) \rangle$
  - $\langle \text{pickup}(c2,A), \text{pickup}(c1,A), \text{drive}(A,B), \text{put}(c1,B), \text{put}(c2,B) \rangle$
  - $\langle \text{pickup}(c1,A), \text{pickup}(c2,A), \text{drive}(A,B), \text{put}(c2,B), \text{put}(c1,B) \rangle$
  - $\langle \text{pickup}(c2,A), \text{pickup}(c1,A), \text{drive}(A,B), \text{put}(c2,B), \text{put}(c1,B) \rangle$



# Partial Orders and Concurrency

- Can be extended to allow concurrent execution
  - Requires a new formal model!
    - Our state transition model *does not define* what happens if  $c1$  and  $c2$  are picked up simultaneously!

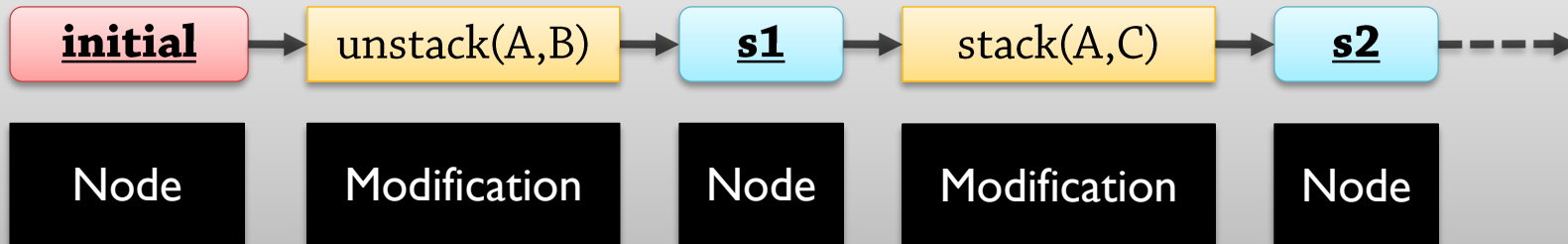


# Partial Order Causal Link: Search Space

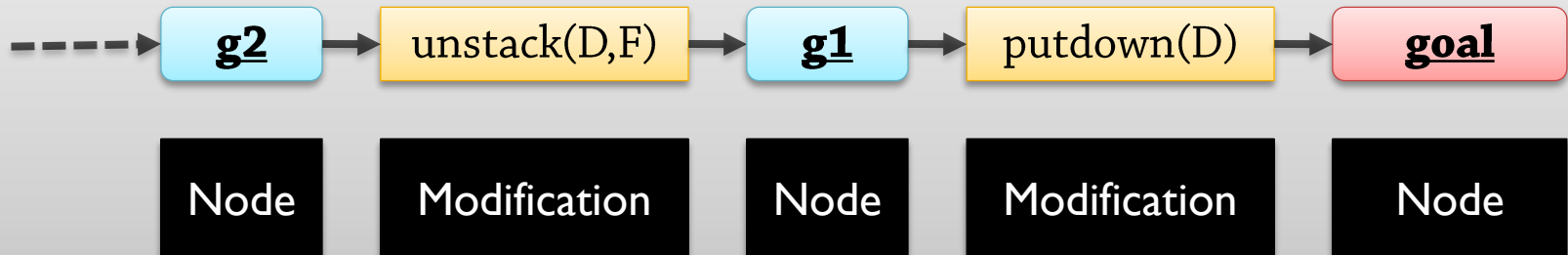


# Context: Forward, Backward

**Forward search:** A search node is a "current state"

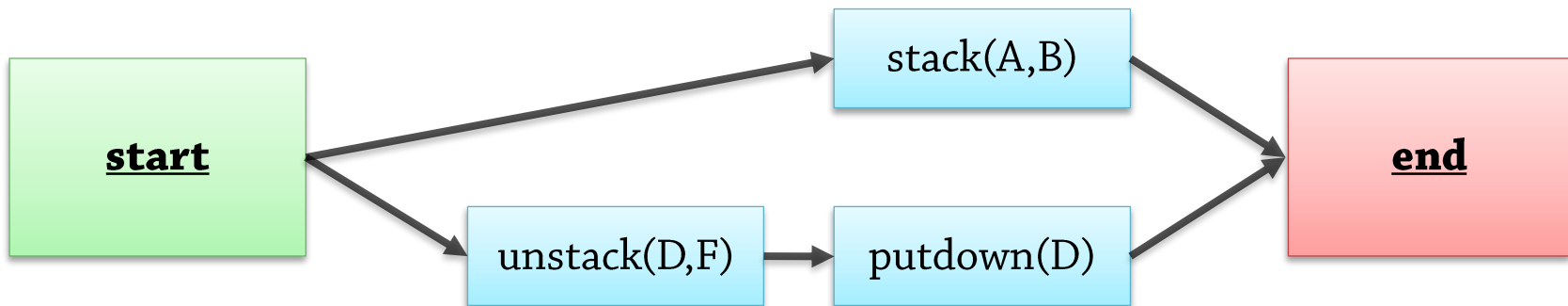


**Backward search:** A search node is a "current goal"



# No Current State during Search!

- With **partial-order plans**: No “current” state or goal!
  - What is true after `stack(A,B)` below?
    - **Depends** on the order in which **other** actions are executed
    - **Changes** if we insert **new** actions **before** `stack(A,B)`!



**A search node can't correspond to a state or goal!**

# Search Nodes are Partial Plans

- [1] Each node must contain *more information*: **The entire plan!**
- [2] The **initial** search node contains an **initial plan**
  - The special **initial** and **goal actions**
  - A single **precedence constraint**

So: this is one form of  
"plan-space" planning!

## Initial node containing initial plan



# Branching Rule

- [3] We need a **branching rule** as well!
  - Forward planning: One successor per action **applicable** in  $s$
  - Backward planning: One successor per action **relevant** to  $g$
  - POCL planning: ???



*Could allow inserting any actions,  
any precedence constraints...*

***Too much freedom, too many alternatives!***

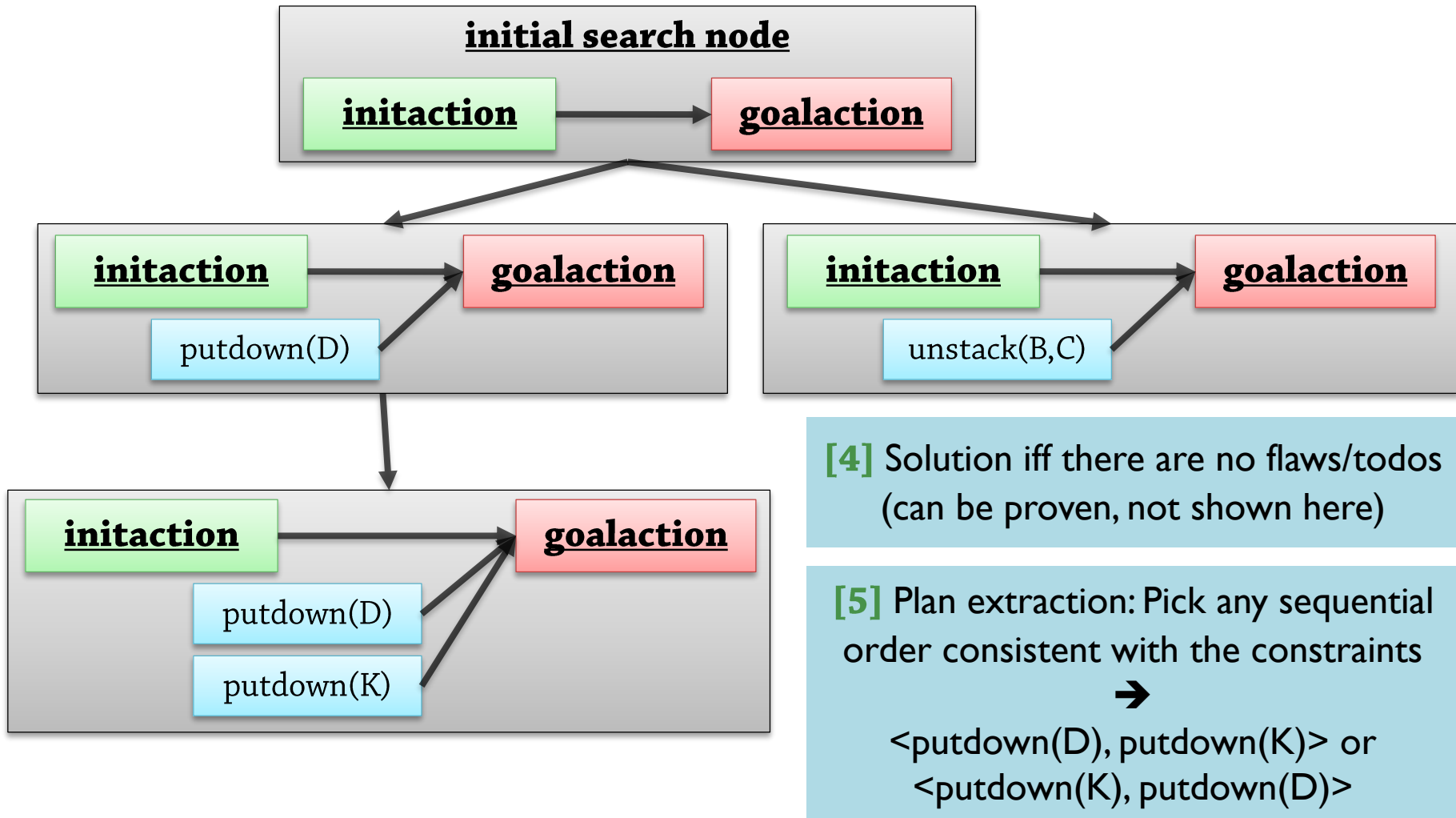
*What do we need for completeness?*

- **[3] Branching rule** for POCL planning
  - Identify specific reasons for modifying the plan, called flaws (basically todo:s)
    - 1) **Open goal**: We haven't decided *how* to achieve a precondition – clear(A)
    - 2) **Threat**: An action may *interfere* with another
  - One successor for each different way of repairing a flaw



- Gives rise to a **search space**

- [6] Use search strategies, backtracking, heuristics, ... to search **this** space!



# Successors Repair Flaws: Open Goals and Threats

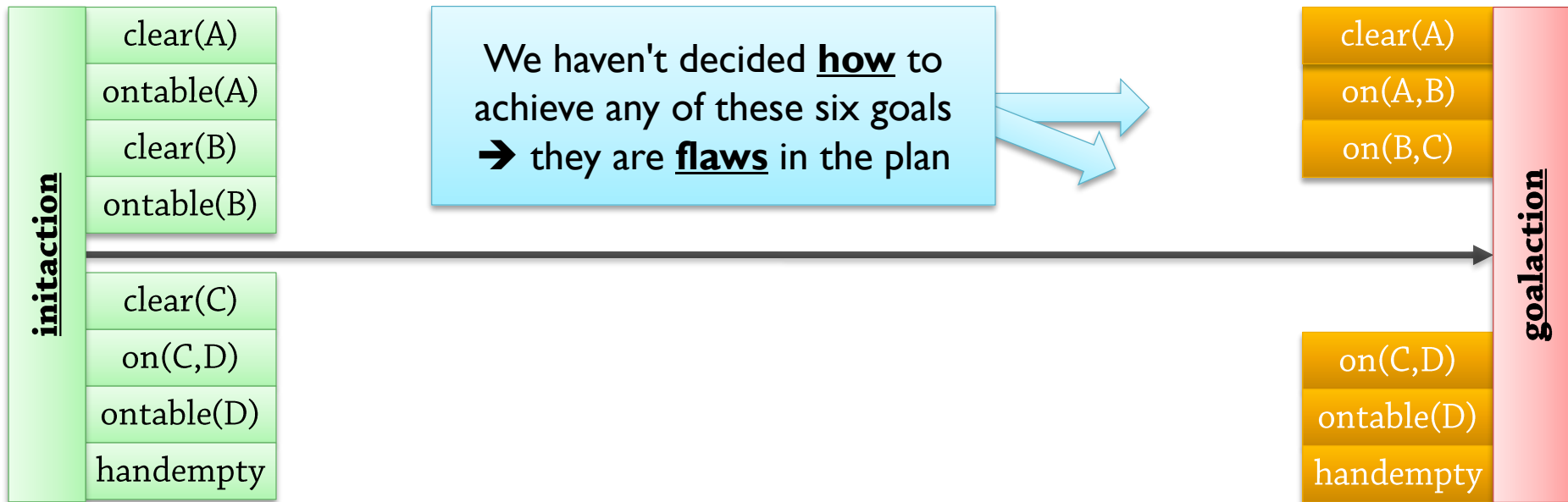


- **Flaw, *noun*:**
  1. a feature that mars the perfection of something; defect; fault: beauty without flaw; the flaws in our plan.
  2. a defect impairing legal soundness or validity.
  3. a crack, break, breach, or rent.
  
- **Flaw, in POCL planning:**
  - Something we **need to take care of** to **complete the plan**
  - Technical definition: *An open goal or a threat*
  
- **Not:**
  - Something that has "gone wrong"
  - A problem during planning
  - A mistake in the final solution
  - ...

*Open Goals*

# Flaw Type 1: Open Goals

- Open goal:
  - An action  $a$  has a precondition  $p$  with no incoming causal link



clear(A) is already true in  $s_0$ , but there is no causal link...

Adding one from  $s_0$  means clear(A) **must never be deleted!**

We need other alternatives too: Delete clear(A), then re-achieve it for goalaction...

# Flaw Type 1: Open Goals

- To **resolve** an open goal :
  - Find an action  $b$  that causes  $p$ 
    - Can be a *new* action
    - Can be an action *already* in the plan, if we can *make* it precede  $a$
  - Add a **causal link**



Partial order! This was not possible in backward search...

## **Essential:**

Even if there **is already** an action that causes  $p$ ,  
you can still add a **new** action that **also** causes  $p$ !

# Resolving Open Goals 1

## ■ Here: Six open goals

- Could choose to find support for clear(A):
  - From *initaction*
  - From a new `unstack(B,A)`, `unstack(C,A)`, or `unstack(D,A)`
  - From a new `stack(A,B)`, `stack(A,C)`, `stack(A,D)`, or `putdown(A)`
- Could choose to find support for on(A,B):
  - Only from a new instance of `stack(A,B)`
- ...

8 distinct  
successors

+1 successor

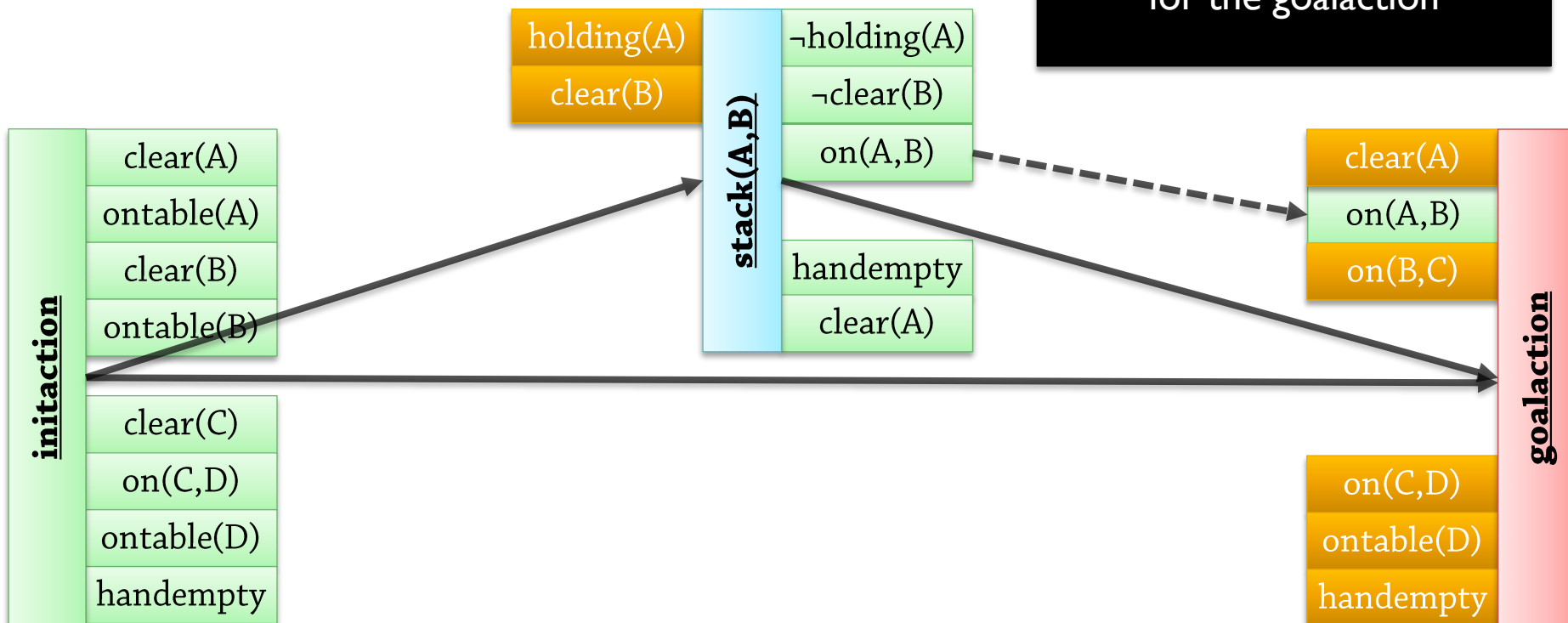


# Resolving Open Goals 2

- Suppose we add stack(A,B) to support (achieve) on(A,B)

- Must add a causal link for on(A,B)
  - Dashed line
- Must also add precedence constraints
- Looks totally ordered
  - Because it actually only has one “real” action...

Causal link says:  
This instance of stack(A,B)  
is responsible for  
achieving on(A,B)  
for the goalaction

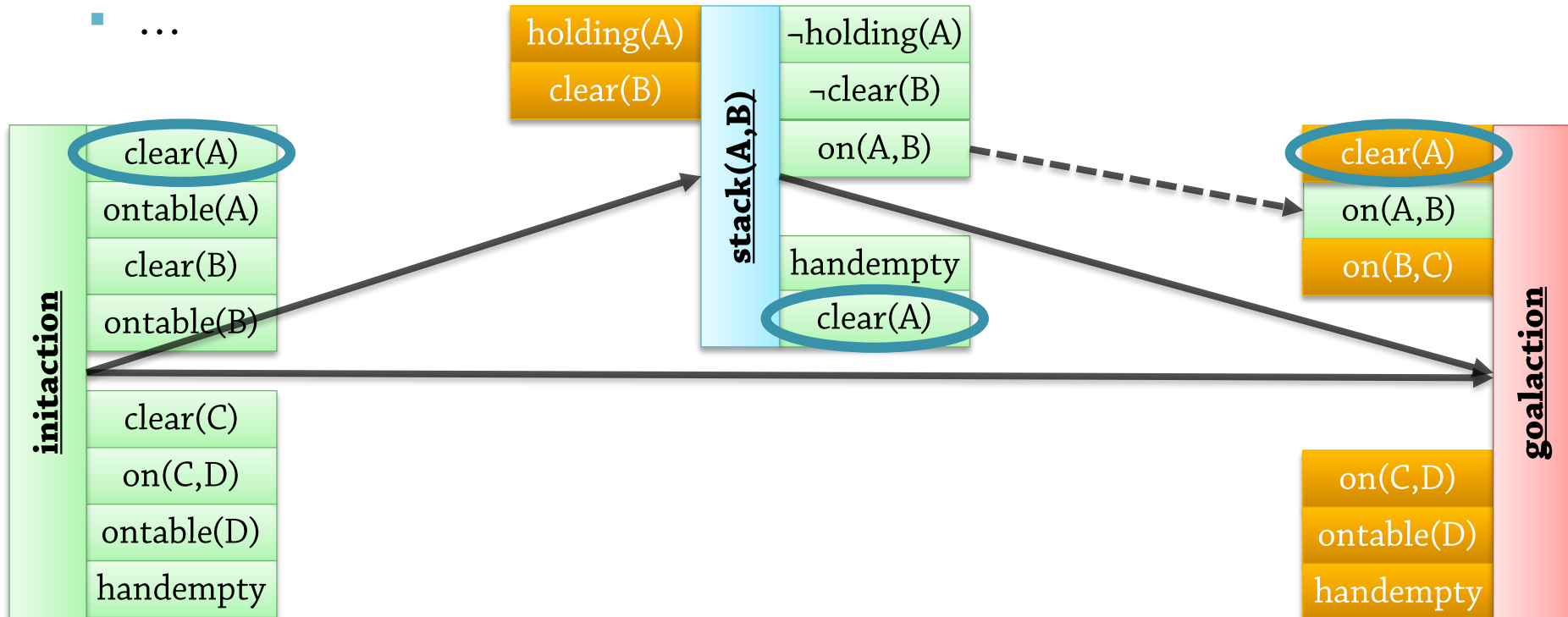


# Resolving Open Goals 3

## ■ Now: **7** open goals (one more!)

- Can choose to find support for `clear(A)`:
  - From the initaction
  - From the instance of **stack(A,B)** that we just added
  - From a **new** instance of **stack(A,B)**, `stack(A,C)`, `stack(A,D)`, or `putdown(A)`
  - From a **new** instance of `unstack(B,A)`, `unstack(C,A)`, or `unstack(D,A)`

■ ...



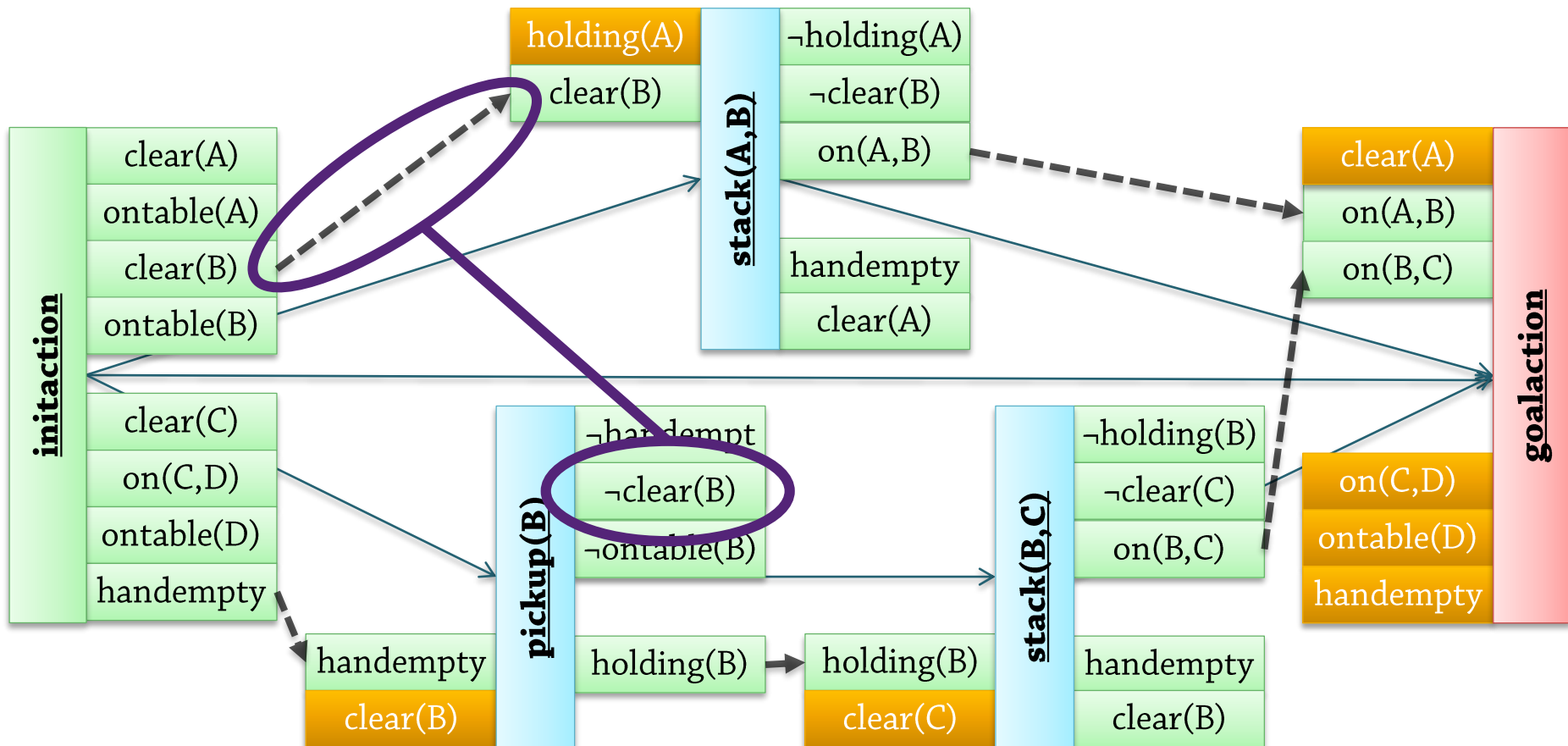
*Threats*



# Flaw Type 2: Threats

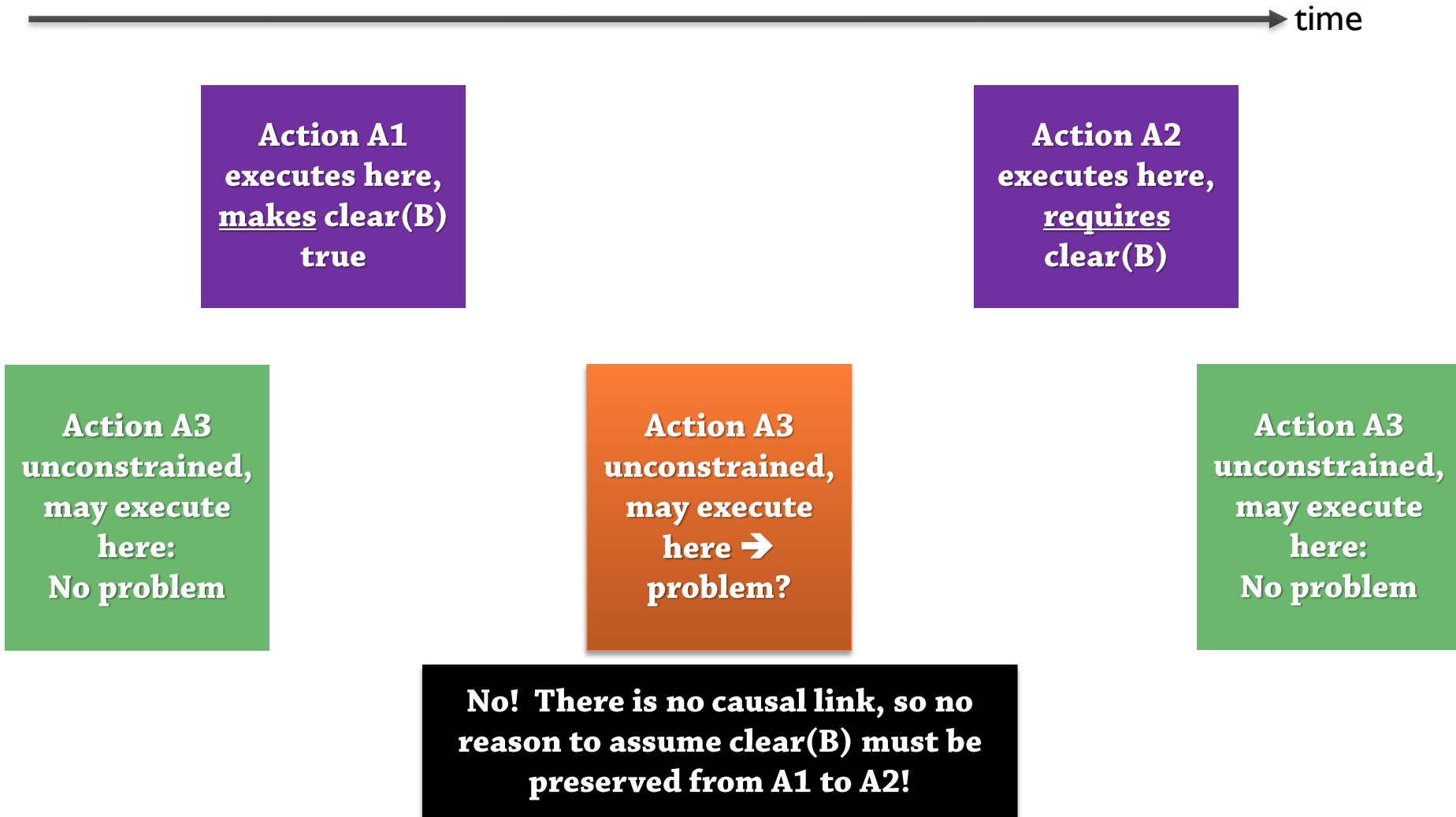
## ■ Second flaw type: A threat against a causal link

- initiation **should support** clear(B) for stack(A,B) – there's a **causal link**
- pickup(B) **deletes** clear(B), and may occur **between** initiation and stack(A,B)
- So we can't be certain that clear(B) still holds when stack(A,B) starts!



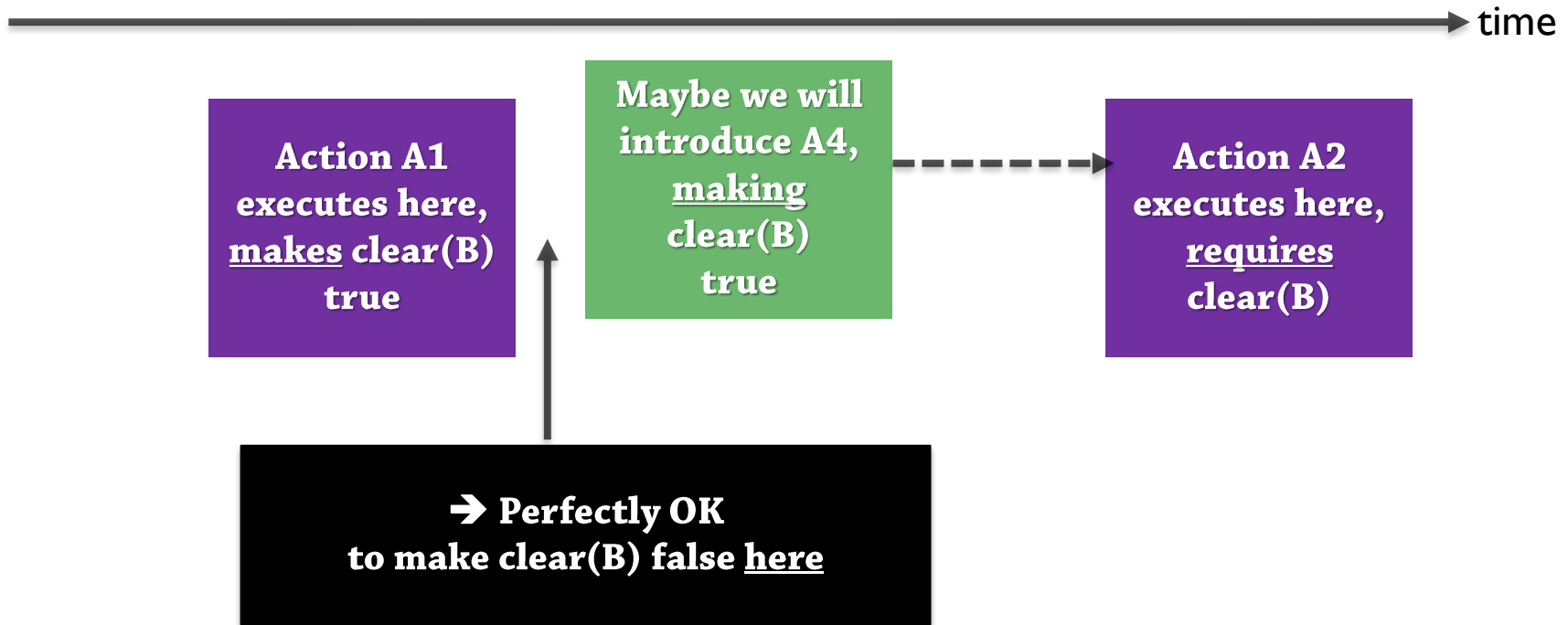
# Flaw Type 2: Threats (2)

- Another way of illustrating a threat, on a “timeline”



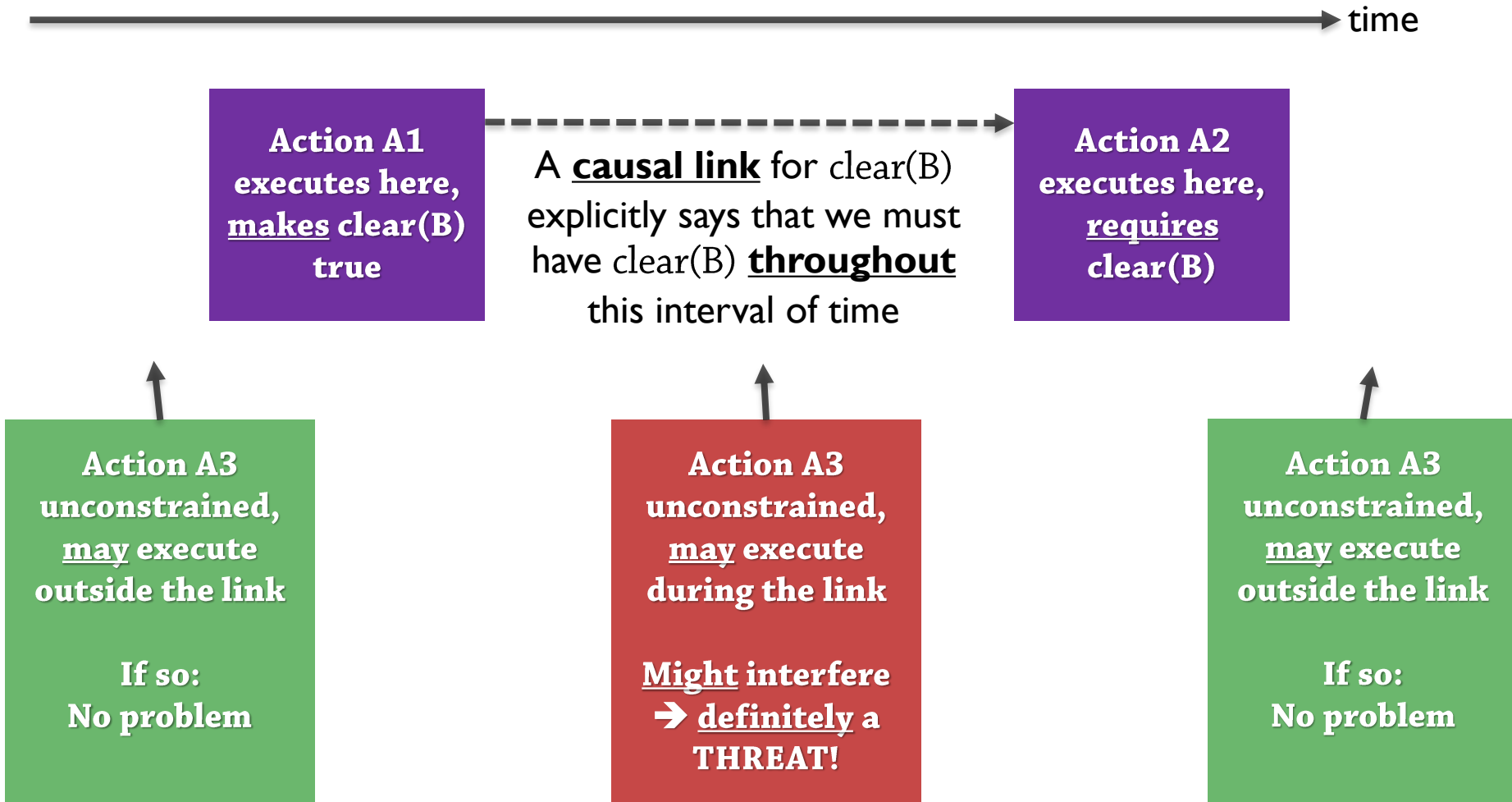
# Flaw Type 2: Threats (3)

- Why no threat without causal link?



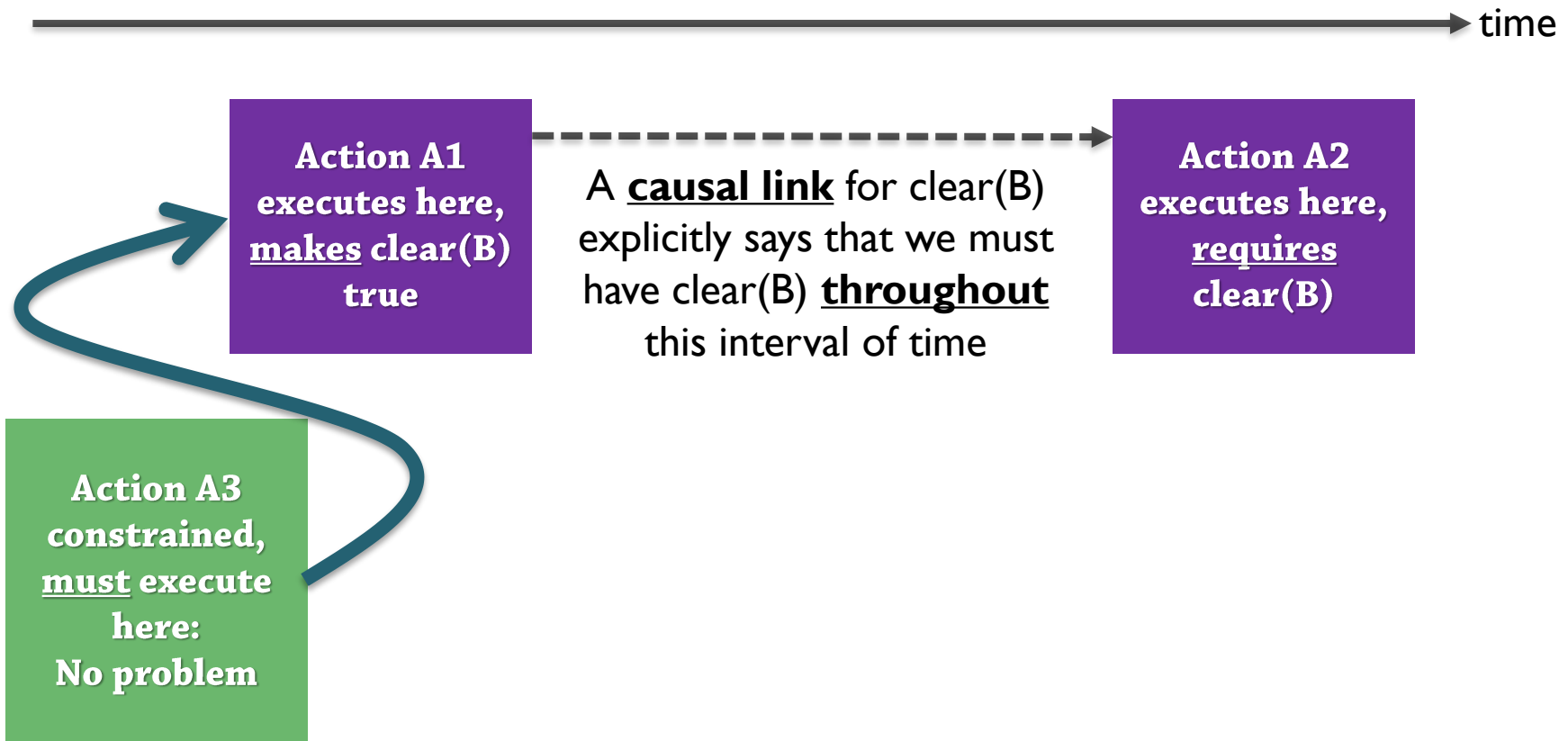
# Flaw Type 2: Threats (4)

- But when we have a causal link:



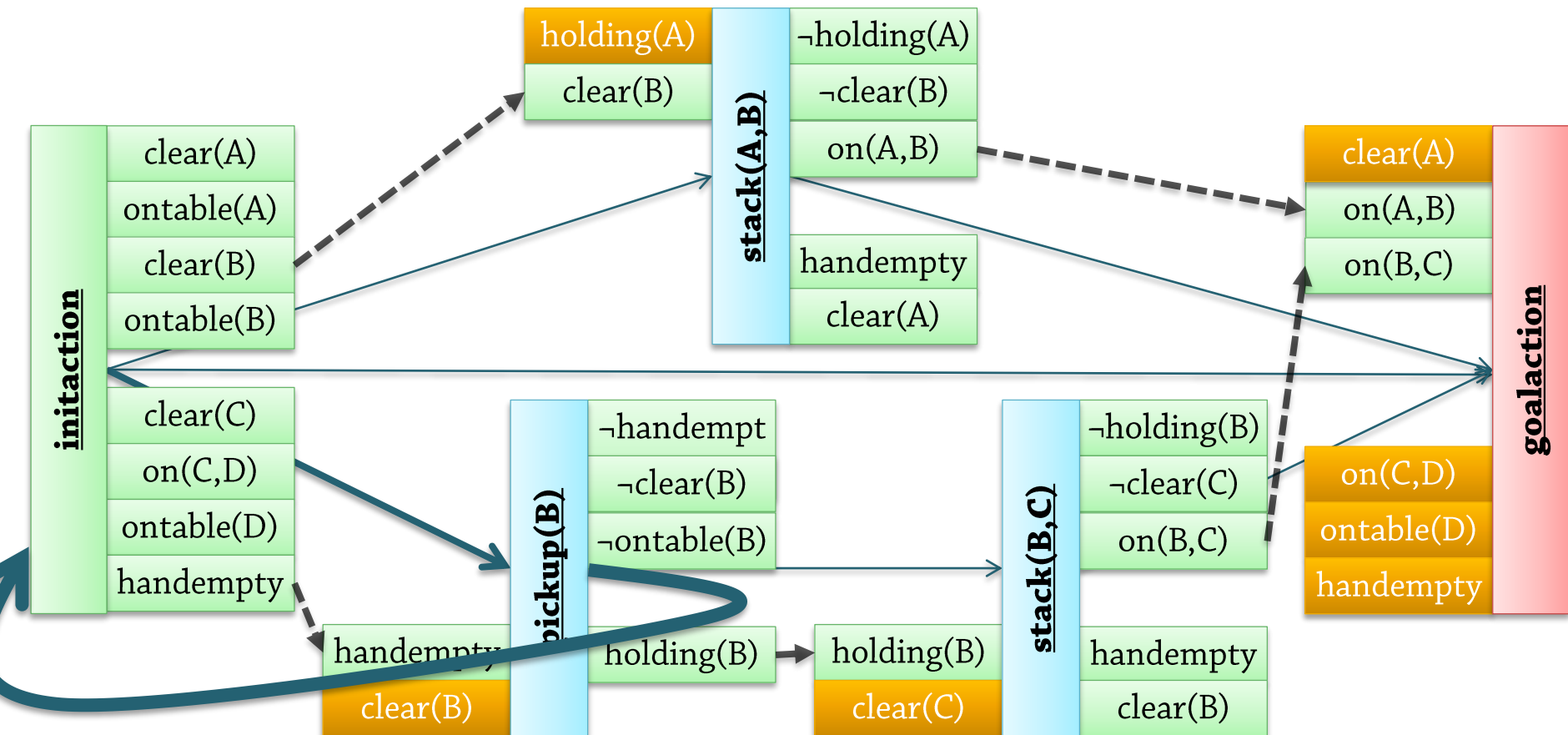
# Resolving Threats (1)

- Resolution 1: The action that **disturbs** the causal link is placed **before** the action that **supports** the precondition
  - Only possible if the resulting partial order is consistent (acyclic)!



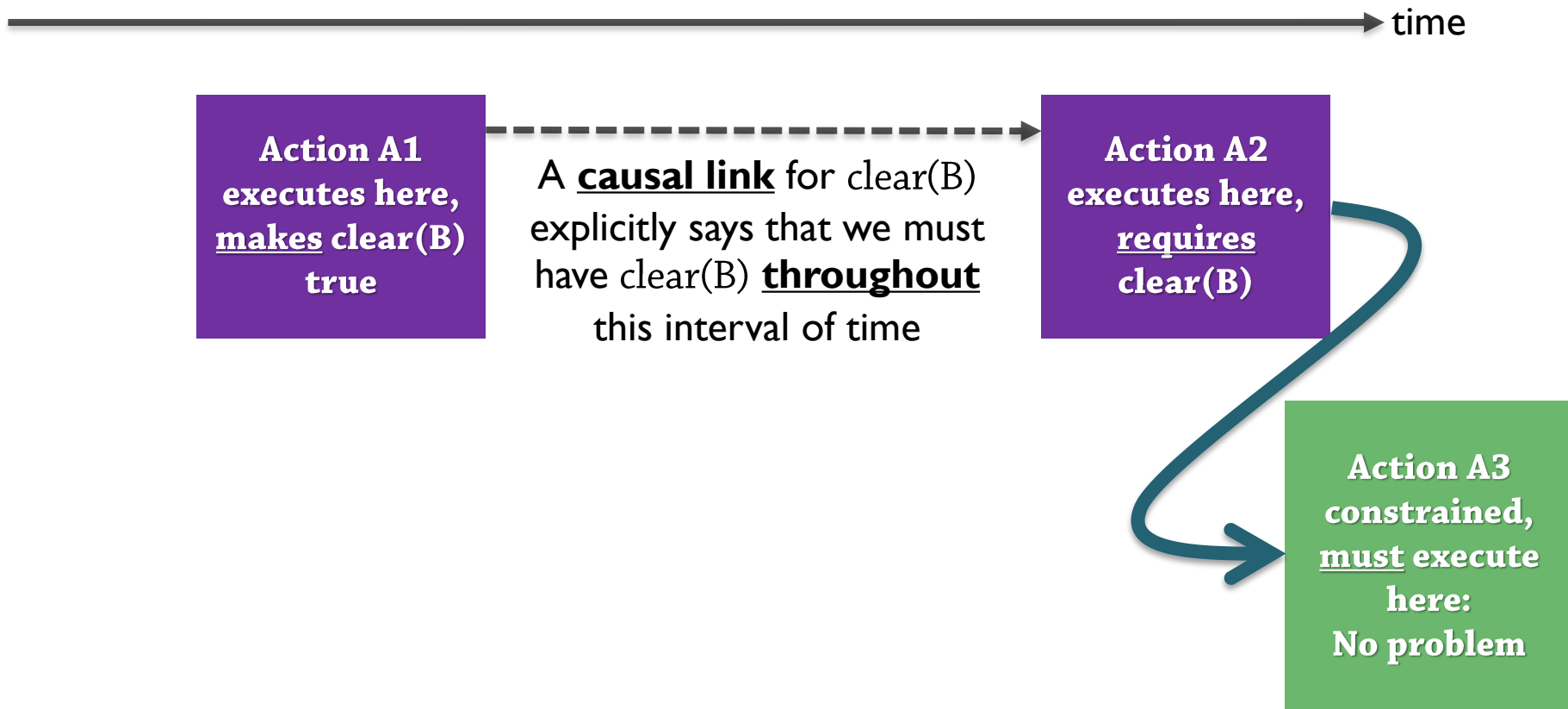
# Resolving Threats (2)

- In this case, not consistent



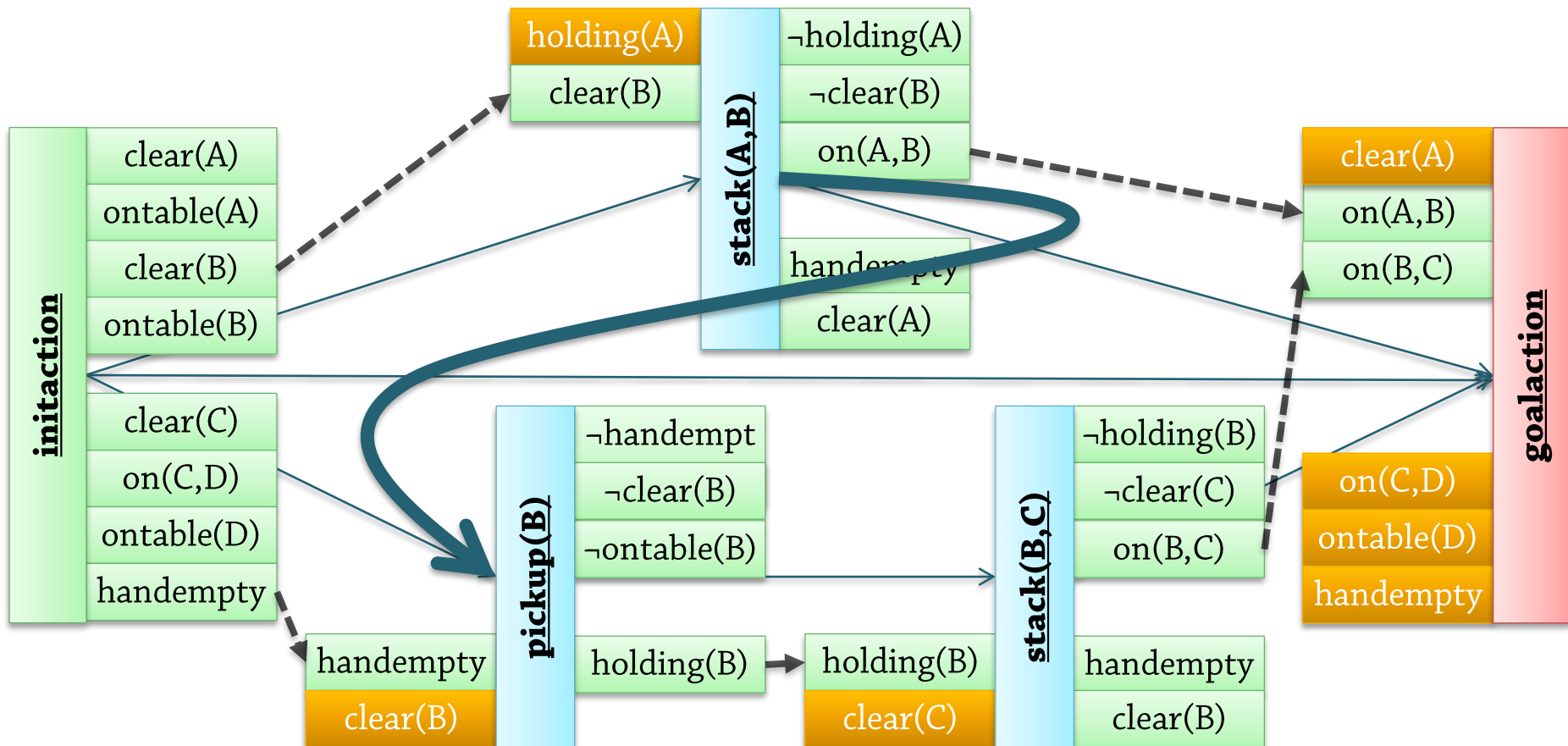
# Resolving Threats (3)

- Resolution 2: The action that **disturbs** the causal link is placed **after** the action that **requires** the precondition
  - Only possible if the resulting partial order is consistent (acyclic)!



# Resolving Threats (4)

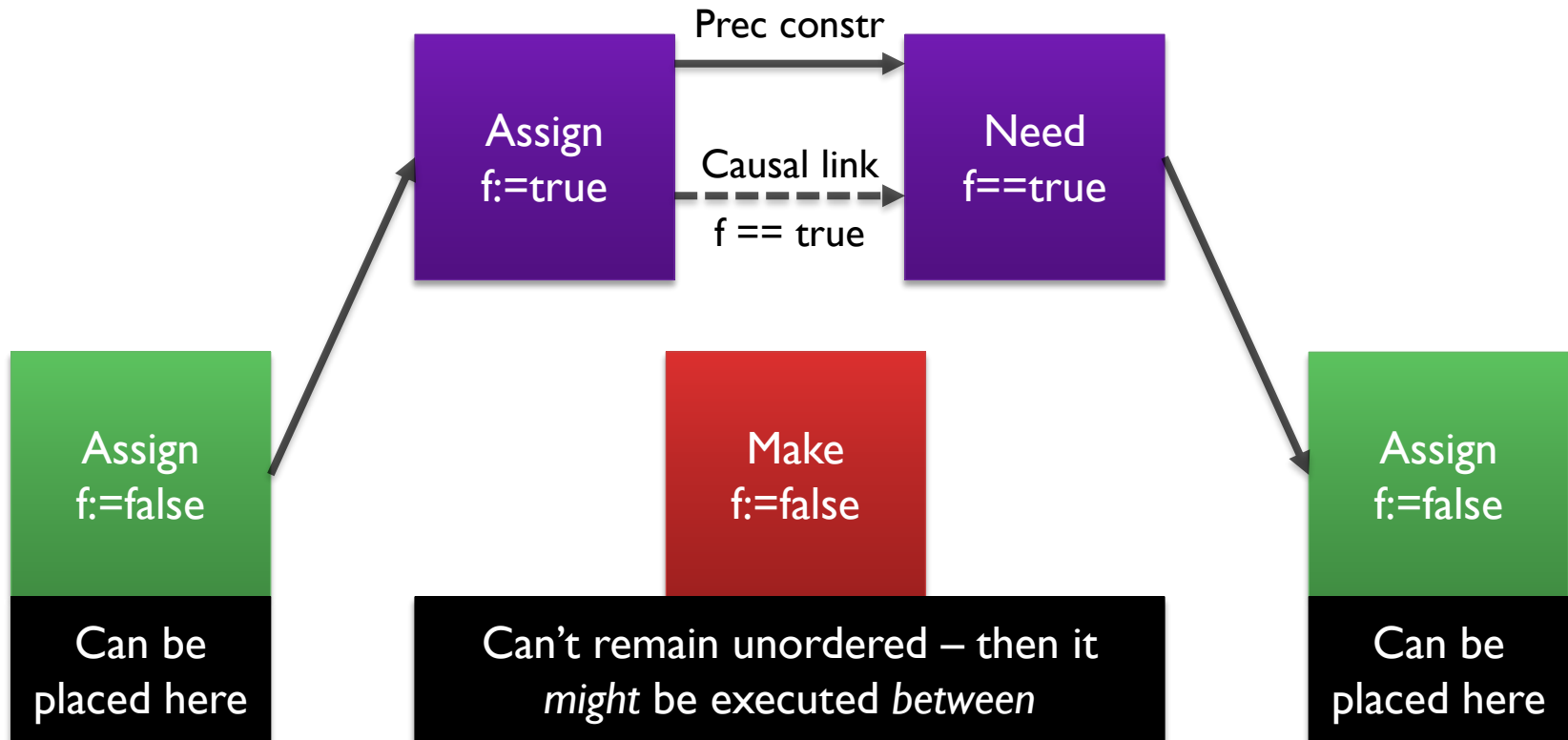
- Works for this example





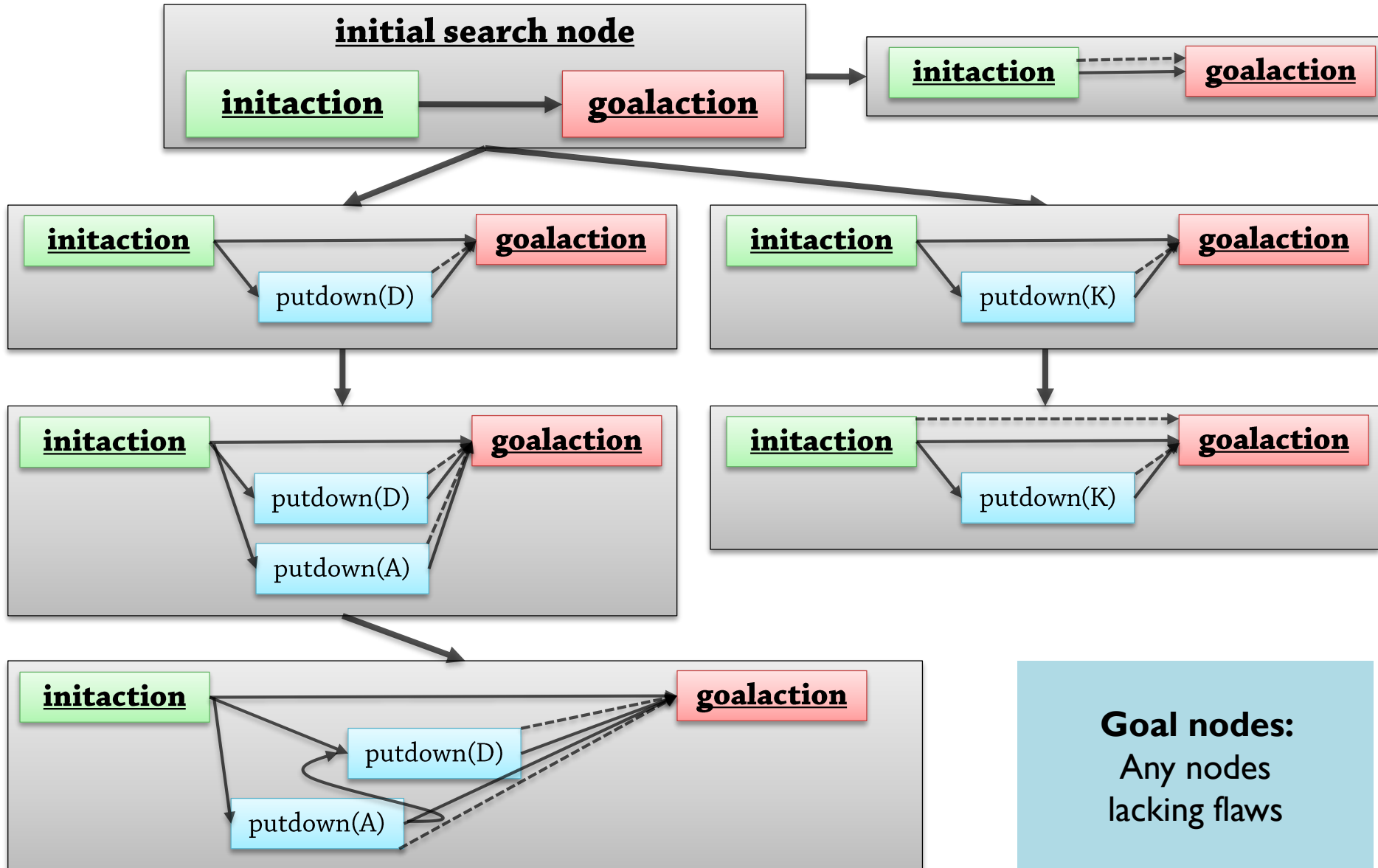
# Resolving Threats (5)

- Summary:



# POCL Algorithms

# Repetition: POCL Search Space



# Repetition: Planning as Search

## ■ Repetition

```
■ search(problem) {  
    initial-node ← make-initial-node(problem) // [2]  
    open ← { initial-node }  
    while (open ≠ ∅) {  
        node ← search-strategy-remove-from(open) // [6]  
        if is-solution(node) then // [4]  
            return extract-plan-from(node) // [5]
```

```
        foreach newnode ∈ successors(node) { // [3]  
            add newnode to open
```

```
        }
```

```
    // Expanded the entire search space without finding a solution  
    return failure
```

```
}
```

[3] Expand  
Successors:  
All ways of  
resolving  
some flaw

- POCL planning – one possible formulation (sound/complete):

```
pocl-search(problem) {  
  initial-node  $\leftarrow$  make-initial-node(problem.initial, problem.goal) // [2]  
  open  $\leftarrow$  { initial-node }  
  while (open  $\neq$   $\emptyset$ ) {  
     $\pi \leftarrow$  search-strategy-remove-from(open) // [6]  
    flaws  $\leftarrow$  OpenGoals( $\pi$ )  $\cup$  Threats( $\pi$ )  
    if flaws =  $\emptyset$  then  $\leftarrow$  [4] Can prove:  $\pi$  is a solution  
      iff there are no remaining flaws  
      return  $\pi$  // [5]  
    select any flaw  $\varphi \in$  flaws  $\leftarrow$  One flaw chosen!  
    resolvers  $\leftarrow$  FindResolvers( $\varphi$ ,  $\pi$ ) // May be the empty set!  
    foreach r in resolvers {  
       $\pi' \leftarrow$  Refine(r,  $\pi$ ) // Actually apply the resolver  
      open  $\leftarrow$  open  $\cup$  {  $\pi'$  }  
    }  
  }  
  return failure  
}
```

[3] Expand Successors: **All** ways of resolving **some** flaw

But **all** resolvers must be tested...

Returns a **partially** ordered solution plan  
Any **total** ordering of this plan will achieve the goals

# Understanding Successors in POCL

At first we said 'every *flaw* leads to successors'.

But it is actually sufficient to try one flaw, any flaw, to resolve. Testing other flaws would be *redundant*. Why?

- 1) Every flaw *has* to be resolved
- 2) Choosing this flaw *later* cannot help us resolve it:  
All possibilities already exist
- 3) Choosing this flaw *later* cannot help us resolve some other flaw

Allows us to use heuristics to select flaws  
(as well as prioritizing open nodes)

select any flaw  $\varphi \in \text{flaws}$   
 $\text{resolvers} \leftarrow \text{FindResolvers}(\varphi, \pi)$

foreach  $r$  in  $\text{resolvers}$ :  
 $\pi' \leftarrow \text{Refine}(\rho, \pi)$   
 $\text{open} \leftarrow \text{open} \cup \{ \pi' \}$

We must allow search to test different resolvers for the chosen flaw. Why?

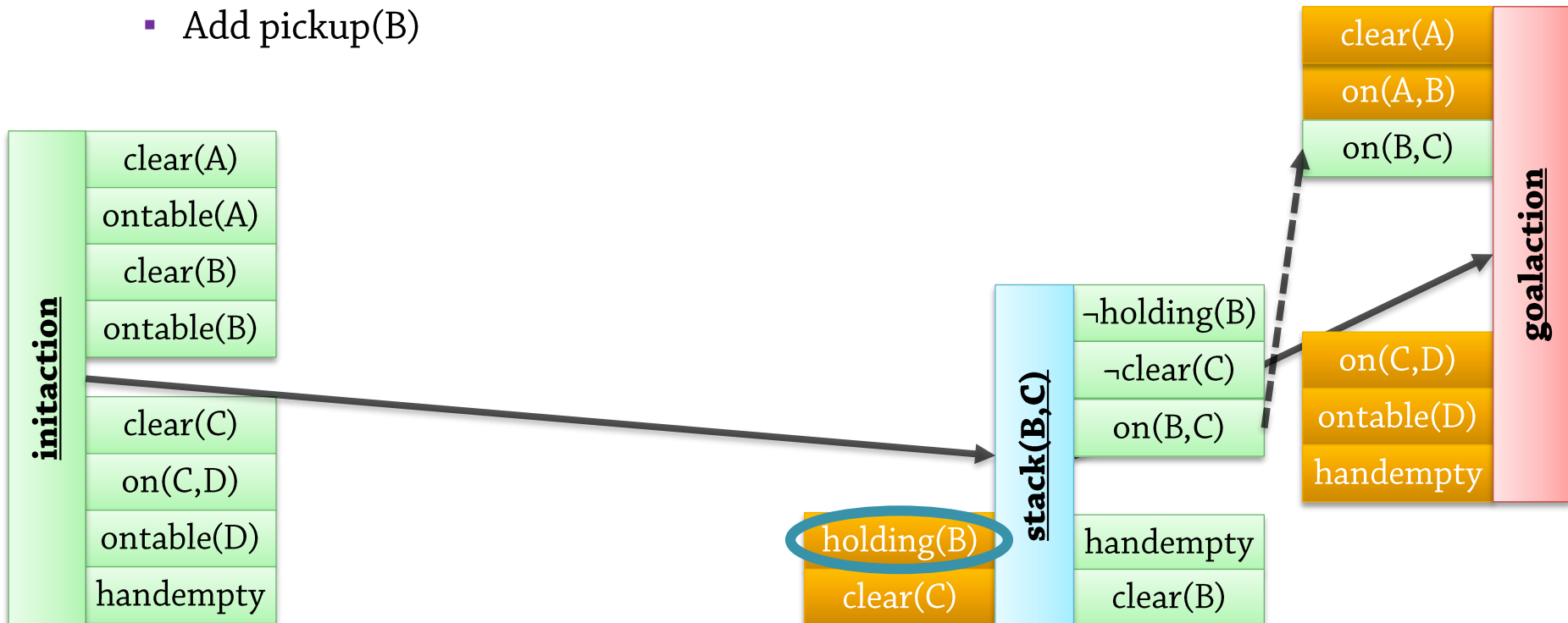
- 1) Choosing one resolver can prevent other problem resolutions.
- 2) Open goal: Use action A or B?
- 3) Threat: Which order to choose?

# Lifted Planning Revisited

# Partial Instantiation

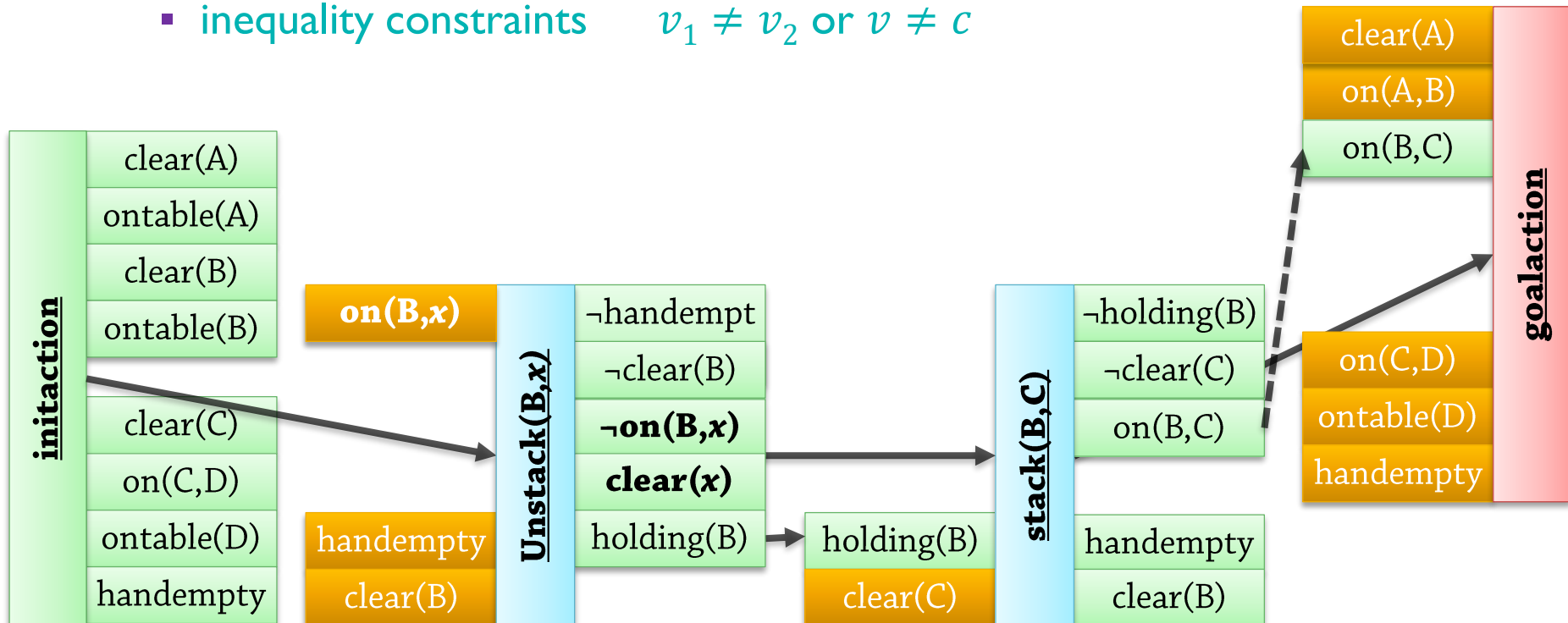
- Suppose we want to achieve `holding(B)`
  - **Ground** search generates **many** alternatives
    - Add `unstack(B,A)`, `unstack(B,F)`, `unstack(B,G)`, ...
    - Add `pickup(B)`
  - **Lifted** search generates two **partially instantiated** alternatives
    - Add **unstack(B, x)**
    - Add `pickup(B)`

So far, we see no reason why we should unstack B from any **specific** block!

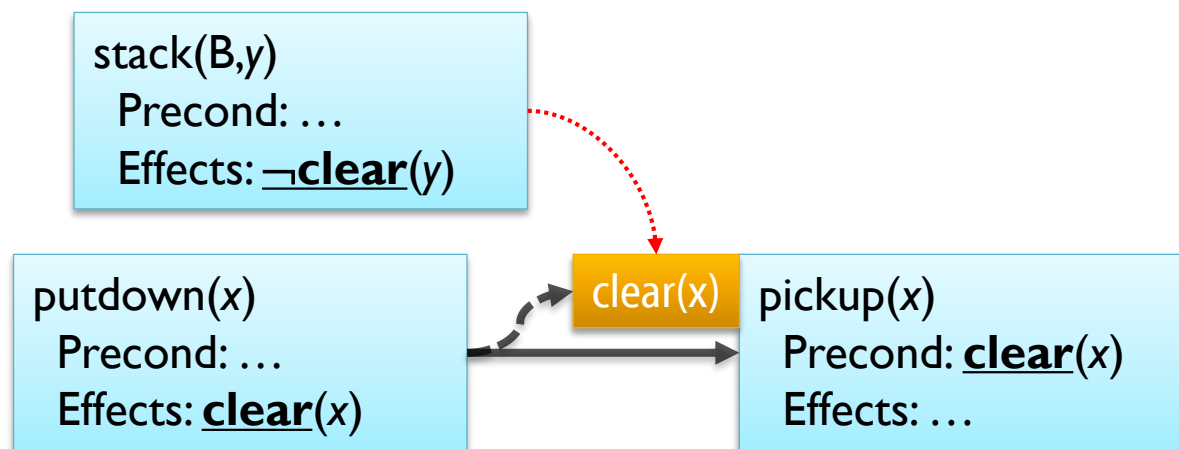




- A lifted partial-order plan consists of:
  - A set of possibly unground actions
  - A set of precedence constraints:  $a$  must precede  $b$
  - A set of causal links: action  $a$  establishes the precondition  $p$  needed by  $b$
  - A set of binding constraints:
    - equality constraints  $v_1 = v_2$  or  $v = c$
    - inequality constraints  $v_1 \neq v_2$  or  $v \neq c$



- Another way of resolving threats for lifted plans:
  - For partly uninstantiated actions, we may find potential threats
    - $\text{stack}(B,y)$  may threaten the causal link, but only if  $x = y$
    - Can be resolved by adding a constraint:  $x \neq y$

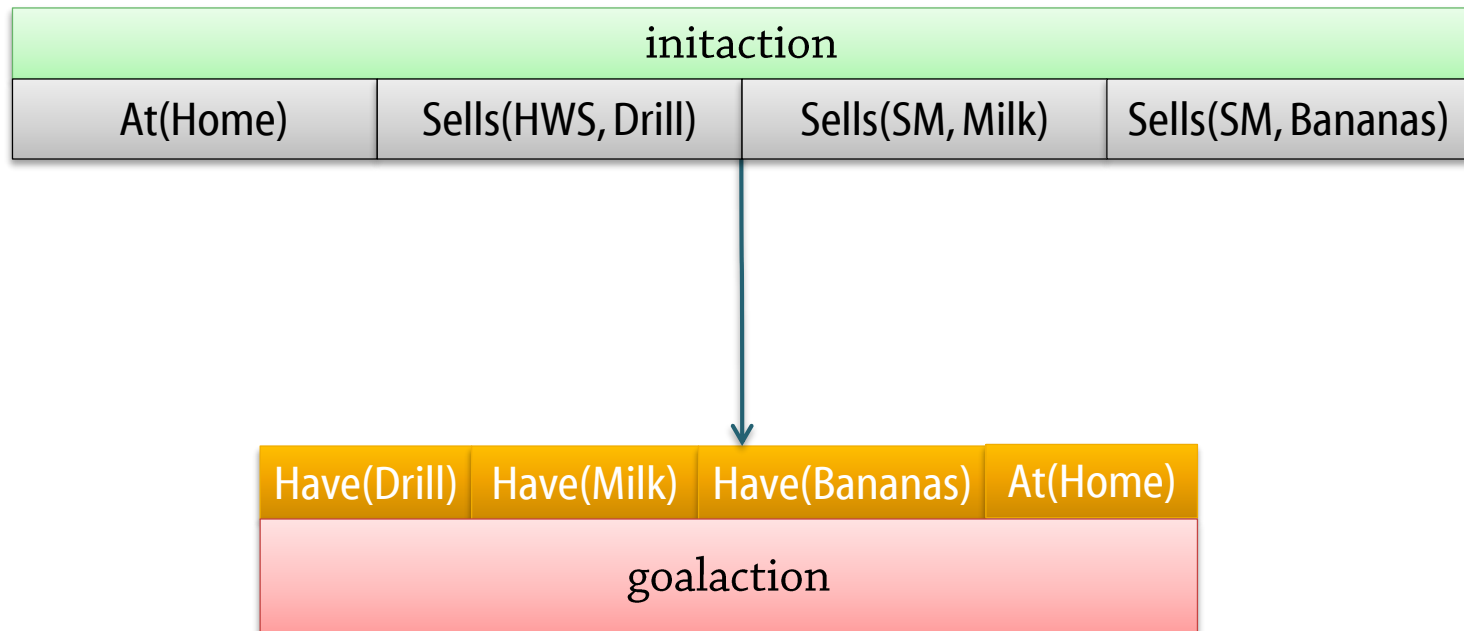


# Complete Example

- Running Example: Similar to an example in ALMA
  - Russell and Norvig's *Artificial Intelligence: A Modern Approach* (1st ed.)
  - Operator **Go(from,to)**
    - Precond:  $\text{At}(\text{from})$
    - Effects:  $\text{At}(\text{to}), \neg \text{At}(\text{from})$
  - Operator **Buy(product, store)**
    - Precond:  $\text{At}(\text{store}), \text{Sells}(\text{store}, \text{product})$
    - Effects:  $\text{Have}(\text{product})$
  - **Initial state**
    - $\text{At}(\text{Home}), \text{Sells}(\text{HWS}, \text{Drill}), \text{Sells}(\text{SM}, \text{Milk}), \text{Sells}(\text{SM}, \text{Bananas})$
  - **Goal**
    - $\text{At}(\text{Home}), \text{Have}(\text{Drill}), \text{Have}(\text{Milk}), \text{Have}(\text{Bananas})$

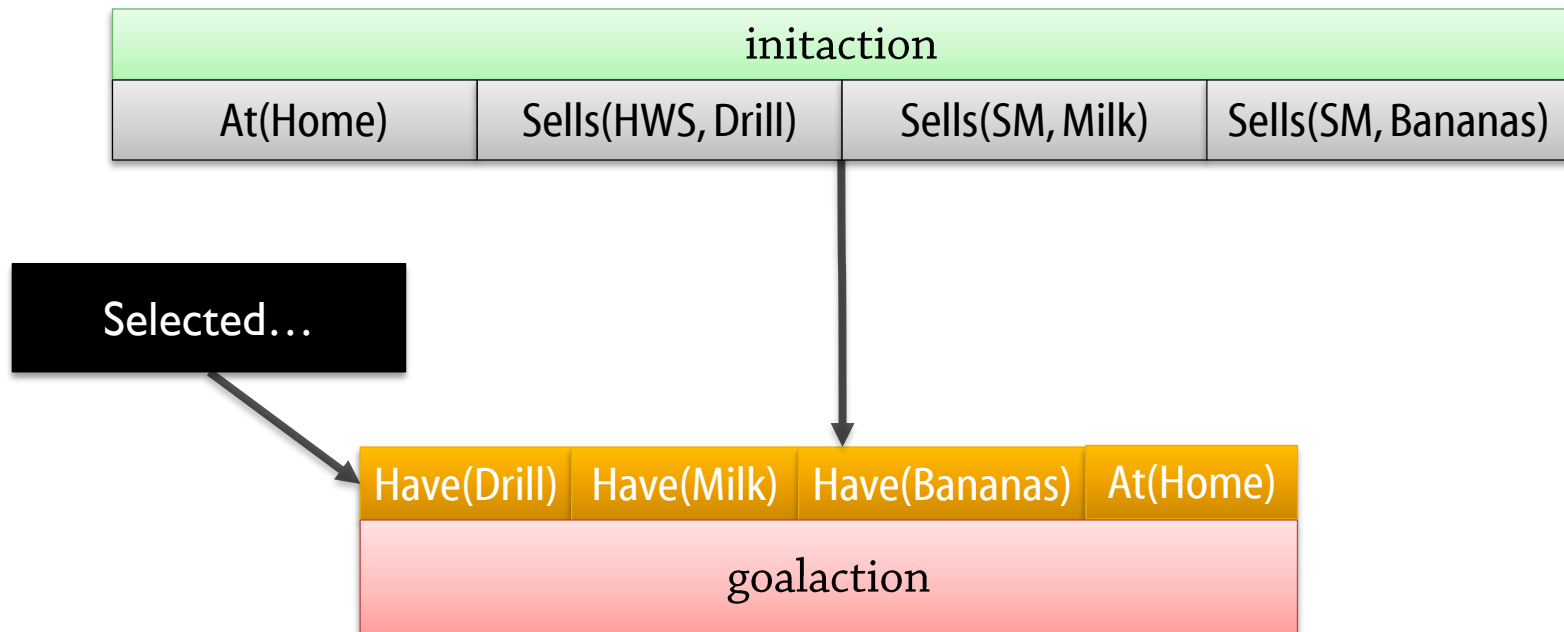
# Example (continued)

- Initial plan: **initaction**, **goalaction**, and a precedence constraint



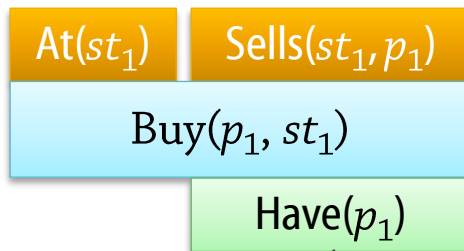
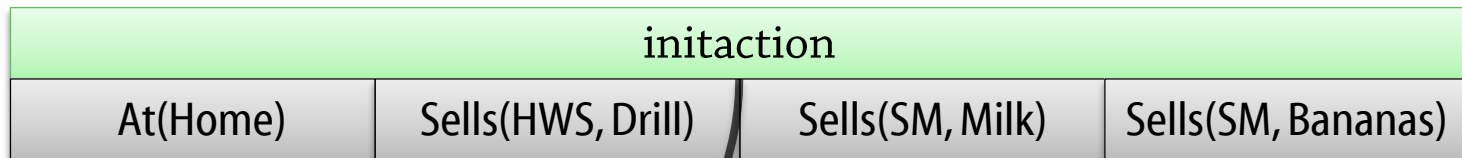
# Example (continued)

- Four **flaws** exist: Open goals
  - Suppose our heuristics tell us to resolve **Have(Drill)** first



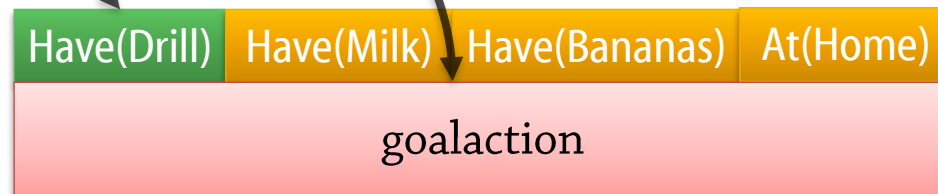
# Example (continued)

- Have(drill) is not achieved by any action in the current plan
- But **Buy(product, store)** achieves Have(product)
  - Partially instantiate:  
**Buy(Drill, store)** (right now we don't care where we buy it)



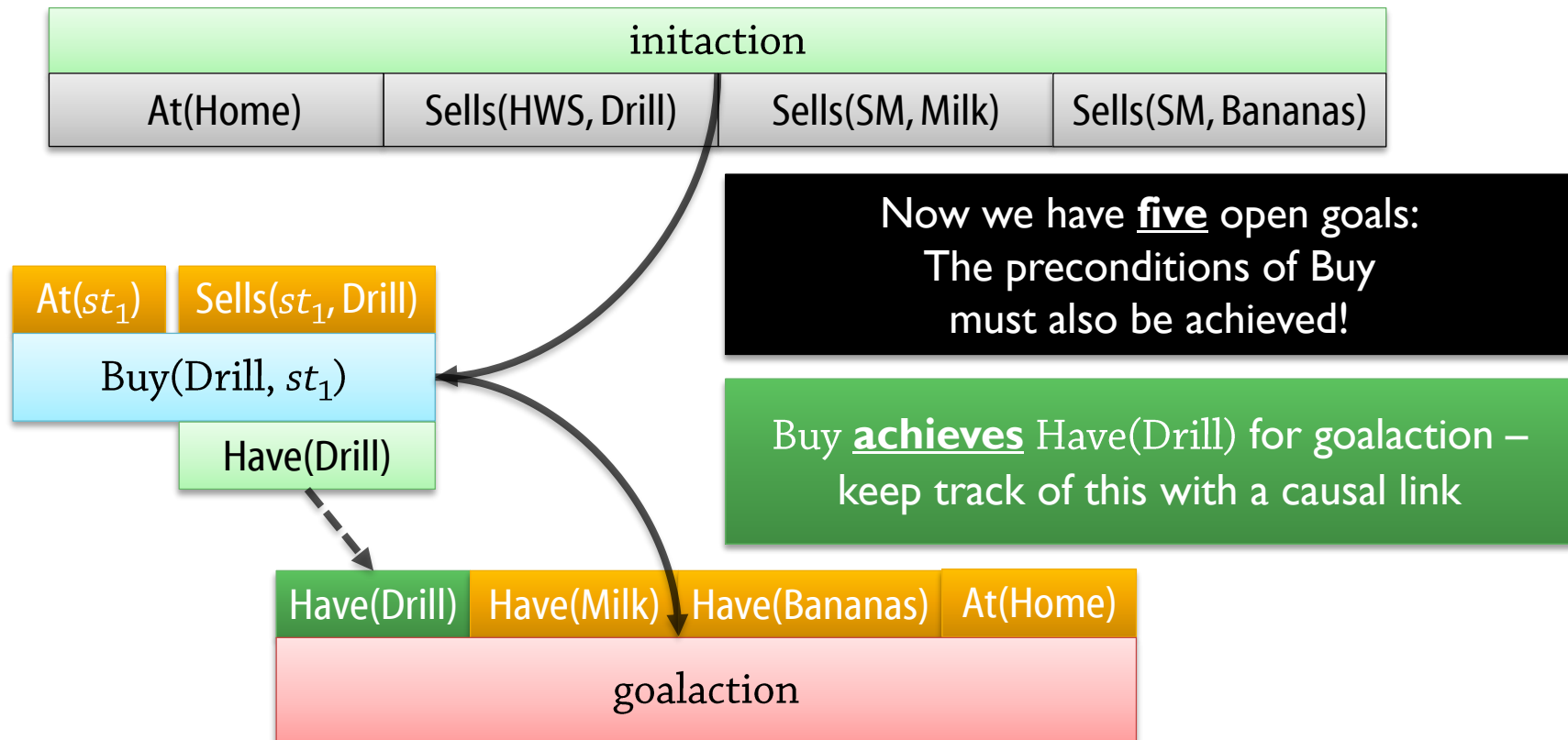
Buy achieves Have(Drill) for goalaction –  
keep track of this with a causal link

**Bindings:**  
 $p_1 = \text{Drill}$



# Example (continued)

- **Alternative Notation** for simplicity
  - Variable bindings are implicit in the diagram

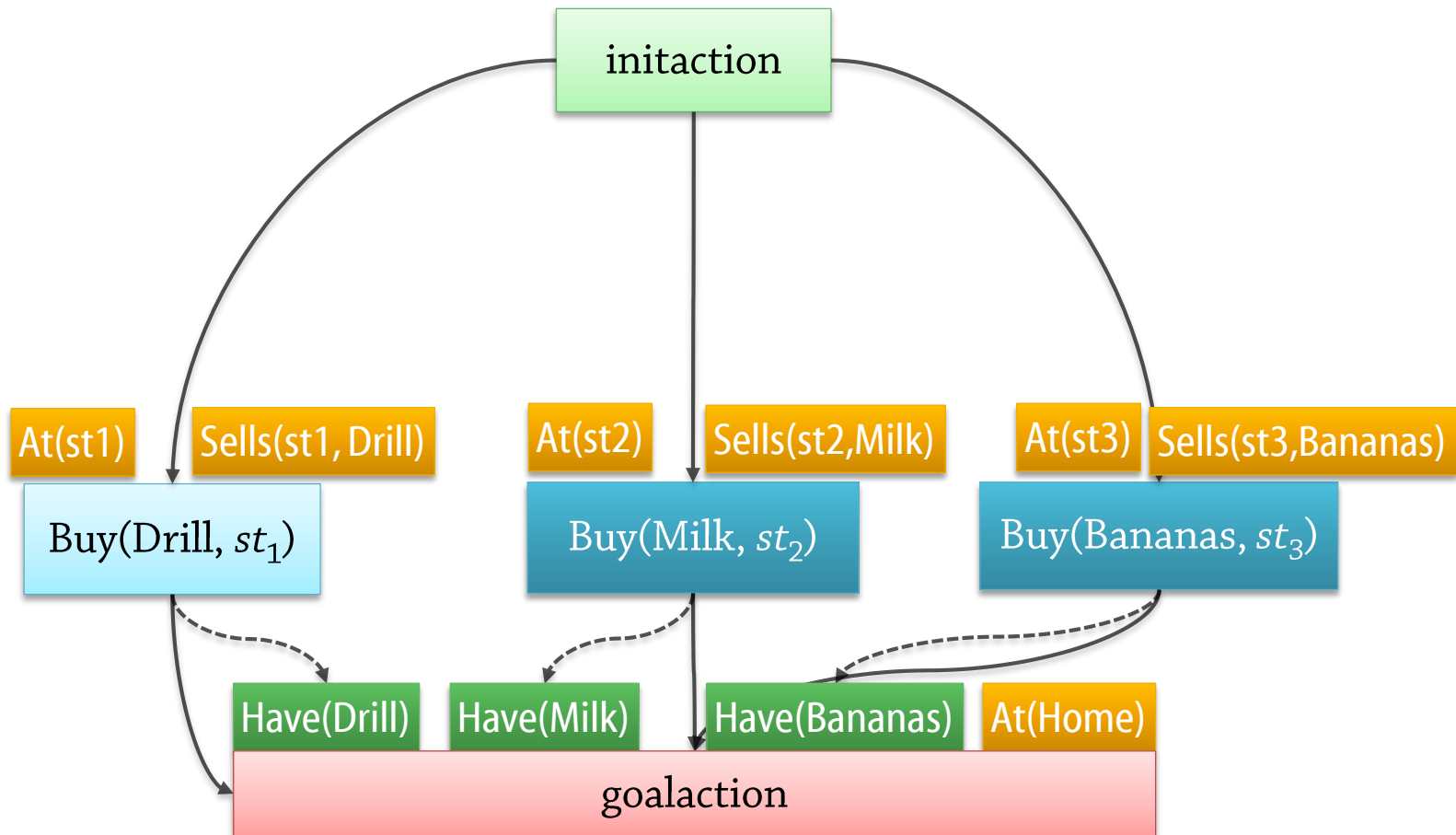




# Example (continued)

- The first **three** refinement steps

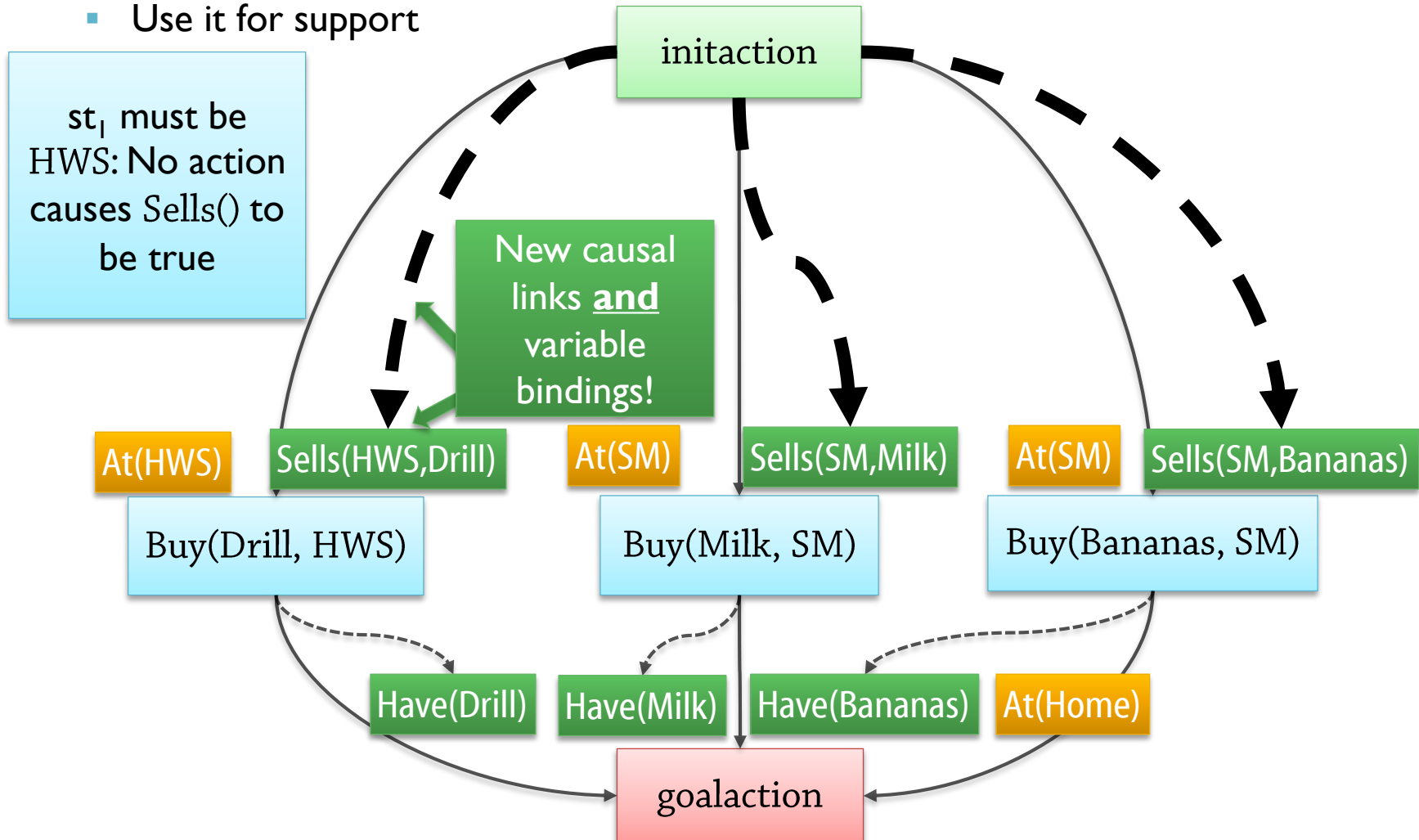
- These are the only possible ways to establish the Have preconditions
- We don't care in which **order** we buy things!



# Example (continued)

- Three more refinement steps

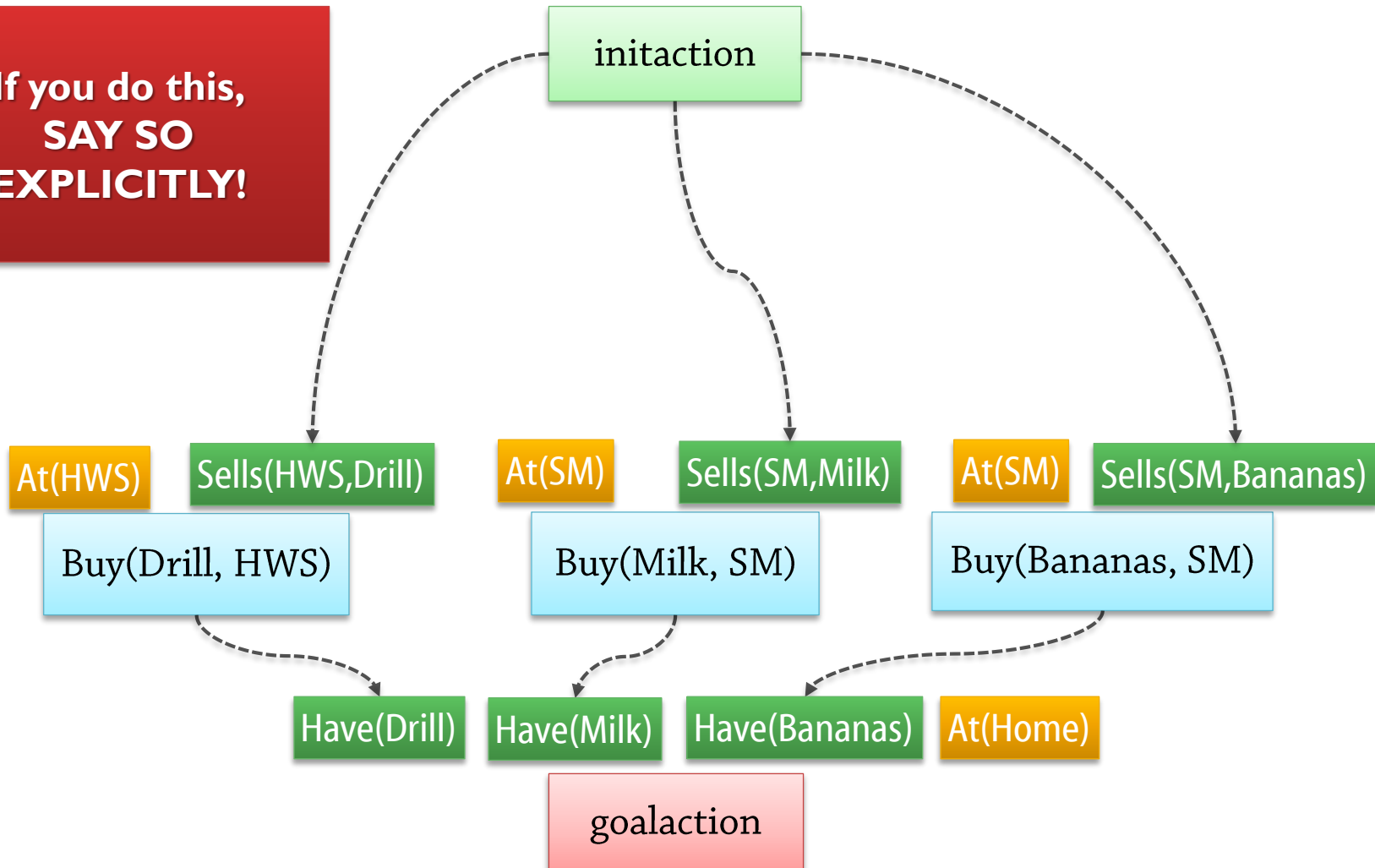
- No action causes Sells(...) to be true – except the “fake” initial action!
- Use it for support



# Example (continued)

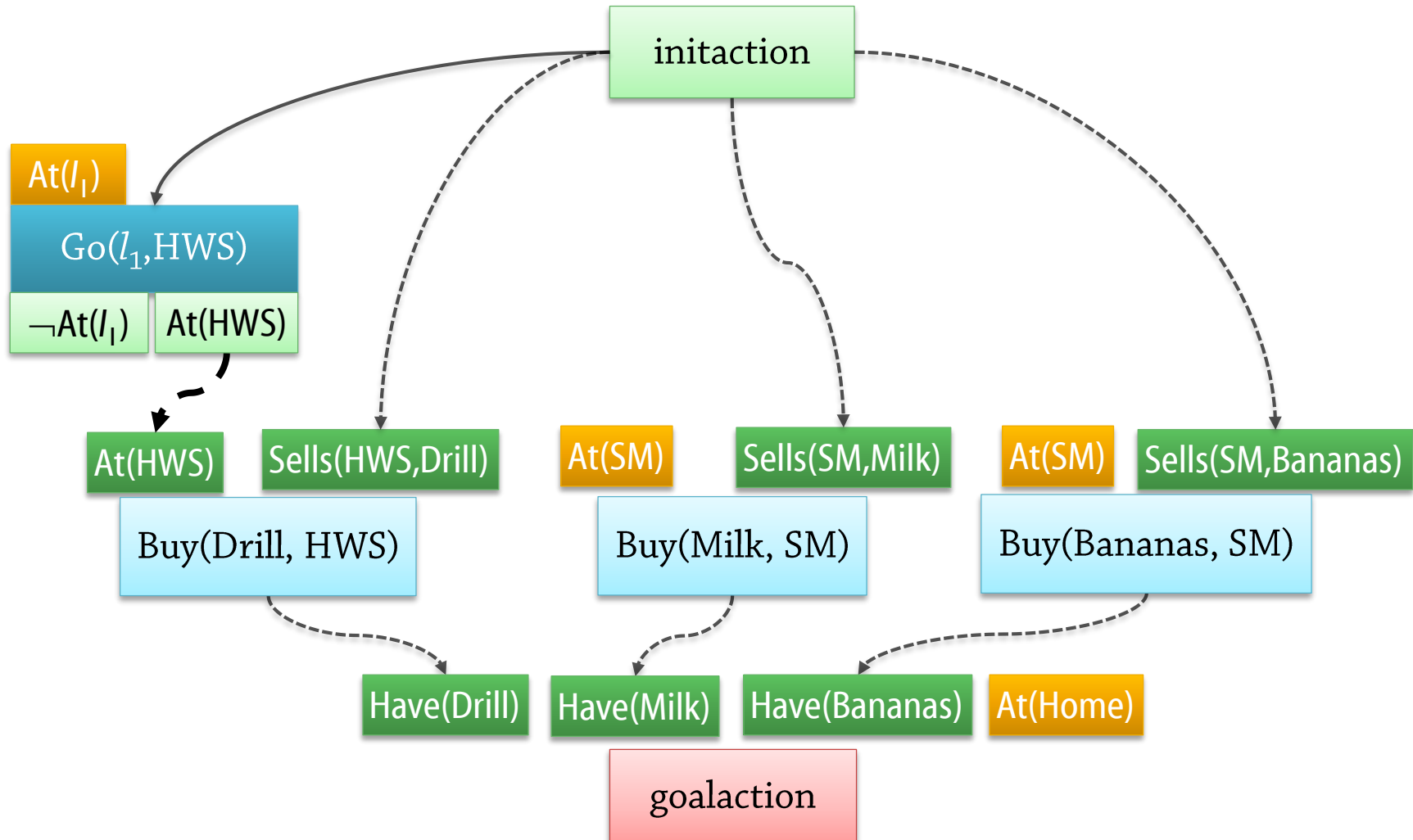
- It's getting messy!
  - Let's omit the precedence constraints that are implicit in causal links...

If you do this,  
**SAY SO  
EXPLICITLY!**



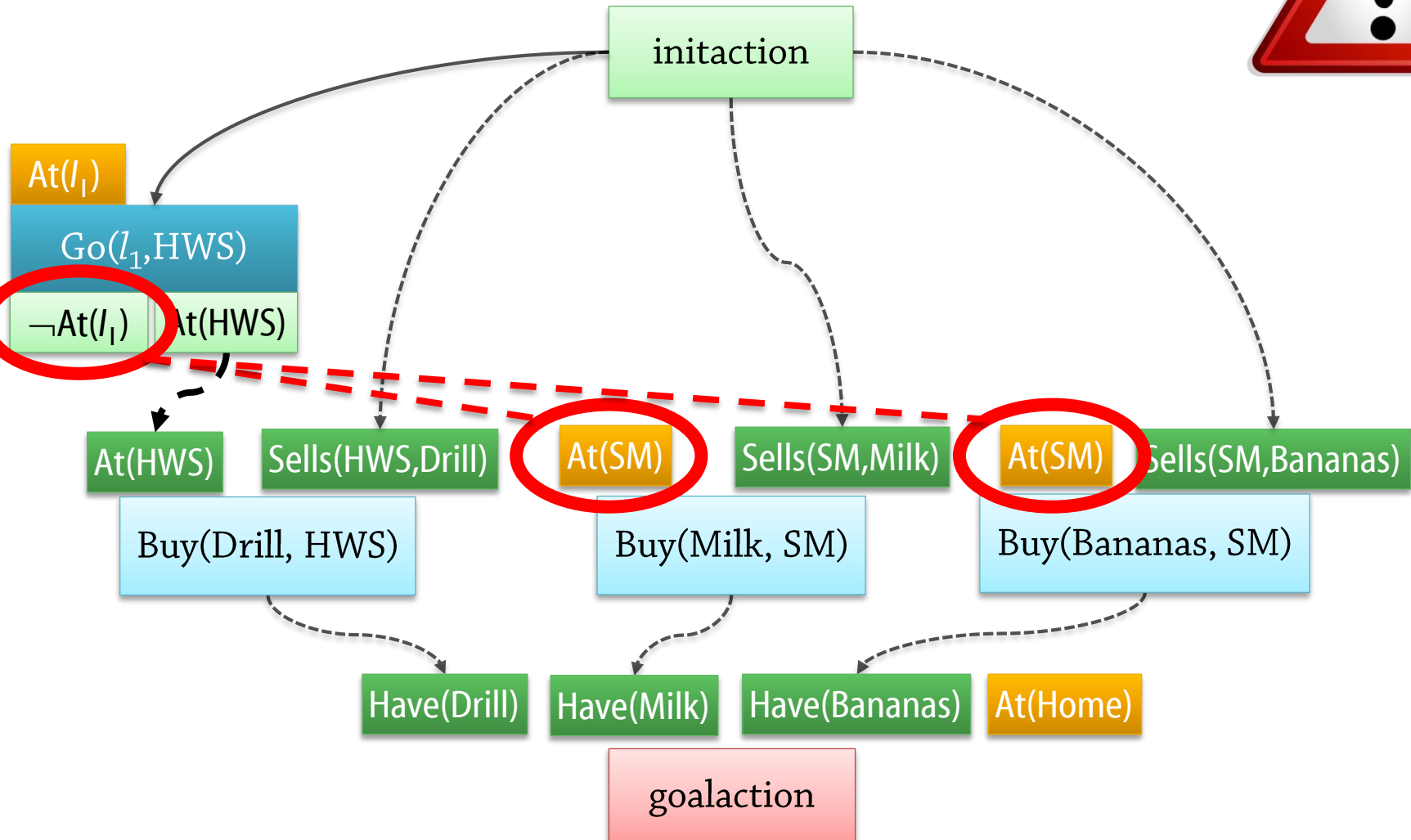
# Example (continued)

- To establish  $\text{At}(\text{HWS})$ : **Must** go there from somewhere



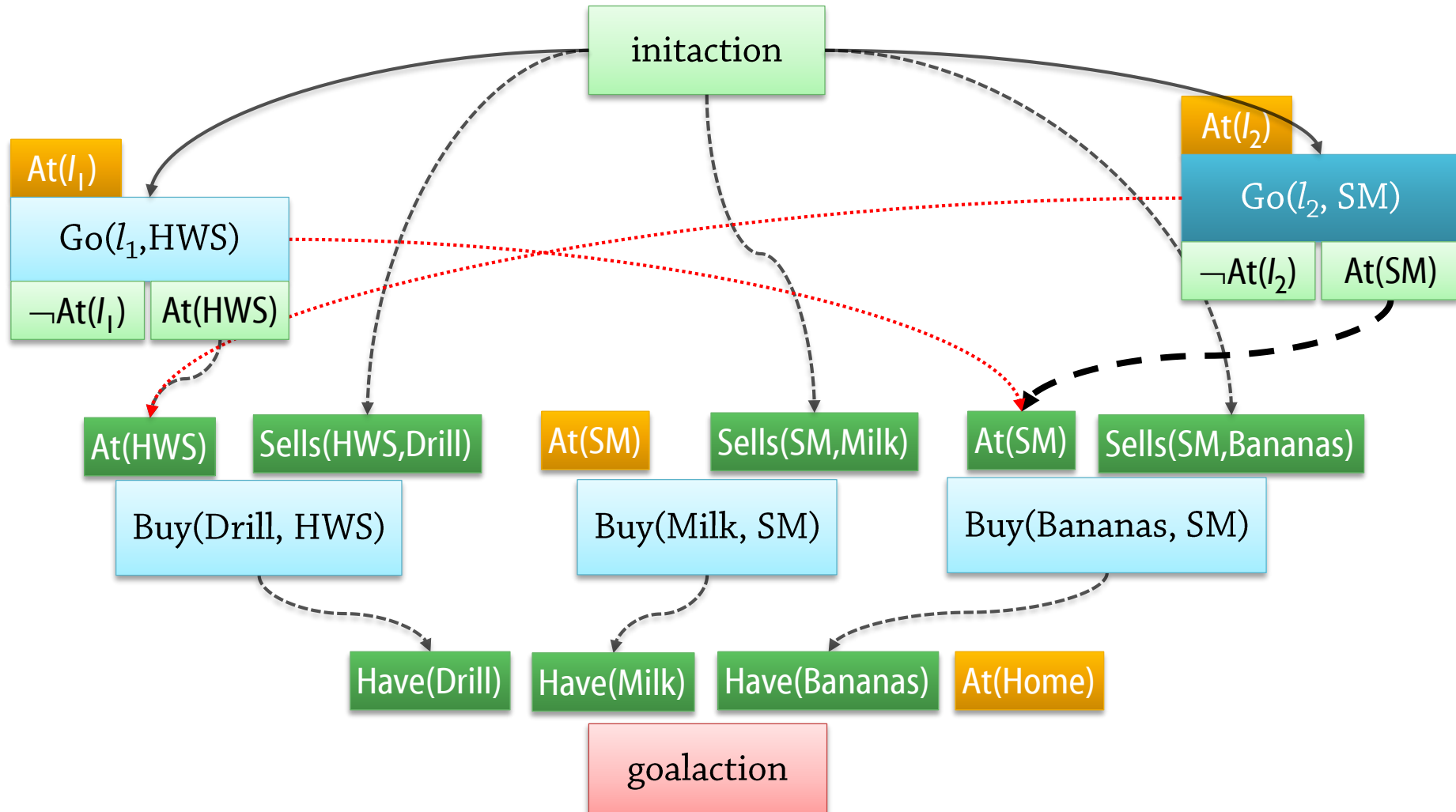
# Example (continued)

- Does  $\neg \text{at}(l_1)$  threaten  $\text{At}(\text{SM})$ ?
  - No! Only a **causal link** to  $\text{At}(\text{SM})$  can be threatened



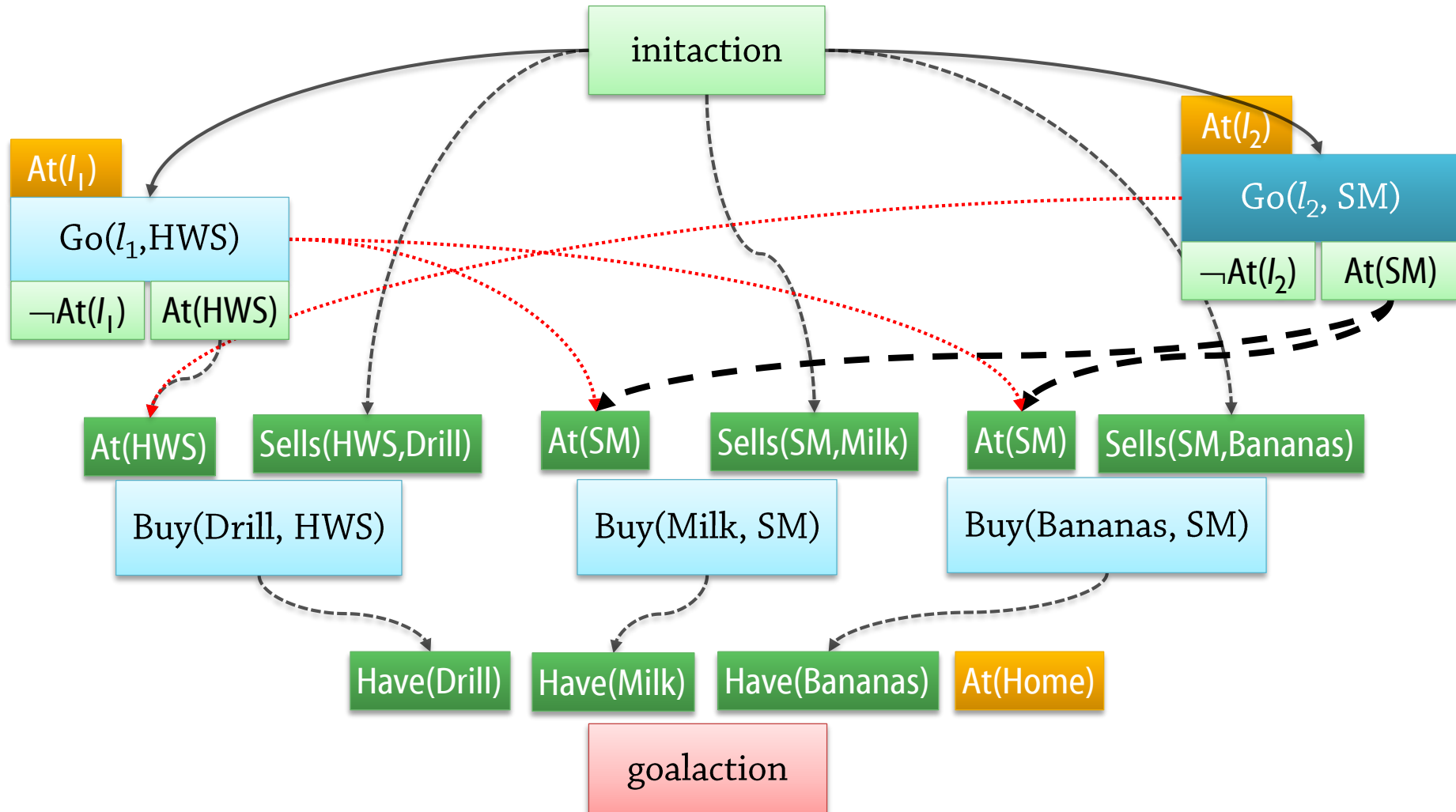
# Example (continued)

- To establish  $At(SM)$ : Must go there from somewhere
  - Mutual threats...



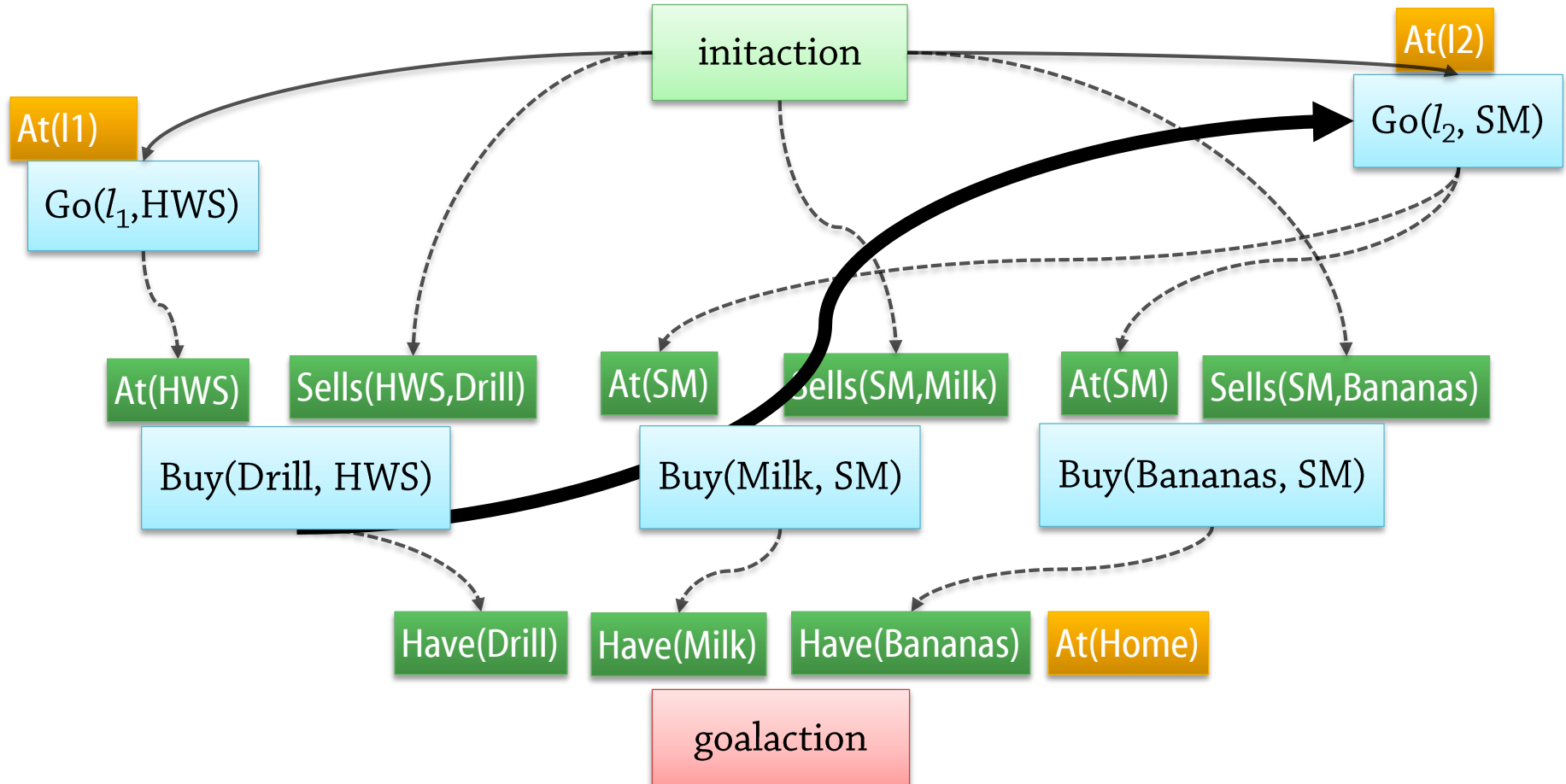
# Example (continued)

- Let's use the same action for **both**  $\text{At}(\text{SM})$  preconditions...
  - More threats – could deal with them now or wait



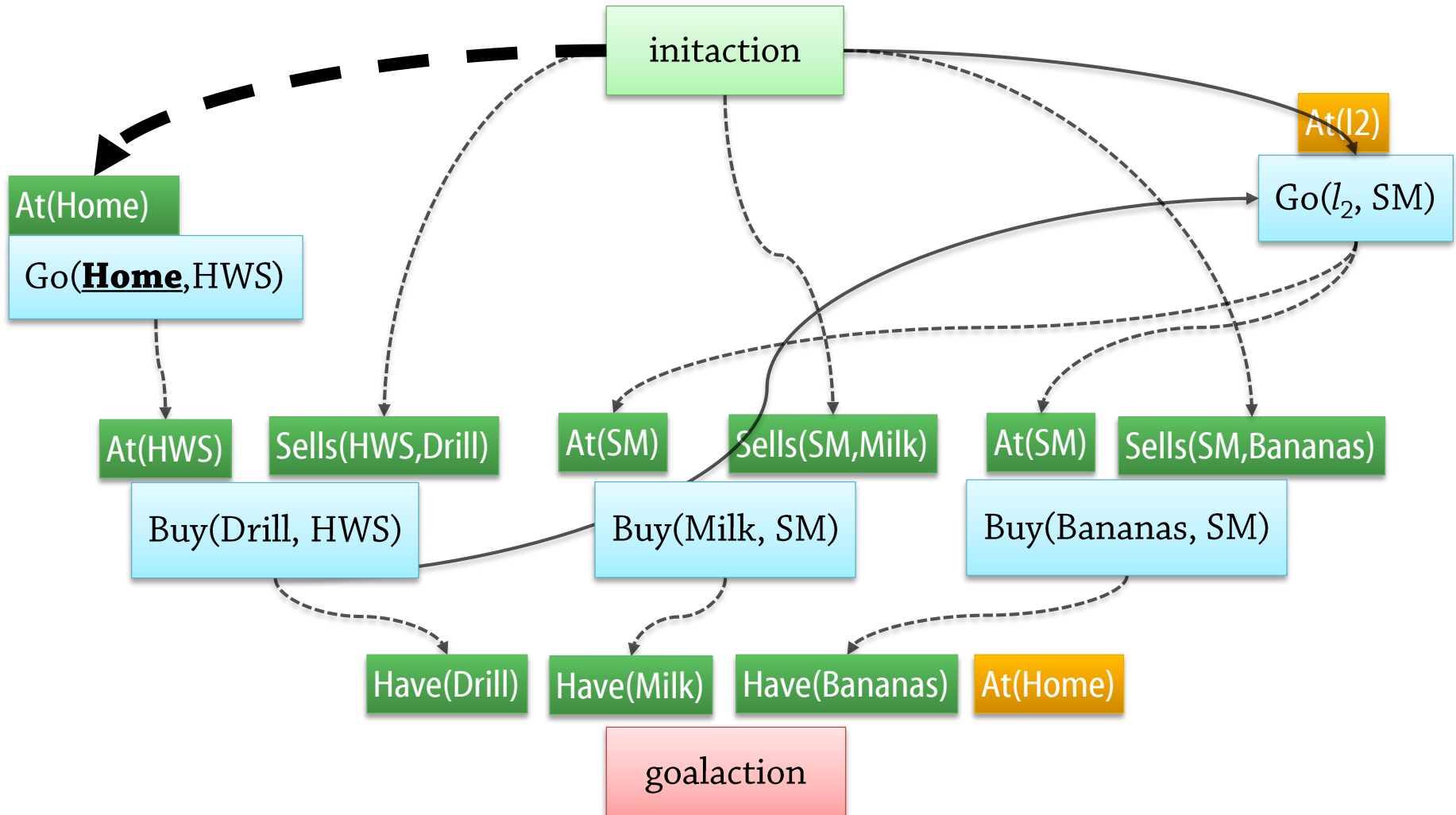
# Example (continued)

- **Nondet. choice**: how to resolve the threat to  $At(HWS)$ ?
  - Our choice: make the “requirer”  $Buy(Drill)$  precede the “threatener”  $Go(l_2, SM)$
  - Also happens to resolve the other two threats
    - “Threatener”  $Go(l_1, HWS)$  before “achiever”  $Go(l_2, SM)$

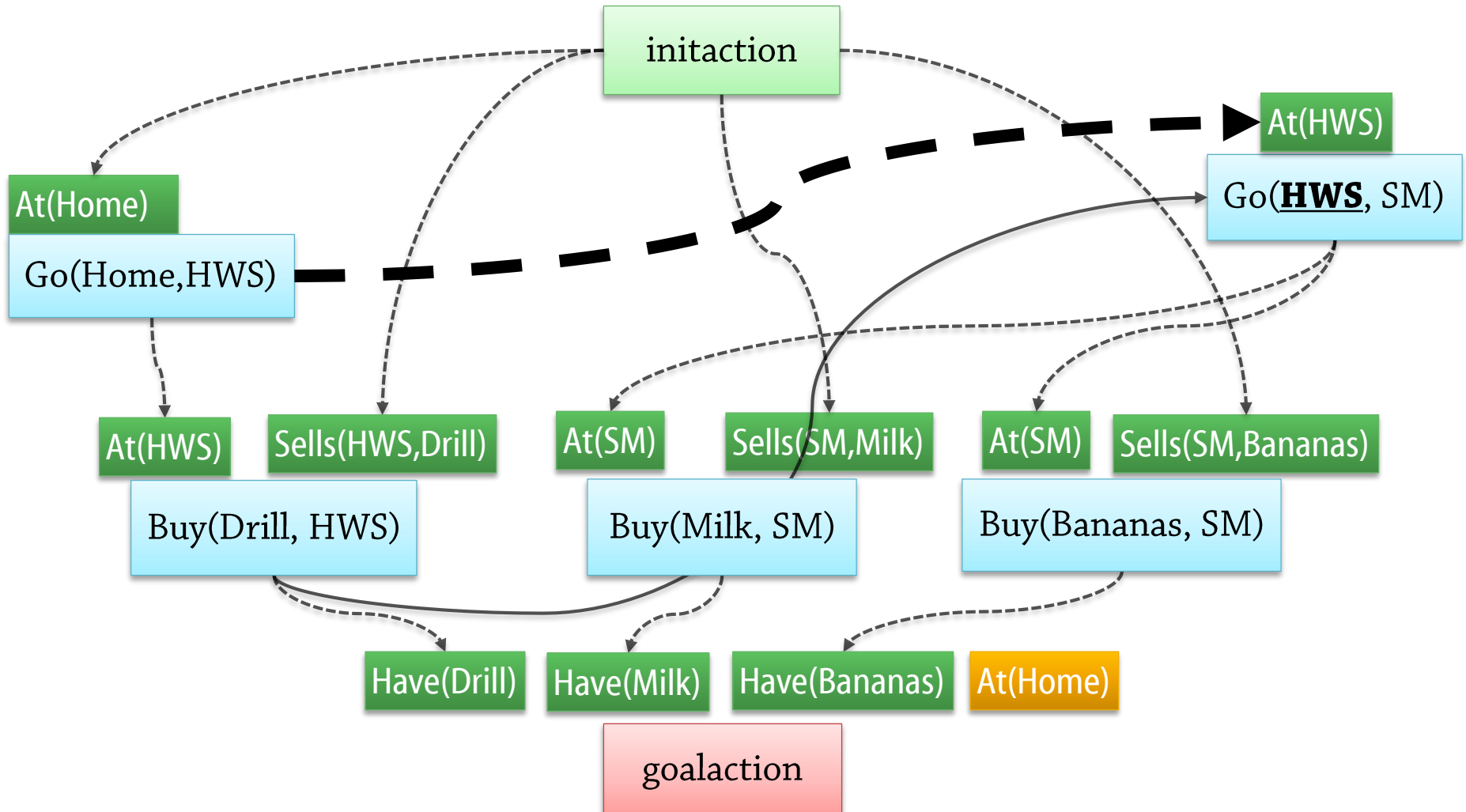




- We'll do it from initaction, with  $l_1$ =Home

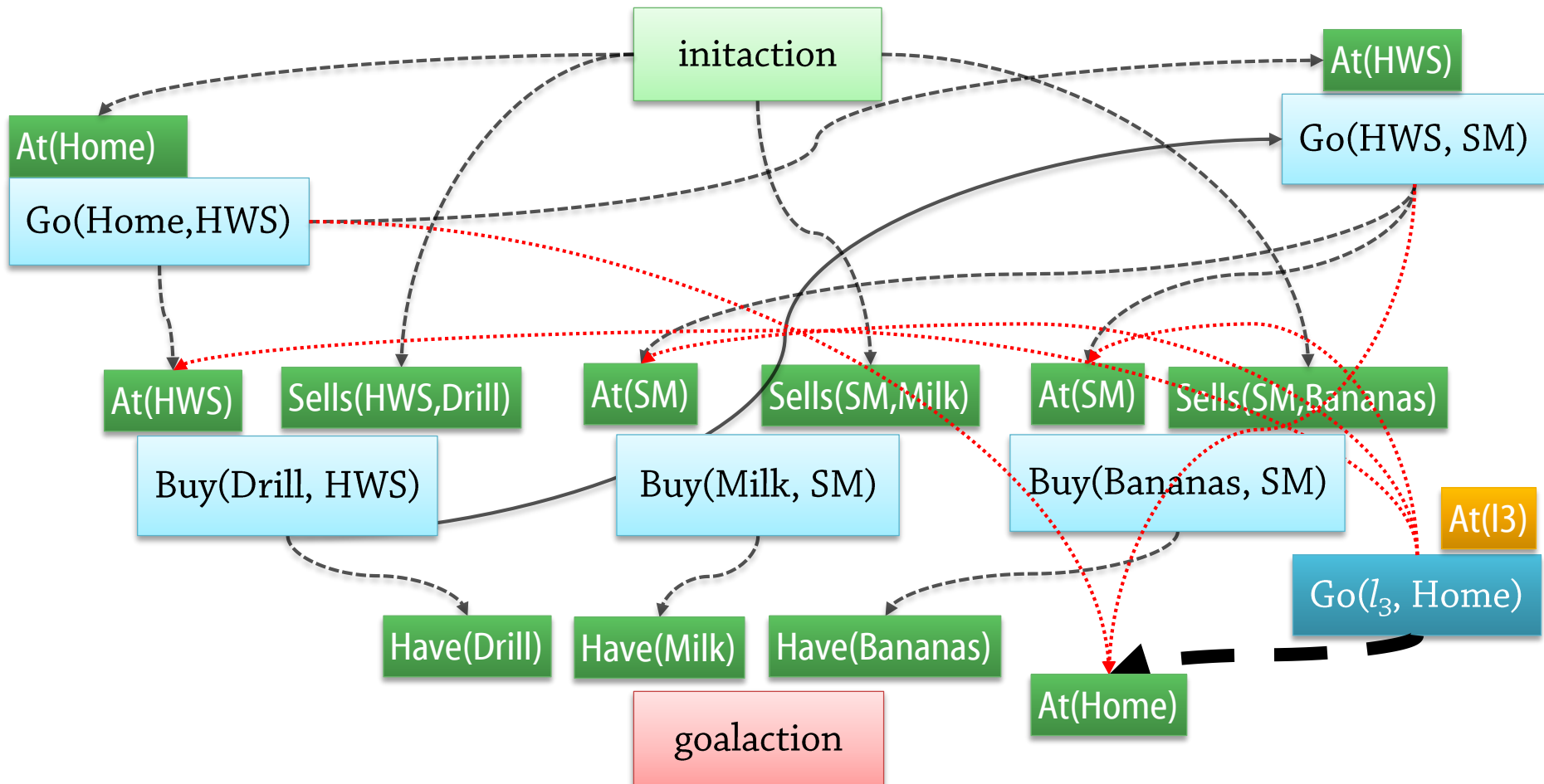


- We'll do it from  $\text{Go}(\text{Home}, \text{HWS})$ , with  $l_2 = \text{HWS}$



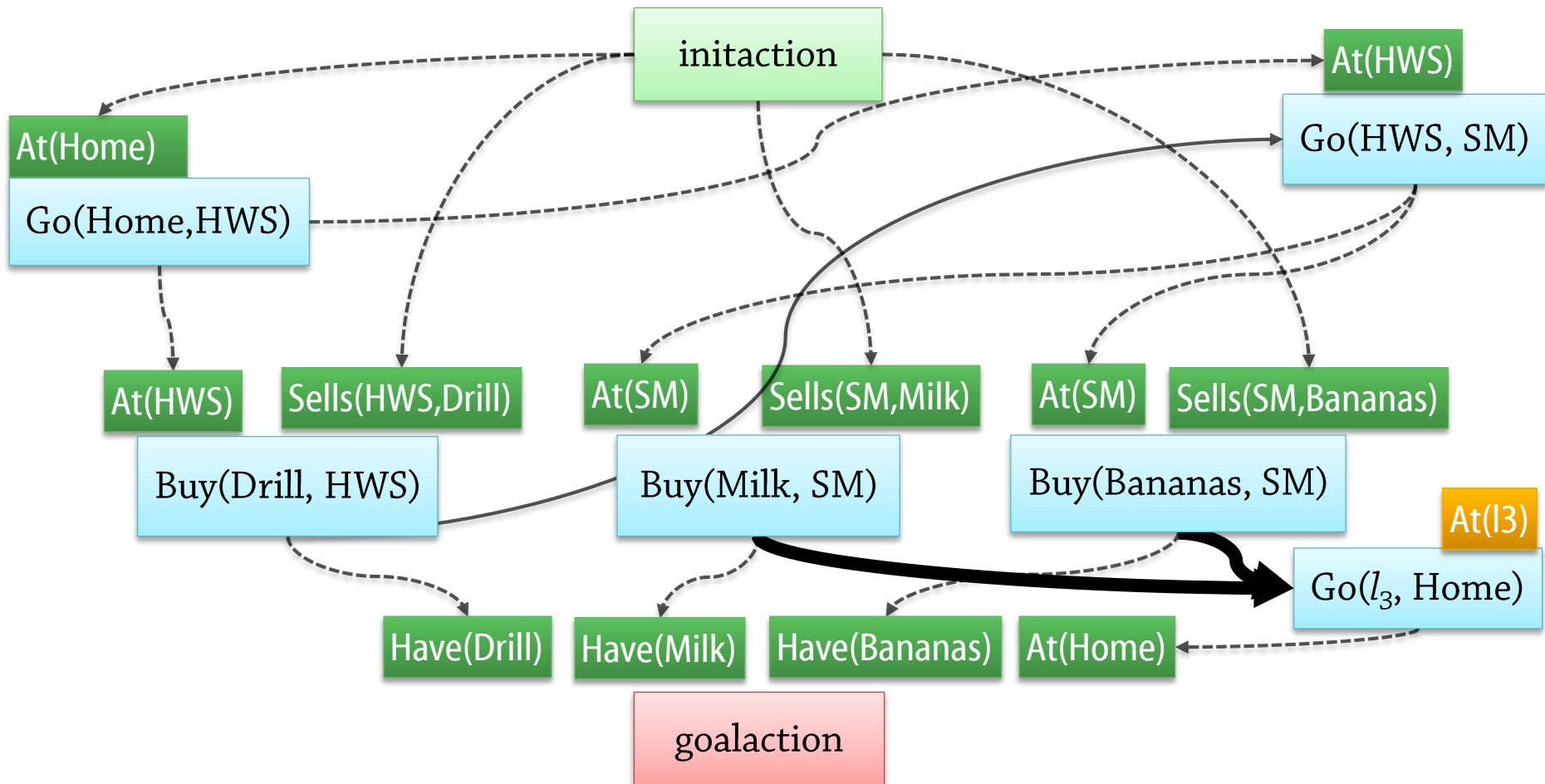
# Example (continued)

- The only possible way to establish  $\text{At}(\text{Home})$  for goalaction
  - This creates several new threats



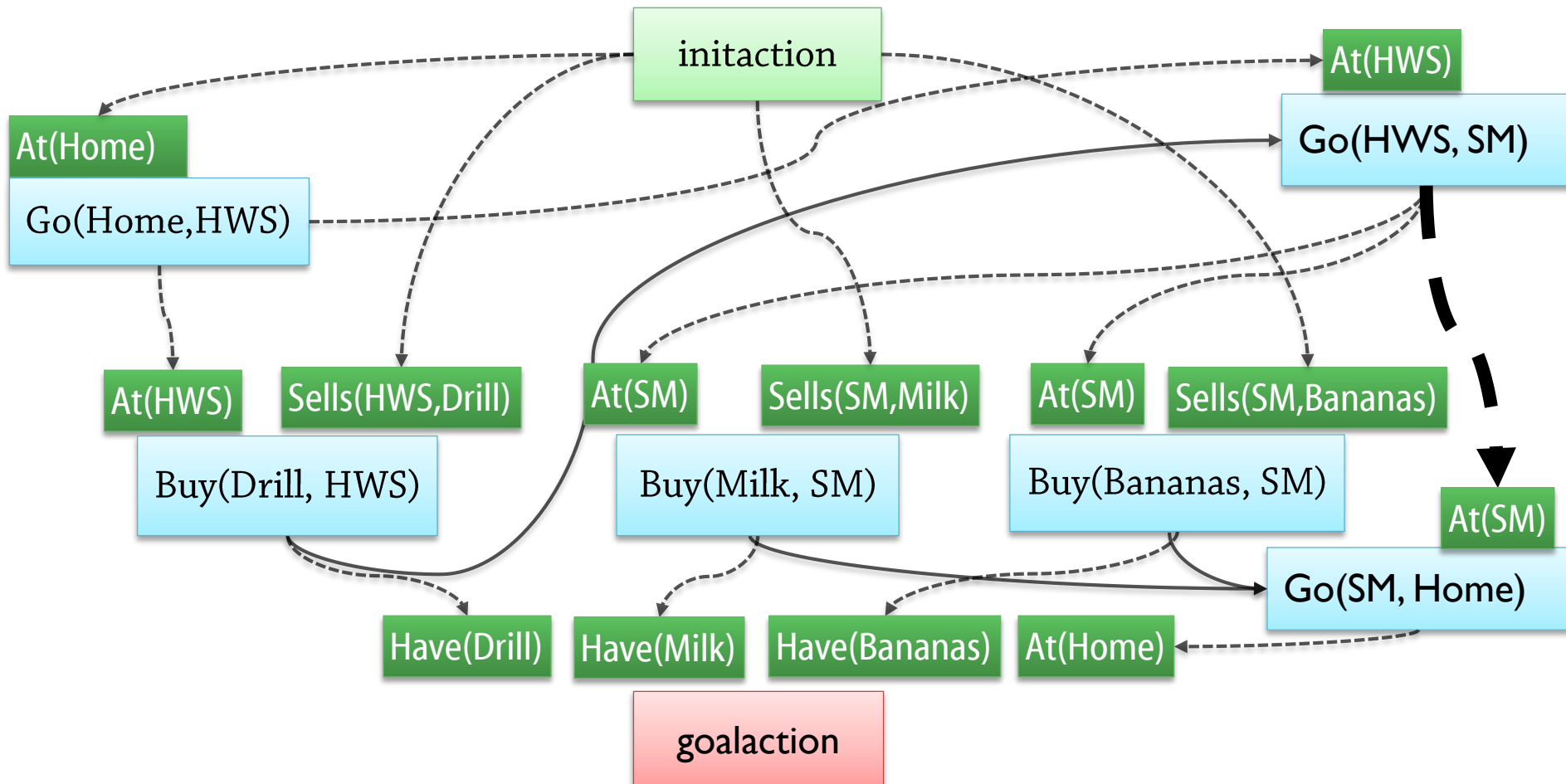
# Example (continued)

- To remove the threats to  $At(SM)$  and  $At(HWS)$ :
  - Make  $go(HWS, SM)$  and  $go(Home, HWS)$  precede  $Go(l_3, Home)$
  - This also removes the other threats

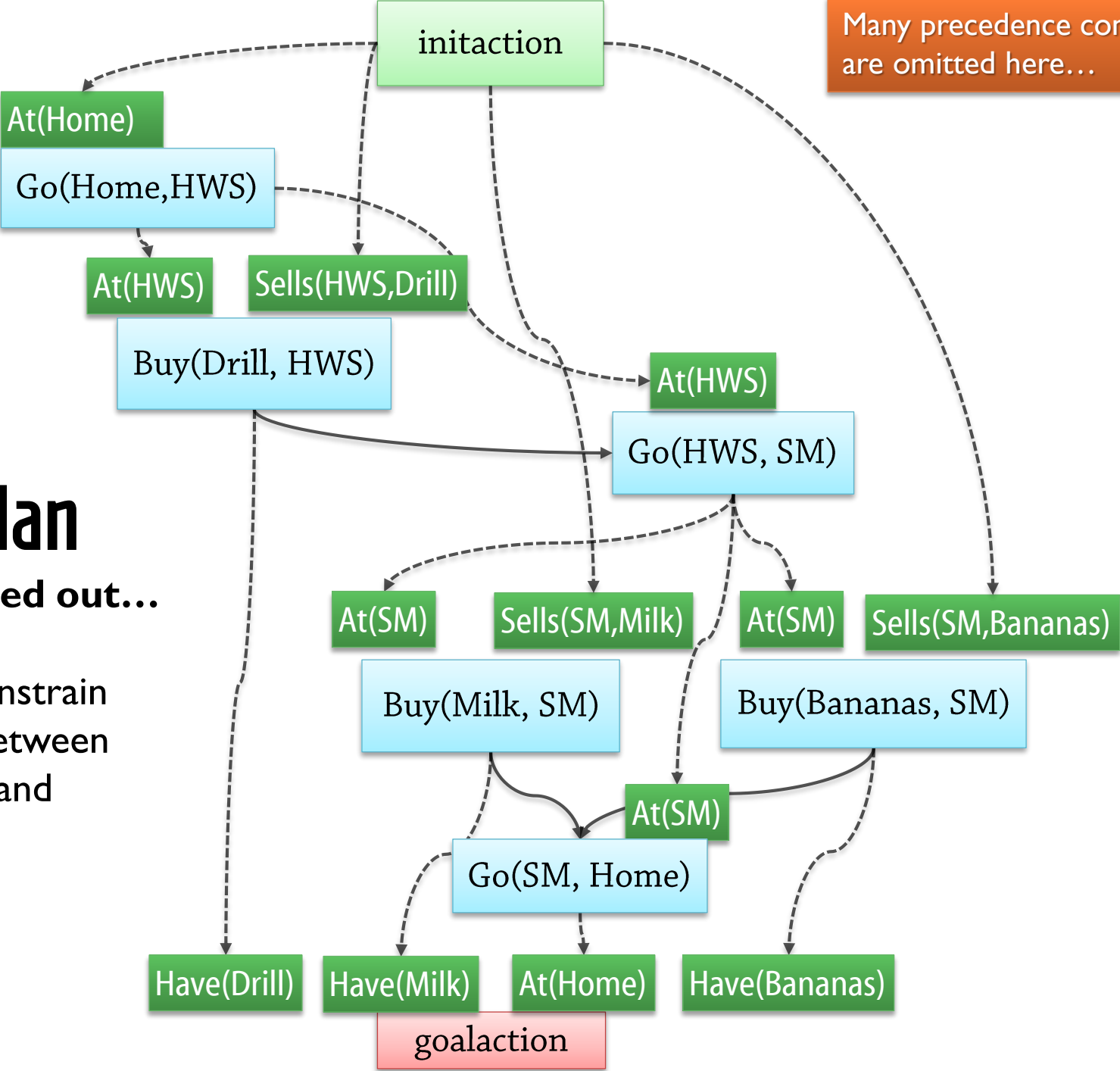


# Final Plan

- Establish  $At(l_3)$  with  $l_3=SM$



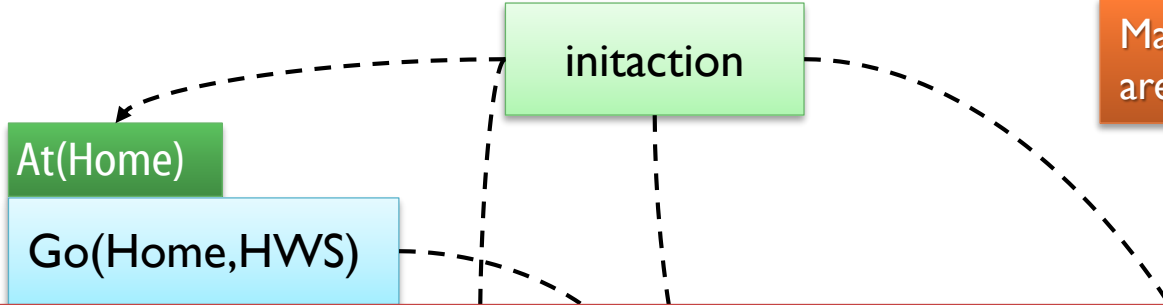
Many precedence constraints are omitted here...



# Final Plan

**Straightened out...**

(Note: Still does not constrain the order between buying milk and bananas)



Many precedence constraints are omitted here...

This sequence assumed optimal choices!

Heuristics are required

Still, planners try many other alternatives, dead ends, etc.

