



Automated Planning

Heuristics for Forward State Space Search: Overview and Examples

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Heuristics in Forward State Space Search: Introduction

■ General Forward State Space Search Algorithm

```
■ forward-search(A, s0, g) {  
    open ← { <s0, ε> }  
    while (open ≠ ∅) {  
        use a strategy to select and remove one n=<s,path> from open  
        if goal g satisfied in state s then  
            return path  
  
        foreach a ∈ A such that γ(s, a) ≠ ∅ {  
            {s'} ← γ(s, a)  
            path' ← append(path, a)  
            add n'=<s', path'> to open  
        }  
    }  
    return failure;  
}
```

- A heuristic strategy bases its decisions on:
 - **Heuristic value $h(n)$**
 - Often other factors, such as **$g(n)$ = cost of reaching n**

Requires a heuristic function

How do we calculate $h(n)$?

$h_1(n), h_2(n), h_{add}(n)$,
landmarks,
pattern databases, ...

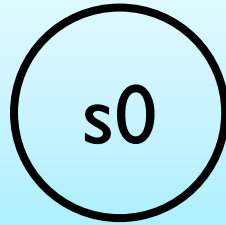
Requires a heuristic search strategy

How do we use $h(n)$?

A*, IDA*, D*, simulated annealing,
hill-climbing, (various forms of)
best first search, ...

Example (1)

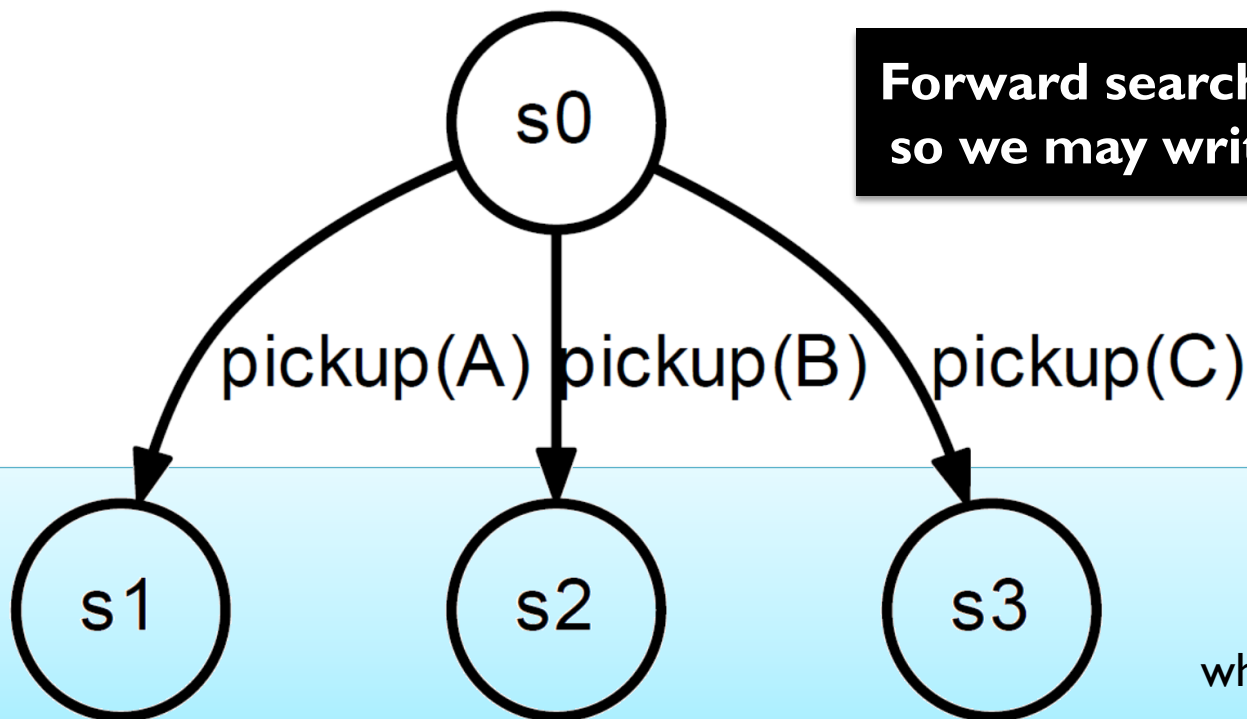
Example: 3 blocks, all on the table in s0



We now have
1 *open node*,
which is *unexpanded*

Example (2)

We visit s_0 and expand it



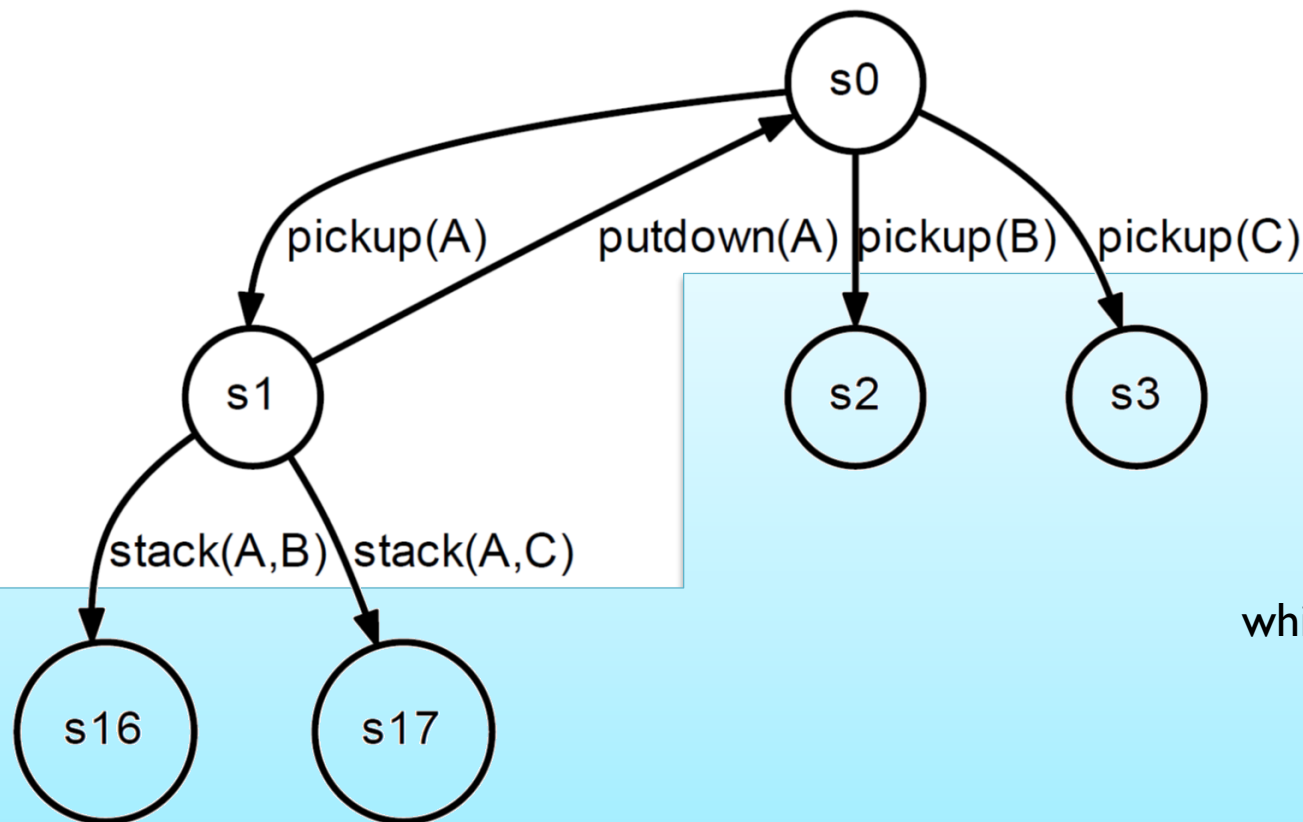
A **heuristic function** estimates the distance from each open node to the goal:

We calculate $h(s_1)$, $h(s_2)$, $h(s_3)$

A **heuristic strategy** uses this value (**and** other info) to *prioritize*

Example (3)

Suppose the strategy chooses to visit s_1 :



2 new heuristic values are calculated: $h(s_{16}), h(s_{17})$
The **search strategy** now has 4 nodes to prioritize

Heuristic Functions: What to Measure?

What to Measure?



Question 1A: What should a heuristic function measure?

- A heuristic strategy bases its decisions on:
 - **Heuristic value $h(s)$**
 - Often other factors, such as **$g(s)$ = cost of reaching s**

Very general definition

→ could measure anything that some strategy might find useful!

Often: $h(s)$ *tries to* approximate the cost of achieving the goal from s

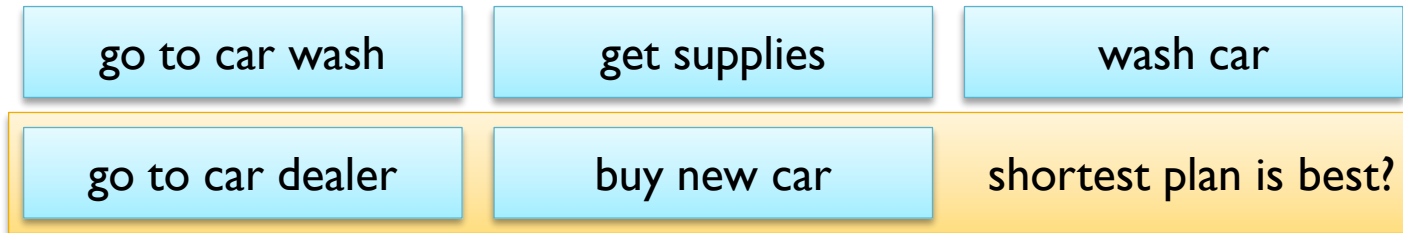
Useful for finding cheap plans –
and often, as a side effect, for finding plans cheaply

→ Question 1B: What is "cost"?

Plan Quality and Action Costs

- Maybe: **Long** plan = **expensive** plan

- $c(\pi) = |\pi|$ (number of actions in plan π)
 - Reasonable in Towers of Hanoi
 - But: How to make sure your car is clean?



Heuristic $h(s)$ estimates:

"How many actions will I need to reach the goal from s ?"

- Would prefer to support different **action costs**

- Supported by most current planners
 - Each action $a \in A$ associated with a cost $c(a)$
- **Total cost:**

$$c(\pi) = \sum_{a \in \pi} c(a)$$

Heuristic $h(s)$ estimates:

"How expensive actions will I need to reach the goal from s ?"

- PDDL: Specify requirements
 - (:requirements :action-costs)

- **Numeric state variable** for the total cost, called **(total-cost)**

- And possibly numeric state variables to *calculate* action costs

- (:functions (total-cost)
(travel-slow-cost ?f1 - count ?f2 - count)
(travel-fast-cost ?f1 - count ?f2 - count)

- number Built-in type
- number supported by
- number cost-based
planners

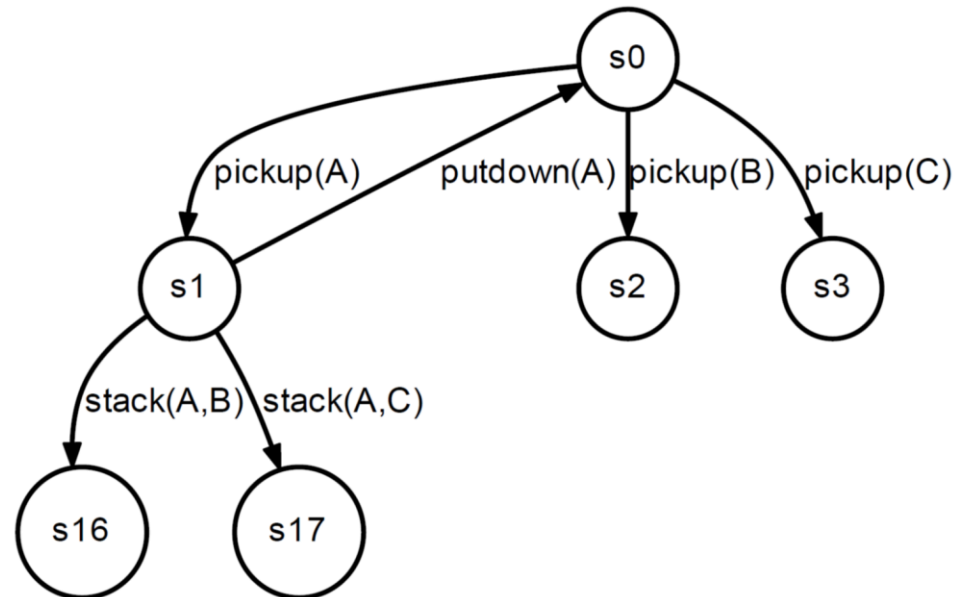
- **Initial state**

- (:init (= (total-cost) 0)
(= (travel-slow-cost n0 n1) 6) (= (travel-slow-cost n0 n2) 7)
(= (travel-slow-cost n0 n3) 8) (= (travel-slow-cost n0 n4) 9)
...)

- Special **increase effects** to increase total cost

- (:action move-up-slow
:parameters (?lift - slow-elevator ?f1 - count ?f2 - count)
:precondition (and (lift-at ?lift ?f1) (above ?f1 ?f2) (reachable-floor ?lift ?f2))
:effect (and (lift-at ?lift ?f2) (not (lift-at ?lift ?f1))
(**increase (total-cost) (travel-slow-cost ?f1 ?f2))))**

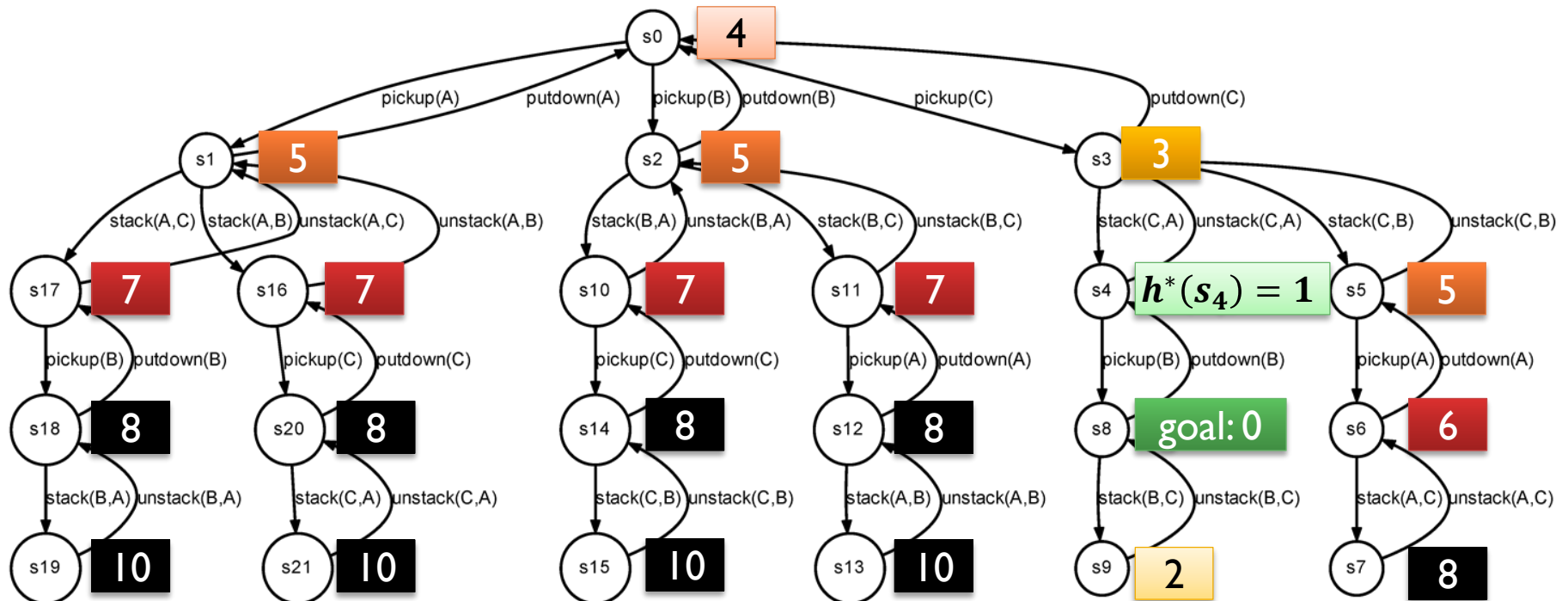
- The **remaining cost** in **any** search state s :
 - The cost of a **cheapest (optimal) solution** starting in s
 - Denoted by $h^*(s)$
 - Star * \rightarrow the best, optimal, estimate: *exact* cost
- The cost of an **optimal solution** to (Σ, s_0, S_g) :
 - $h^*(s_0)$



True Remaining Costs (1)

True Cost of Reaching a Goal from n : $h^*(n)$

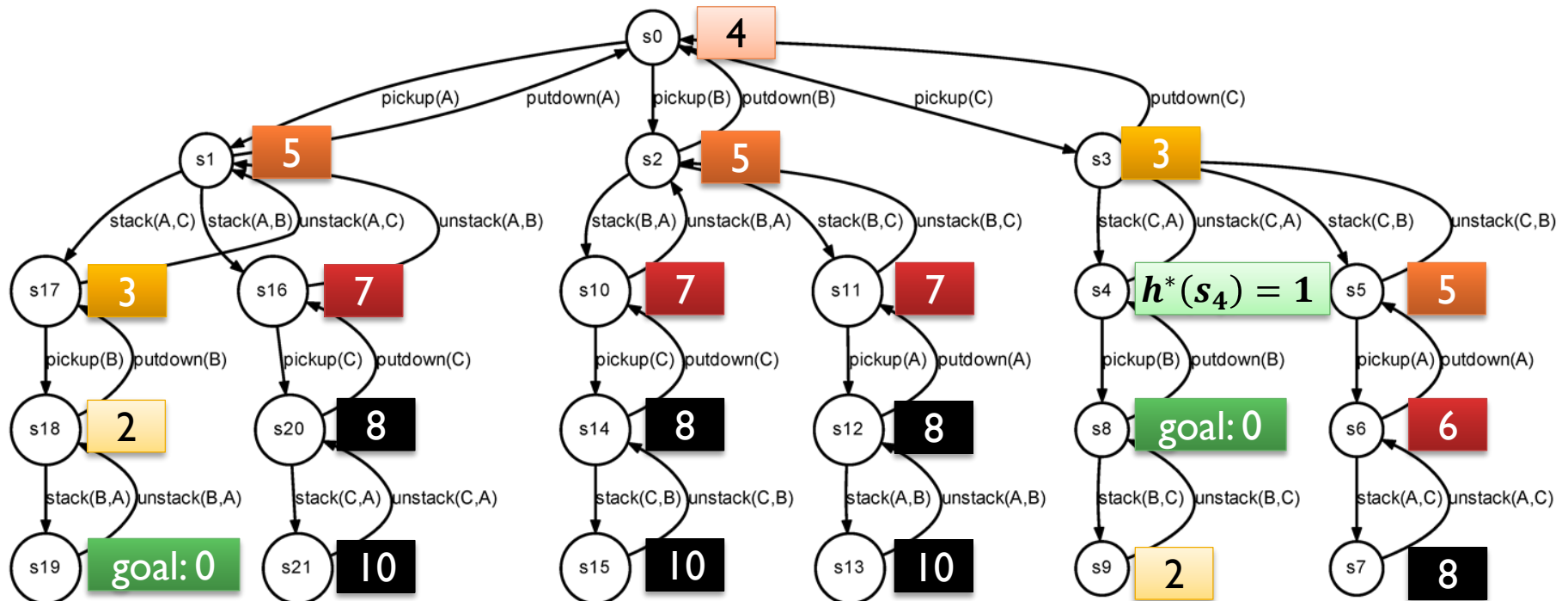
Initially: A,B,C on the table
pickup, putdown cost 1
stack, unstack cost 2 (must be more careful)



True Remaining Costs (2)

True Cost of Reaching a Goal: $h^*(n)$

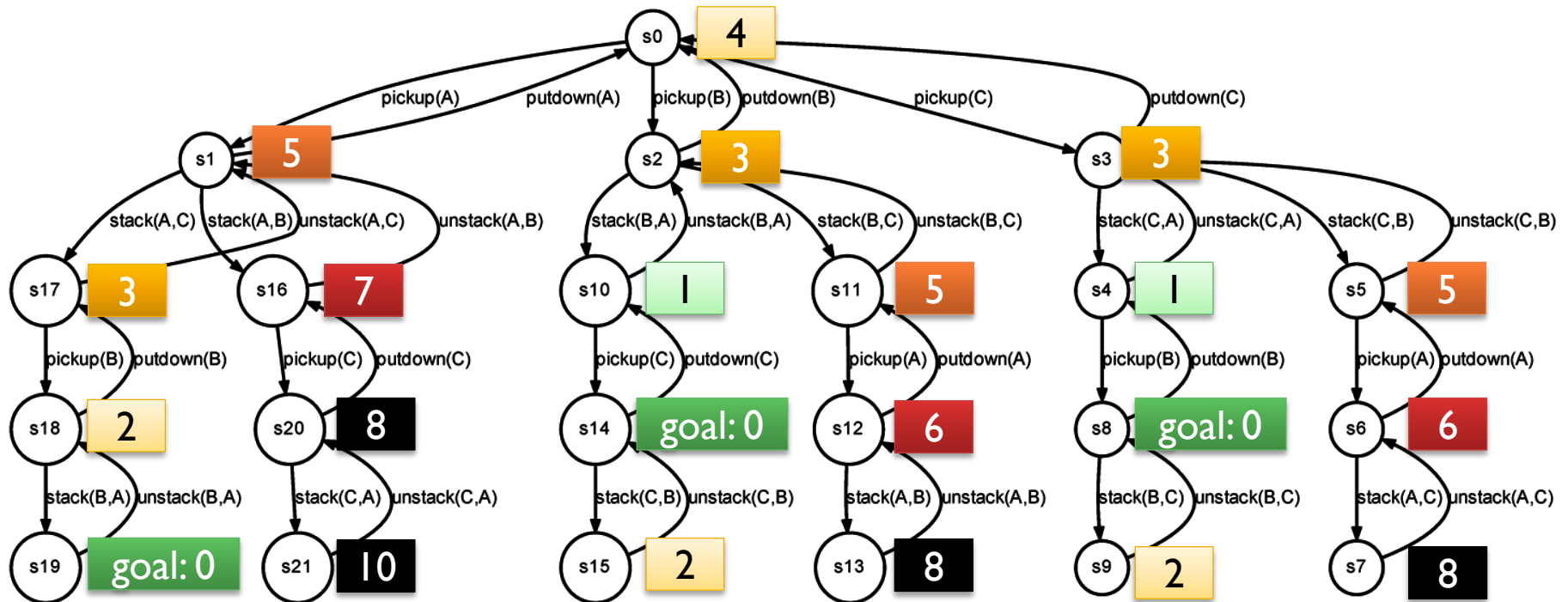
Two reachable goal states



True Remaining Costs (3)

True Cost of Reaching a Goal: $h^*(n)$

Three reachable goal states
(there can be many)



True Remaining Costs (4)

If we *knew* the true remaining cost $h^*(n)$ for every node:

Algorithm simplePlan:

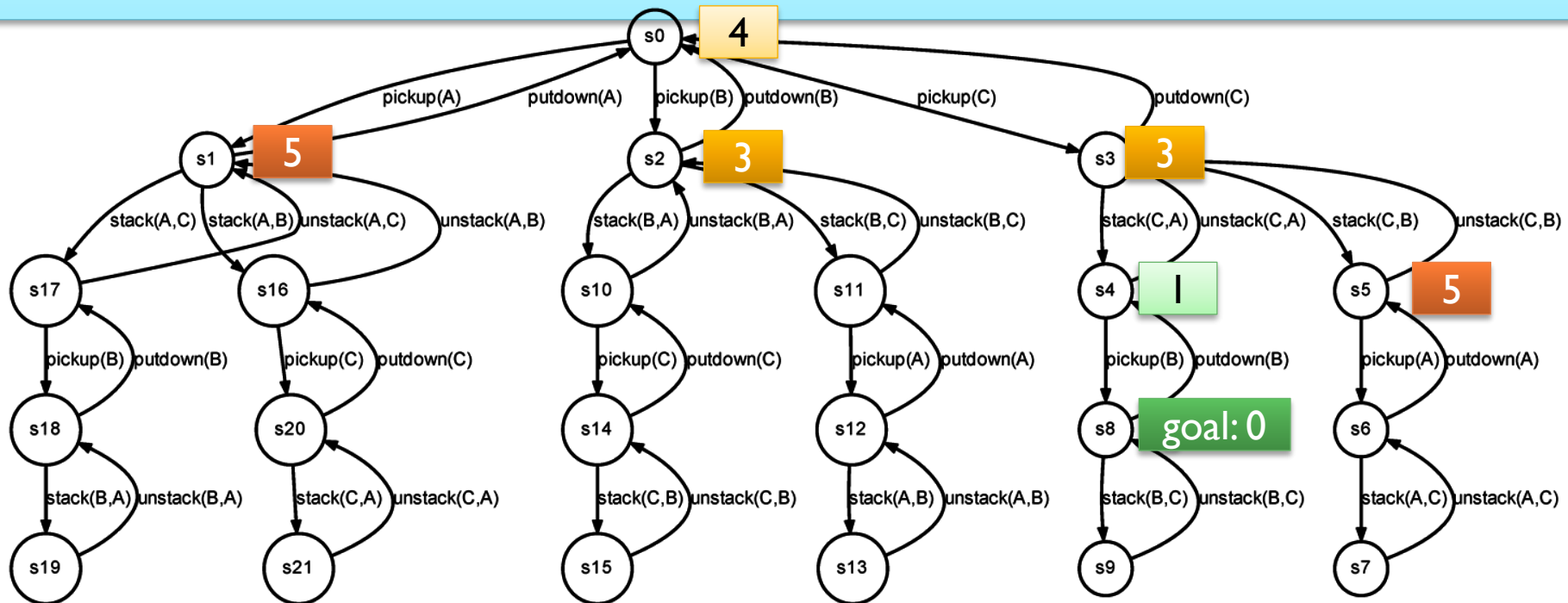
$node \leftarrow \text{initstate}$

while (not reached goal) {

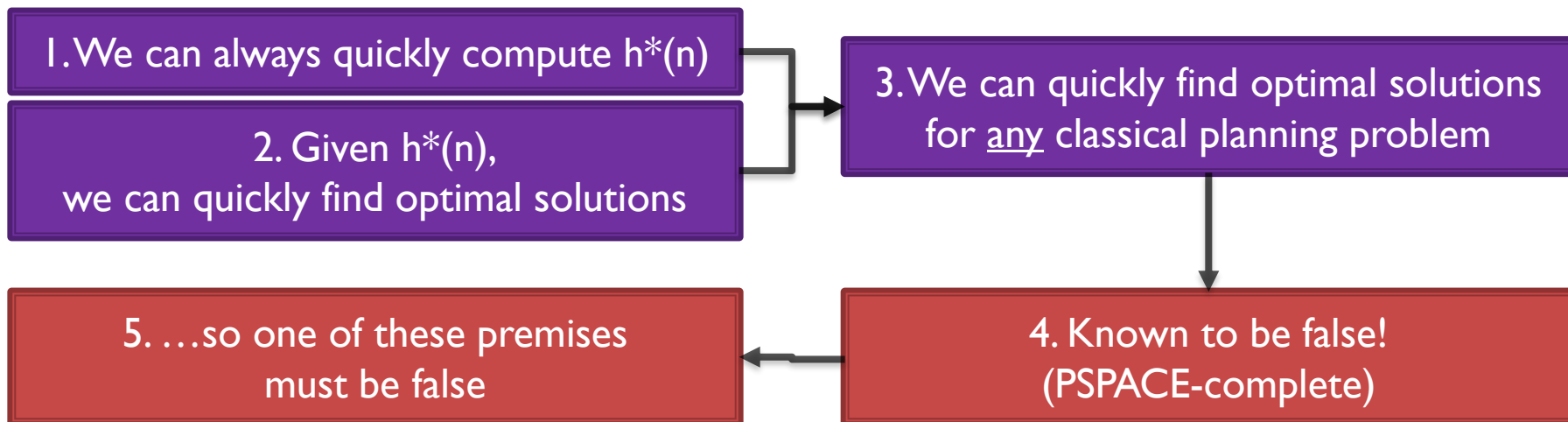
$node \leftarrow$ a successor of $node$ with minimal $h^*(n)$

}

Trivial straight-line path
minimizing h^* values
gives an *optimal* solution!



- What does this mean?
 - Calculating $h^*(n)$ is a good idea, because then we can easily find optimal plans?
- No – because we can prove that finding optimal plans is hard!
 - So the hard part must be calculating $h^*(n)$...

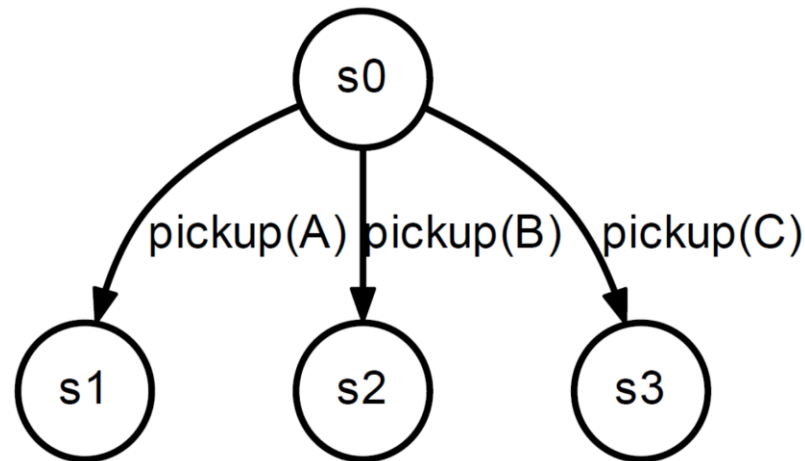


Must settle for an estimate that helps us search less than otherwise

Heuristic Functions:

What properties should an estimate have?

Example Strategy: Depth first search; select a child with minimal $h(s)$



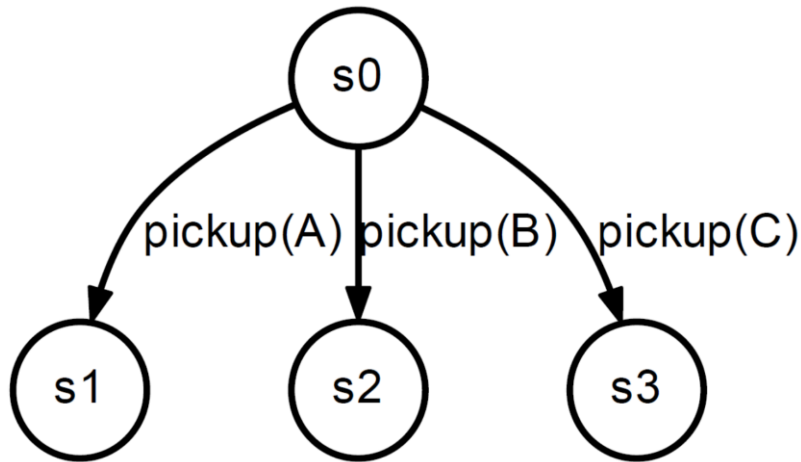
$h^*(s1)=55$ $h^*(s2)=57$ $h^*(s3)=62$

If I start with pickup(A),
then make optimal choices:
Plan cost = 55

If I start with pickup(C),
then make optimal choices:
Plan cost = 62

Minimization, case 1

Strategy: Depth first search; select a child with minimal $h(s)$



$h^*=55$	$h^*=57$	$h^*=62$
$hA=50$	$hA=53$	$hA=55$
$hB=4$	$hB=20$	$hB=21$

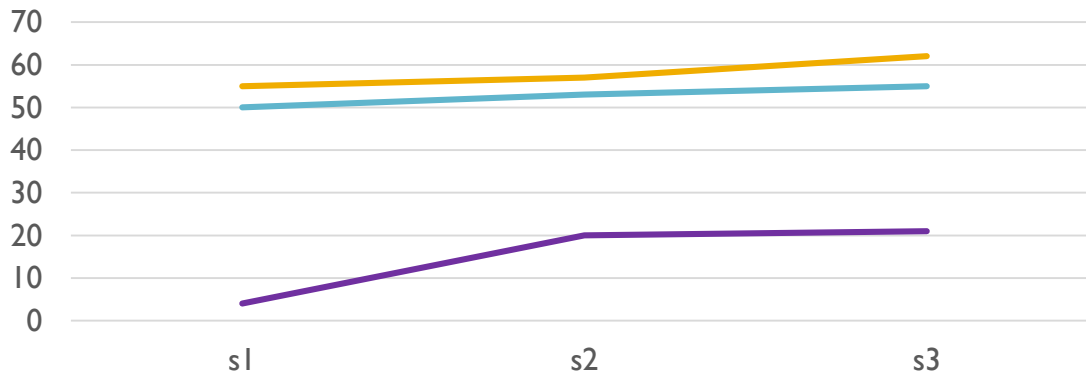
Close!

Far from the truth...

Which is best?

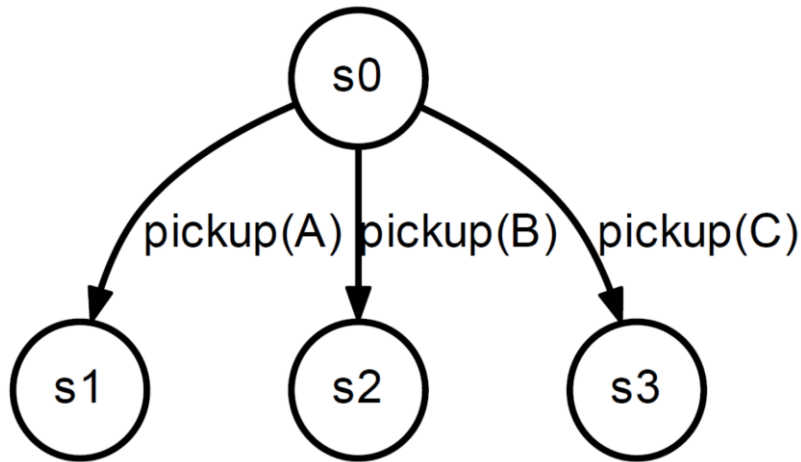
The strategy only cares about relative values

h^* , hA , hB result in identical choices: s_1 first!



Minimization, case 2

Strategy: Depth first search; select a child with minimal $h(s)$



$h^*=55$	$h^*=57$	$h^*=62$
$h_A=50$	$h_A=53$	$h_A=55$
$h_B=107$	$h_B=258$	$h_B=522$

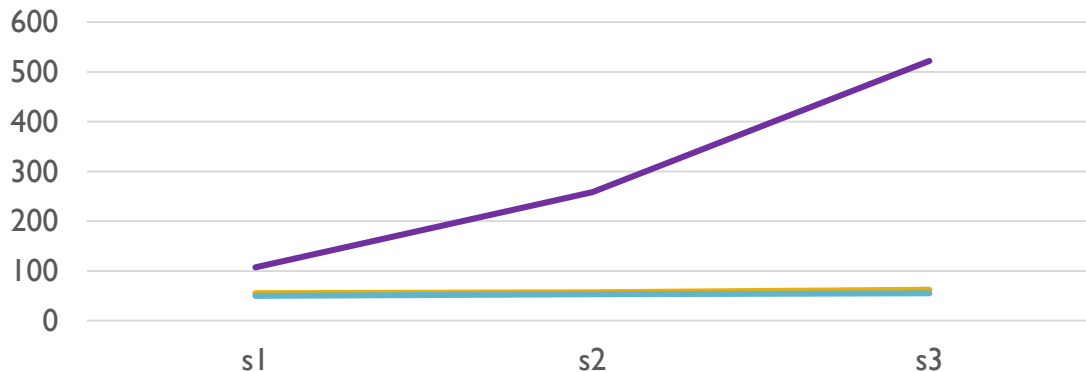
Close!

Large overestimate!

Which is best?

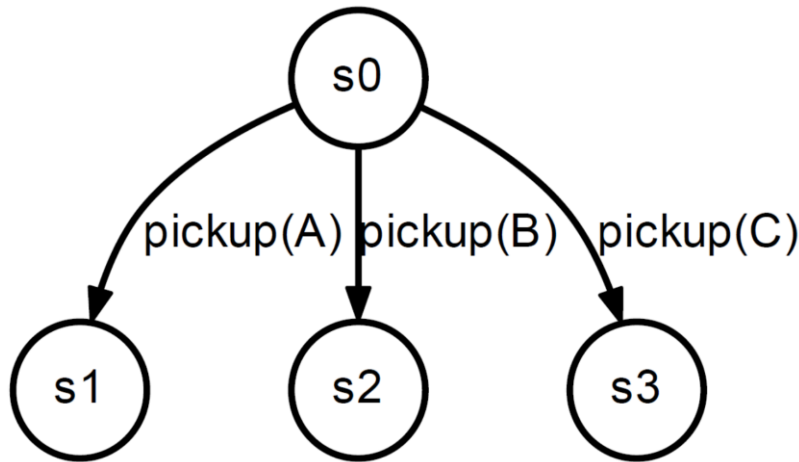
The strategy only cares about relative values

h^* , h_A , h_B result in identical choices: s_1 first!

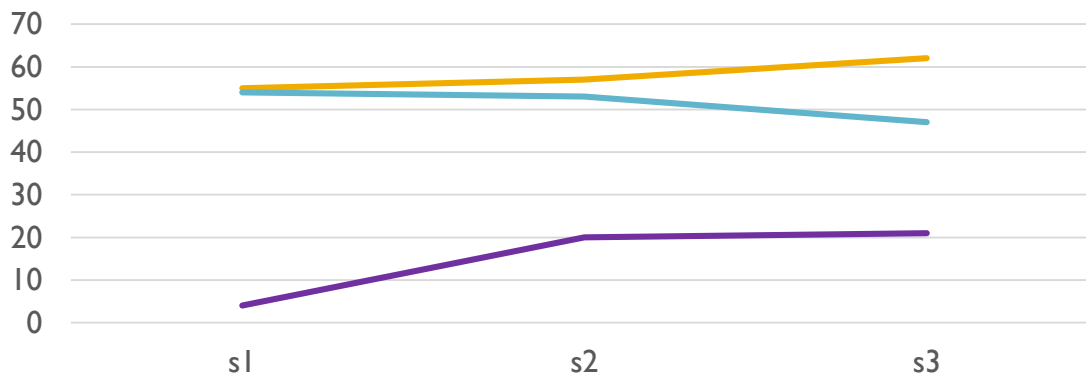


Minimization, case 3

Strategy: Depth first search; select a child with minimal $h(s)$



$h^*=55$	$h^*=57$	$h^*=62$
$hA=54$	$hA=53$	$hA=47$
$hB=4$	$hB=20$	$hB=21$



Which is best?

h^* and hB result in identical choices

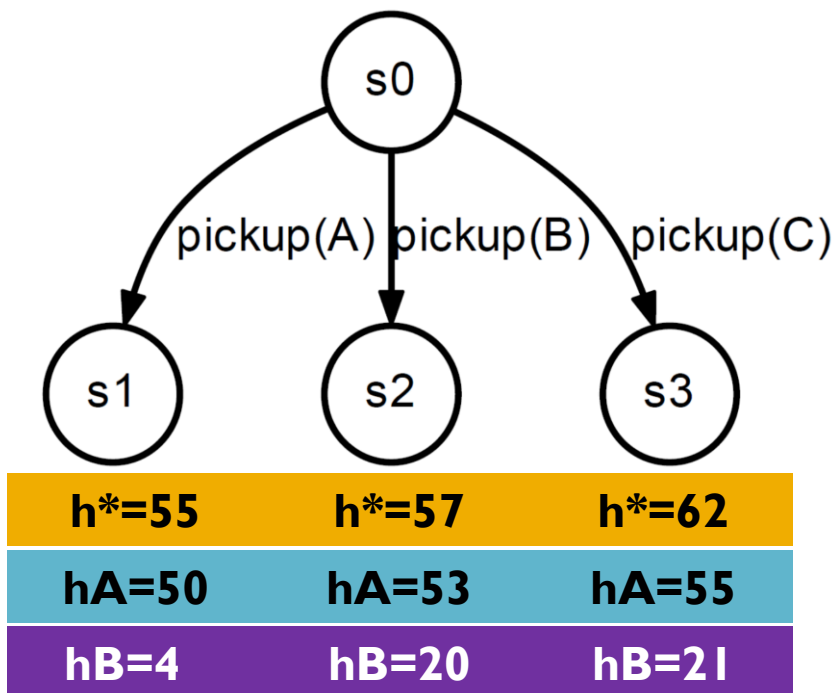
hA is worse for this strategy, despite being closer to h^* :

Goes to s_3 first

Even if we continue optimally, cost ≥ 62 !

A*, case 1

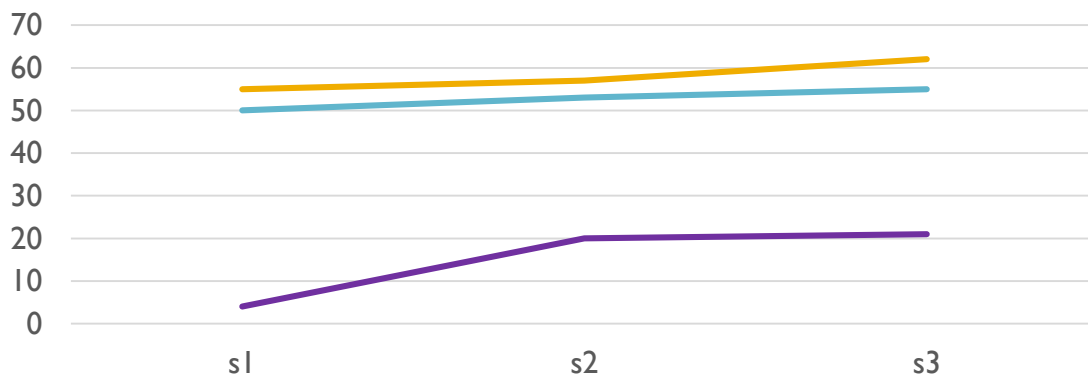
Back to case 1 – but suppose the strategy is A*



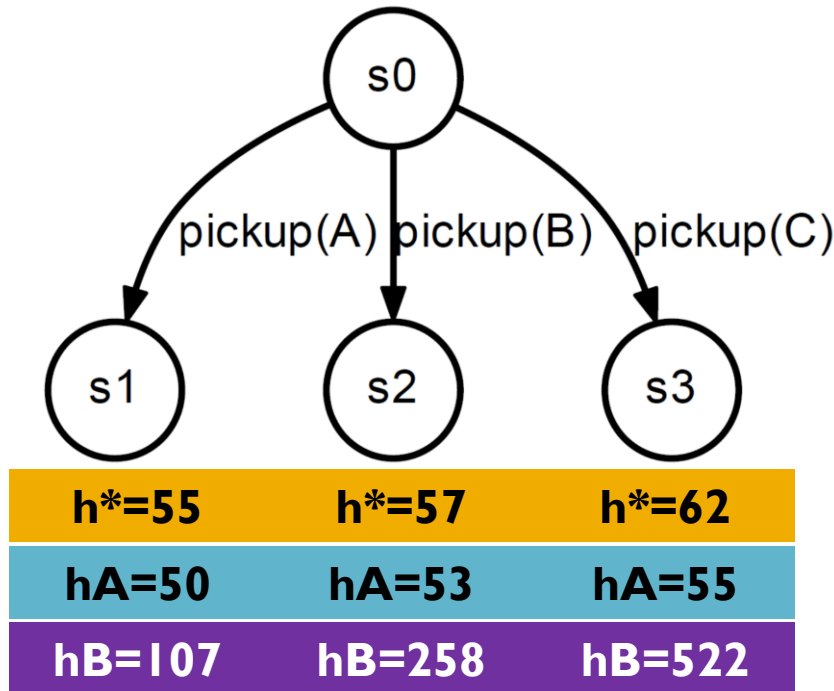
Which is best?

A* expands all nodes
where $g(s) + h(s) \leq \text{optcost}$

As long as h is admissible
 $[\forall s: h(s) \leq h^*(s)]$,
increasing it is always better



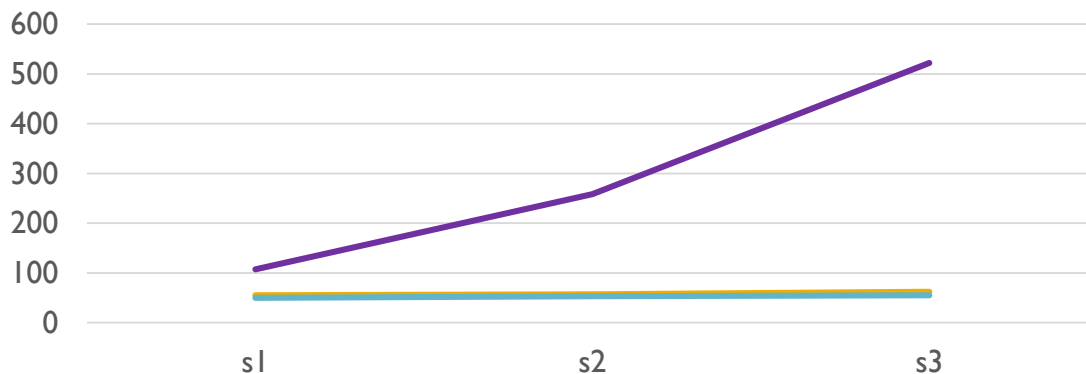
Case 2: Suppose the strategy is A*



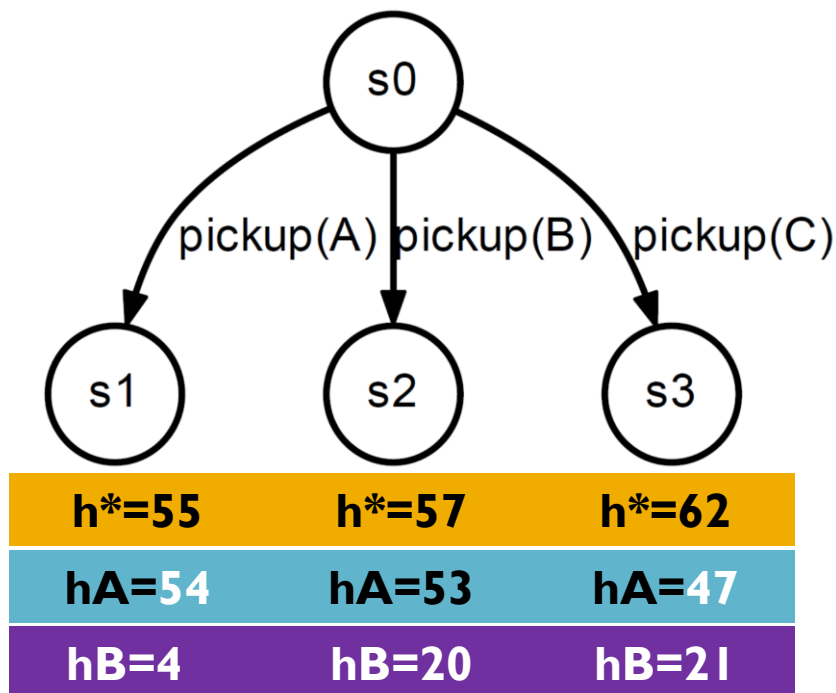
Which is best?

A* expands all nodes where $g(s) + h(s) \leq \text{optcost}$

Because h_B is not admissible, optimal solutions may be missed!



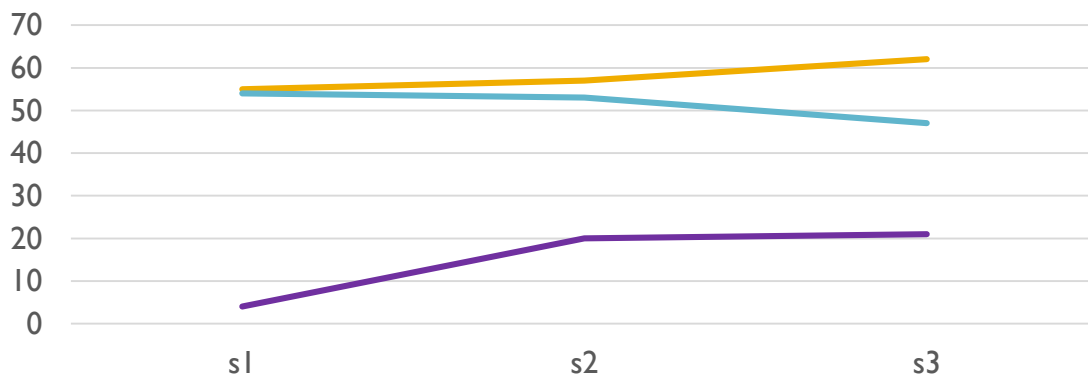
Case 3: Suppose the strategy is A*



Which is best?

A* expands all nodes
where $g(s) + h(s) \leq \text{optcost}$

As long as $h(s)$ is admissible
[$h(s) \leq h^*(s)$],
increasing it is always better
 h_A better than h_B



Two Requirements for Heuristic Guidance



- Heuristic planners must consider **two** requirements

Define a **search strategy**
able to take guidance into account

Examples:

A* uses a heuristic function
Hill-climbing uses a heuristic... differently!

Find a **heuristic function**
suitable for **the selected strategy**

Example:

Find a heuristic function
suitable specifically for A* or hill-climbing

Can be **domain-specific**,
given as input in the planning problem

Can be **domain-independent**,
generated automatically by the planner
given the problem domain

We will consider both – heuristics more than strategies

Some Desired Properties (1)

- What properties do **good heuristic functions** have?
 - **Informative**, of course:
Provide good guidance to the specific search strategy we use
 - Admissible?
 - Close to $h^*(n)$?
 - Correct "ordering"?
 - ...

Some Desired Properties (2)

- What properties do **good heuristic functions** have?
 - **Efficiently computable!**
 - Spend as little time as possible deciding which nodes to expand
 - **Balanced...**
 - Many planners spend almost all their time calculating heuristics
 - But: Don't spend more time computing h than you gain by expanding fewer nodes!
 - Illustrative (made-up) example:

Heuristic quality	Nodes expanded	Expanding one node	Calculating h for one node	Total time
Worst	100000	100 μ s	1 μ s	10100 ms
Better	20000	100 μ s	10 μ s	2200 ms

Some Desired Properties (3)

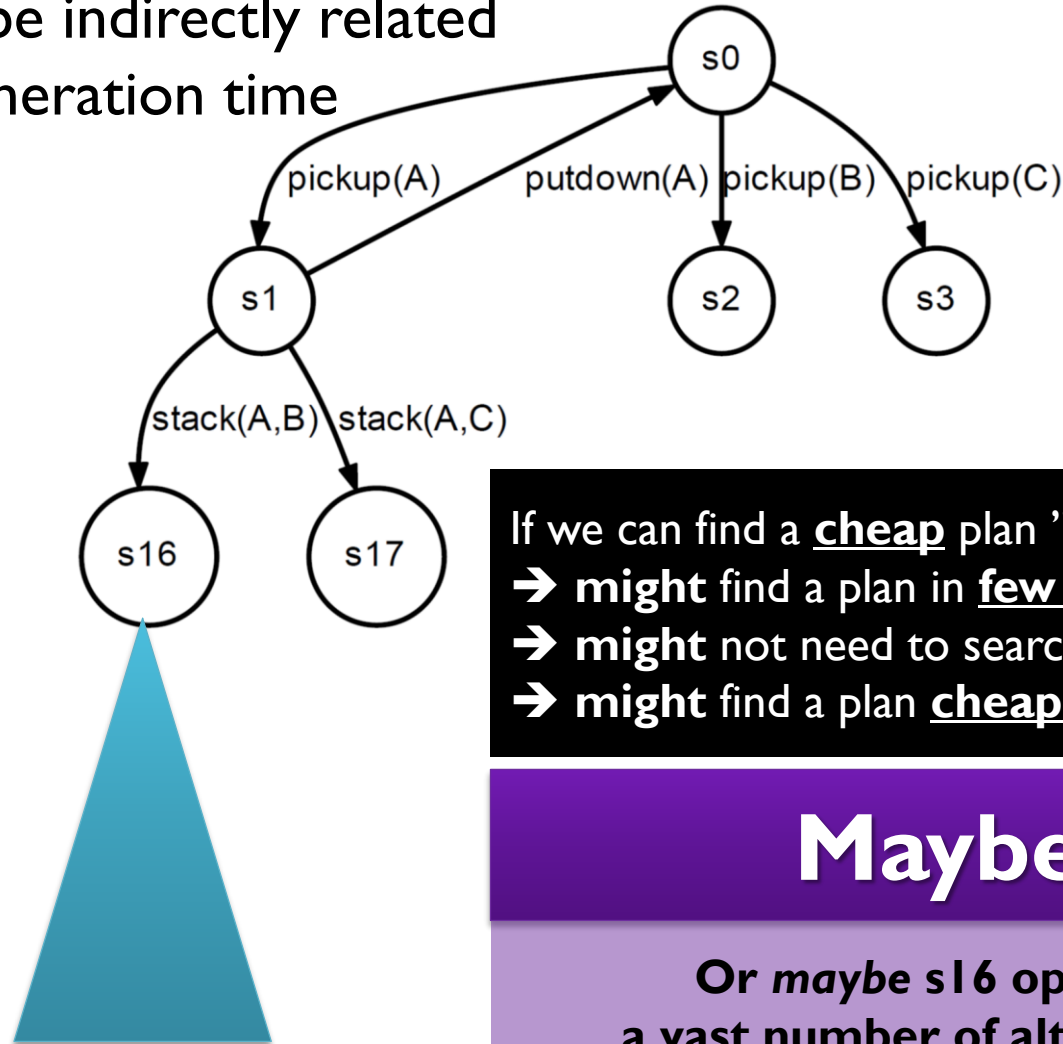
- [Table copy for the online lecture notes!]

Heuristic quality	Nodes expanded	Expanding one node	Calculating h for one node	Total time
Worst	100000	100 μ s	1 μ s	10100 ms
Better	20000	100 μ s	10 μ s	2200 ms
...	5000	100 μ s	100 μ s	1000 ms
...	2000	100 μ s	1000 μ s	2200 ms
...	500	100 μ s	10000 μ s	5050 ms
Best	200	100 μ s	100000 μ s	20020 ms

Speed vs. Cost

Cheap Plans, Cheap Planning?

- Cost can be indirectly related to plan generation time



If we can find a **cheap** plan "under" s_{16}
→ might find a plan in **few steps**
→ might not need to search so many nodes
→ might find a plan **cheaply**

Maybe!

Or *maybe* s_{16} opens up
a vast number of alternatives,
so finding a solution takes more time...

Prioritizing Speed or Plan Cost

Can design strategies to prioritize speed or plan cost

Find a solution quickly

Expand nodes where you think you can easily find a way to a goal node

Should prefer

Find a good solution

Expand nodes where you think you can find a way to a good (high quality) solution, even if finding it will be difficult

Should prefer

Open nodes

Accumulated plan cost $g(n)=50$,
estimated "cost distance" $h(n)=10$

Accumulated plan cost $g(n)=5$,
estimated "cost distance" $h(n)=30$

Often one strategy+heuristic can achieve *both* reasonably well, but for optimum performance, the distinction can be important!

A Simple Domain-Independent Heuristic and Search Strategy

Heuristics given Structured States

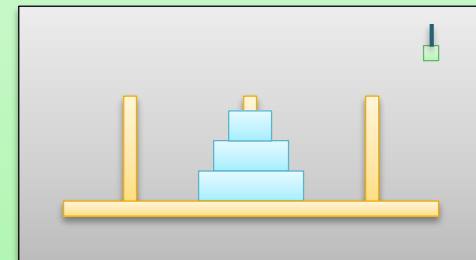
- In planning, we often want domain-independent heuristics
 - Should work for any planning domain – how?
- Take advantage of structured high-level representation!

■ Plain state transition system

- We are in state
572,342,104,485,172,012
- The goal is to be in one of the 10^{47} states in $S_g = \{ s[482,293], s[482,294], \dots \}$
- Should we try action
A297,295,283,291
leading to state
572,342,104,485,172,016?
- Or maybe action A297,295,283,292
leading to state
572,342,104,485,175,201?

■ Classical representation

- We are in a state where
disk 1 is on top of disk 2
- The goal is for all disks to be
on peg C
- Should we try take(B), leading to a
state where we are holding disk 1?
- ...

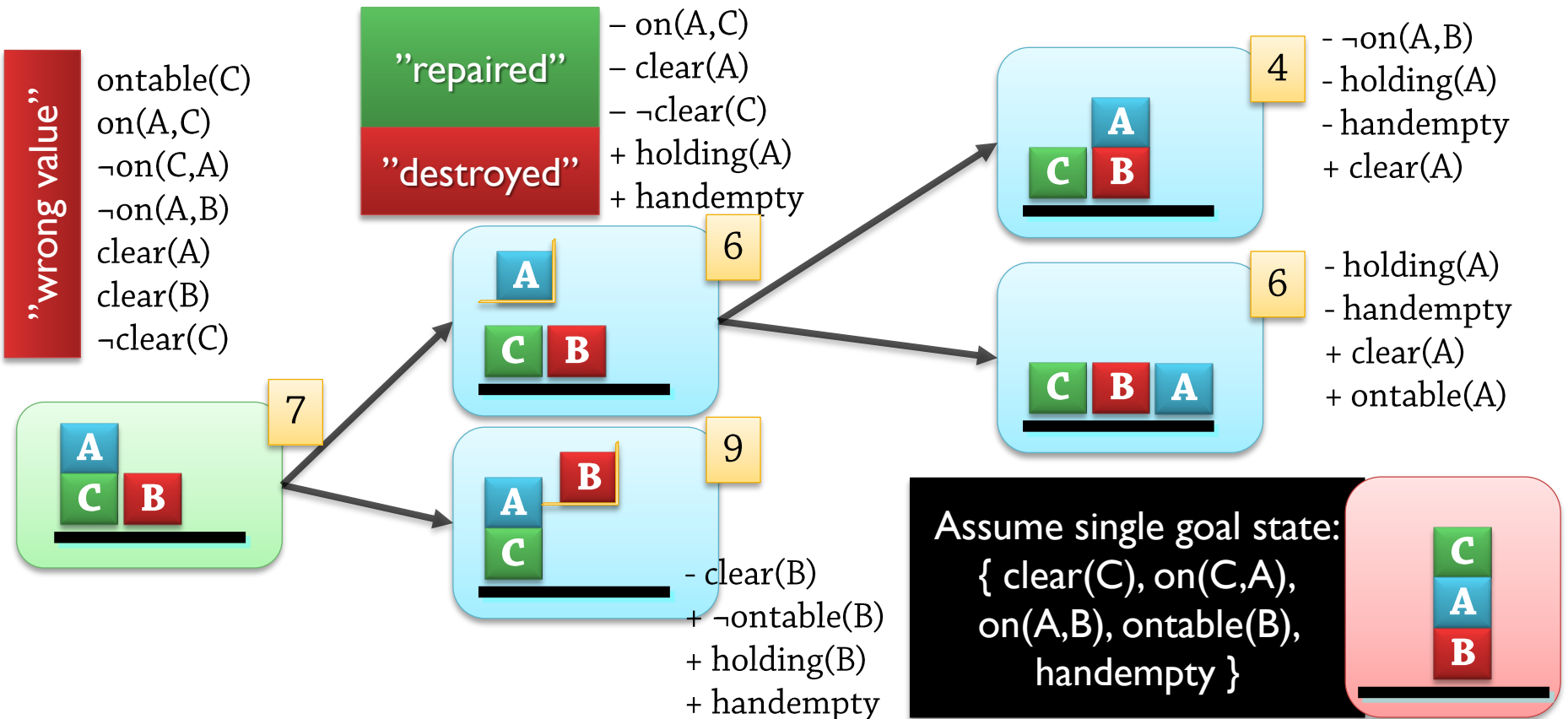


- An intuitive idea:
 - Number of steps required to reach the goal from s should be ***approximately proportional to*** how many goal requirements are not yet achieved in s
 - Let $h(s) =$ number of unsatisfied goals in s
- An associated search strategy:
 - Suppose we want to *minimize planning time*
 - Choose an open node with *minimal* $h(s)$
 - Greedy: Only care about *apparent amount of planning left to do*

Counting Remaining Goals

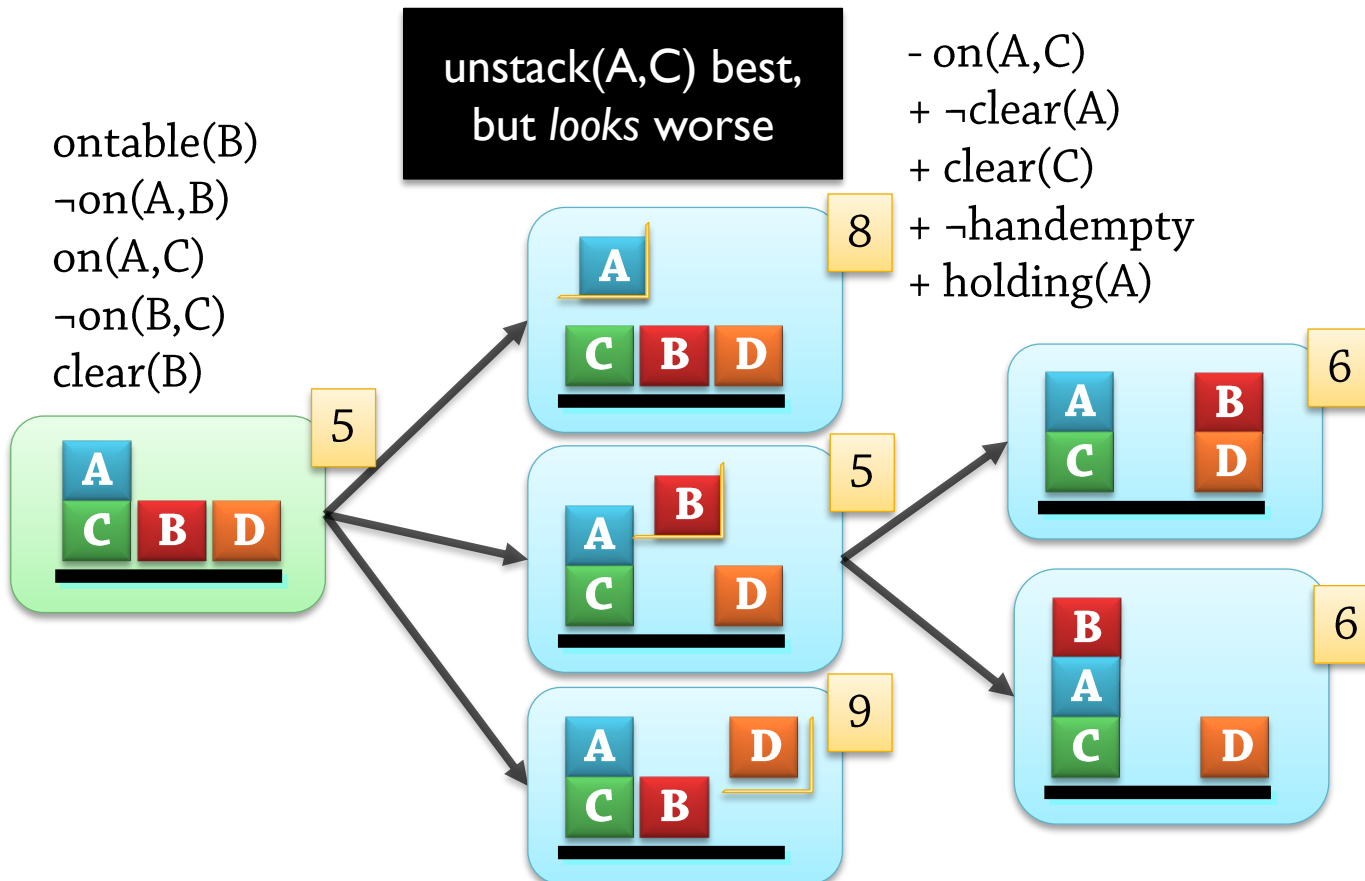
- **Count** the number of facts that are “wrong”
 - Requires that *states and goals are sets of facts*
 - (Conjunctions – not disjunctions)

Optimal:
`unstack(A,C)`
`stack(A,B)`
`pickup(C)`
`stack(C,A)`

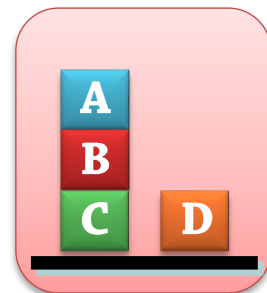


Counting Remaining Goals (2)

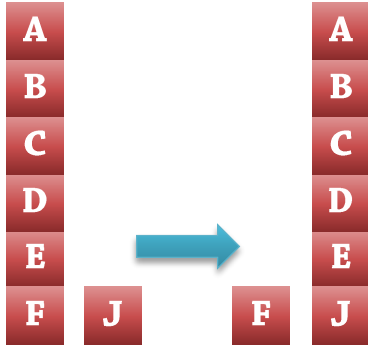
- A perfect solution? No!
 - We must often "unachieve" individual goal facts to get closer to a goal state!



Optimal:
unstack(A,C)
putdown(A)
pickup(B)
stack(B,C)
pickup(A)
stack(A,B)



bw-tower07-astar-gc: Only 7 blocks, A* search, based on goal count

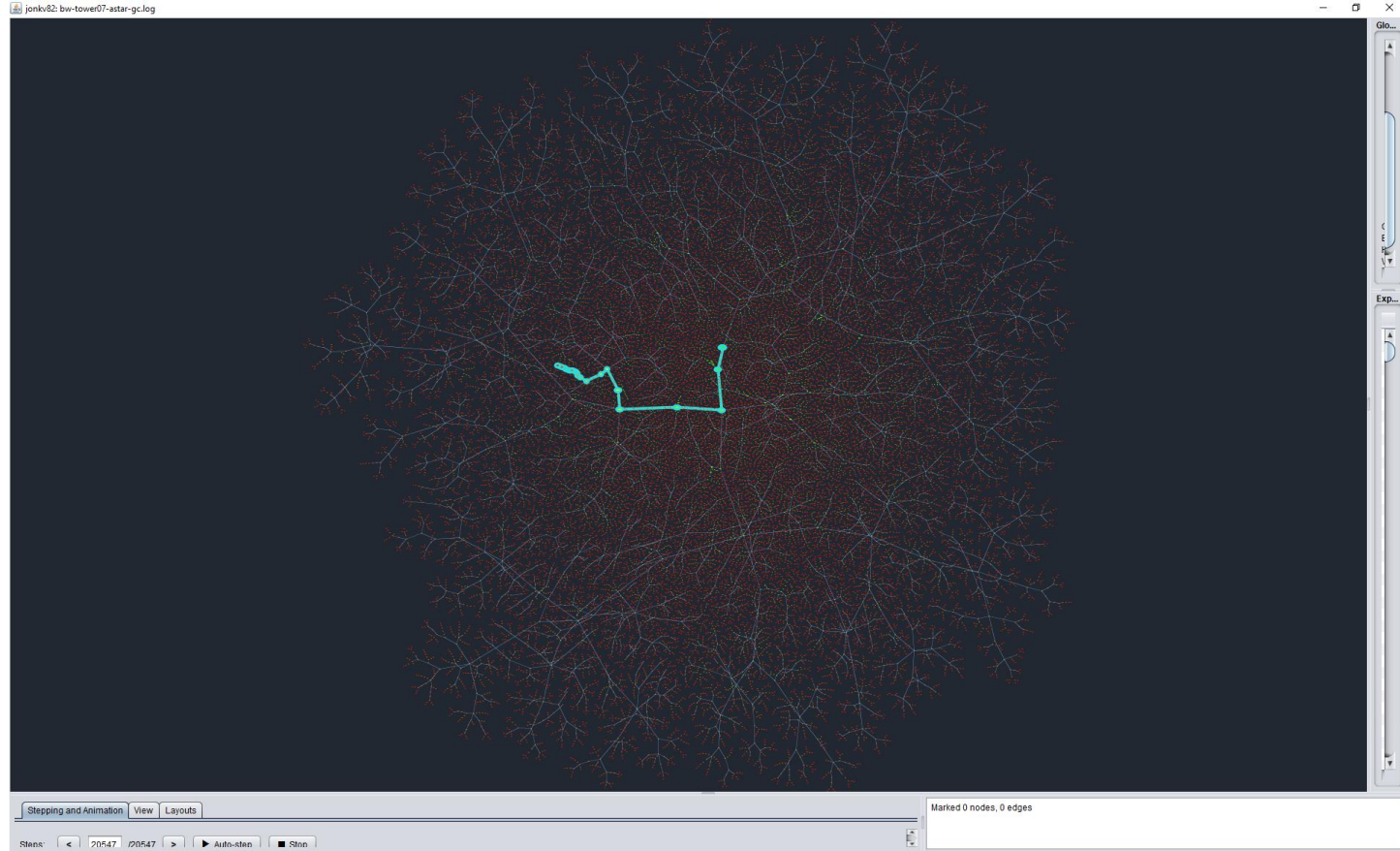


18 actions in π

States:

6463 calculated,
3222 visited

(With Dijkstra,
43150 / 33436 –
improved, but we
can do better!)



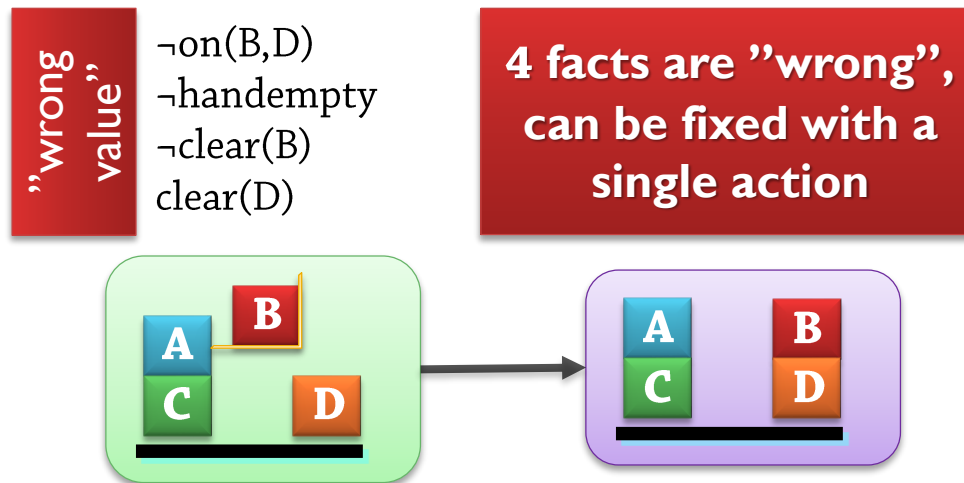
- $h(s_0) = 1$: Only one “missing” fact
- For a long time, all **useful** successors appear to **increase** remaining cost
 - Removing a block that must be moved
- And many **useless** successors appear to **decrease** remaining cost
 - Building towers that will need to be torn down

Not very
informative!

Counting Remaining Goals (3)

■ Admissible?

- No!
- (Doesn't matter in our chosen search strategy)



■ Can we make it admissible?

- Yes: Divide by the maximum number of facts modified by any action

Counting Remaining Goals (4): Analysis

- What we see from this example...
 - Not very much: **All heuristics have weaknesses!**

Even the best planners will make “strange” choices, visit **tens, hundreds** or even **thousands** of “unproductive” nodes for every action in the final plan

The heuristic should make sure we don’t need to visit **millions, billions** or even **trillions** of “unproductive” nodes for every action in the final plan!

- But a thorough empirical analysis *would* tell us:
 - This heuristic is far from sufficient!

- Planning Competition 2011: Elevators domain, problem 1
 - A* with goal count heuristics
 - States: 108922864 generated, gave up
 - LAMA 2011 planner, good heuristics, other strategy:
 - Solution: 79 steps, 369 cost
 - States: 13236 generated, 425 evaluated/expanded
- Elevators, problem 5
 - LAMA 2011 planner:
 - Solution: 112 steps, 523 cost
 - States: 41811 generated, 1317 evaluated/expanded
- Elevators, problem 20
 - LAMA 2011 planner:
 - Solution: 354 steps, 2182 cost
 - States: 1364657 generated, 14985 evaluated/expanded

Important insight:

Even a
state-of-the-art planner
can't go *directly* to a goal
state!

Generates *many* more
states than those
actually on the path to
the goal...

Search Strategies and Heuristics for Optimal Forward State Space Planning

- **Optimal** plan generation:
 - There is a **quality measure** for plans
 - Minimal number of actions
 - Minimal sum of action costs
 - ...
 - We **must** find an optimal plan!
- Suboptimal plans
(0.5% more expensive):



Irrelevant

A Well Known Heuristic Search Algorithm: A*

Used in many optimal planners

- Optimal Plan Generation: Often uses **A***
 - A* focuses entirely on optimality
 - Slowly expand from the initial state, systematically checking possibilities
 - No point in trying to find a "reasonable" plan *before* the optimal one!
 - Requires admissible heuristics to guarantee optimality
 - Reason: Heuristic used for *pruning* (skipping some search nodes + all descendants)
 - Search queue ordered by $f(n) = g(n)$ [actual cost] + $h(n)$ [heuristic]:

$$11 = 10 + 1$$

Pop – not a
solution

$$12 = 10 + 2$$

Pop – not a
solution

$$12 = 12 + 0$$

Pop –
solution!

$$12 = 11 + 1$$

$$13 = 11 + 2$$

Ignore:
 g is *known*, h is an *underestimate*,
so solutions found by expanding
these nodes will cost $\geq g+h$
(and we *have* one of cost $\leq g+h$)

■ Dijkstra vs. A*: The essential difference

Dijkstra

- Selects from *open* a node n with minimal $g(n)$
 - Cost of reaching n from initial node

Uninformed (blind)

A*

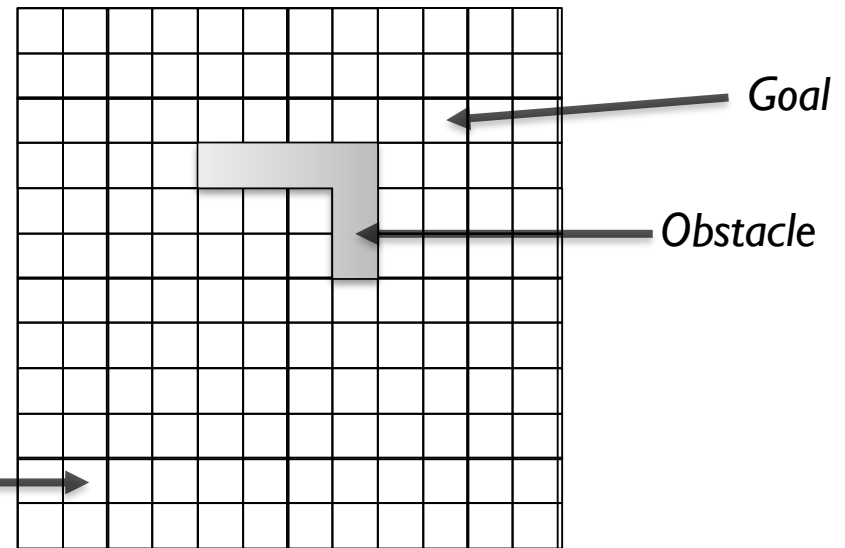
- Selects from *open* a node n with minimal $g(n) + h(n)$
 - + underestimated cost of reaching a goal from n

Informed

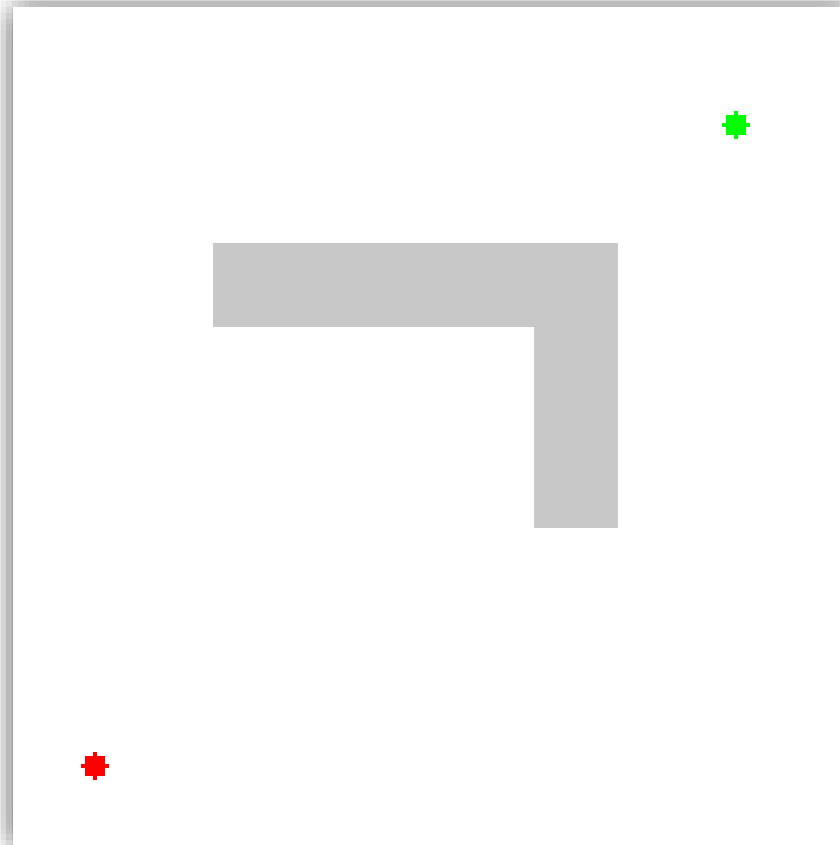
■ Example:

- **Hand-coded** heuristic function
- Can move diagonally →
 $h(n) = \text{Chebyshev distance}$
from n to goal =
 $\max(\text{abs}(n.x - \text{goal}.x), \text{abs}(n.y - \text{goal}.y))$
- Related to **Manhattan Distance** =
 $\text{sum}(\text{abs}(n.x - \text{goal}.x), \text{abs}(n.y - \text{goal}.y))$

Start →



- A* Search:

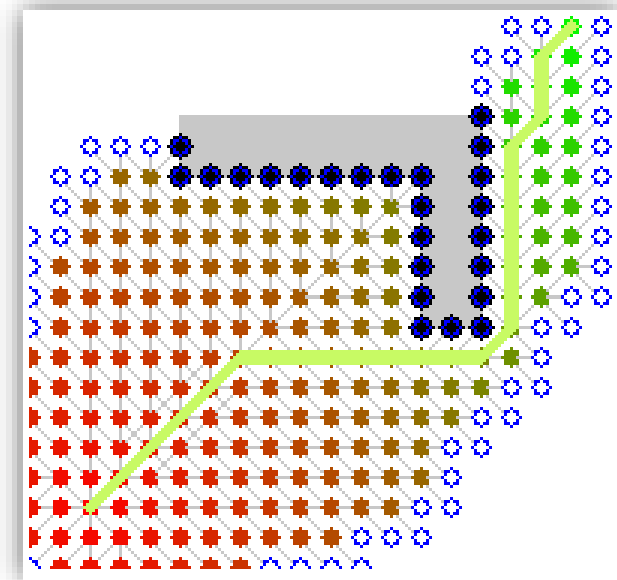


Here:
A single
physical obstacle

In general:
Many states where
all available actions
will increase $g+h$
(cost + heuristic)

Investigate *all* states
where $g+h=15$,
then all states
where $g+h=16, \dots$

- Given an admissible heuristic h , A^* is optimal in two ways
 - Guarantees an optimal plan
 - Expands the minimum number of nodes required to *guarantee optimality* with the given heuristic
- Still expands many "unproductive" nodes in the example
 - Because the heuristic is not perfectly informative
 - Even though it is hand-coded
 - Does not take obstacles into account
 - If we knew $h^*(n)$:
 - Expand optimal path to the goal



- What is an **informative** heuristic for A*?
 - Basic requirement: **Must be admissible** $\rightarrow \forall n. h(n) \leq h^*(n)$
 - As always, $h(n) = h^*(n)$ would be perfect – but not attainable...
 - As indicated before: The *closer* $h(n)$ is to $h^*(n)$, the *better*
 - Suppose **hA** and **hB** are both **admissible**
 - Suppose **$\forall n. hA(n) \geq hB(n)$** : hA is at least close to true costs as hB
 - Then A* with hA *cannot* expand more nodes than A* with hB

Problem

Given an **arbitrary** planning problem

$$P = (\Sigma, s_0, g),$$

find an admissible heuristic function $h(s)$

Creating Admissible Heuristic Functions: The General Relaxation Principle

- **We have:**
 - An arbitrary planning problem $P = \langle \Sigma, s_0, S_g \rangle$

- **We want:**
 - A way to compute an **admissible heuristic $h(s)$**
 - Given P and some state s

What do we do?
Where do we start?
How do we think?

Fundamental Ideas (1)

- **One obvious method:**

Every time we need $h(s)$ for some state s ...

1. **Solve P optimally** starting in s , resulting in an *actual* solution $\pi^*(s)$

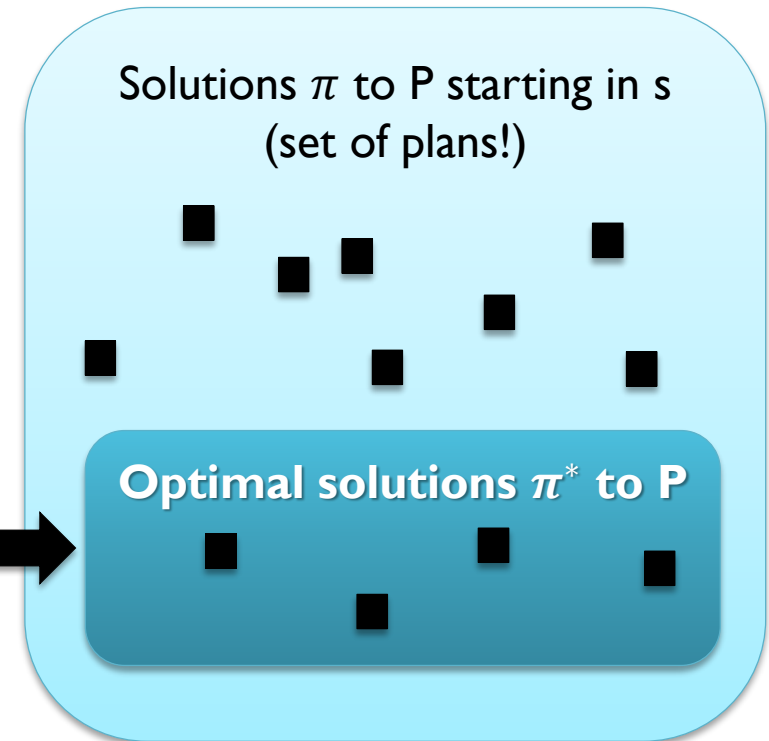
2. Let $h(s) = h^*(s) = \text{cost}(\pi^*(s))$

- Admissible – why?

- Obvious, but stupid

- If we find $\pi(s)$, we're already done!

Also: These are hard to find
(or we wouldn't need
a heuristic)



Fundamental Ideas (2)

- Let's modify the obvious idea:

- **Change / transform** P to make it easy (quick) to solve

- But make sure optimal solutions cannot become more expensive!
- Example: Add more goal states to P
→ easier to reach!

Relaxation will be one specific way of (1) **finding** a simplifying transformation, and (2) **proving** "not-more-expensive"!

- **Compute** an admissible heuristic:

- Solve the modified planning problem optimally
- $h(s)$ = cost of optimal solution for modified problem
 \leq
 $h^*(s)$ = cost of optimal solution for original problem
- Definition of admissibility!

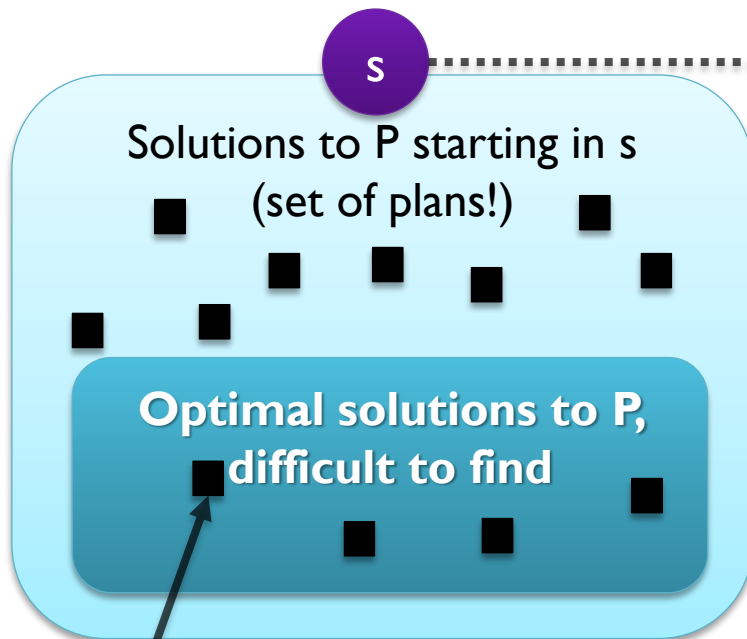
- **Preferably:**

- Keep $h(s)$ as close as possible to $h^*(s)$ – we want *strong cost information*!

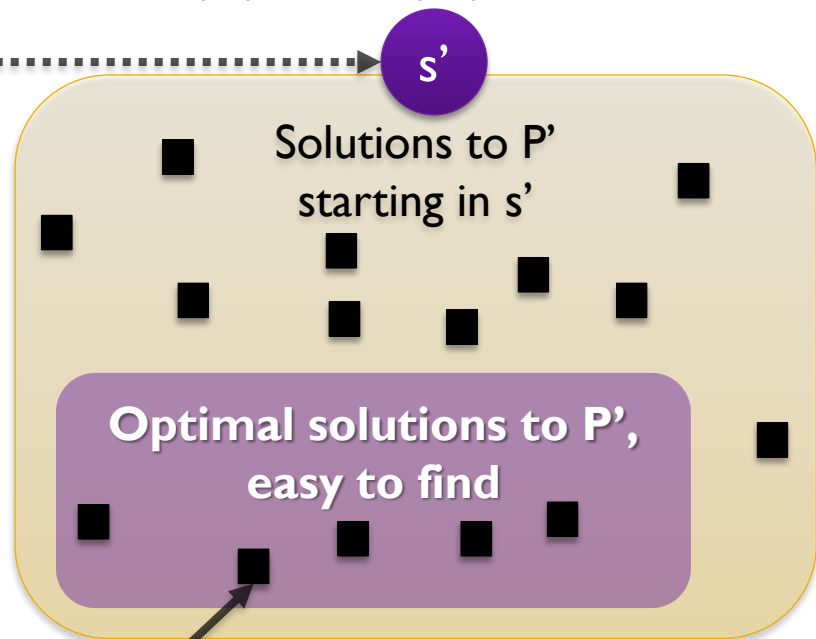
Fundamental Ideas (3)

■ More formally:

- Before planning, **find** a **simpler** problem P' , such that in every state s (of P):
 - We can **quickly** transform s into a state s' for P'
 - Then we can **quickly** find an optimal solution π' for P' starting in s'
 - The solution is **never more expensive**: $\text{cost}(\pi') \leq \text{cost}(\pi^*)$



π^* : An optimal *plan* for P'



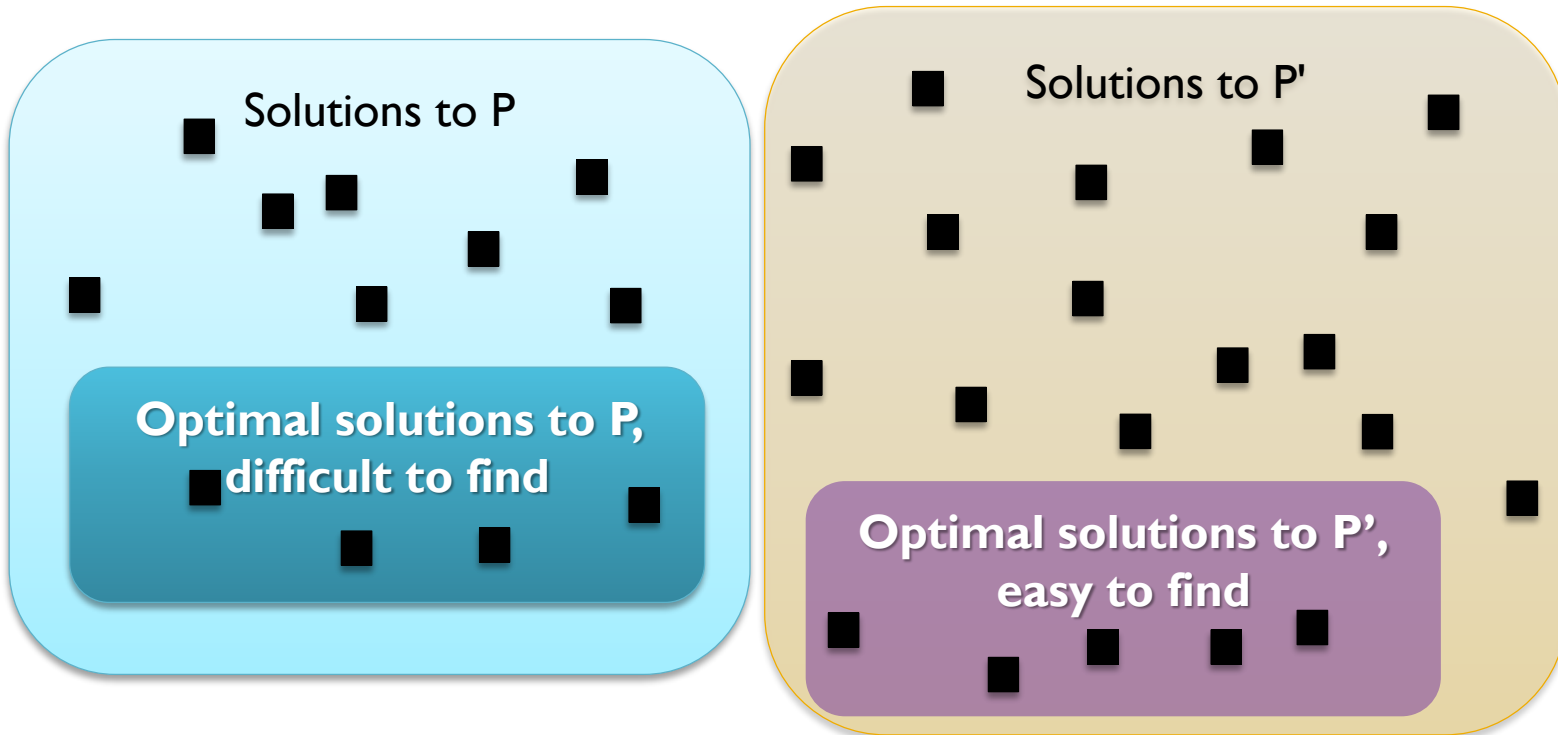
π' : solution to another problem; we only use it to compute a heuristic

- **During** planning:
 - Every time we need $h(s)$ for some state s :
 - Transform s to s'
 - **Quickly solve** problem P' **optimally** starting in s' , resulting in solution π' – for the *transformed* problem
 - Let $h(s) = \text{cost}(\pi')$
 - Throw away π' : It isn't interesting in itself
- We then know:
 - $h(s) = \text{cost}(\pi'(s)) = \text{cost}(\text{optimal-solution}(P')) \leq \text{cost}(\text{optimal-solution}(P))$
 - $h(s)$ is admissible

Fundamental Ideas (5)

- Important:

- What we **need**: $\text{cost}(\text{optimal-solution}(P')) \leq \text{cost}(\text{optimal-solution}(P))$
- **Could** use a transformation yielding **completely disjoint** solution sets + a **proof** that optimal solutions to P' are not more expensive

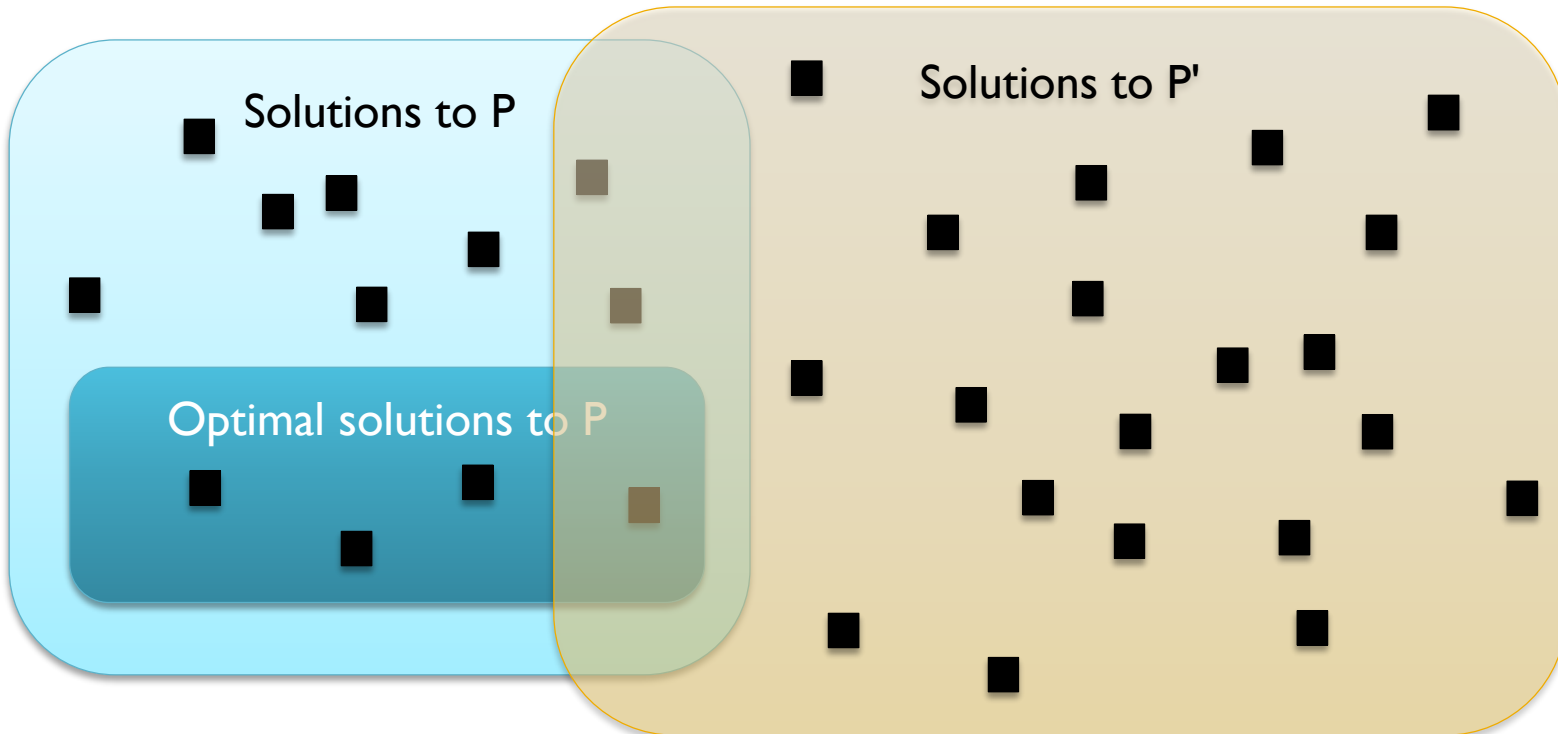


Difficult to find transformations, prove correctness – we need a *method*

Fundamental Ideas (6)

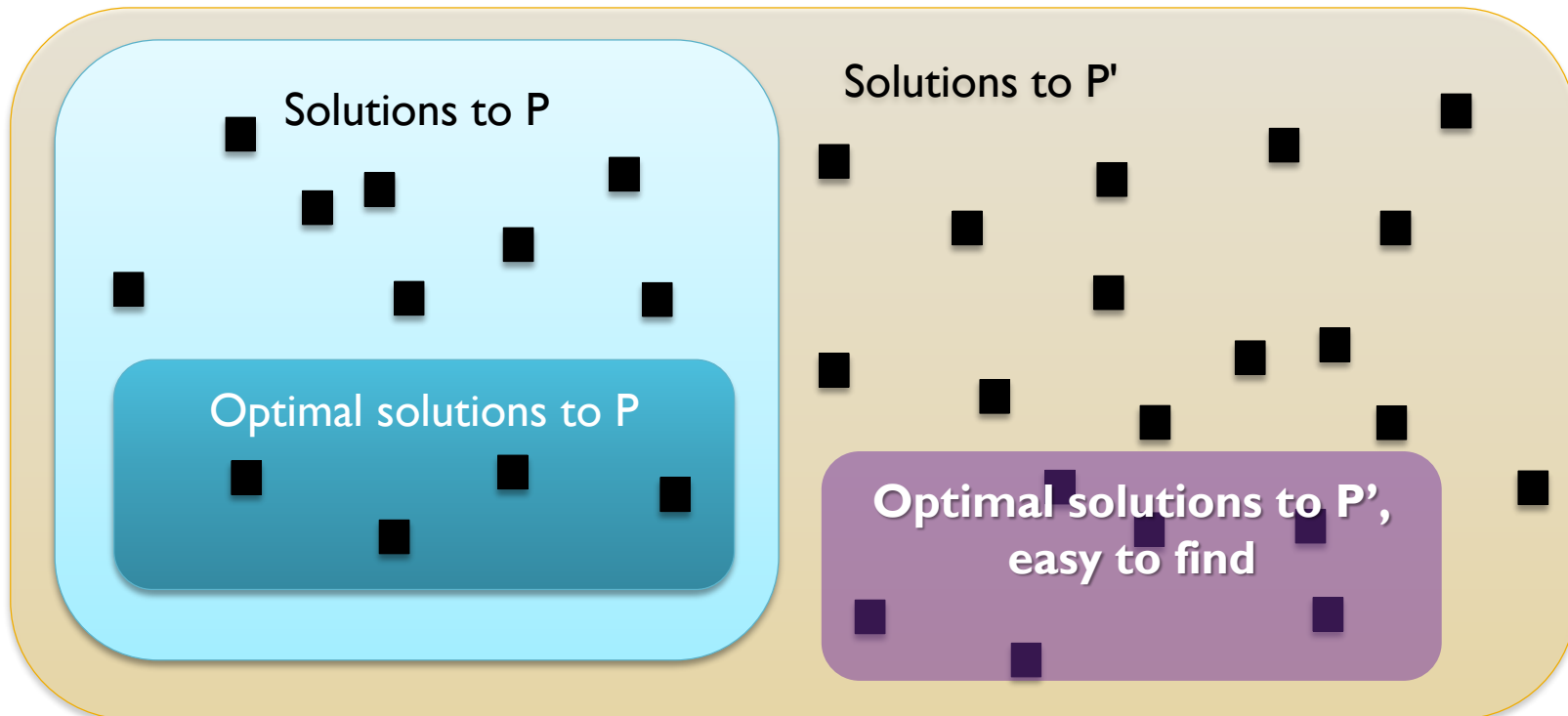
- How to prove $\text{cost}(\text{optimal-solution}(P')) \leq \text{cost}(\text{optimal-solution}(P))$?
 - **Sufficient** criterion: One optimal solution to P remains a solution for P'
 - $\text{cost}(\text{optimal-solution}(P')) = \min \{ \text{cost}(\pi) \mid \pi \text{ is any solution to } P' \} \leq \text{cost}(\text{optimal-solution}(P))$

Includes the optimal solutions to P,
so $\min \{ \dots \}$ cannot be greater



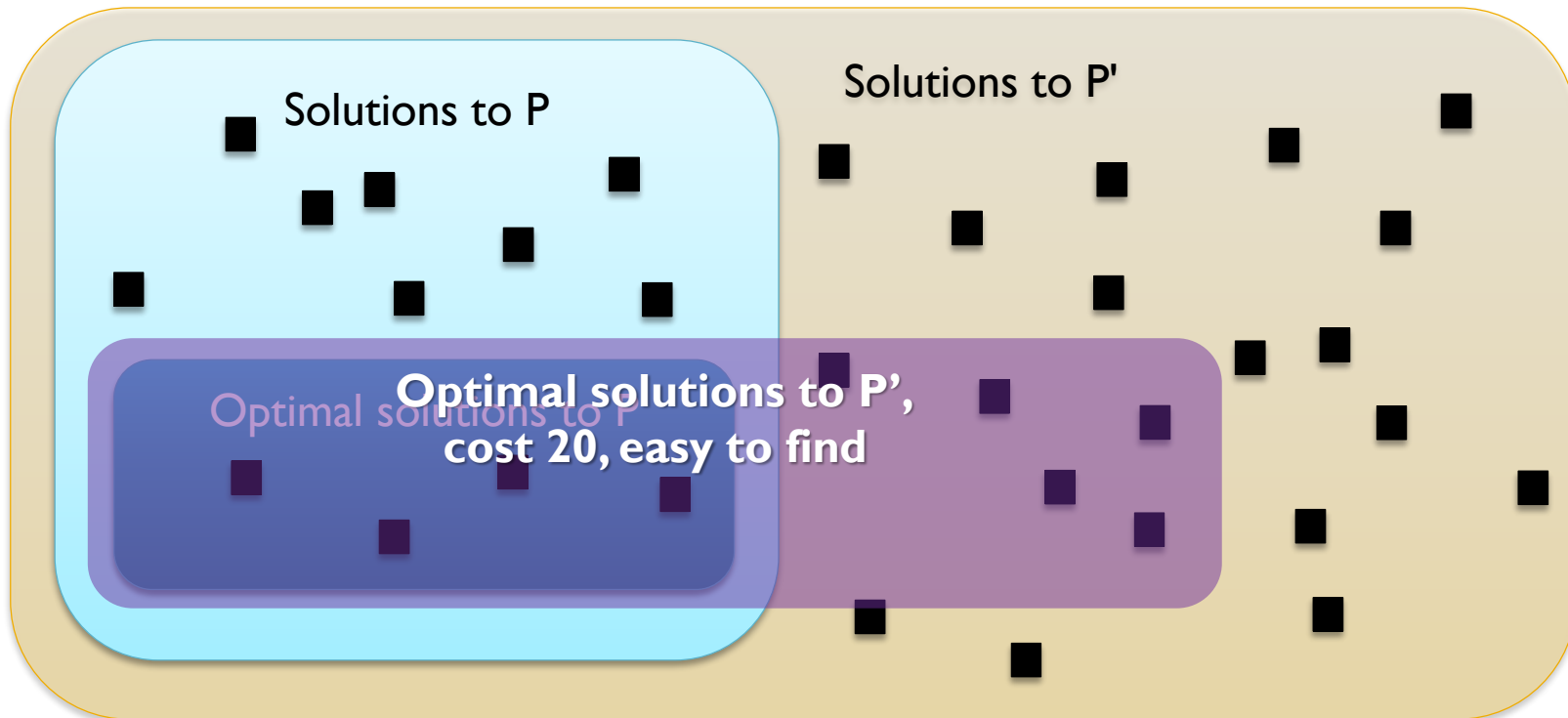
Fundamental Ideas (7)

- Another sufficient criterion: **All solutions** to P **remain** solutions for P'
 - Stronger, but often **easier to prove**
 - **This** is called **relaxation**: P' is a relaxed version of P
 - **Relaxes** the constraint on what is accepted as a solution:
The **is-solution(plan)?** test is "expanded, relaxed" to cover additional plans



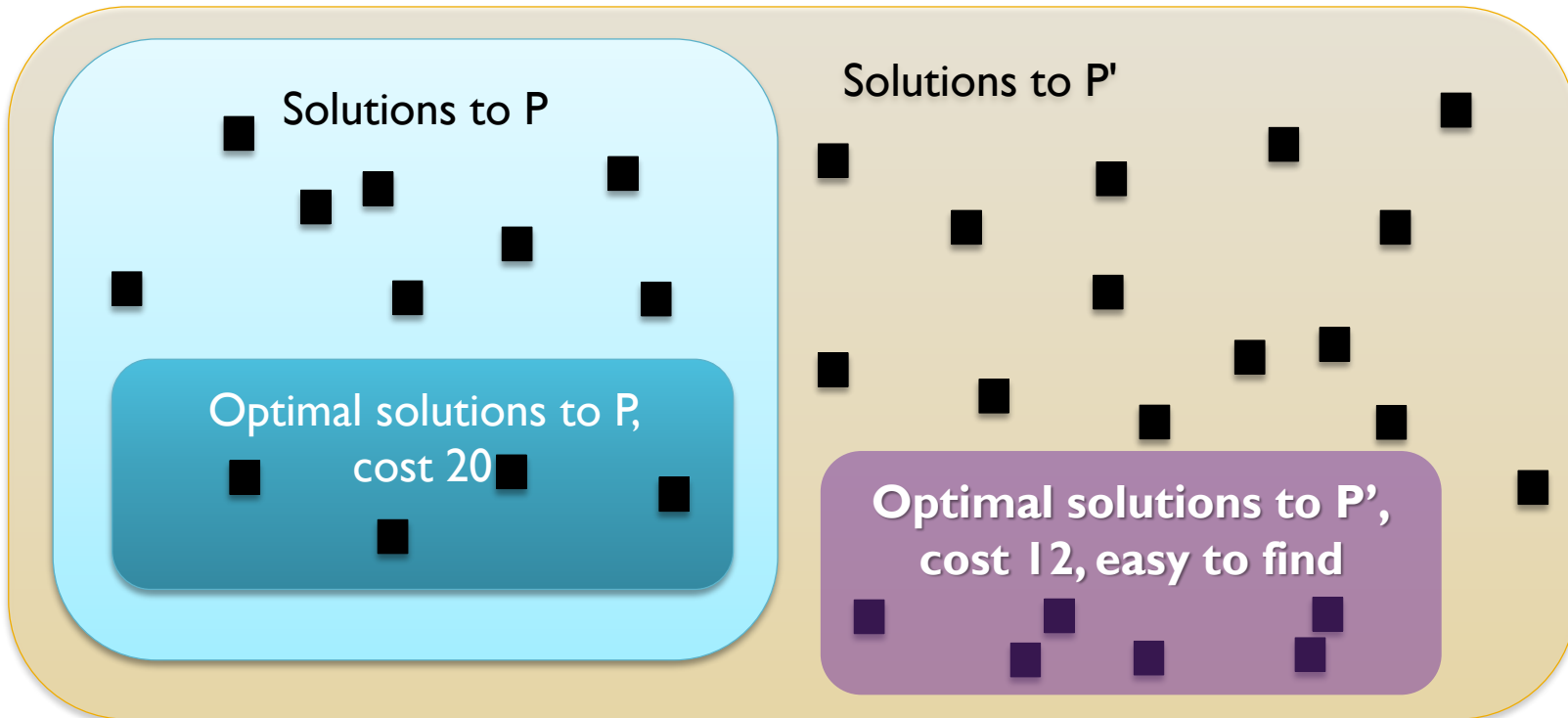
Fundamental Ideas (8)

- Case I: P' has identical cost (for some starting state s)
 - Unlikely!



Fundamental Ideas (9)

- Case 2: P' has lower cost (for some starting state s)



Relaxation:

Definition and Examples

Relaxation for Planning Problems

- A classical planning problem $P = (\Sigma, s_0, S_g)$ has a set of solutions
 - $Solutions(P) = \{ \pi : \pi \text{ is an executable action sequence leading from } s_0 \text{ to a state in } S_g \}$
- Suppose that:
 - $P = (\Sigma, s_0, S_g)$ is a classical planning problem
 - $P' = (\Sigma', s_0', S_g')$ is another classical planning problem
 - $Solutions(P) \subseteq Solutions(P')$
- Then (and only then): P' is a relaxation of P

Solutions for P:

Sol1, cost 10
Sol2, cost 12
Sol3, cost 27

Optimal in P

Solutions for P':

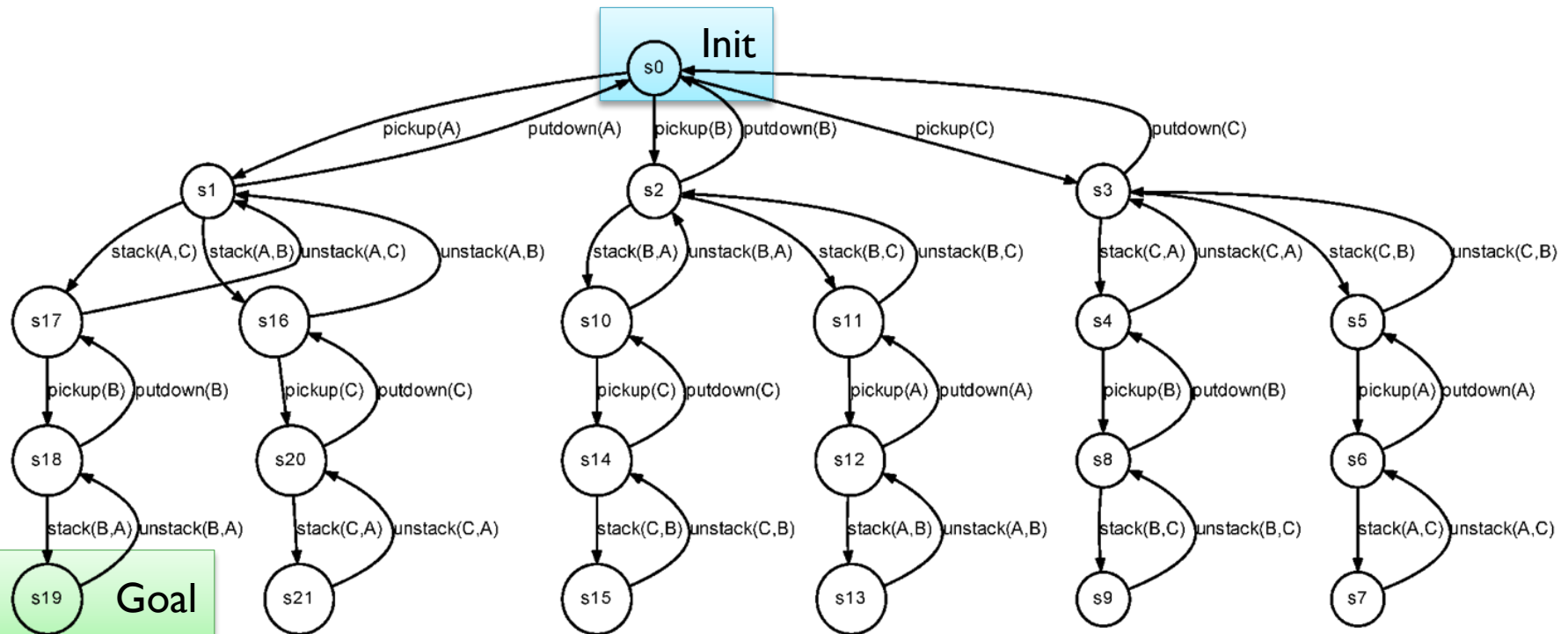
Sol1, cost 10
Sol2, cost 12
Sol3, cost 27
Sol4, cost 8
Sol5, cost 42

**All old solutions
remain solutions!**

Now **sol4** is optimal

Relaxation Example: Basis

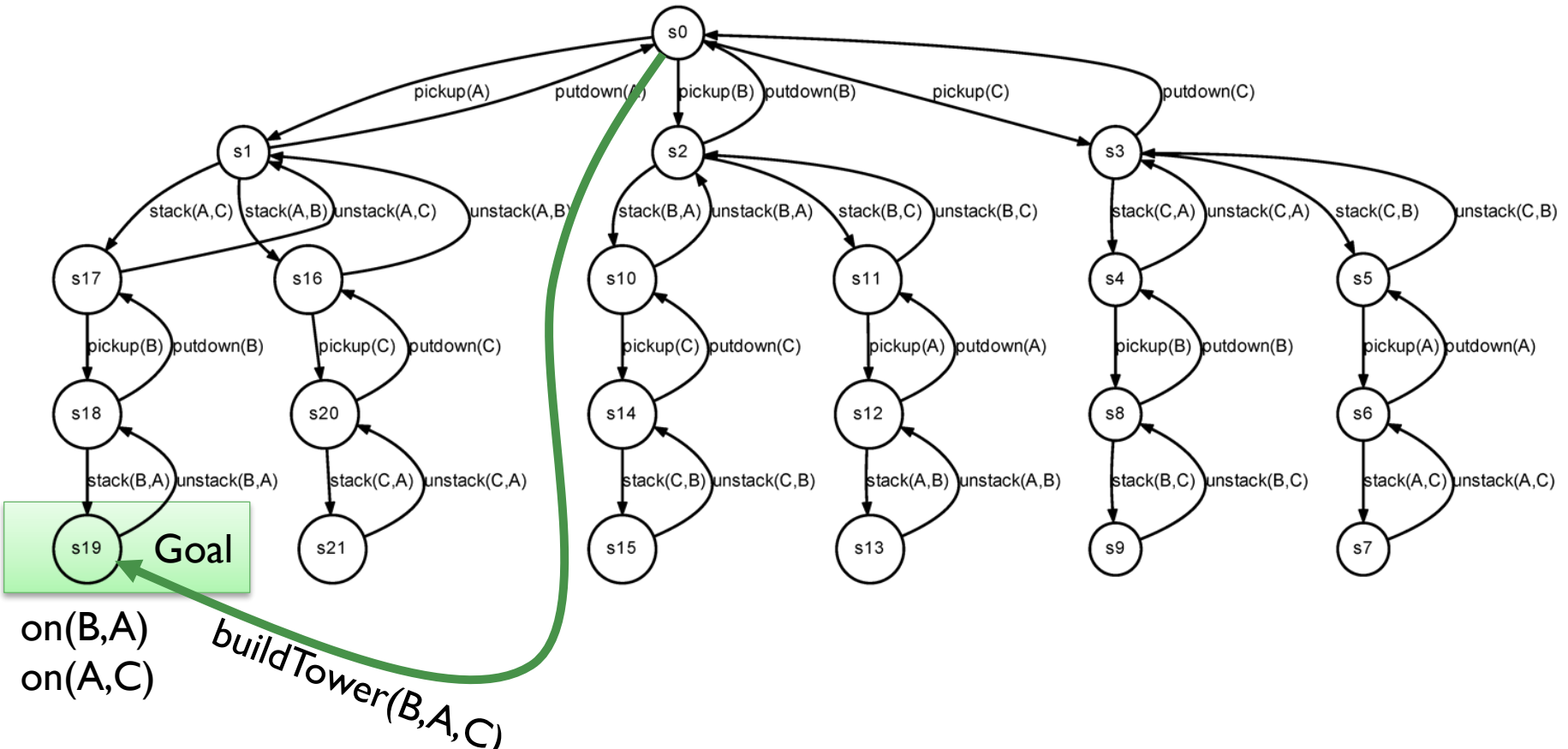
- A simple planning problem (domain + instance)
 - Blocks world, 3 blocks
 - Initially all blocks on the table
 - Goal: (and (on B A) (on A C)) (only satisfied in s19)
 - Solutions: **All** paths from init to goal (infinitely many – can have cycles)



Relaxation Example 1

■ Example 1: Adding new operators to the domain

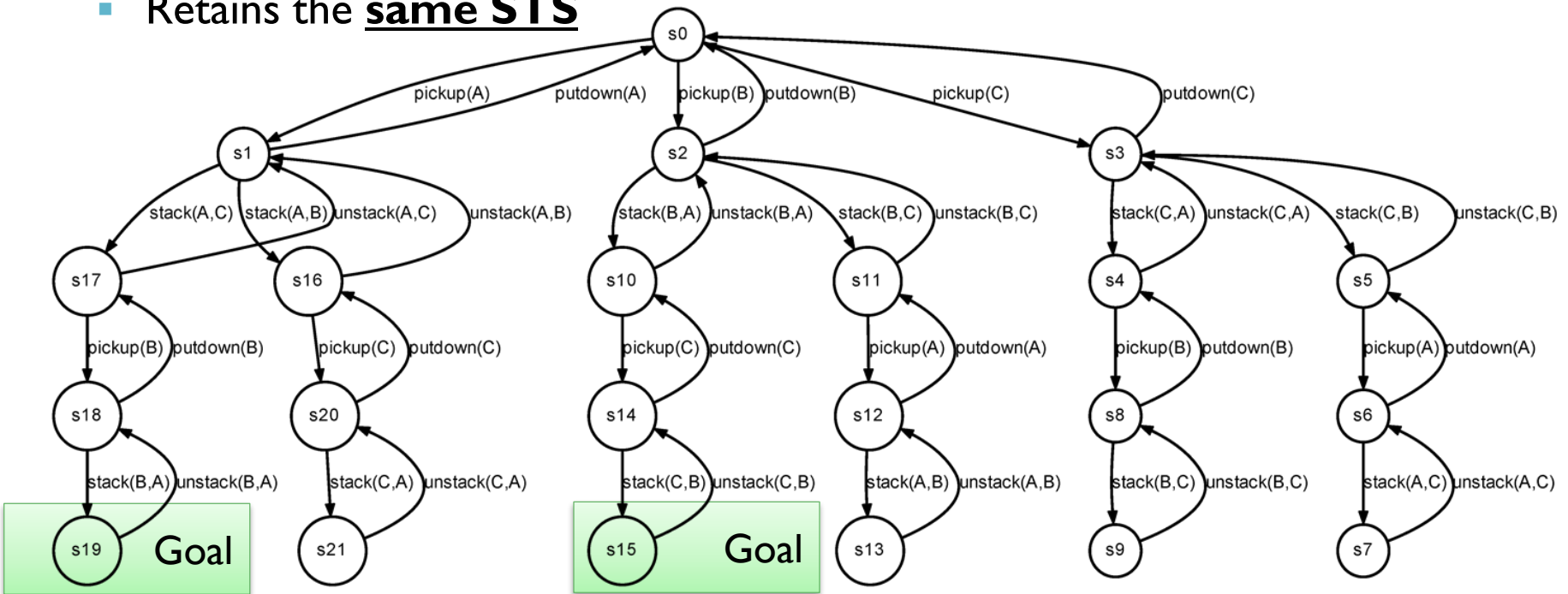
- All old solutions still valid, but new solutions may exist
- Modifies the STS by adding new edges / transitions
- This particular example: *shorter* solution exists



Relaxation Example 2

■ Example 2: Adding goal states

- New goal formula: (and (on B A) (**or (on A C) (on C B)**))
- All old solutions still valid, but new solutions may exist
- This particular example: Optimal solution **from s_0** retains the same length
- Retains the **same STS**



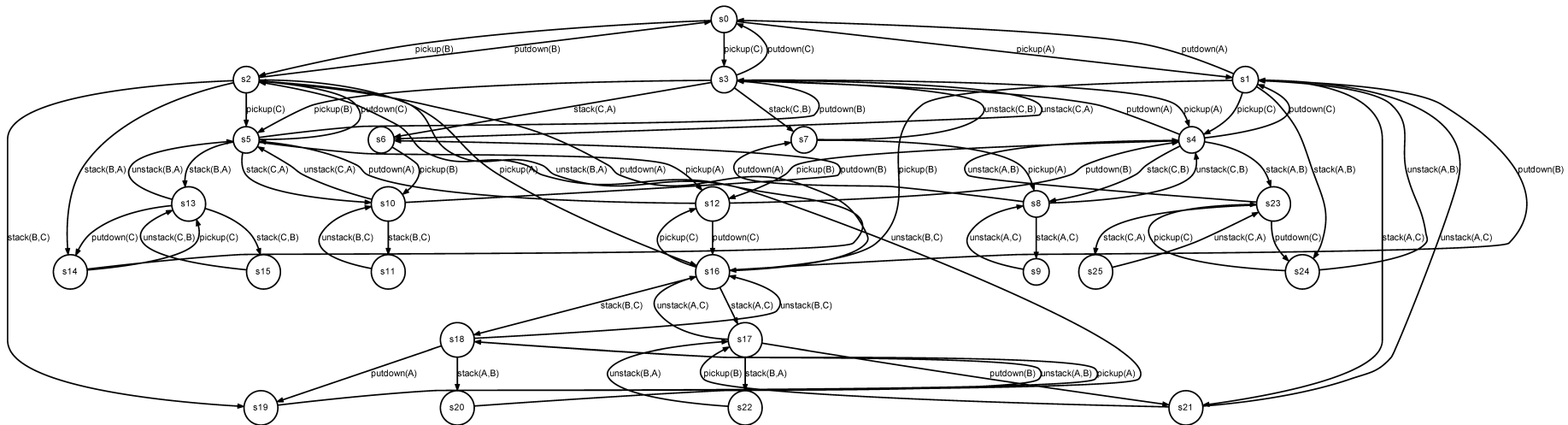
on(B,A)
on(A,C) or on(C,B)

on(B,A)
on(A,C) or on(C,B)

Relaxation Example 3

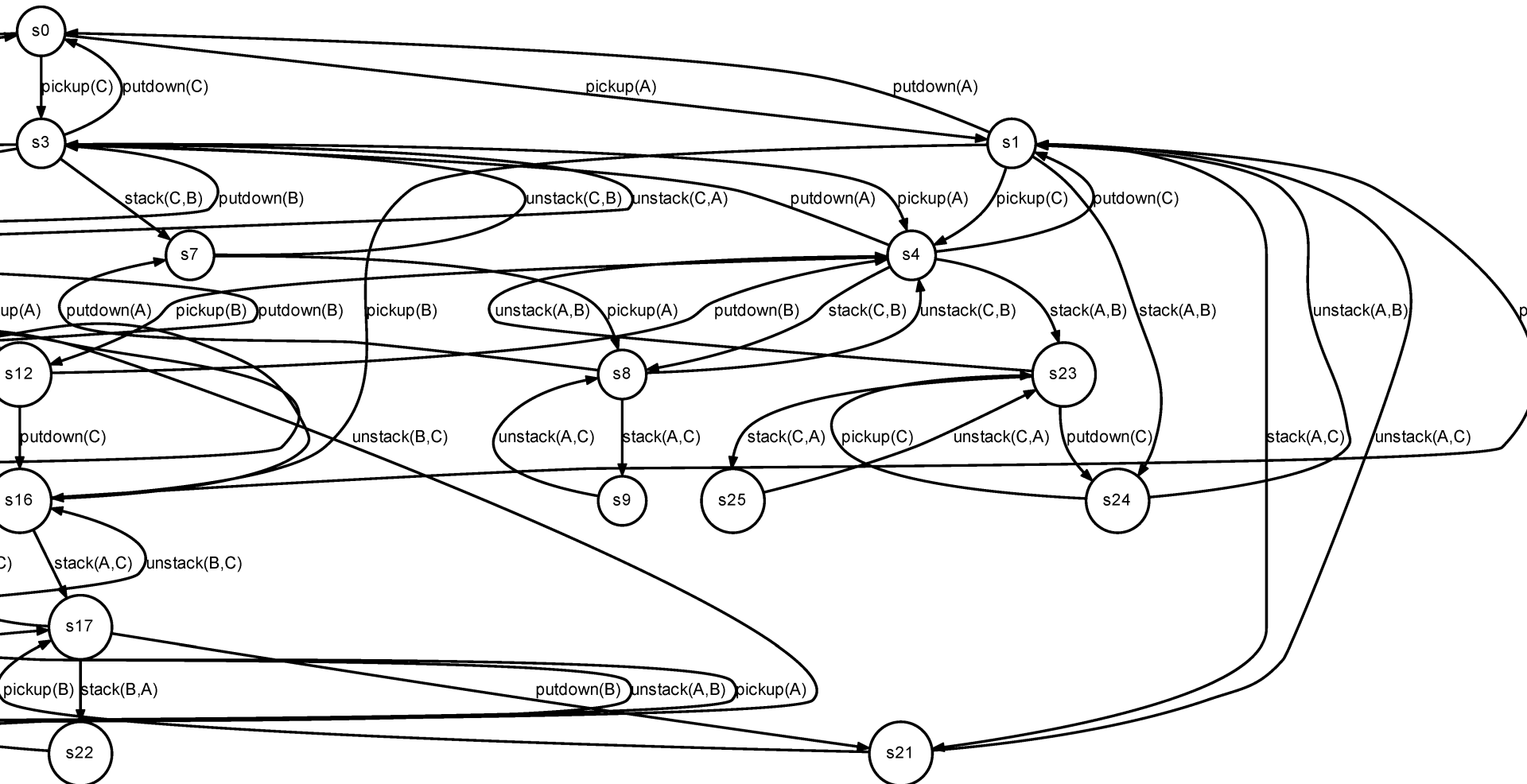
■ Example 3: Ignoring state variables

- Ignore the *handempty* fact in preconditions and effects
- **Different** state space, no simple addition or removal, **but** all the old solutions (paths) still lead to goal states!
 - 22 reachable states → 26
 - 42 transitions → 72



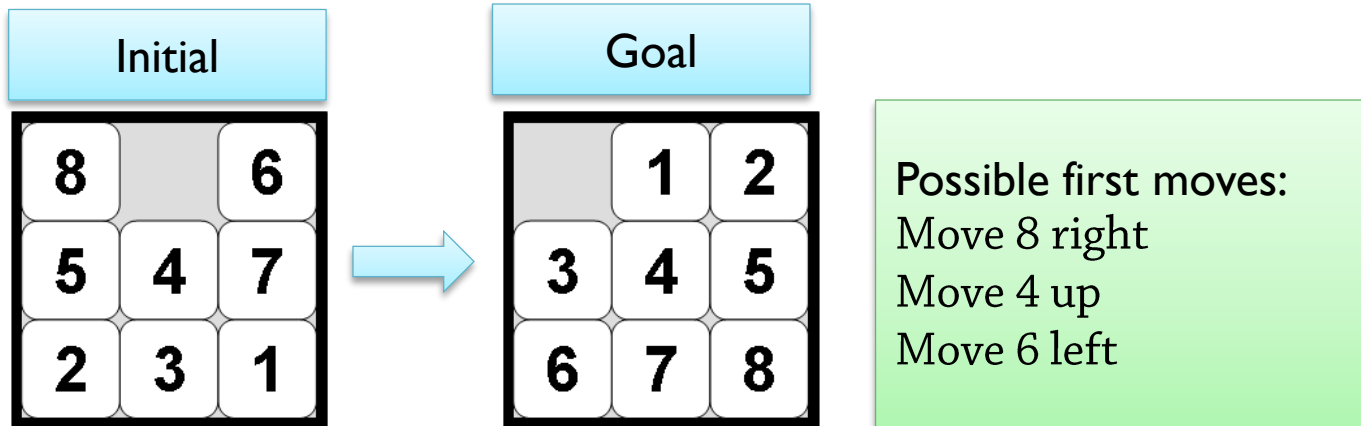
Relaxation Example 3b

■ Example 3, enlarged



Relaxation Example 4

- Example 4: **Weakening preconditions** of existing actions



- Precondition relaxation: **Tiles can be moved across each other**
 - Now we have 21 possible first moves: **New transitions** added to the STS
- All **old solutions are still valid**, but new ones are added
 - To move “8” into place:
 - Two steps to the right, two steps down, ends up in the same place as “1”

Can still be **solved** through **search**
The **optimal** solution for the *relaxed 8-puzzle*
can **never** be more expensive than the optimal solution for *original 8-puzzle*

Relaxation Heuristics: Summary



- **Relaxation: One general principle**
for designing **admissible** heuristics for **optimal** planning
 - Find a way of transforming planning problems, so that given a problem instance P :
 - **Computing its transformation** P' is easy (polynomial)
 - **Finding an optimal solution** to P' is easier than for P
 - **All solutions to P are solutions to P'** ,
but the new problem can have additional solutions as well
 - Then the cost of an optimal solution to P'
is an admissible heuristic for the original problem P

This is only *one* principle!
There are others, *not* based on relaxation

Relaxation: Search or Direct Computation?

Search or Direct Computation (1)



- As stated:
 - Compute an actual solution π' for the relaxed problem P'
 - Compute $\text{cost}(\pi')$
- Example: The **8-puzzle**...
 - Ignore **blank(x,y)** in preconditions and effects
 - Run the problem through an optimal planner
 - Compute the cost of the resulting plan π'

```
(:action move-up
:parameters (?t ?px ?py ?by)
:precondition (and
  (tile ?t) (position ?px) (position ?py) (position ?by)
  (dec ?by ?py) (blank ?px ?by) (at ?t ?px ?py))
:effect (and (not (blank ?px ?by)) (not (at ?t ?px ?py))
  (blank ?px ?py) (at ?t ?px ?by)))
```

Search or Direct Computation (2)



- But we only use π' to compute its cost!

- Let's analyze the problem...

- Each piece has to be moved to the intended row
- Each piece has to be moved to the intended column
- These are exactly the required actions given the relaxation!

- → optimal cost for relaxed problem = sum of Manhattan distances
- → admissible heuristic for *original* problem = sum of Manhattan distances
- → Cost of any optimal solution π' can be computed efficiently *without* π' :

$$\sum_{p \in \text{pieces}} x\text{distance}(p) + y\text{distance}(p)$$

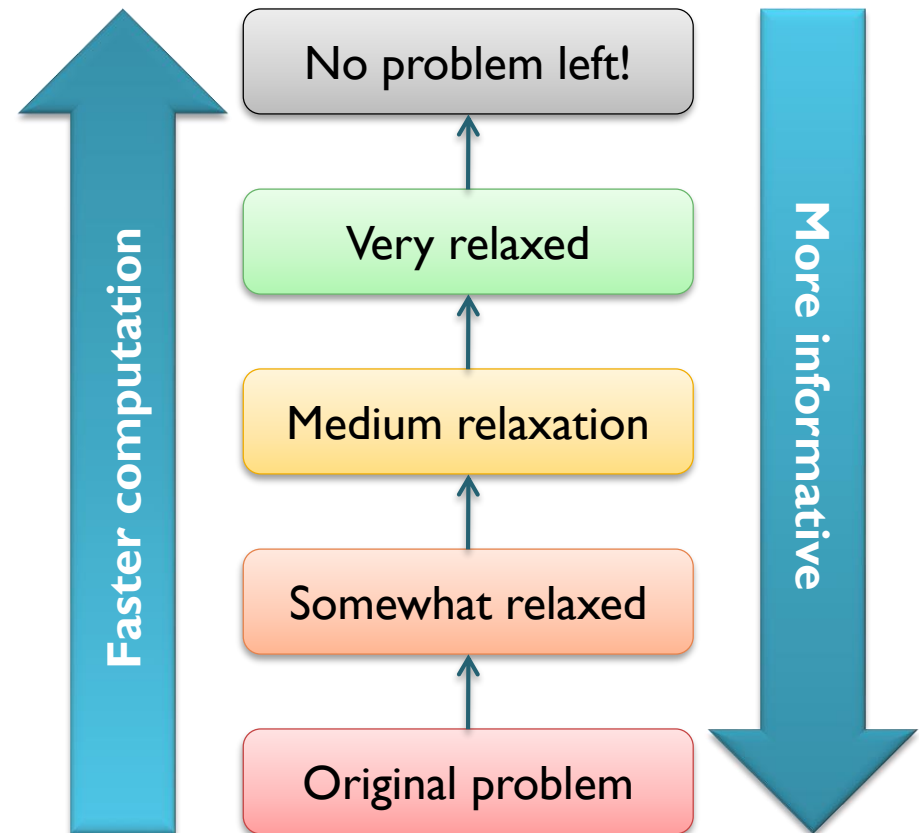
But now we had to analyze the problem:
(1) Decide to ignore "blank"
(2) Find "sum of manhattan distances"

Soon: How do we *automatically* find
good relaxations + computation methods?

Relaxation: Essential Facts

Relaxation Heuristics: Balance

- The reason for relaxation is rapid calculation
 - Shorter solutions are an *unfortunate side effect*:
Leads to less informative heuristics
 - Relax too much → not informative
 - Example: Any piece can teleport into the desired position
→ $h(n) = \text{number of pieces left to move}$

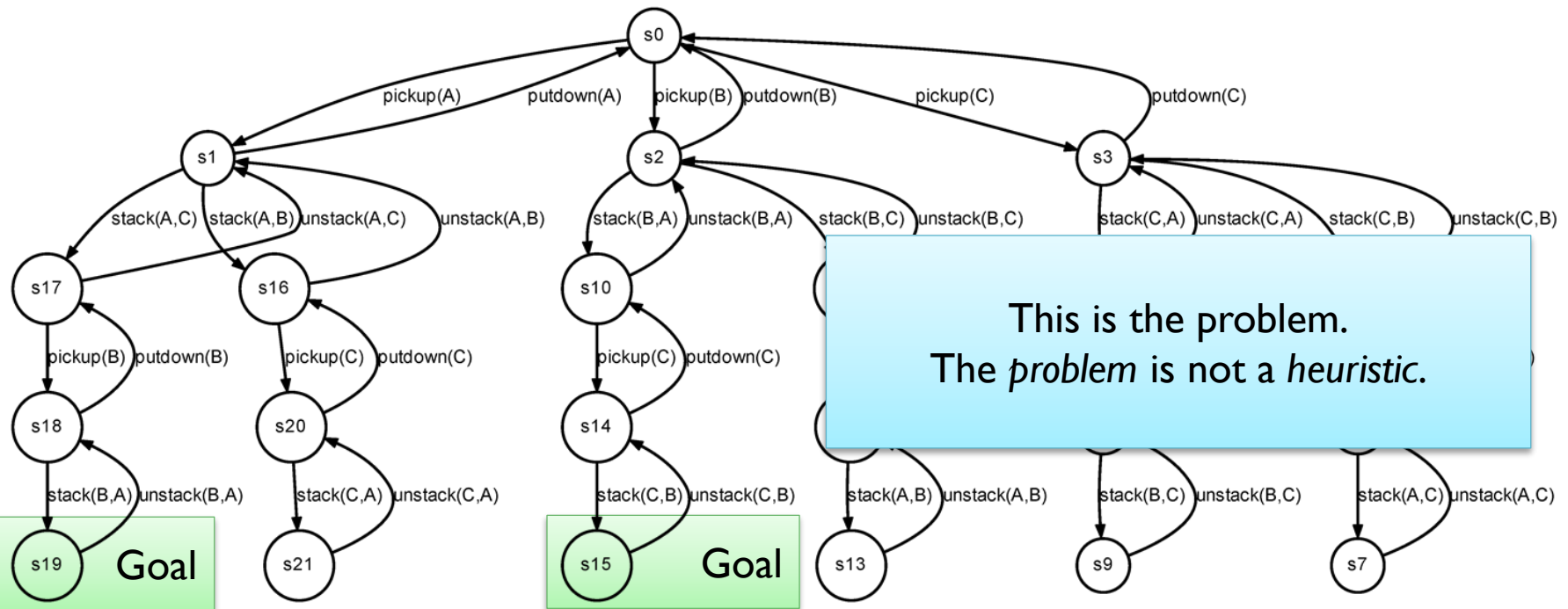


Relaxation Heuristics: Important Issues!

You **cannot** "use a relaxed problem as a heuristic".

What would that mean?

You use the **cost** of an **optimal solution** to the relaxed problem as a heuristic.



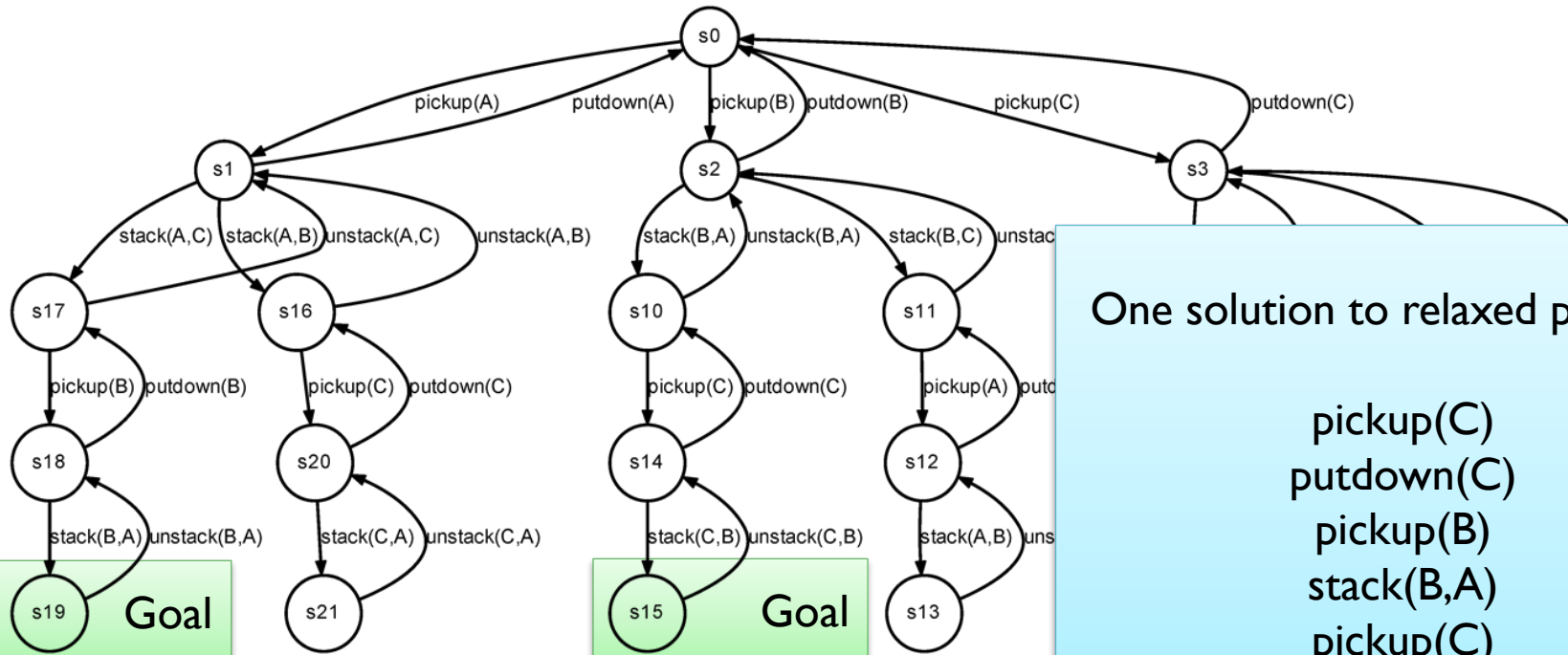
on(B,A)
on(A,C) or on(C,B)

on(B,A)
on(A,C) or on(C,B)

Relaxation Heuristics: Important Issues!

Solving the relaxed problem
can result in a more expensive solution
→ inadmissible!

You have to solve it optimally to get the admissibility guarantee.



One solution to relaxed problem:

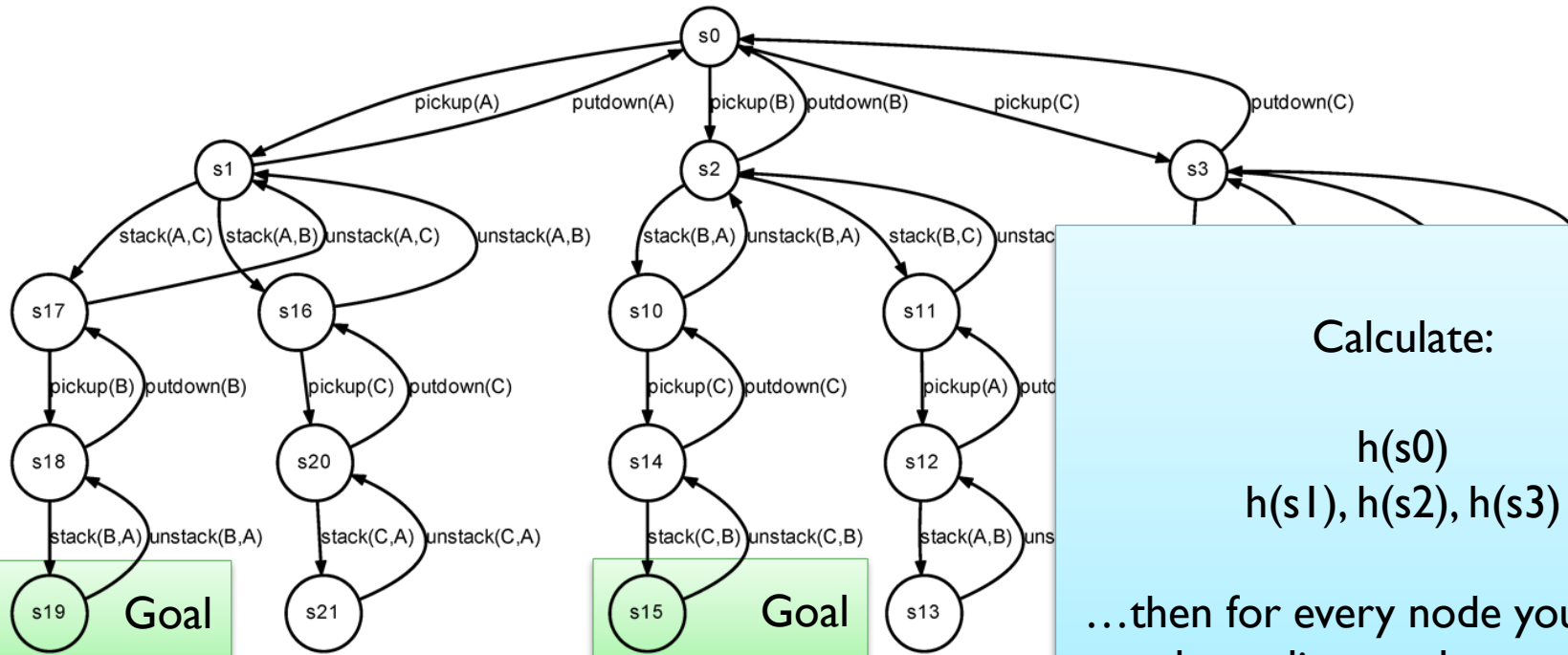
pickup(C)
putdown(C)
pickup(B)
stack(B,A)
pickup(C)
stack(C,B)

on(B,A)
on(A,C) or on(C,B)

on(B,A)
on(A,C) or on(C,B)

Relaxation Heuristics: Important Issues!

You don't just solve the relaxed problem once.
Every time you reach a new state and want to calculate a heuristic,
you have to solve the relaxed problem
of getting from that state to the goal.



on(B,A)
on(A,C) or on(C,B)

on(B,A)
on(A,C) or on(C,B)

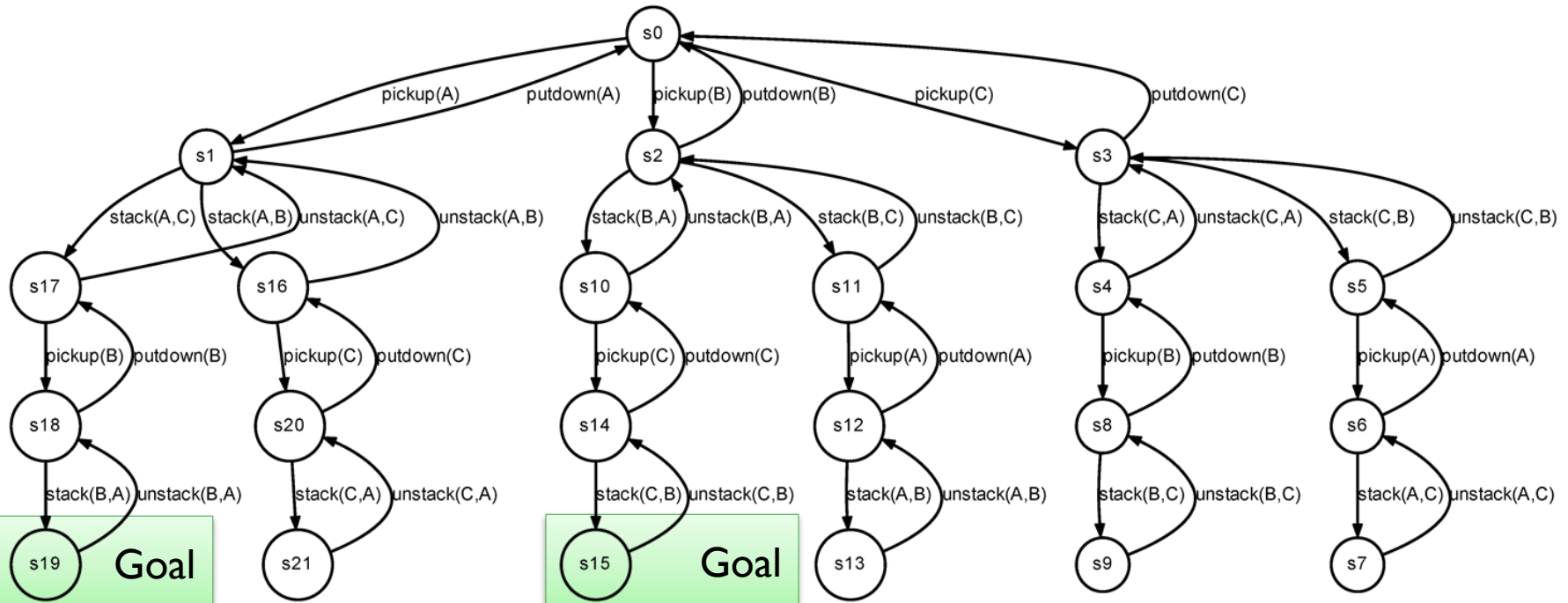
Calculate:

$h(s_0)$
 $h(s_1), h(s_2), h(s_3)$

...then for every node you create,
depending on the strategy

Relaxation Heuristics: Important Issues!

Relaxation does **not** always mean "**removing constraints**" in the sense of *weakening preconditions* (moving across tiles, removing walls, ...) Sometimes we get new *goals*. Sometimes the entire *state space* is transformed. Sometimes action *effects* are modified, or some other change is made. What defines relaxation: **All old solutions are valid, new solutions may exist.**



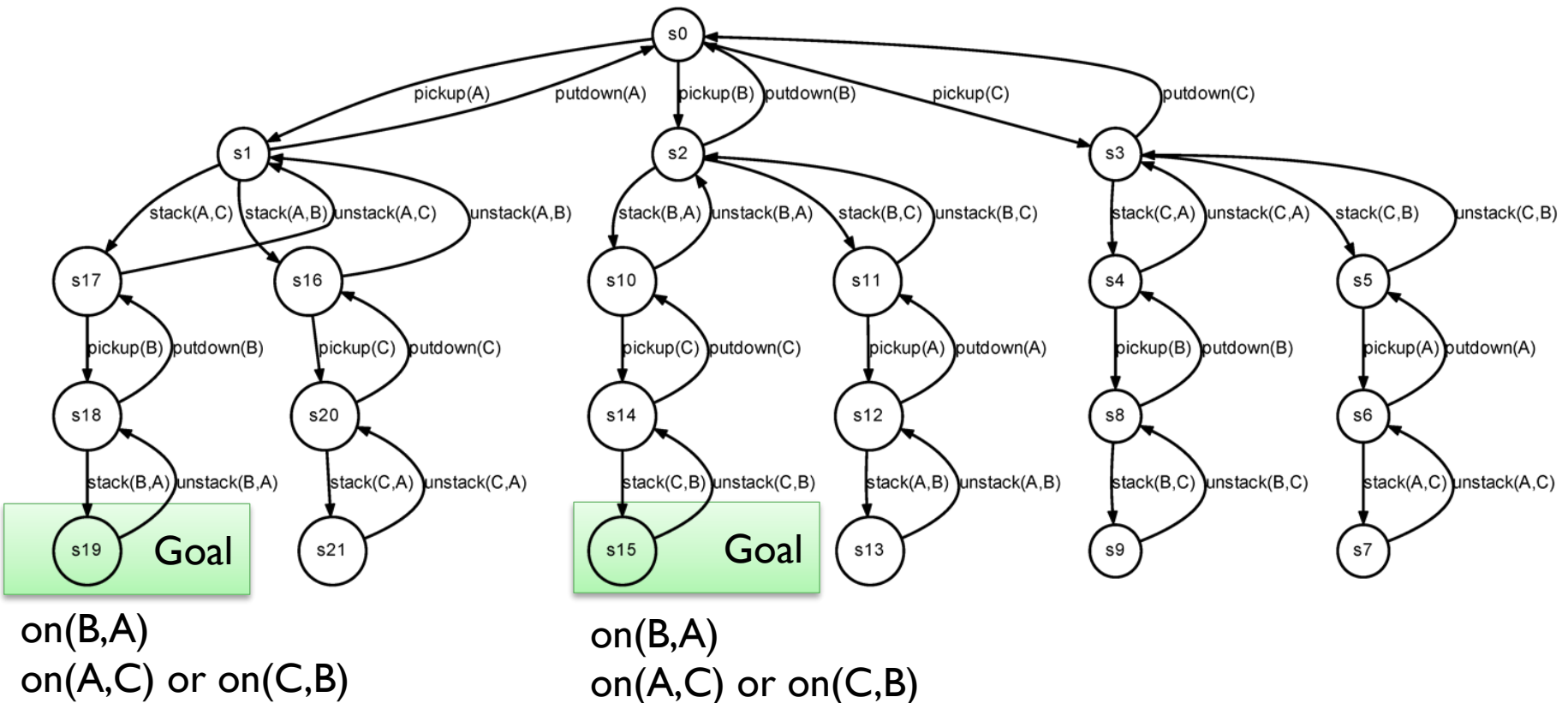
on(B,A)
on(A,C) or on(C,B)

on(B,A)
on(A,C) or on(C,B)

Admissibility: Important Issues!

Relaxation is useful for finding admissible heuristics.

A heuristic cannot be admissible for some states.
Admissible == does not overestimate costs for *any* state!



Admissibility: Important Issues!

If you are asked "why is a relaxation heuristic admissible?", don't answer "because it cannot overestimate costs". This is the *definition* of admissibility!

"Why is it admissible?" == "Why can't it overestimate costs?"

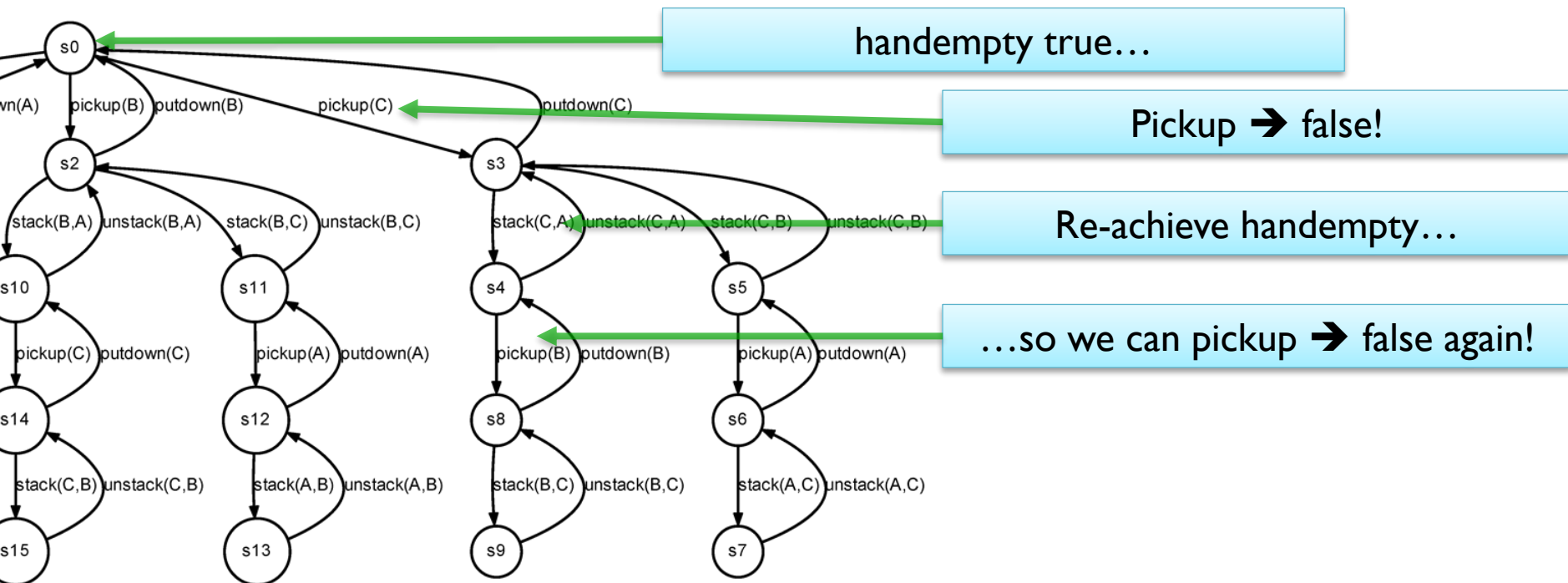
Admissible heuristics *can* "lead you astray" and you *can* "visit" suboptimal solutions.

But with the right search strategy, such as A^* ,
the planner will eventually get around to finding an optimal solution.
This is not the case with A^* + non-admissible heuristics.

Delete Relaxation

Delete Relaxation (1)

- In classical planning:
 - **Negative effects** can "un-achieve" goals or preconditions
 - A plan may have to achieve the same fact many times
- Example: If **handempty** is a goal



Delete Relaxation (2)

- Suppose we remove all negative effects

- **Example:** (unstack ?x ?y)

- **Before transformation:**

:precondition (and (handempty) (clear ?x) (on ?x ?y))

:effect (and (not (handempty)) (holding ?x) (not (clear ?x)) (clear ?y)
(not (on ?x ?y)))

- **After transformation:**

:precondition (and (handempty) (clear ?x) (on ?x ?y))

:effect (and (holding ?x) (clear ?y))

- A fact that is achieved stays achieved

Is this a relaxation?

Delete Relaxation (3)

- Suppose we use the book's classical representation:
 - Precondition = set of literals that must be true
 - Goal = set of literals that must be true
 - Effects = set of literals (making atoms true or false)
- Suppose we have a solution $\langle \mathbf{A1}, \mathbf{A2} \rangle$:
 - Initially handempty
 - Action A1 \rightarrow handempty := false
 - Action A2 \rightarrow **requires** (not handempty)
- Remove all negative effects:
 - Initially handempty
 - Action A1 \rightarrow no effect
 - Action A2 \rightarrow **requires** (not handempty), not executable
- $\langle \mathbf{A1}, \mathbf{A2} \rangle$ is no longer a solution; can't be a relaxation

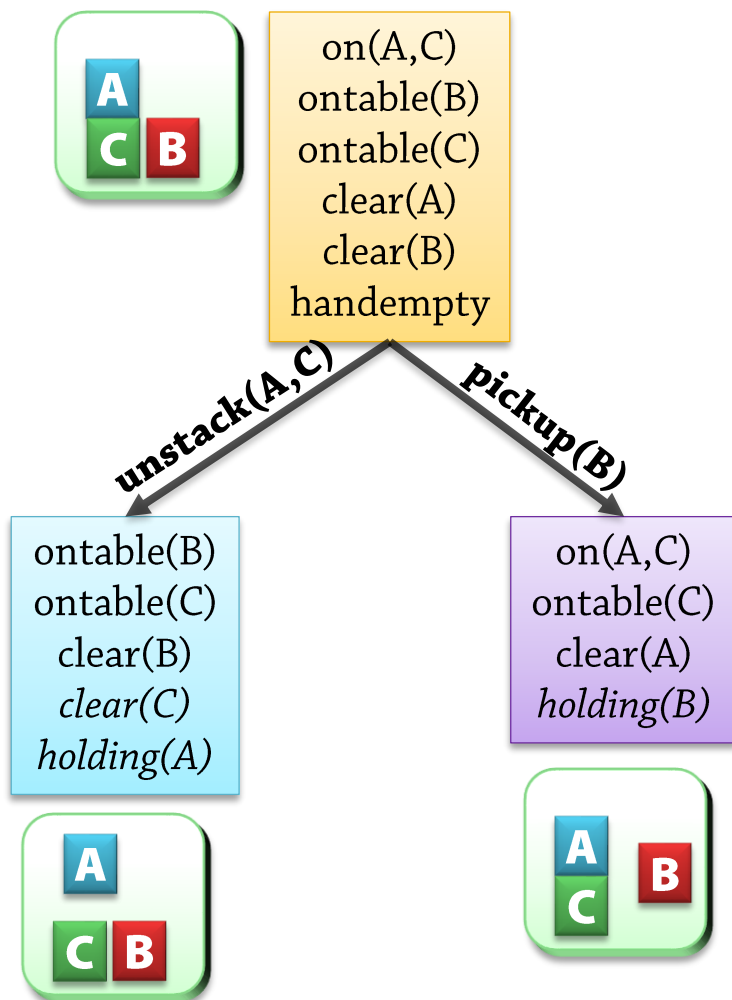
Delete Relaxation (4)

- Suppose we use PDDL's plain **:strips** level
 - **Forbids negative preconditions / goals**
 - Precondition = set of **atoms** (no negations!)
 - Goal = set of **atoms** (no negations!)
 - Effects = set of **literals** (making **atoms** true or false)
 - No solution can ***depend on*** a fact being false in a visited state
 - No solution can ***disappear*** because we stop making facts false

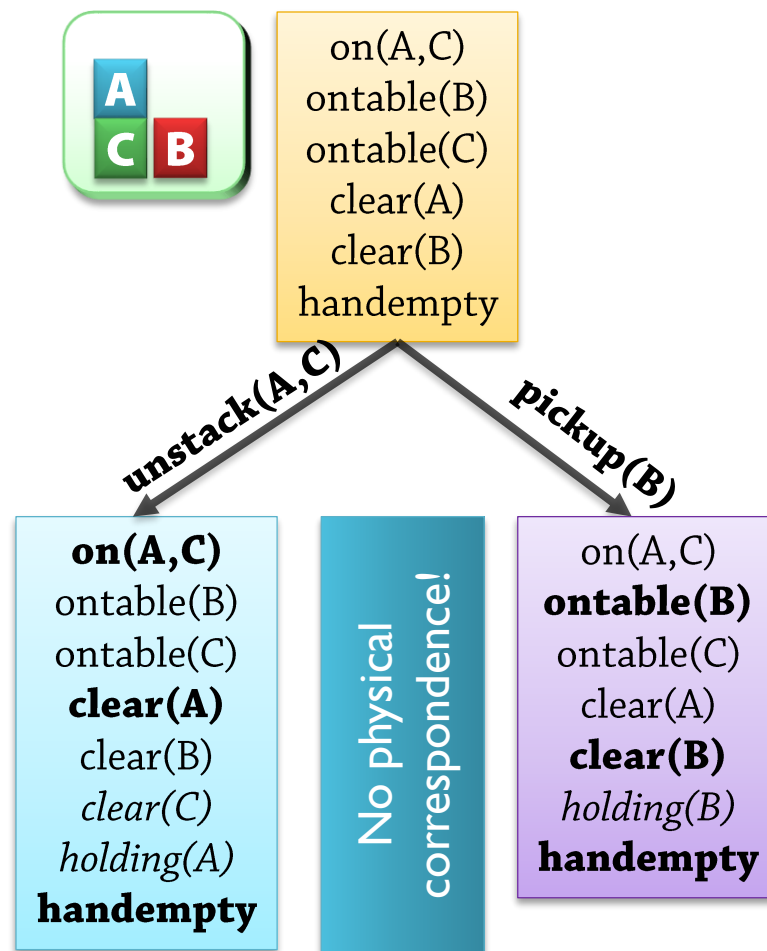
This is a **relaxation** if **the problem lacks negative preconditions / goals!**

Delete Relaxation (5): Example

STS for the original problem

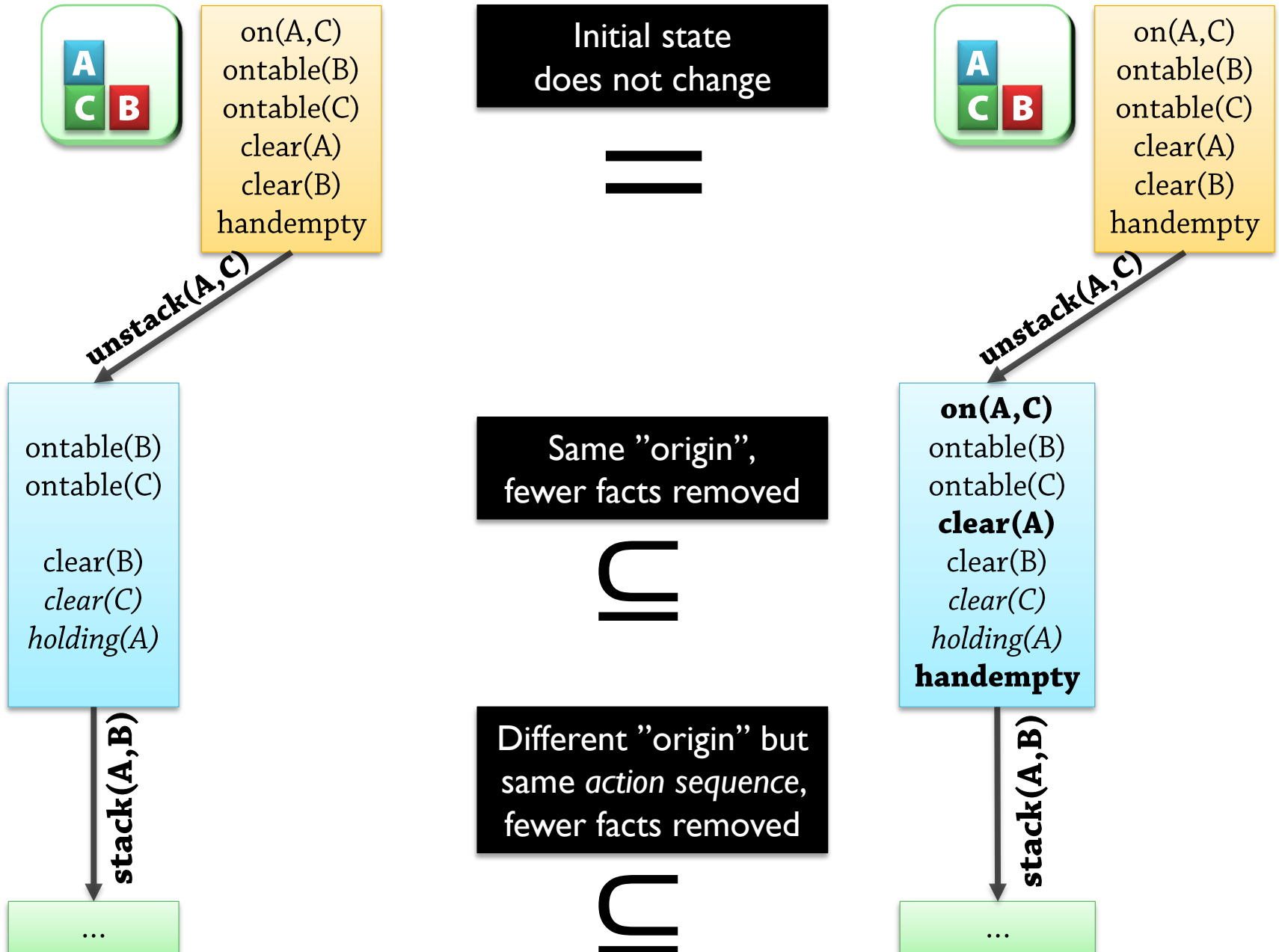


Delete-relaxed STRIPS problem



STS for the original problem

Delete-relaxed STRIPS problem



STS for the original problem



on(A,C)
ontable(B)
ontable(C)
clear(A)
clear(B)
handempty

unstack(A,C)

ontable(B)
ontable(C)

clear(B)
clear(C)
holding(A)

Applicable
actions: app_1

Delete-relaxed STRIPS problem



on(A,C)
ontable(B)
ontable(C)
clear(A)
clear(B)
handempty

unstack(A,C)

on(A,C)
ontable(B)
ontable(C)
clear(A)
clear(B)
clear(C)
holding(A)
handempty



Applicable
actions: app_2



No **action** requires
the *absence* of a fact



Delete Relaxation (8): Example

STS for the original problem



on(A,C)
ontable(B)
ontable(C)
clear(A)
clear(B)
handempty

unstack(A,C)

ontable(B)
ontable(C)
clear(B)
clear(C)
holding(A)

No **goal** requires the
absence of a fact

Satisfies the
goal?

Delete-relaxed STRIPS problem

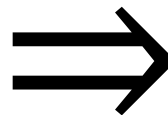


on(A,C)
ontable(B)
ontable(C)
clear(A)
clear(B)
handempty

unstack(A,C)

on(A,C)
ontable(B)
ontable(C)
clear(A)
clear(B)
clear(C)
holding(A)
handempty

Also satisfies
the goal



Delete Relaxation (9)



- **Negative effects** are also called "**delete effects**"
 - They delete facts from the state
- So this is called **delete relaxation**
 - "*Relaxing the problem by getting rid of the delete effects*"

**Delete relaxation does not mean
that we "delete the relaxation" (anti-relax)!**

**Delete relaxation is only a relaxation
if preconditions and goals are positive!**

Delete Relaxation (10)

- Since solutions are preserved when facts are added:

A state where additional facts are true can never be "worse"!
(Given positive preconds/goals)

$$h^*\left(\begin{array}{l} \text{ontable(B)} \\ \text{ontable(C)} \\ \text{clear(B)} \\ \text{clear(C)} \\ \text{holding(A)} \\ \text{handempty} \end{array}\right) \leq h^*\left(\begin{array}{l} \text{ontable(B)} \\ \text{ontable(C)} \\ \text{clear(B)} \\ \text{clear(C)} \\ \text{holding(A)} \end{array}\right)$$

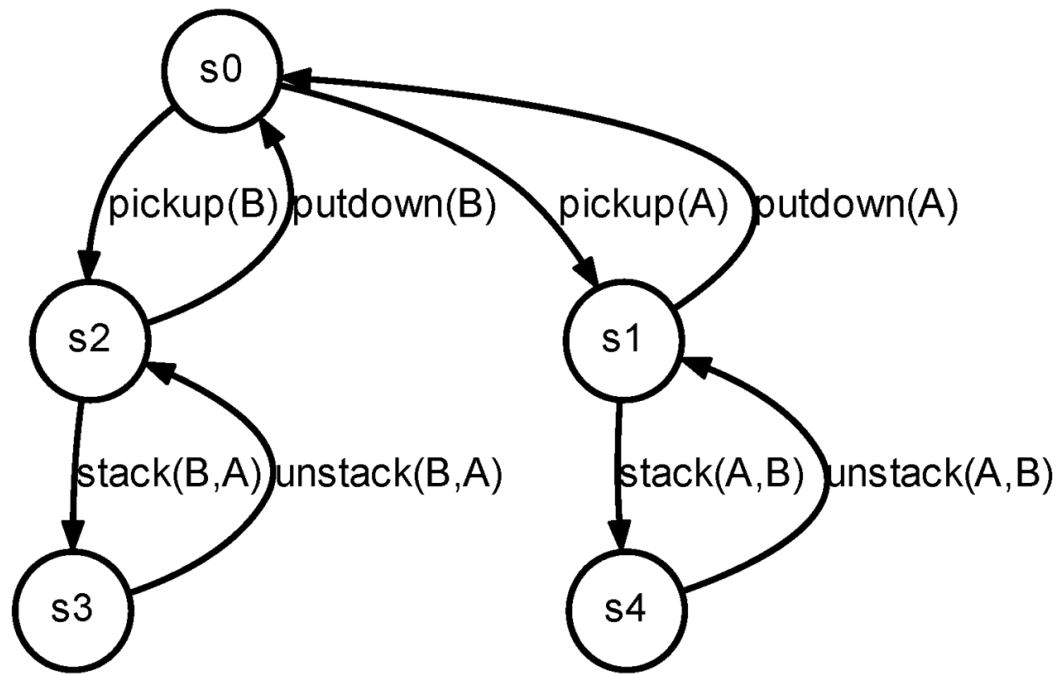
Given two states (sets of true atoms) s, s' :
 $s \supset s' \rightarrow h^*(s) \leq h^*(s')$

Delete Relaxation:

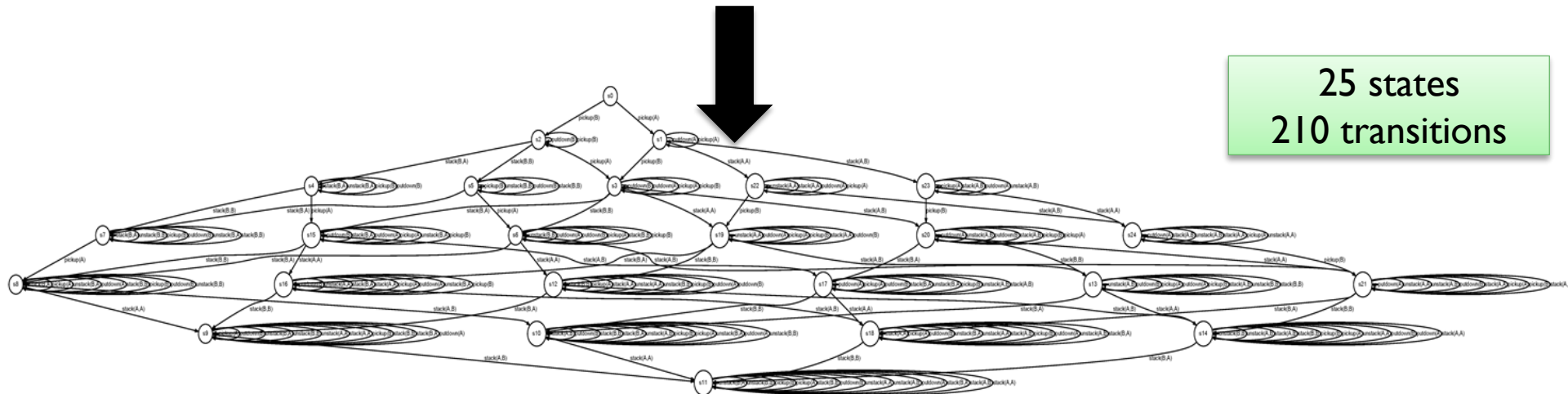
State Space Examples

Reachable State Space: BW size 2

5 states
8 transitions

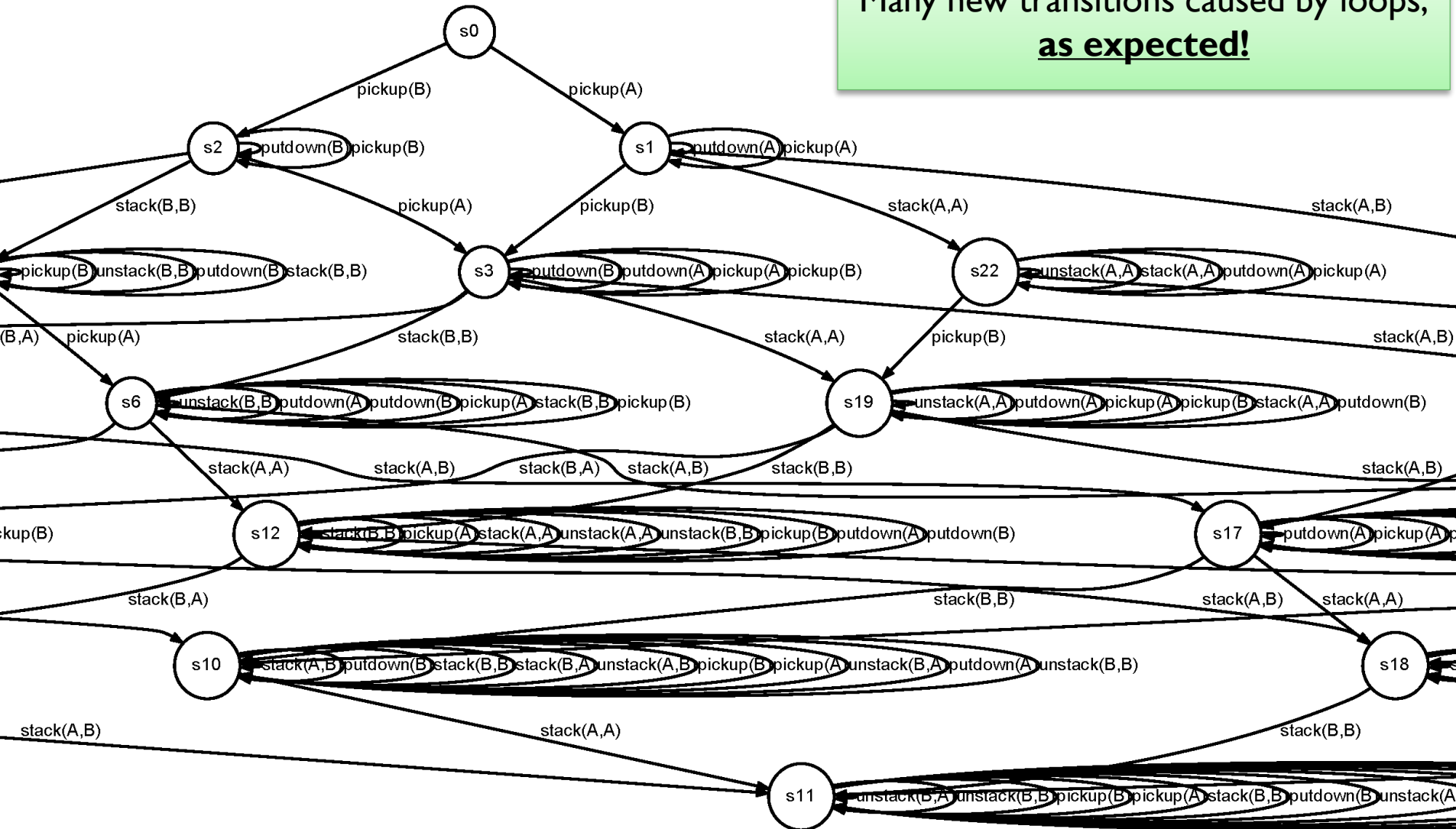


25 states
210 transitions



Delete-Relaxed BW size 2: Detail View

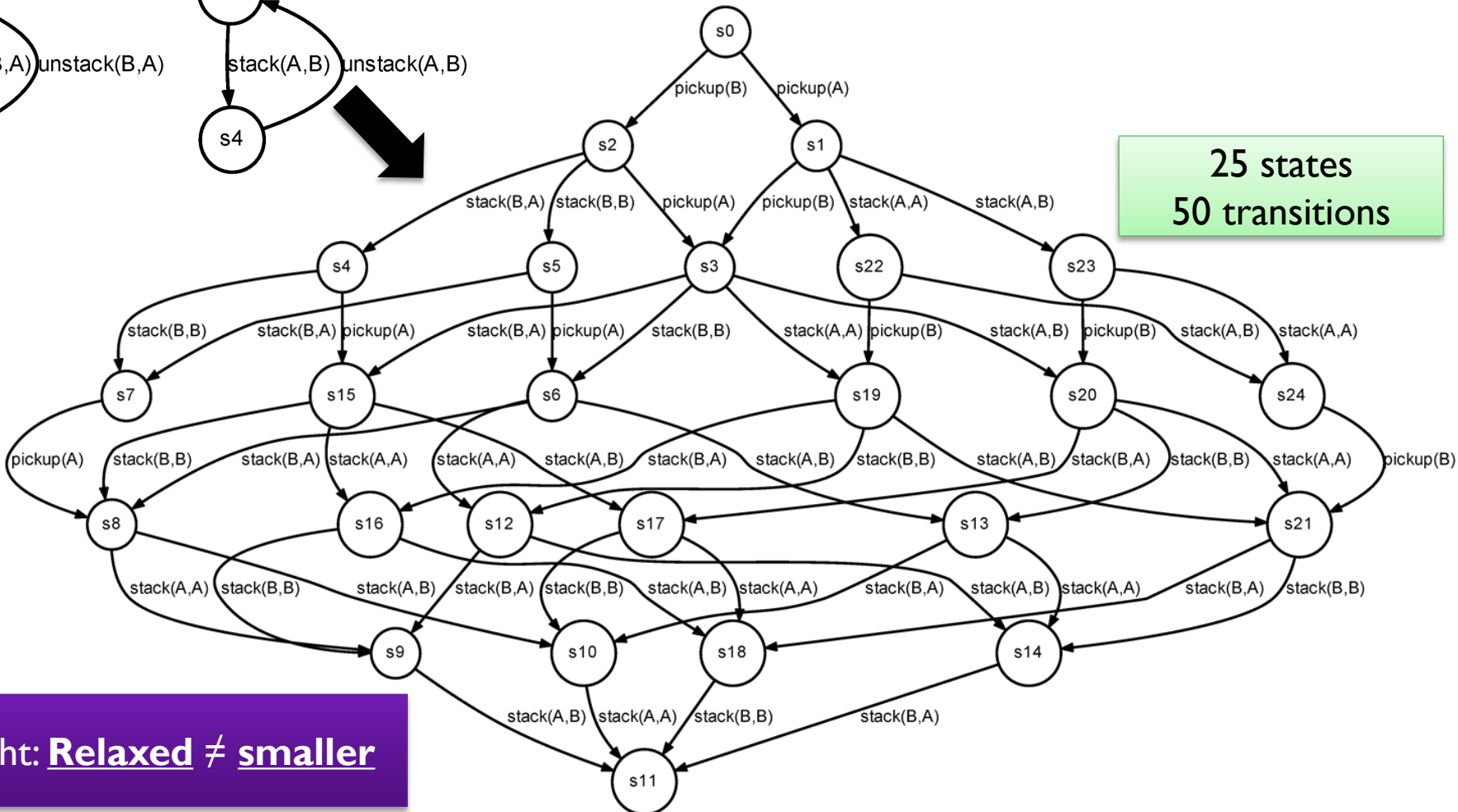
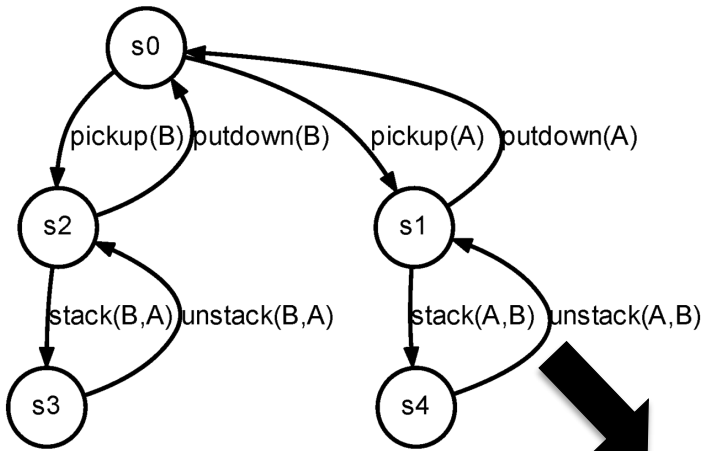
Many new transitions caused by loops,
as expected!



Delete-Relaxed: "Loops" Removed

5 states
8 transitions

25 states
50 transitions



Insight: Relaxed \neq smaller

The Optimal Delete Relaxation Heuristic

Optimal Delete Relaxation Heuristic

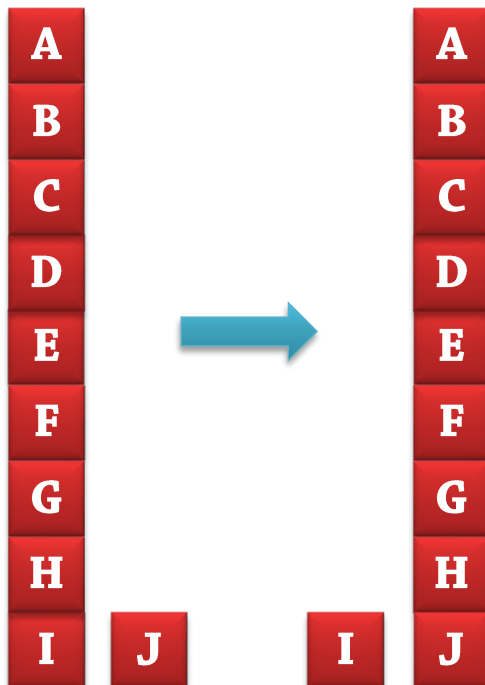


- If **only** delete relaxation is applied:
 - We can calculate the **optimal delete relaxation heuristic**, $h^+(n)$
 - $h^+(n) =$ the cost of an **optimal solution** to a **delete-relaxed** problem starting in node n

Accuracy of h^+ in Selected Domains

- **How close** is $h^+(n)$ to the true goal distance $h^*(n)$?
 - **Worst case asymptotic accuracy** as problem size approaches infinity:
 - Blocks world: $1/4 \rightarrow h^+(n) \geq \frac{1}{4}h^*(n)$

Optimal plans in delete-relaxed Blocks World
can be down to 25% of the length of optimal plans in "real" Blocks World



Standard:

unstack(A,B)	pickup(G)
putdown(A)	stack(G,H)
unstack(B,C)	pickup(F)
putdown(B)	stack(F,G)
unstack(C,D)	pickup(E)
putdown(C)	stack(E,F)
...	pickup(D)
unstack(H,I)	stack(D,E)
stack(H,J)	...

Relaxed:

unstack(A,B)
unstack(B,C)
unstack(C,D)
unstack(D,E)
unstack(E,F)
unstack(F,G)
unstack(G,H)
unstack(H,I)
stack(H,J)
DONE!

Accuracy of h^+ in Selected Domains (2)

- **How close** is $h^+(n)$ to the true goal distance $h^*(n)$?
 - **Worst case asymptotic accuracy** as problem size approaches infinity:
 - Blocks world: $1/4 \rightarrow h^+(n) \geq \frac{1}{4}h^*(n)$
 - Gripper domain: $2/3$ (single robot moving balls)
 - Logistics domain: $3/4$ (move packages using trucks, airplanes)
 - Miconic-STRIPS: $6/7$ (elevators)
 - Miconic-Simple-ADL: $3/4$ (elevators)
 - Schedule: $1/4$ (job shop scheduling)
 - Satellite: $1/2$ (satellite observations)

- Details:
 - Malte Helmert and Robert Mattmüller
*Accuracy of Admissible Heuristic Functions
in Selected Planning Domains*

Accuracy of Admissible Heuristic Functions in Selected Planning Domains

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Abstract

The efficiency of optimal planning algorithms based on heuristic search crucially depends on the accuracy of the heuristic function used to guide the search. Often, we are interested in domain-independent heuristics for planning. In order to assess the limitations of domain-independent heuristic planning, we analyze the (in)accuracy of common domain-independent planning heuristics in the 18th benchmark domains. For a selection of these domains, we analytically investigate the accuracy of the h^+ heuristic, the h^* family of heuristics, and certain (additive) pattern database heuristics, compared to the perfect heuristic h^* . Whereas h^+ and additive pattern database heuristics usually return cost estimates proportional to the true cost, non-additive h^* and non-additive pattern database heuristics can yield results underestimating the true cost by arbitrarily large factors.

Introduction

Heuristic search with h^* and similar algorithms remains the most popular method for optimal sequential planning, with significant effort spent on perfecting old heuristic estimators (Holmström et al. 2007) or deriving new ones (Helmert, Helmert, and Hoffmann 2007). While methods not based on state-space search have achieved remarkable success in addressing related problems, such as optimal parallel planning (Kantor and Schaefer 1996, 1999), the state of the art in optimal sequential planning is still defined by heuristic search almost exclusively. Symbolic state-space exploration (Edelkamp and Helmert 2003) is the only non-classical approach that sometimes outperforms heuristic search.

Considering the important role of admissible heuristics for optimal sequential planning, or search in general, the question arises how to evaluate the quality of a given heuristic. A popular method is to run a search algorithm against some benchmark tasks and count the number of node expansions. The fewer nodes an algorithm expands, the better.

While experiments of this kind are certainly useful, there are some questions they cannot address. In particular, their results can almost exclusively be interpreted with relative, i.e., comparative statements: “Heuristic h expands fewer nodes than heuristic h' for benchmark suite S .” Unless experiments show polynomial scaling behavior on a family of

benchmark tasks of growing size, which they very rarely do, the data usually does not lend itself to absolute statements of the type “Heuristic h is well-suited for solving tasks from benchmark suite S .” In this contribution, we address this issue by providing absolute quality results for certain popular planning heuristics on some popular benchmark domains taken from the first four International Planning Competitions (McDermott 2000; Burchard 2003; Long and Fox 2005; Hoffmann and Edelkamp 2005), in the form of comparisons to the perfect heuristic function h^* .

Planning Domains

We consider the planning domains GRIPPER, LOGISTICS, BLOCKSWORLD, MICONIC-STRIPS, MICONIC-SIMPLE-ADL, SCHEDULE and SATELLITE. Familiarity with the domains is assumed. For an in-depth treatment, we refer to the literature (Helmert 2006).

Heuristics

We compare the accuracy of h^+ , h^* , non-additive and additive pattern database (PDB) heuristics relative to the perfect heuristic h^* . An admissible heuristic h maps states s to optimistic estimates of the true cost of reaching a goal state from s . Whenever the planning task to which h belongs, including the available operators and the goal description, is clear from the context, we will simply write $h(s)$ without explicitly mentioning operators or goal.

The h^* heuristic. The perfect heuristic h^* assigns to each state s the length of a shortest plan from s to a goal state. Computing h^* for the initial state of a planning task is PSPACE-equivalent in general (Bylander 1994), but can be easier for fixed domains. For the domains we consider, the problem is NP-equivalent, with the exception of GRIPPER and SCHEDULE, where it is polynomial (Helmert 2006).

The h^+ heuristic. The h^+ heuristic (McDermott 1990; Bonet and Geffner 2001; Edelkamp 2005) assigns to each state s a value $h^+(s)$ the length of a shortest plan leading from s to a goal state in the relaxed task T^- . Evaluating h^+ is NP-equivalent in general (Bylander 1994), but easier for many of the domains we consider.

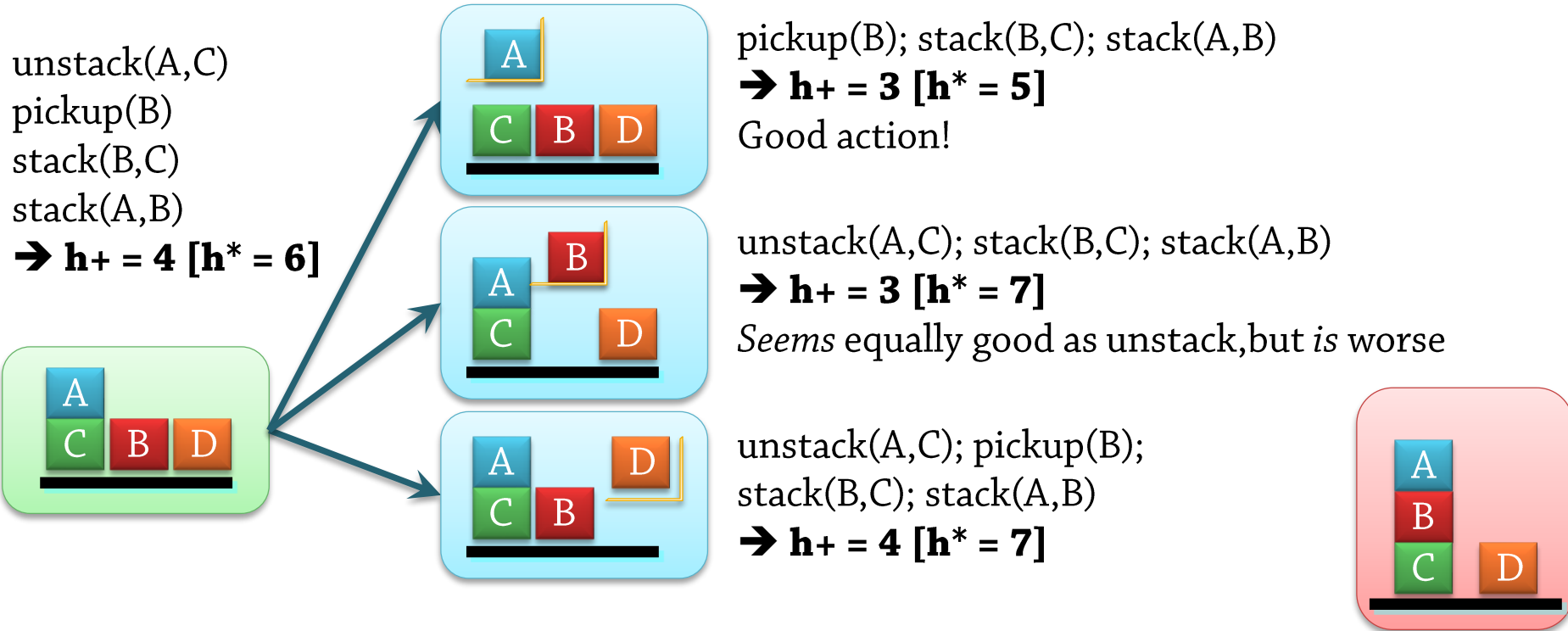
The h^* Family of Heuristics. The h^* on $\{1, 2, \dots\}$ family of heuristics (Helmert and Geffner 2000) is based on

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Example of Accuracy

- How close is $h^+(n)$ to the true goal distance $h^*(n)$?

- In practice: Also depends on the problem instance!

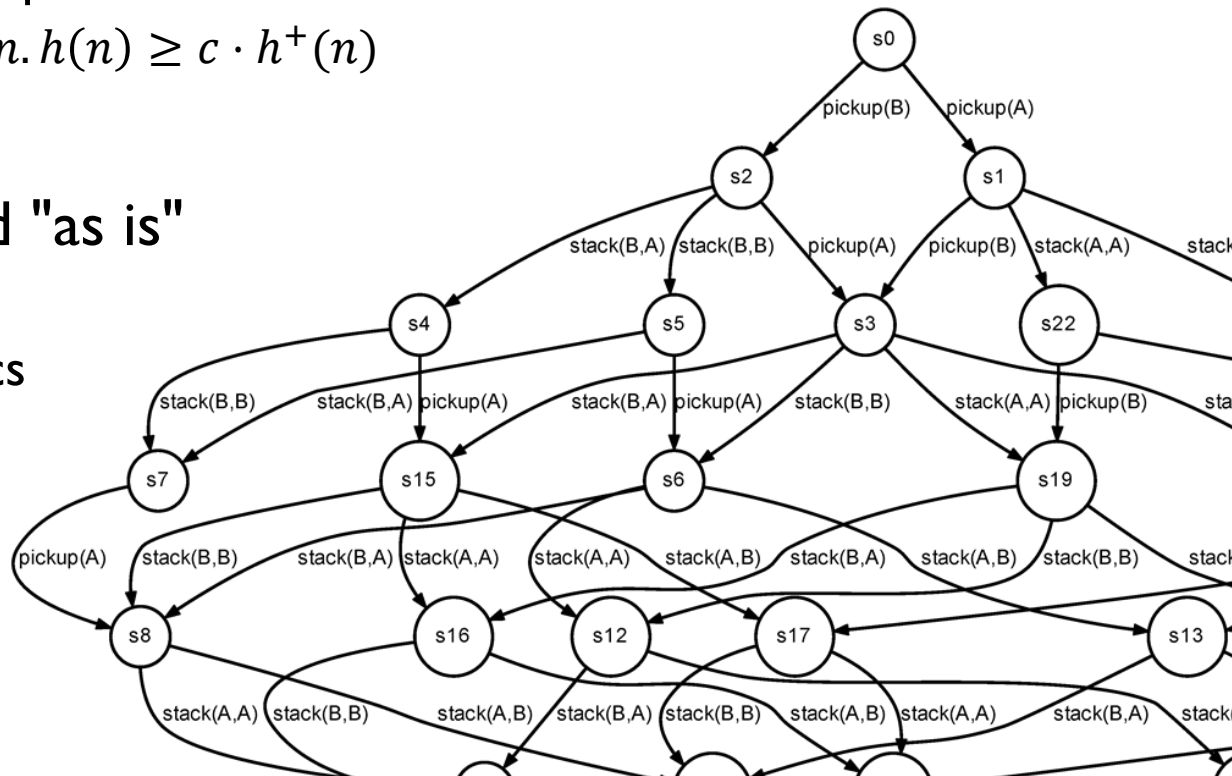


- Performance also depends on the search strategy
 - How sensitive it is to specific types of inaccuracy

Computing the Optimal Delete Relaxation Heuristic

- Is $h^+(n)$ easier to compute than $h^*(n)$?
 - $h^*(n)$ = length of optimal plan for **arbitrary planning problem**
 - Supports negative effects
 - If we can execute either $a1;a2$ or $a2;a1$:
 - $a1$ removes p , $a2$ adds $p \rightarrow$ net result: add p
 - $a2$ adds p , $a1$ removes $p \rightarrow$ net result: remove p
 - **Both orders** must be considered
 - $h^+(n)$ = length of optimal plan after removing negative effects
 - If we can execute either $a1;a2$ or $a2;a1$:
 - Must lead to the same state (add $p1$ before $p2$, or $p2$ before $p1$)
 - Sufficient to consider **one order** – simpler?
 - Incomplete analysis
 - But the worst case for $h^+(n)$ **is** easier than the worst case for $h^*(n)$

- Still **difficult** to calculate in general!
 - NP-equivalent (reduced from PSPACE-equivalent)
 - Since you must find **optimal** solutions to the relaxed problem
 - Even a constant-factor approximation is NP-equivalent to compute!
 - Finding $h(n)$ so that $\forall n. h(n) \geq c \cdot h^+(n)$
- Therefore, rarely used "as is"
 - But forms the **basis** of many other heuristics



Optimal Classical Planning: The Admissible h_1 Heuristic

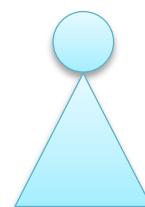
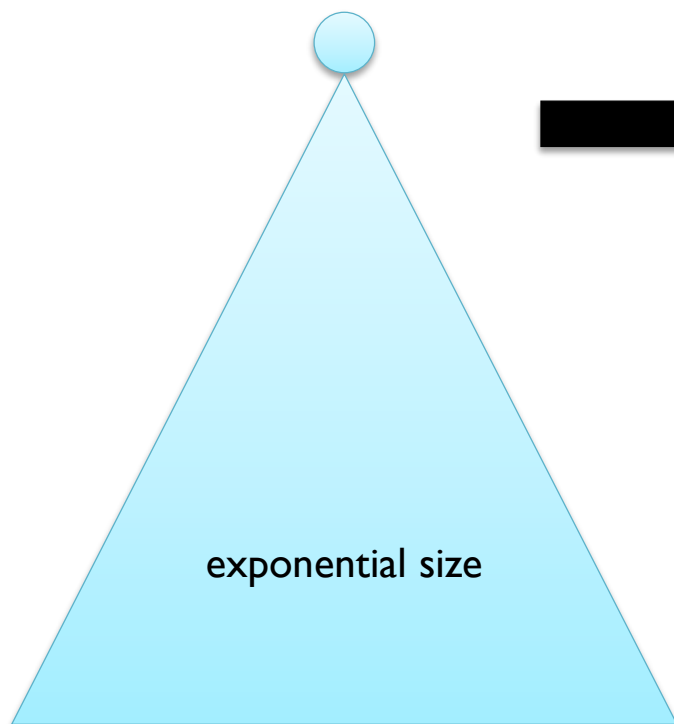
- Why is $h^+(n)$ so "slow"?

Must compute the exact cost
of an optimal plan
achieving all goals

s0

As problem sizes grow,
the number of goals will grow
→ plan lengths grow (even delete-relaxed!)
→ number of plans to check (directly or indirectly) grows *exponentially*

- Suppose we delete-relax, then only consider **one goal fact**
 - Remove **goal requirements** → add new **goal states** in S_g
- Relaxation!
 - "Old" plans achieving *all* goals are still valid solutions
 - **Also has much shorter solutions**, much faster to compute



Too relaxed!
And which goal to choose?

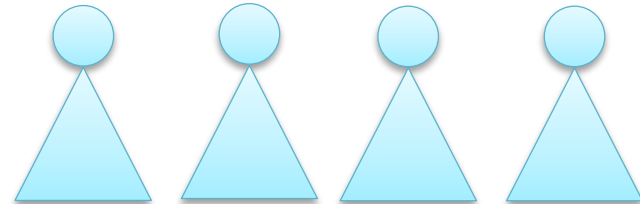
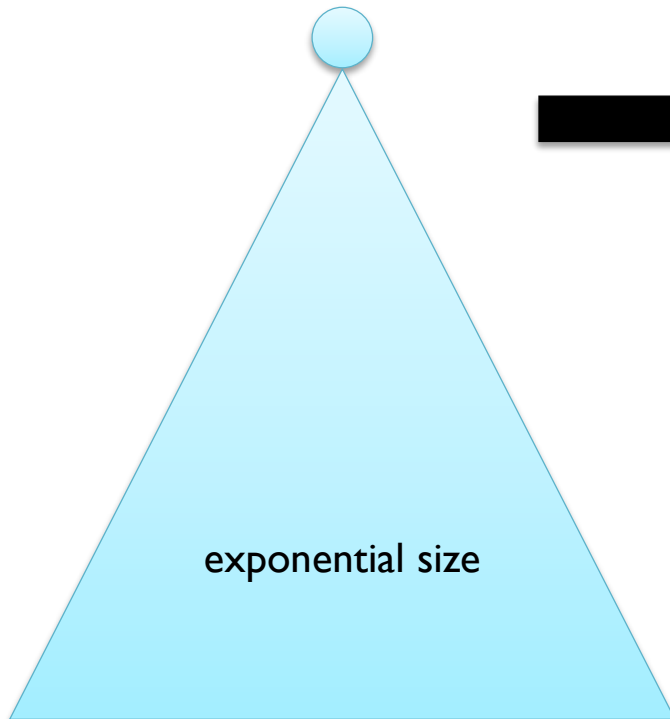
Intuitions (3)

- Given **two admissible heuristics** $h_A(n)$ and $h_B(n)$:
 - $h_{AB}(n) = \max(h_A(n), h_B(n))$ is admissible
 - If neither heuristic overestimates, their maximum cannot overestimate

The h_1 Heuristic

- Idea (from HSP^{*}): Consider one goal atom at a time

Treat each goal atom separately
Take the maximum of the costs



Uses a set of relaxations!

Computing $h_1(n)$

The h_1 Heuristic: Example (action cost = 1)

Goal:

clear(A)

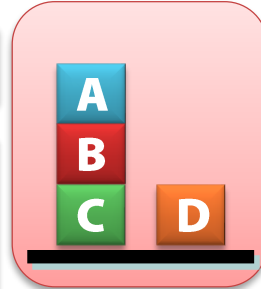
on(A,B)

on(B,C)

ontable(C)

clear(D)

ontable(D)



Don't find the best way to achieve *all goal atoms*:
{ clear(A), on(A,B), on(B,C), on(B,C), ontable(C), clear(D), ontable(D) }

Avoid interactions:
Find the best way to achieve **clear(A)**
Then find the best way to achieve **on(A,B)**

...

Use backward search, starting with the goals



s_0 : clear(A), on(A,C), ontable(C), clear(B), ontable(B), clear(D), ontable(D), handempty

The h_1 Heuristic: Example (action cost = 1)



Goal: `clear(A)` `on(A,B)` `on(B,C)` `ontable(C)` `clear(D)` `ontable(D)`
cost 0

First goal atom:
`clear(A)`

Already achieved,
cost 0

stack(A,B)
holding(A) clear(B)

How to achieve `on(A,B)`?
Not true in the initial state.
Check *all actions* having `on(A,B)`
as an effect...
Here: Only `stack(A,B)`!

We have two preconditions to achieve.

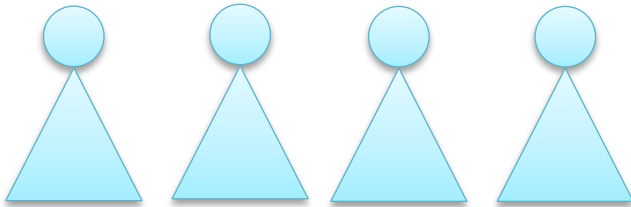
Reduce interactions even more:
Consider each of *these* as a separate "subgoal"!
First `holding(A)`, then `clear(B)`.



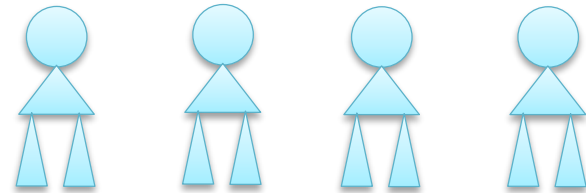
s_0 : `clear(A)`, `on(A,C)`, `ontable(C)`, `clear(B)`, `ontable(B)`, `clear(D)`, `ontable(D)`, `handempty`

The h_1 Heuristic: Intuitions (2)

Idea: Treat each **goal atom** separately
Take the maximum of the costs



$h_1(n)$: Split the problem even further;
consider *individual subgoals* at every "level"



The h_1 Heuristic: Example (continued)

Goal:

clear(A)

cost 0

on(A,B)

cost 2

on(B,C)

cost 2

ontable(C)

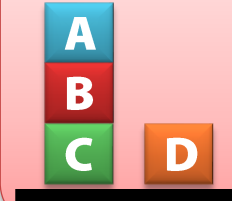
cost 0

clear(D)

cost 0

ontable(D)

cost 0



$$h_1(s_0) = \max(2, 2) = 2$$

stack(A,B)

holding(A)

clear(B)

cost 1

cost 0

stack(B,C)

holding(B)

clear(C)

cost 1

cost 1

unstack(A,C)

handempty

clear(A)

on(A,C)

cost 0

cost 0

cost 0

Search continues: This is cheaper!

pickup(B)

handempty

clear(B)

cost 0

cost 0

unstack(A,D)

handempty

clear(A)

on(A,D)

More calculations show:
This is expensive...

unstack(A,C)

handempty

clear(A)

on(A,C)



s_0 : *clear(A)*, *on(A,C)*, *ontable(C)*, *clear(B)*, *ontable(B)*, *clear(D)*, *ontable(D)*, *handempty*

The h_1 Heuristic: Important Property 1

on(B,C)
cost 2

Each goal considered separately!



We don't search for a **valid plan** achieving on(B,C)!

Then we would need putdown(A)...

The heuristic considers individual subgoals *at all levels*, misses interactions *at all levels*

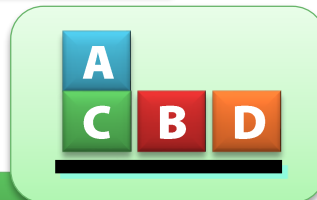
<u>stack(B,C)</u>	
holding(B)	clear(C)
cost 1	cost 1

Each precondition considered separately!

<u>pickup(B)</u>	
handempty	clear(B)
cost 0	cost 0

Each precondition considered separately!

<u>unstack(A,C)</u>		
handempty	clear(A)	on(A,C)



This is why it is fast! No need to consider interactions → **no combinatorial explosion**

The h_1 Heuristic: Important Property 2



$\text{on}(B,C)$
cost 2

Given a problem
using **:strips** expressivity,
we ignore negative effects!

(Given a goal atom,
find an action achieving it,
without considering
any other effects)

<u>stack(B,C)</u>	
holding(B)	clear(C)
cost 1	cost 1

<u>pickup(B)</u>	
<i>handempty</i>	<i>clear(B)</i>
cost 0	cost 0

<u>unstack(A,C)</u>		
<i>handempty</i>	<i>clear(A)</i>	<i>on(A,C)</i>

h_1 takes the delete relaxation heuristic, relaxes it further

The h_1 Heuristic: Important Property 3

Goal:

clear(A)

cost 0

on(A,B)

cost 2

on(B,C)

cost 2

ontable(C)

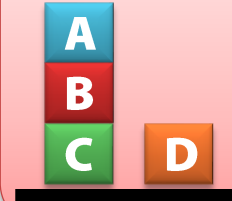
cost 0

clear(D)

cost 0

ontable(D)

cost 0



stack(A,B)

holding(A)

clear(B)

cost 1

cost 0

stack(B,C)

holding(B)

clear(C)

cost 1

cost 1

unstack(A,C)

handempty

clear(A)

on(A,C)

cost 0

cost 0

cost 0

The same action can be counted twice!

Doesn't affect admissibility,
since we take the **maximum** of subcosts,
not the **sum**

unstack(A,C)

handempty

clear(A)

on(A,C)

The h_1 Heuristic: Formal Definition

$h_1(s) = \Delta_1(s, g)$ – the heuristic depends on the goal g

- For a **goal**, a **set** g of facts to achieve:
 - $\Delta_1(s, g)$ = the cost of achieving the **most expensive** proposition in g
 - $\Delta_1(s, g) = 0$ (zero) if $g \subseteq s$ // Already achieved entire goal
 - $\Delta_1(s, g) = \max \{ \Delta_1(s, p) \mid p \in g \}$ otherwise // Part of the goal not achieved

The cost of each
atom in goal g

Max: The entire goal
must be at least as
expensive as the most
expensive subgoal

Implicit delete relaxation:
Cheapest way of
achieving $p1 \in g$
may actually delete $p2 \in g$

So how expensive is it to achieve a single proposition?

The h_1 Heuristic: Formal Definition

$h_1(s) = \Delta_1(s, g)$ – the heuristic depends on the goal g

■ For a single proposition p to be achieved:

■ $\Delta_1(s, p)$ = the cost of achieving p from s

- $\Delta_1(s, p) = 0$ if $p \in s$ // Already achieved p
- $\Delta_1(s, p) = \infty$ if $\forall a \in A. p \notin \text{effects}^+(a)$ // Unachievable
- Otherwise:

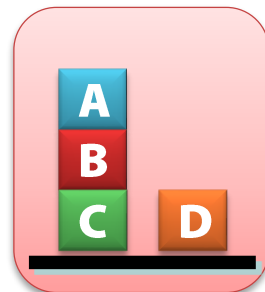
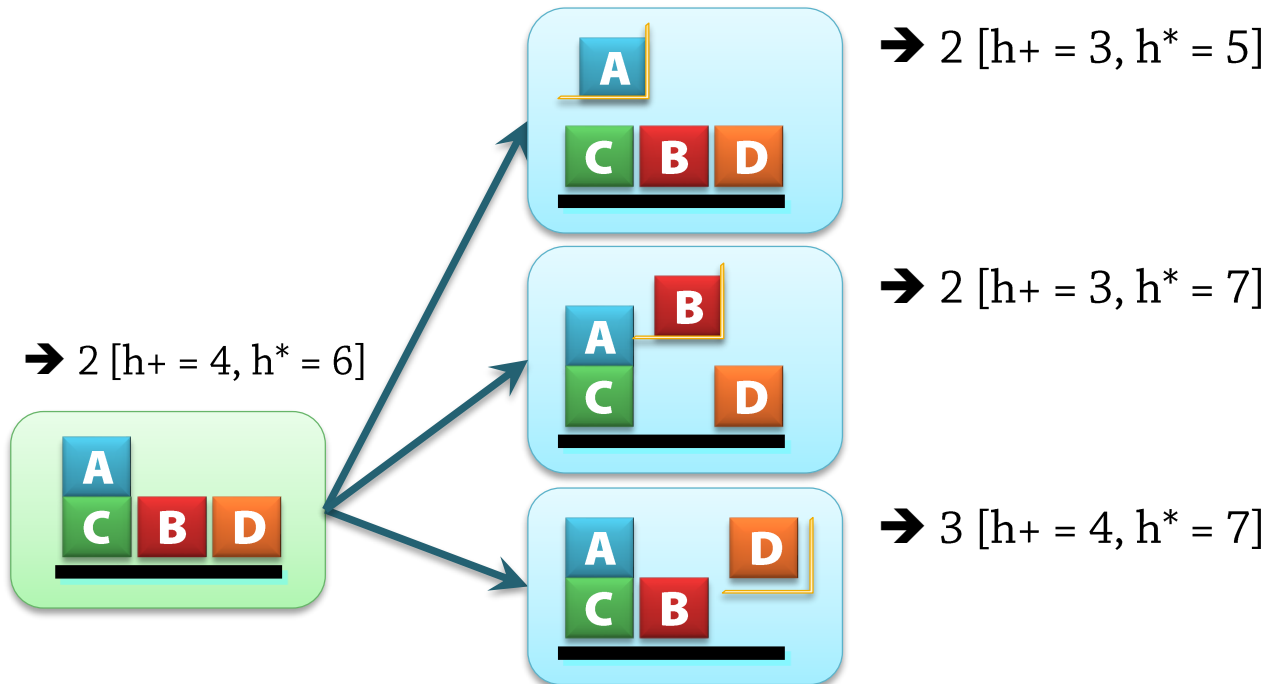
$$\Delta_1(s, p) = \min \{ \text{cost}(a) + \Delta_1(s, \text{precond}(a)) \mid a \in A \text{ and } p \in \text{effects}^+(a) \}$$

Must execute an action $a \in A$ that achieves p ,
and before that, *acheive its preconditions*

Min: Choose the action
that lets you achieve the proposition p as cheaply as possible

The h_1 Heuristic: Examples

- In the problem below:
 - $g = \{ \text{ontable}(C), \text{ontable}(D), \text{clear}(A), \text{clear}(D), \text{on}(A,B), \text{on}(B,C) \}$
- So for any state s :
 - $\Delta_1(s, g) = \max \{ \Delta_1(s, \text{ontable}(C)), \Delta_1(s, \text{ontable}(D)), \Delta_1(s, \text{clear}(A)), \Delta_1(s, \text{clear}(D)), \Delta_1(s, \text{on}(A,B)), \Delta_1(s, \text{on}(B,C)) \}$
- With unit action costs:



The h_1 Heuristic: Properties

- $h_1(s)$ is:
 - **Easier** to calculate than the optimal delete relaxation heuristic h^+
 - Somewhat **useful** for this simple BW problem instance
 - **Not sufficiently informative** in general
- Example:
 - Forward search in Blocks World using Fast Downward planner, A^*

Blocks	nodes blind	nodes h_1
5	1438	476
6	6140	963
7	120375	24038
8	1624405	392065
9	25565656	14863802
10	>84 million (out of mem)	208691676

Optimal Classical Planning: The Admissible h_m Heuristics

The h_m Heuristics



- Next idea: Could we generalize to multiple but few atoms?
 - $h_1(s) = \Delta_1(s, g)$: The most expensive atom
 - $h_2(s) = \Delta_2(s, g)$: The most expensive pair of atoms
 - $h_3(s) = \Delta_3(s, g)$: The most expensive triple of atoms
 - ...
 - → A family of admissible heuristics $h_m = h_1, h_2, \dots$ for optimal classical planning

The h_2 Heuristic

- $h_2(s) = \Delta_2(s, g)$: The most expensive **pair** of goal propositions

Goal
(set)

- $\Delta_2(s, g) = 0$ if $g \subseteq s$ // Already achieved
- $\Delta_2(s, g) = \mathbf{max} \{ \Delta_2(s, p, q) \mid p, q \in g \}$ otherwise // Can have $p=q$!

Pair of
propo-
sitions

(maybe
 $p=q$)

- $\Delta_2(s, p, q) = 0$ if $p, q \in s$ // Already achieved
- $\Delta_2(s, p, q) = \infty$ if $\forall a \in A. p \notin \text{effects}^+(a)$
or $\forall a \in A. q \notin \text{effects}^+(a)$
- $\Delta_2(s, p, q) = \mathbf{min} \{$

$\min \{ \text{cost}(a) + \Delta_2(s, \text{precond}(a))$	$\mid a \in A \text{ and } p, q \in \text{effects}^+(a) \},$
$\min \{ \text{cost}(a) + \Delta_2(s, \text{precond}(a) \cup \{q\})$	$\mid a \in A, p \in \text{effects}^+(a), q \notin \text{effects}^-(a) \},$
$\min \{ \text{cost}(a) + \Delta_2(s, \text{precond}(a) \cup \{p\})$	$\mid a \in A, q \in \text{effects}^+(a), p \notin \text{effects}^-(a) \}$

- $h_2(s)$ is more informative than $h_1(s)$, requires non-trivial time
- $m > 2$ rarely useful

The h_2 Heuristic and Delete Effects

- In this definition of h_2 :

- $\Delta_2(s, p, q) = \min \{$
 $\text{cost}(a) + \min \{ \Delta_2(s, \text{precond}(a)) \mid a \in A \text{ and } p, q \in \text{effects}^+(a) \},$
 $\text{cost}(a) + \min \{ \Delta_2(s, \text{precond}(a) \cup \{q\}) \mid a \in A, p \in \text{effects}^+(a), q \notin \text{effects}^-(a) \},$
 $\text{cost}(a) + \min \{ \Delta_2(s, \text{precond}(a) \cup \{p\}) \mid a \in A, q \in \text{effects}^+(a), p \notin \text{effects}^-(a) \}$
 $\}$

Takes into account some delete effects

So h_2 is not a *delete* relaxation heuristic (but it is admissible)!

- Misses other delete effects

- Goal: $\{p, q, r\}$
- A1: Adds $\{p, q\}$ Deletes $\{r\}$
- A2: Adds $\{p, r\}$ Deletes $\{q\}$
- A3: Adds $\{q, r\}$ Deletes $\{p\}$
- $\Delta_2(s, p, q), \Delta_2(s, q, r), \Delta_2(s, p, r) = 1$: Any pair can be achieved with a single action
- $\Delta_2(s, g) = \max(\Delta_2(s, p, q), \Delta_2(s, q, r), \Delta_2(s, p, r)) = \max(1, 1, 1) = 1,$
but the problem is unsolvable!

The h_2 Heuristic and Pairwise Mutexes



- If $\Delta_2(s_0, p, q) = \infty$:
 - Starting in s_0 , can't reach a state where p and q are true
 - Starting in s_0 , p and q are *mutually exclusive (mutex)*
- One-way implication!
 - Can be used to find *some* mutex relations, not necessarily *all*

The h_2 Heuristic and Delete Relaxation

- In the book:
 - $\Delta_2(s, p, q) = \underline{\mathbf{min}} \{$

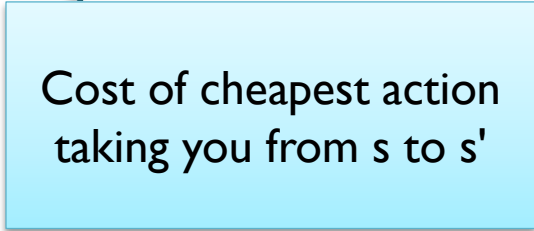
$1 + \min \{ \Delta_2(s, \text{precond}(a))$	$ a \in A \text{ and } p, q \in \text{effects}^+(a) \},$
$1 + \min \{ \Delta_2(s, \text{precond}(a) \cup \{q\})$	$ a \in A, p \in \text{effects}^+(a) \},$
$1 + \min \{ \Delta_2(s, \text{precond}(a) \cup \{p\})$	$ a \in A, q \in \text{effects}^+(a) \}$

 $\}$
- This is **not** how the heuristic is normally presented!
 - Corresponds to applying (full) delete relaxation
 - Uses constant action costs (1)

The h_m Heuristics: Calculating

- Calculating $h_m(s)$ **in practice**:
 - Characterized by Bellman equation over a specific search space
 - Solvable using variation of Generalized Bellman-Ford (GBF)
 - (Not part of the course)

$$h^m(s) = \begin{cases} 0 & \text{if } s \subseteq I \\ \min_{s' \in \text{succ}(s)} h^m(s') + \delta(s, s') & \text{if } |s| \leq m \\ \max_{s' \subseteq s, |s'| \leq m} h^m(s') & \end{cases}$$



Cost of cheapest action
taking you from s to s'

Accuracy of h_m in Selected Domains

- **How close** is $h_m(n)$ to the true goal distance $h^*(n)$?
 - **Asymptotic** accuracy as problem size approaches infinity:
 - Blocks world: $0 \rightarrow h_m(n) \geq 0 h^*(n)$
 - For any constant m !

Accuracy of h_m in Selected Domains (2)

- Consider a constructed **family of problem instances**:
 - $10n$ blocks, all on the table
 - Goal: n specific towers of 10 blocks each
- What is the **true cost** of a solution from the initial state?
 - For each tower, 1 block in place + 9 blocks to move
 - 2 actions per move
 - $9 * 2 * n = 18n$ actions
- $h_1(\text{initial-state}) = 2$ – regardless of n !
 - All instances of clear, ontable, handempty already achieved
 - Achieving a single on(...) proposition requires two actions
- $h_2(\text{initial-state}) = 4$
 - Achieving two on(...) propositions
- $h_3(\text{initial-state}) = 6$
- ...

A1	A2
B1	B2
C1	C2
D1	D2
E1	E2
F1	F2
G1	G2
H1	H2
I1	I2
J1	J2

As problem sizes grow,
the number of goals can grow
and plan lengths can grow indefinitely

But $h_m(n)$ only considers a constant
number of goal facts!

Each individual set of size m does not
necessarily become harder to achieve,
and we only calculate *max*, not *sum*...

Accuracy of h_m in Selected Domains (3)

- **How close** is $h_m(n)$ to the true goal distance $h^*(n)$?

- **Asymptotic** accuracy as problem size approaches infinity:

- Blocks world: 0 $\rightarrow h_m(n) \geq 0 h^*(n)$
- Gripper domain: 0
- Logistics domain: 0
- Miconic-STRIPS: 0
- Miconic-Simple-ADL: 0
- Schedule: 0
- Satellite: 0

But this is a **worst-case** analysis
for the **worst possible problem instance**
as **sizes approach infinity!**
+ Variations such as additive h_m exist

- For any constant m !

- Details:

- Malte Helmert, Robert Mattmüller

Accuracy of Admissible Heuristic Functions in Selected Planning Domains

The h_2 Heuristic Accuracy

- **Experimental** accuracy of h_2 in a few classical problems:

Instance	Opt.	$h(\text{root})$
blocks-9	6	5
blocks-11	9	7
blocks-15	14	11
eight-1	31	15
eight-2	31	15
eight-3	20	12
grid-1	14	14
gripper-1	3	3
gripper-2	9	4
gripper-3	15	4

Seems to work well
for the blocks world...

Less informative for the
gripper domain!

The h_m Heuristic: Nodes Expanded

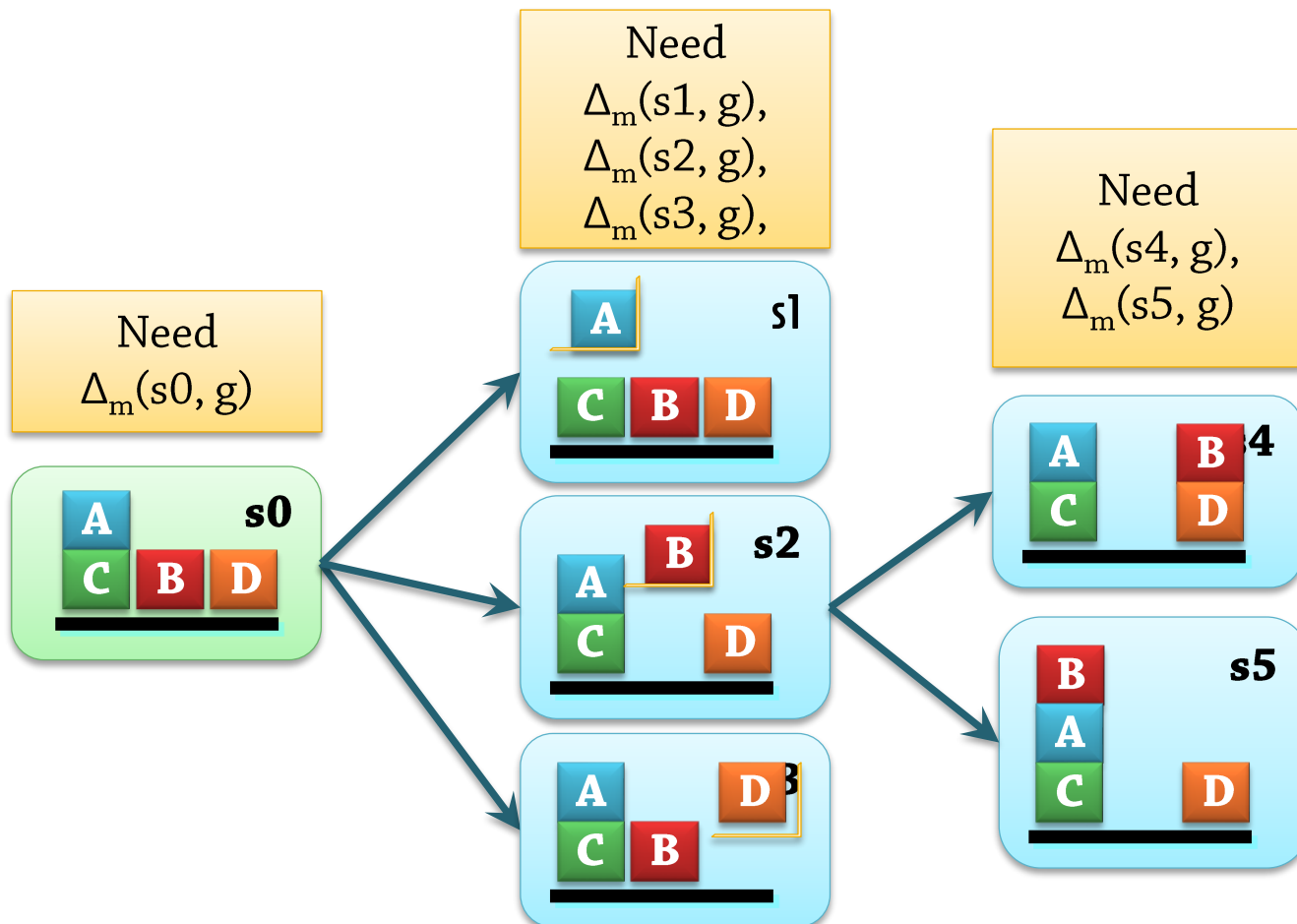


Blocks/length	nodes blind	nodes h1	nodes h2	nodes h3	nodes h4
5	1438	476	112	18	13
6	6140	963	78	23	
7	120375	24038	1662	36	
8	1624405	392065	35971		
9	25565656 (25.2s)	14863802			
10	>84 million (out of mem)	208691676			

Backward Search and h_m Heuristics

Forward Search with h_m

- Consider h_m heuristics using forward search:



Forward Search with h_m : Illustration

Goal:

clear(A)

cost 0

on(A,B)

cost 2

on(B,C)

cost 2

ontable(C)

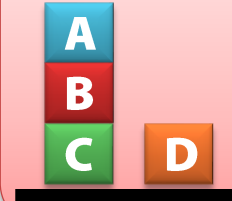
cost 0

clear(D)

cost 0

ontable(D)

cost 0



stack(A,B)

holding(A)

clear(B)

cost 1

cost 0

stack(B,C)

holding(B)

clear(C)

cost 1

cost 1

unstack(A,C)

handempty

clear(A)

on(A,C)

cost 0

cost 0

cost 0

Search continues: This is cheaper!

pickup(B)

handempty

clear(B)

cost 0

cost 0

unstack(A,C)

handempty

clear(A)

on(A,C)



unstack(A,D)

handempty

clear(A)

on(A,D)

More calculations show:

This is expensive...

current: *clear(A)*, *on(A,C)*, *ontable(C)*, *clear(B)*, *ontable(B)*, *clear(D)*, *ontable(D)*, *handempty*

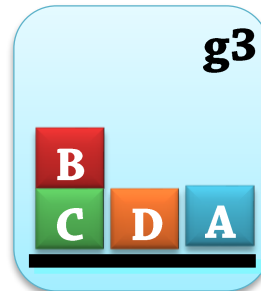
Calculations depend very much on the entire current state!

New search node \rightarrow new current state \rightarrow recalculate Δ_m from scratch

Backward Search with h_m

- In backward search:

Need
 $\Delta_m(s0, g3)$,
 $\Delta_m(s0, g4)$,
 $\Delta_m(s0, g5)$

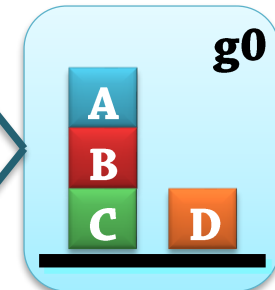
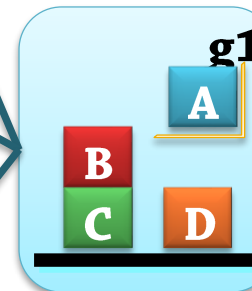


$\Delta_1(s0, g1)$
is the max of
 $\Delta_1(s0, \text{ontable}(C))$,
 $\Delta_1(s0, \text{ontable}(D))$,
 $\Delta_1(s0, \text{clear}(A))$,
...,
 $\Delta_1(s0, \text{holding}(A))$

Need
 $\Delta_m(s0, g1)$,
 $\Delta_m(s0, g2)$

$\Delta_1(s0, g0)$
is the max of
 $\Delta_1(s0, \text{ontable}(C))$,
 $\Delta_1(s0, \text{ontable}(D))$,
 $\Delta_1(s0, \text{clear}(A))$,
 $\Delta_1(s0, \text{clear}(D))$,
 $\Delta_1(s0, \text{on}(A, B))$,
...

Need
 $\Delta_m(s0, g0)$



New search node →
same starting state →
use the old Δ_m values
for previously
encountered
goal subsets



- Results:
 - Faster calculation of heuristics
 - **Not applicable for *all* heuristics!**
 - Many other heuristics work better with forward planning

Heuristics for Satisficing Forward State Space Planning

Optimal and Satisficing Planning

- Optimal planning often uses admissible heuristics + A^*
 - Are there worthwhile alternatives?

- If we need optimality:
 - Can't use non-admissible heuristics
 - Can't expand fewer nodes than A^*

- But we are not limited to optimal plans!
 - High-quality non-optimal plans can be quite useful as well
 - **Satisficing** planning
 - Find a plan that is sufficiently good, sufficiently quickly
 - Handles larger problems

Investigate many different points on the efficiency/quality spectrum!

- What's **sufficiently good, sufficiently quick?**
 - **Strict** definition of satisficing:
 - Searching until you satisfy a quality threshold
 - In automated **planning**:
 - Usually no well-defined threshold that is tested during planning
 - *Try to find strategies and heuristics that seem reasonably quick and give reasonable results in our tests*

The h_{add} Heuristic Function

Also called h_0

- h_m heuristics are admissible, but not very informative
 - Only measure the most expensive goal subsets
- For satisficing planning, we do not need admissibility
 - What if we use the sum of individual plan lengths for each atom!
 - Result: h_{add} , also called h_0

The h_{add} Heuristic: Example

Goal:

clear(A)

cost 0

on(A,B)

cost 2

on(B,C)

cost 3

ontable(C)

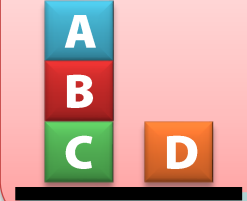
cost 0

clear(D)

cost 0

ontable(D)

cost 0



$$h_{add}(s_0) = \text{sum}(2,3) = 5$$

stack(A,B)

holding(A)

clear(B)

cost 1

cost 0

stack(B,C)

holding(B)

clear(C)

cost 1

cost 1

unstack(A,C)

handempty

clear(A)

on(A,C)

cost 0

cost 0

cost 0

Cheaper!

pickup(B)

handempty

clear(B)

cost 0

cost 0

unstack(A,D)

handempty

clear(A)

on(A,D)

More calculations → expensive...

unstack(A,C)

handempty

clear(A)

on(A,C)



s_0 : *clear(A)*, *on(A,C)*, *ontable(C)*, *clear(B)*, *ontable(B)*, *clear(D)*, *ontable(D)*, *handempty*

The h_{add} Heuristic: Formal Definition

$h_{add}(s) = h_0(s) = \Delta_0(s, g)$ – the heuristic depends on the goal g

- For a **goal**, a **set** g of facts to achieve:
 - $\Delta_0(s, g) =$ the cost of achieving the **most expensive** proposition in g
 - $\Delta_0(s, g) = 0$ if $g \subseteq s$ // Already achieved entire goal
 - $\Delta_0(s, g) = \text{sum } \{ \Delta_0(s, p) \mid p \in g \}$ otherwise // Part of the goal not achieved

The cost of each
atom p in goal g

Sum: We assume we
have to achieve
every subgoal
separately

So how expensive is it to achieve a single proposition?

The h_{add} Heuristic: Formal Definition

$h_{add}(s) = h_0(s) = \Delta_0(s, g)$ – the heuristic depends on the goal g

■ For a **single proposition** p to be achieved:

■ $\Delta_0(s, p)$ = the cost of **achieving p from s**

■ $\Delta_0(s, p) = 0$ if $p \in s$ // *Already achieved p*

■ $\Delta_0(s, p) = \infty$ if $\forall a \in A. p \notin \text{effects}^+(a)$ // *Unachievable*

■ Otherwise:

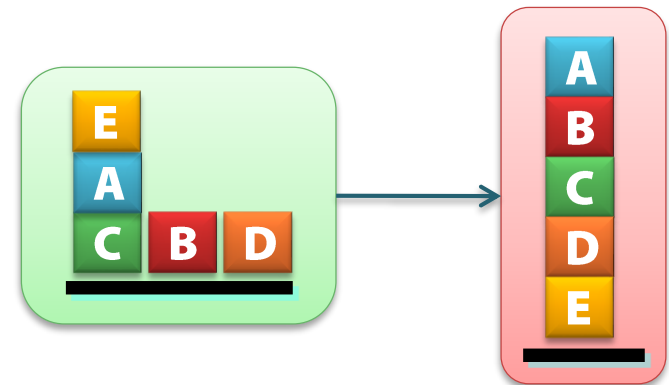
$$\Delta_0(s, p) = \min \{ \text{cost}(a) + \Delta_1(s, \text{precond}(a)) \mid a \in A \text{ and } p \in \text{effects}^+(a) \}$$

Must execute an action $a \in A$ that achieves p ,
and before that, *acheive its preconditions*

Min: Choose the action
that lets you achieve p as cheaply as possible

The h_{add} Heuristic: Example

- $h_{add}(s) = \Delta_0(s, g)$
 - For another example:
 - **ontable(E)**: unstack(E,A), putdown(E) $\rightarrow 2$
 - **clear(A)**: unstack(E,A) $\rightarrow 1$
 - **on(A,B)**: unstack(E,A), unstack(A,C), stack(A,B) $\rightarrow 3$
 - **on(B,C)**: unstack(E,A), unstack(A,C), pickup(B), stack(B,C) $\rightarrow 4$
 - **on(C,D)**: unstack(E,A), unstack(A,C), pickup(C), stack(C,D) $\rightarrow 4$
 - **on(D,E)**: pickup(D), stack(D,E) $\rightarrow 2$
 - \rightarrow sum is 16 [$h^+ = 10$, $h^* = 12$]



Can underestimate but also overestimate, not admissible!

The h_{add} Heuristic: Admissibility

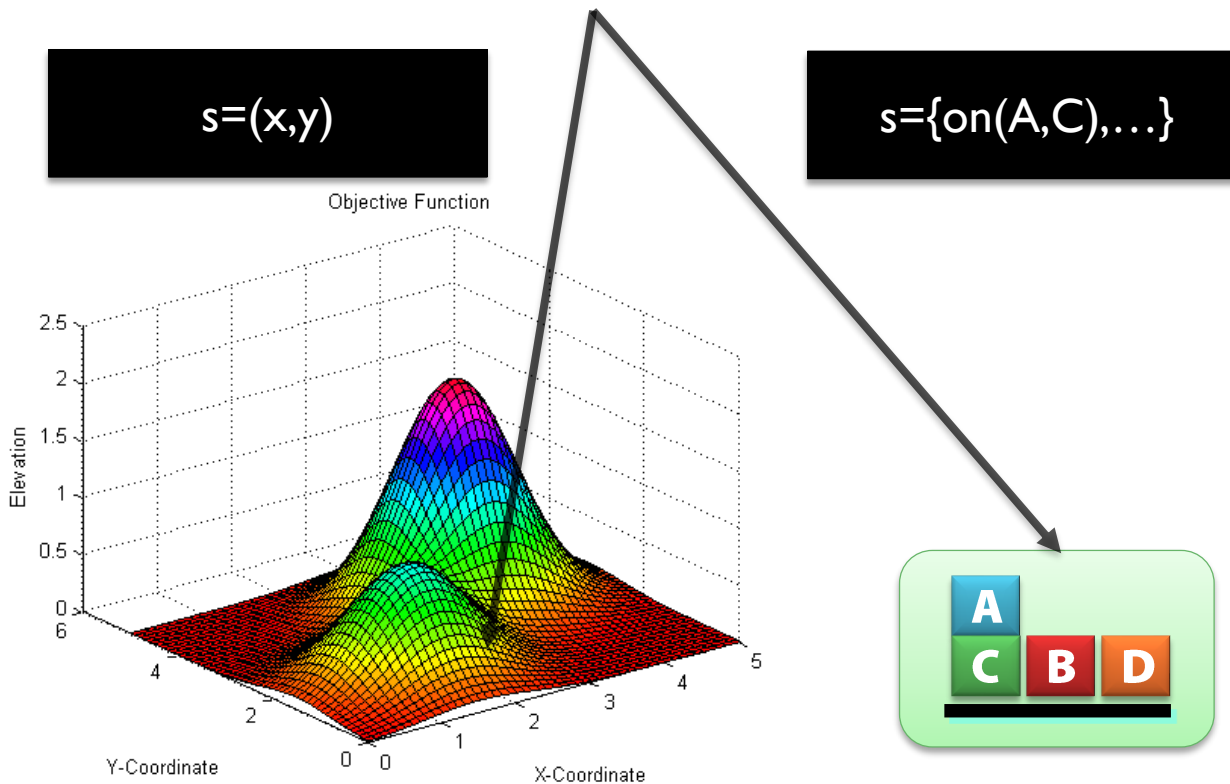
- Why not admissible?
 - Does not take into account interactions between goals
 - Simple case: Same action used
 - on(A,B): unstack(E,A); unstack(A,C); stack(A,B) → 3
 - on(B,C): unstack(E,A); unstack(A,C); pickup(B); stack(B,C) → 4
 - More complicated to detect:
 - Goal: p and q
 - A1: effect p
 - A2: effect q
 - A3: effect p and q
- To achieve p: Use A1 – No specific action used twice
- To achieve q: Use A2 – Still misses interactions

Hill Climbing in HSP (Heuristic Search Planner)

Satisficing planning, in a nutshell:

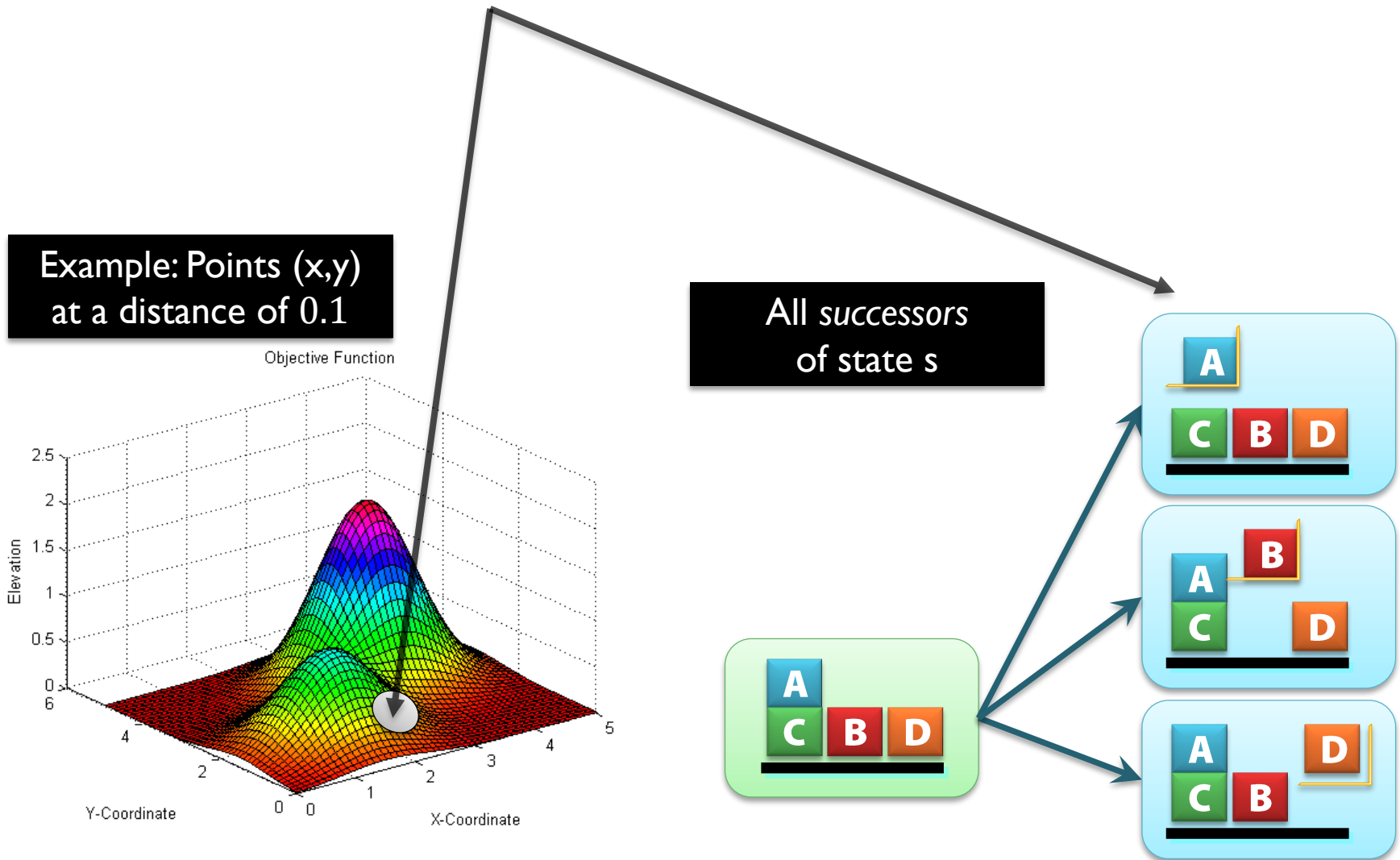
Try to move quickly towards a reasonably good solution

- What about **Steepest Ascent Hill Climbing?**
 - **Greedy local search** algorithm for **optimization problems**
 - (I) Start in some **current location**



Hill Climbing (2)

- (2) Find the **local neighborhood**, which can easily be reached



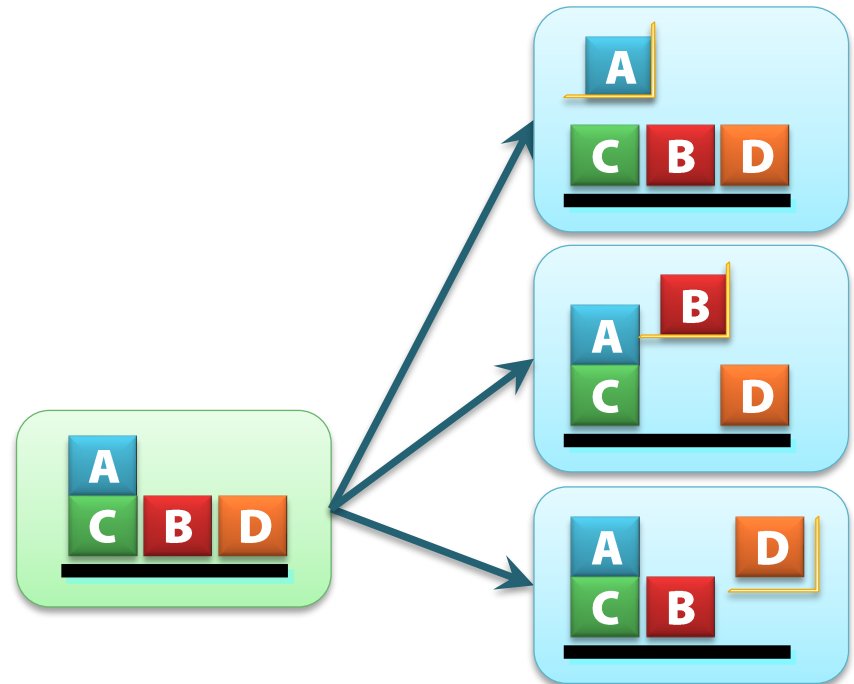
Hill Climbing (3)

- (3) Make a **locally optimal** choice at each step:
Chooses the successor/neighbor that is *best in this step*
(doesn't care about the *future*)



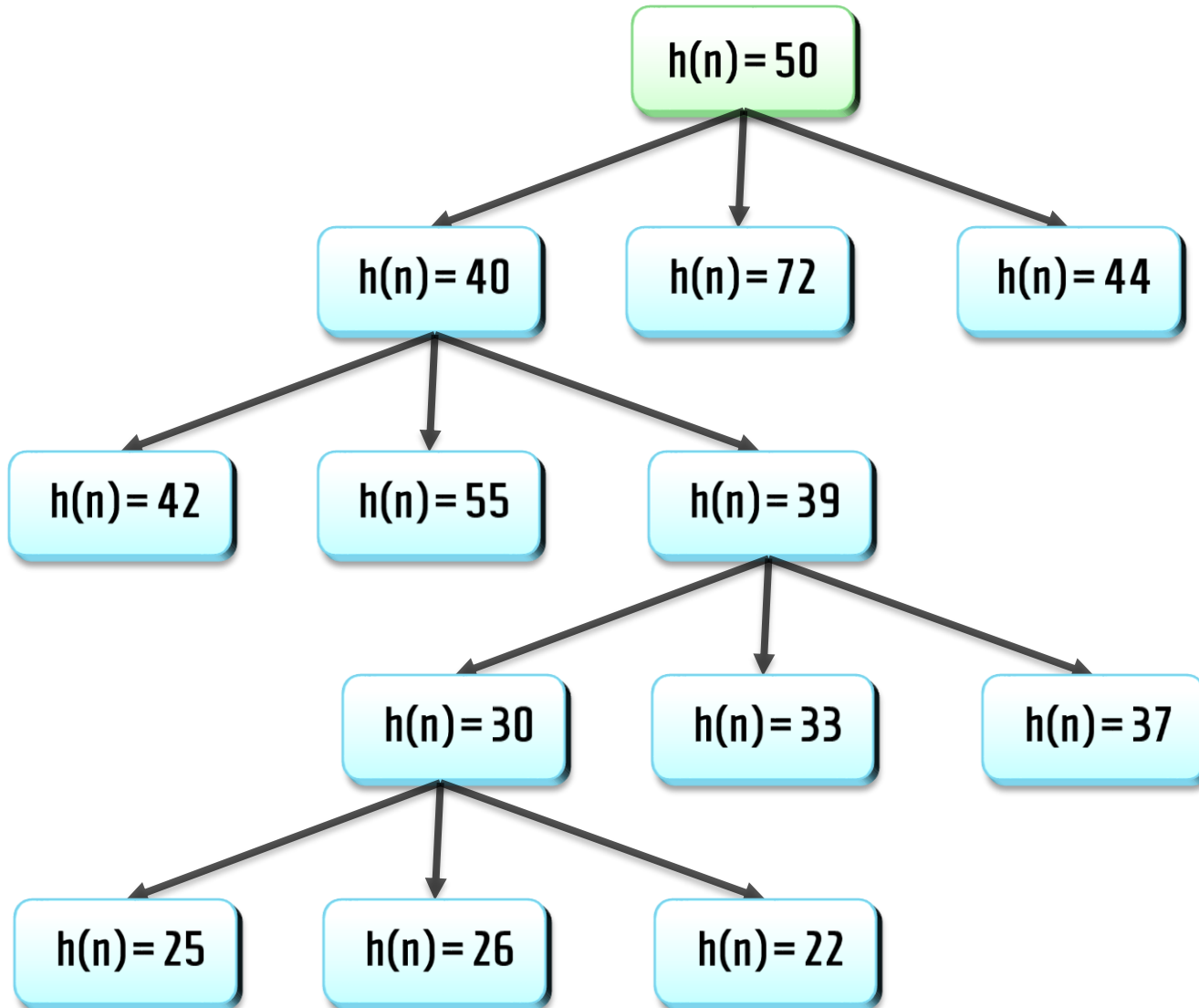
Hill Climbing (4)

- We don't have a metric *state quality* measure!
 - Goal states are *perfect*, other states are *not solutions at all*
- But **minimizing heuristic value** might *lead* to a goal state...
 - (Minimize $h(n)$ = maximize $-h(n)$)
 - → A good heuristic should **order children in the best way**



Hill Climbing (5)

- Example of hill climbing search:



Hill Climbing (6)

A* search:

$n \leftarrow$ initial state

$open \leftarrow \emptyset$

loop

if n is a solution **then return** n

expand children of n

calculate h for children

add children to $open$

$n \leftarrow$ node in $open$

minimizing $f(n) = g(n) + h(n)$

end loop

Steepest Ascent Hill-climbing

$n \leftarrow$ initial state

loop

if n is a solution **then return** n

expand children of n

calculate h for children

if (some child decreases $h(n)$):

$n \leftarrow$ child with minimal $h(n)$

else stop // local optimum

end loop

Be stubborn:

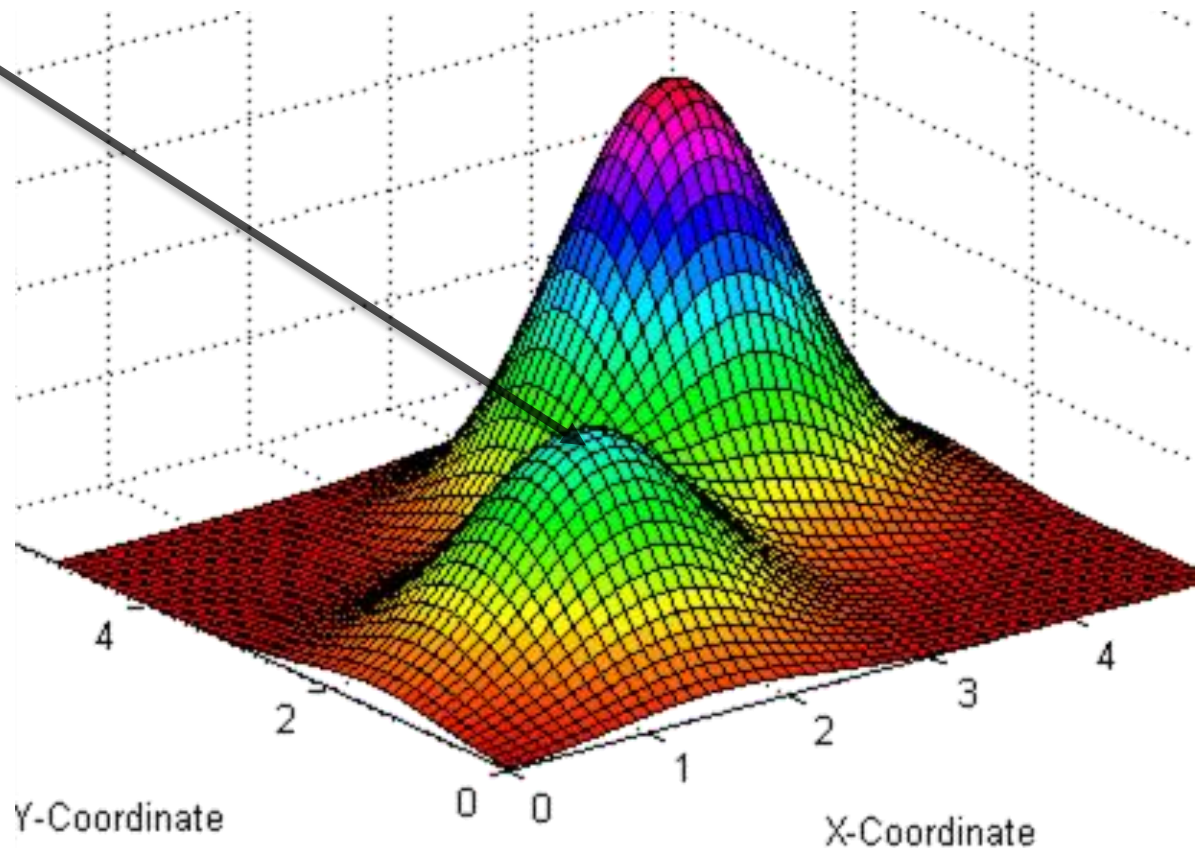
Only consider children of this node, don't even keep track of other nodes to return to

Ignore $g(n)$: prioritize finding a plan quickly over finding a good plan

Local Optima and Plateaus

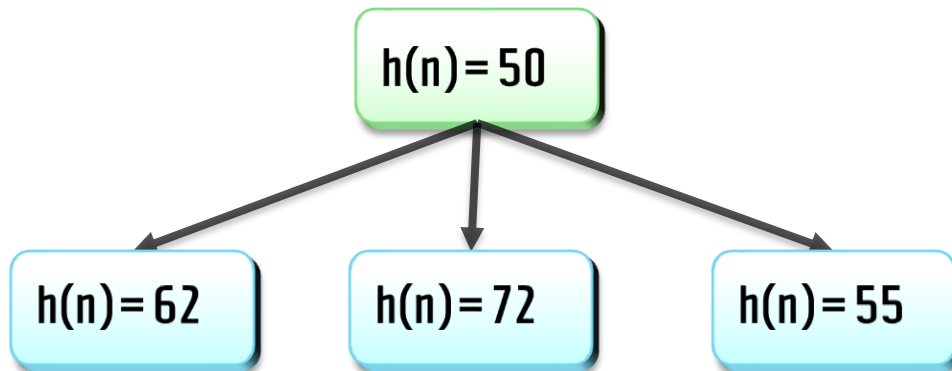
Local Optima (1)

- (4) When there is **nothing better** nearby: Stop!
 - HC is used for *optimization*
 - Any point is a *solution*, we search for the *best* one
 - Might find a *local optimum*:
The top of a hill



Local Optima (2)

- Classical planning \rightarrow *absolute goals*
 - Even if we can't decrease $h(n)$, we can't simply *stop*



Steepest Ascent

Hill-climbing

$n \leftarrow$ initial state

loop

if n is a solution **then return** n

expand children of n

calculate h for children

if (some child decreases $h(n)$):

$n \leftarrow$ child with minimal $h(n)$

else stop // local optimum

end loop

Local Optima (3)

- Standard solution to local optima:
Random restart

- Randomly choose another node/state
- Continue searching from there
- Hope you find a global optimum eventually

- Can *planners* choose arbitrary random states?

Steepest Ascent

Hill-climbing with Restarts

$n \leftarrow$ initial state

loop

if n is a solution **then return** n

expand children of n

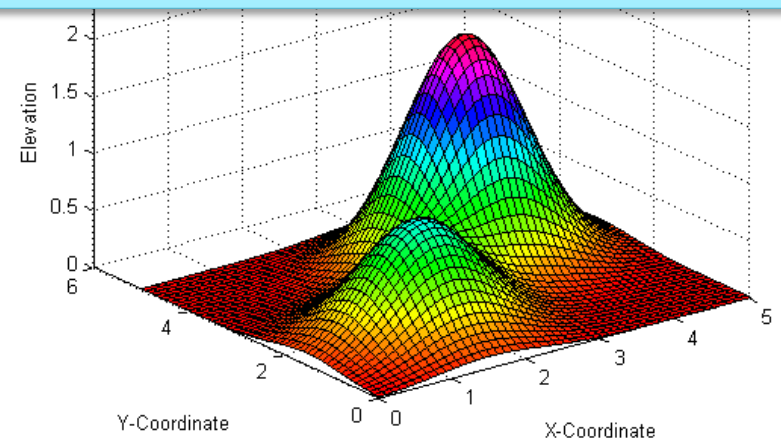
calculate h for children

if (some child decreases $h(n)$):

$n \leftarrow$ child with minimal $h(n)$

else $n \leftarrow$ some random state

end loop



Local Optima (4)

- In planning:
 - The solution is not a *state* but the *path to the state*
 - Random states may not be reachable from the initial state
- So:
 - Randomly choose another *already visited* node/state
 - This node *is* reachable!

Steepest Ascent

Hill-climbing with Restarts (2)

$n \leftarrow$ initial state

loop

if n is a solution **then return** n

expand children of n

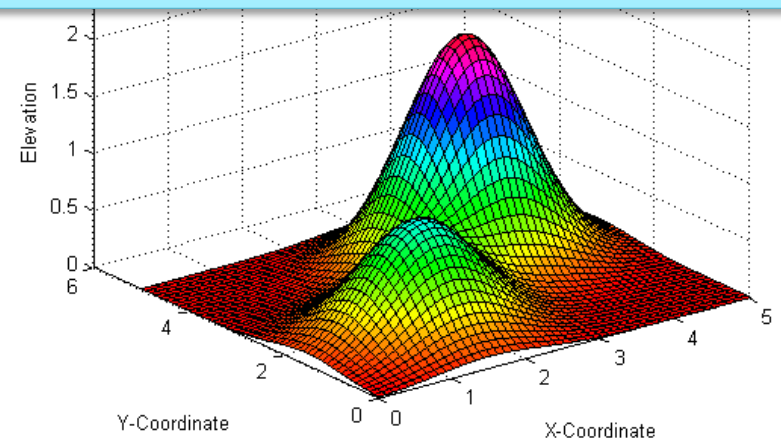
calculate h for children

if (some child decreases $h(n)$):

$n \leftarrow$ child with minimal $h(n)$

else $n \leftarrow$ some rnd. *visited* state

end loop



Hill Climbing with h_{add} : Plateaus

▪ (on A B)	2	1	3	3
▪ (on B C)	3	3	4	4
▪ (clear A)	0	1	0	0
▪ (clear D)	0	0	0	1
▪ (ontable C)	0	0	0	0
▪ (ontable D)	0	0	0	1
▪ $h(n)=\text{sum}$	5	5	7	9

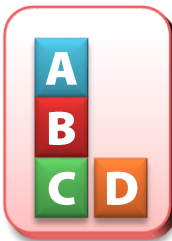
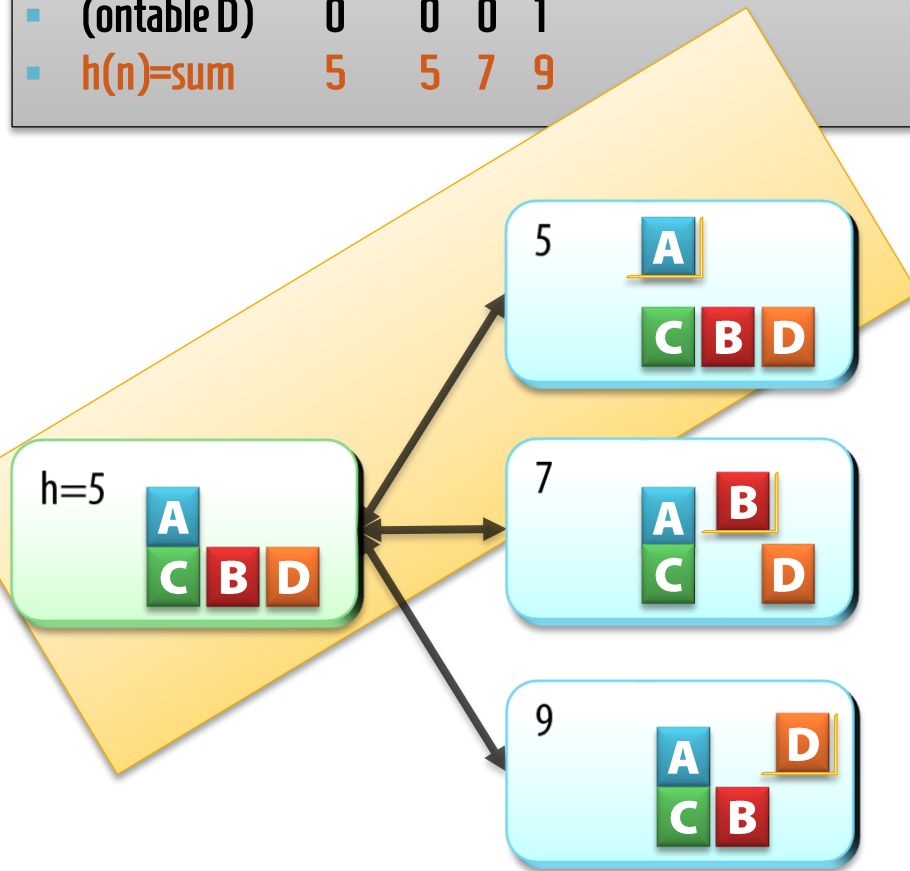
No successor improves the heuristic value; some are equal!

We have a **plateau**...

Jump to a random state immediately?

No: the heuristic is not so accurate – maybe some child is closer to the goal even though $h(n)$ isn't lower!

→ Let's keep exploring:
Allow a small number of consecutive **moves across plateaus**



Plateaus

- A plateau...

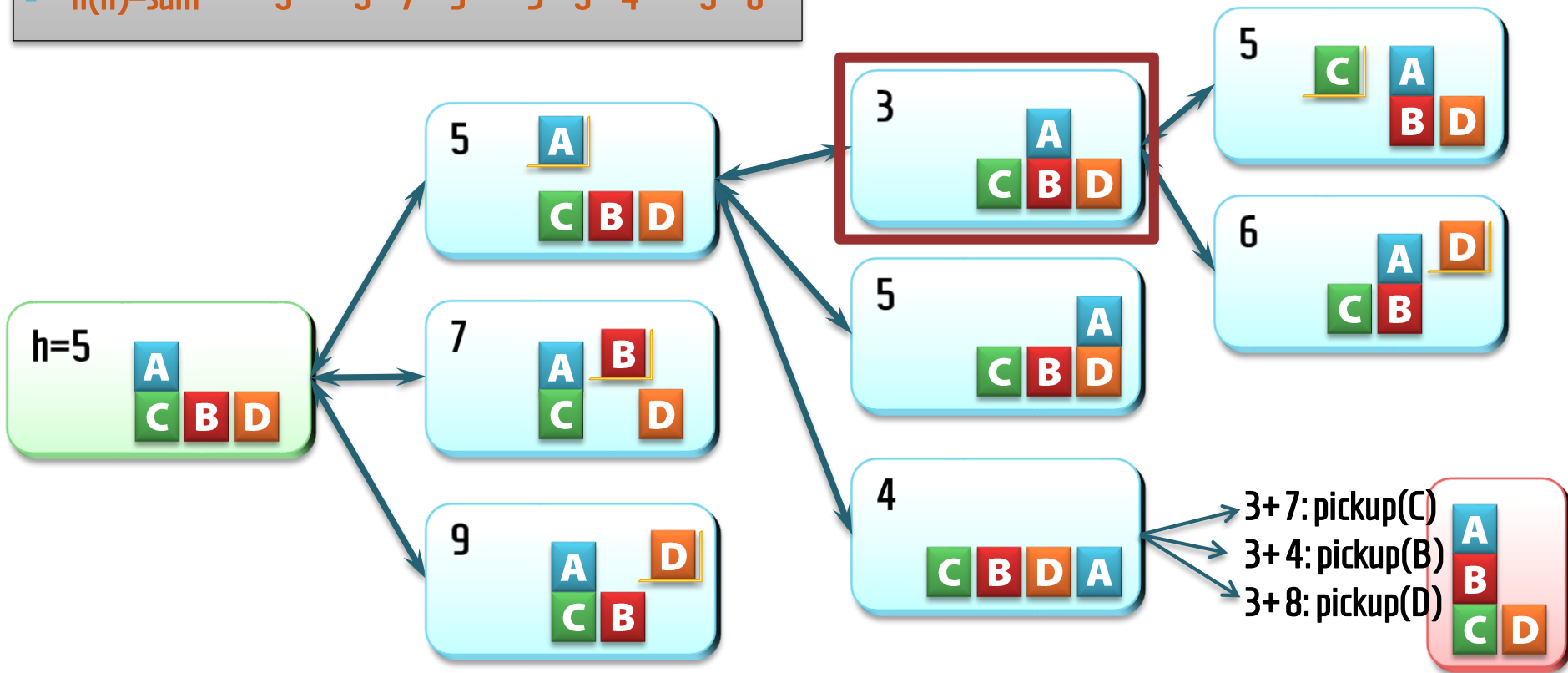


Hill Climbing with h_{add} : Local Optima

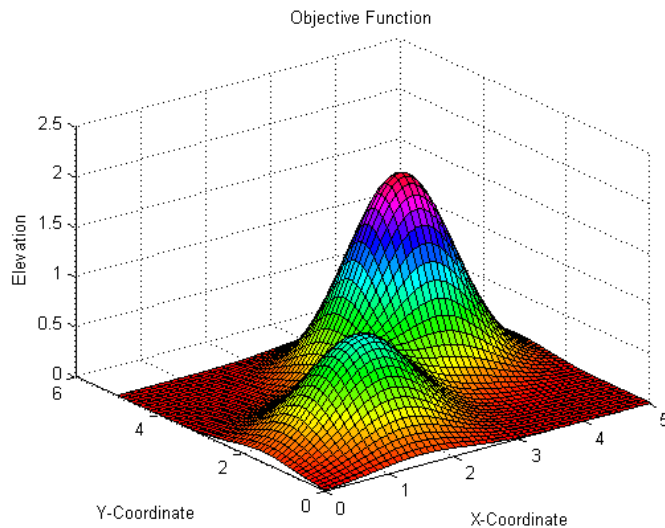
▪ (on A B)	2	1	3	3	0	2	2	0	0
▪ (on B C)	3	3	4	4	3	2	2	4	4
▪ (clear A)	0	1	0	0	0	0	0	0	0
▪ (clear D)	0	0	0	1	0	1	0	0	1
▪ (ontable C)	0	0	0	0	0	0	0	1	0
▪ (ontable D)	0	0	0	1	0	0	0	0	1
▪ $h(n)=\text{sum}$	5	5	7	9	3	5	4	5	6

If we continue, all successors have higher heuristic values!

We have a **local optimum**...
Impasse = optimum or plateau
Some impasses allowed



- Local optimum: You can't improve the *heuristic function* in one step
 - But maybe you can still get *closer to the goal*:
The heuristic only *approximates* our real objectives



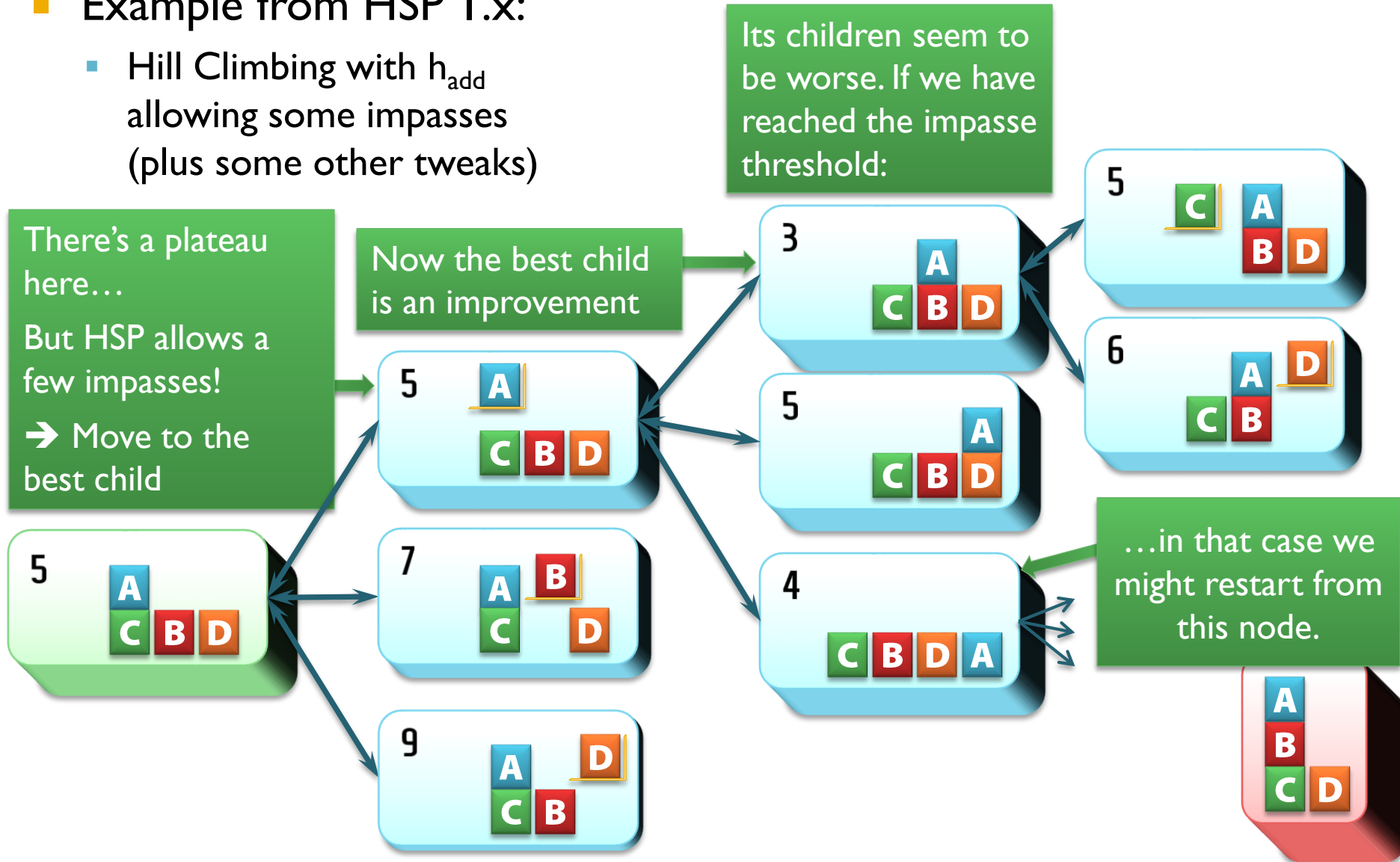
Impasses and Restarts



- What if there are **many** impasses?
 - Maybe we *are* in the wrong part of the search space after all...
 - Misguided by h_{add} at some earlier step
 - ➔ Select another *promising* expanded node where search continues

HSP Example

- Example from HSP 1.x:
 - Hill Climbing with h_{add} allowing some impasses (plus some other tweaks)



HSP 1: Heuristic Search Planner

- HSP 1.x: h_{add} heuristic + hill climbing + modifications
 - Works **approximately** like this (some intricacies omitted):

```
    impasses = 0;
    unexpanded = { };
    current = initialNode;
    while (not yet reached the goal) {
        children = expand(current); // Apply all applicable actions
        if (children ==  $\emptyset$ ) {
            current = pop(unexpanded);
        } else {
            bestChild = best(children); // Child with the lowest heuristic value
            add other children to unexpanded in order of  $h(n)$ ; // Keep for restarts!
            if ( $h(\text{bestChild}) \geq h(\text{current})$ ) {
                impasses++;
                if (impasses == threshold) {
                    current = pop(unexpanded); // Restart from another node
                    impasses = 0;
                }
            }
        }
    }
```

Dead end →
restart

Essentially
hill-climbing, but
not all steps have
to move "up"

Too many
downhill/plateau
moves → escape

Simple structure,
but highly competitive at its introduction
(using h_{add} as a heuristic)

Heuristics part III

Pattern Database Heuristics

Admissible, but useful for both optimal and satisficing planning

- Main idea behind pattern databases:
 - Let's **ignore some facts** – everywhere
 - In goals
 - In preconditions or effects
 - Compute costs **as if those facts didn't matter**

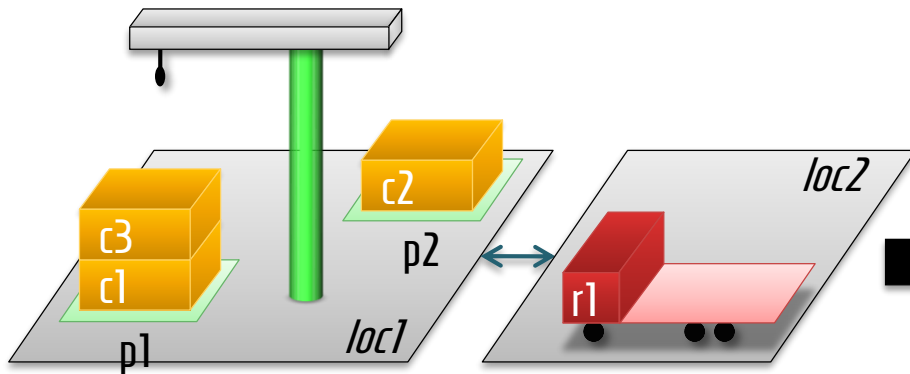


PDB 2: Dock Worker Robots

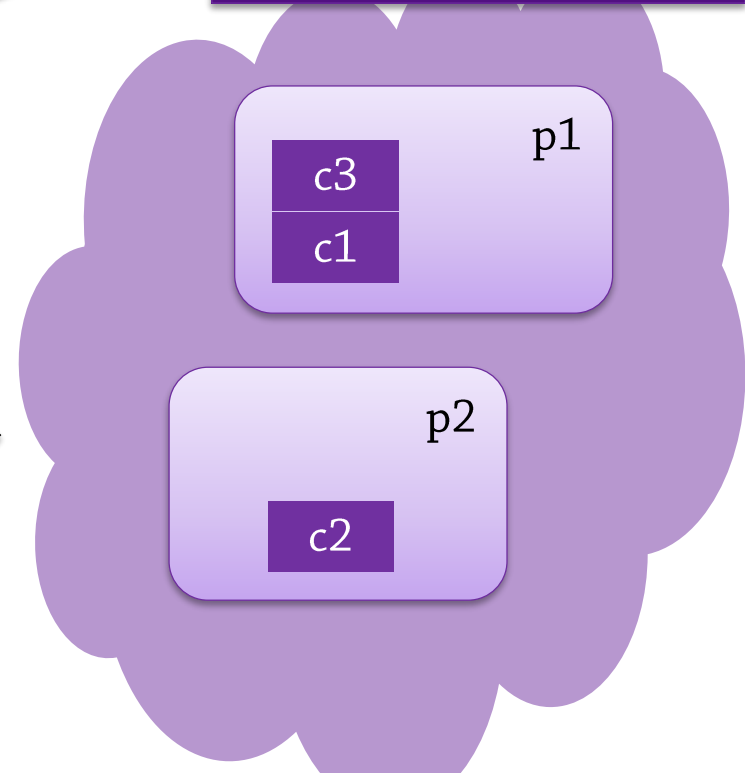
■ Example: Dock Worker Robots

- **Care about** facts related to ***container locations***
 - $in(container, pile)$, $top(container, pile)$, $on(c1, c2)$, ...
- **Ignore** robot locations, crane locations, ...
- Original states are **grouped together**

Ordinary state in P,
all facts included



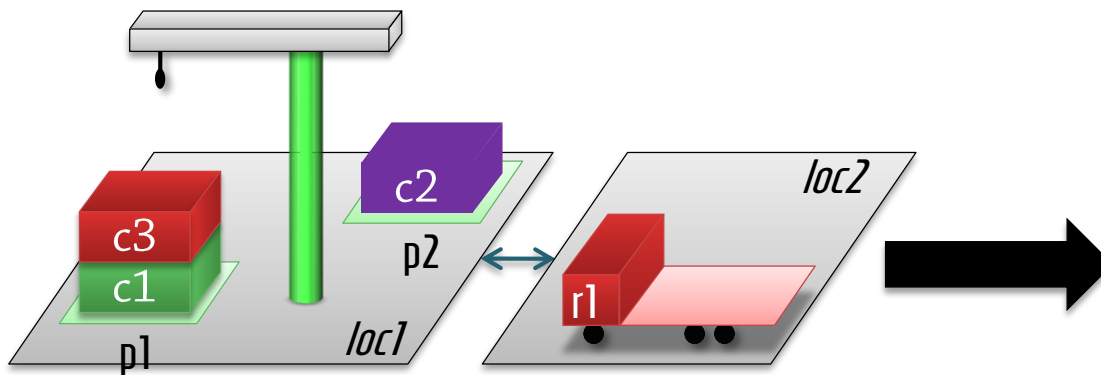
Abstract state in P',
represents
many states in P
where
c3 is on c1 in p1,
...



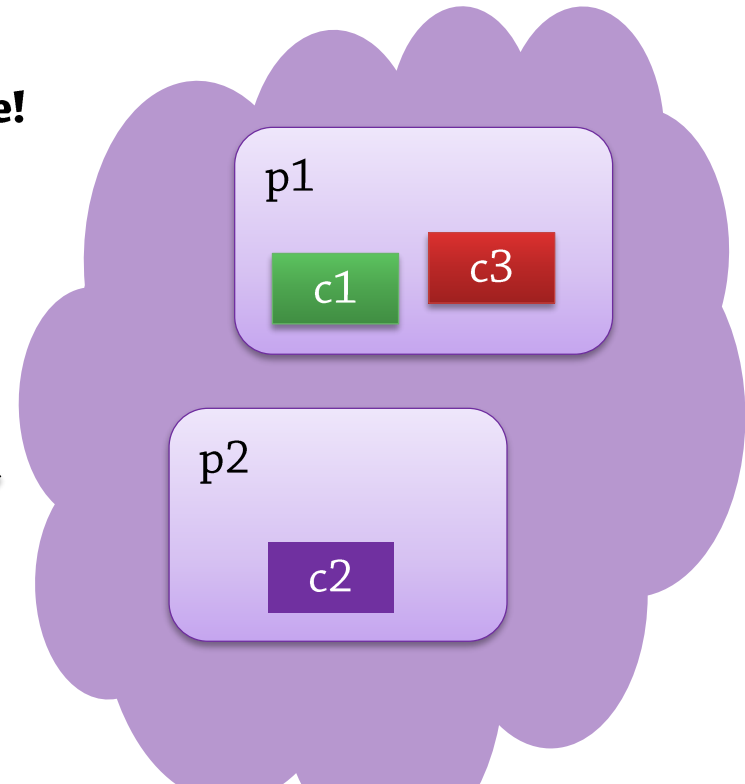
PDB 3: Planning in Patterns

- In P' we (pretend that we) can use the crane at $p1$ to:
 - **pick up** $c3$ (as we should)
 - **place** something on $r1$ (too far away, but we don't care)
 - **place** five containers on one truck
- But we can't:
 - pick up $c1$ (we do care about pile ordering)
 - immediately place $c1$ below $c2$, ...
 - ➔ **Still a planning problem P' left to solve!**

New paths
to the goal!



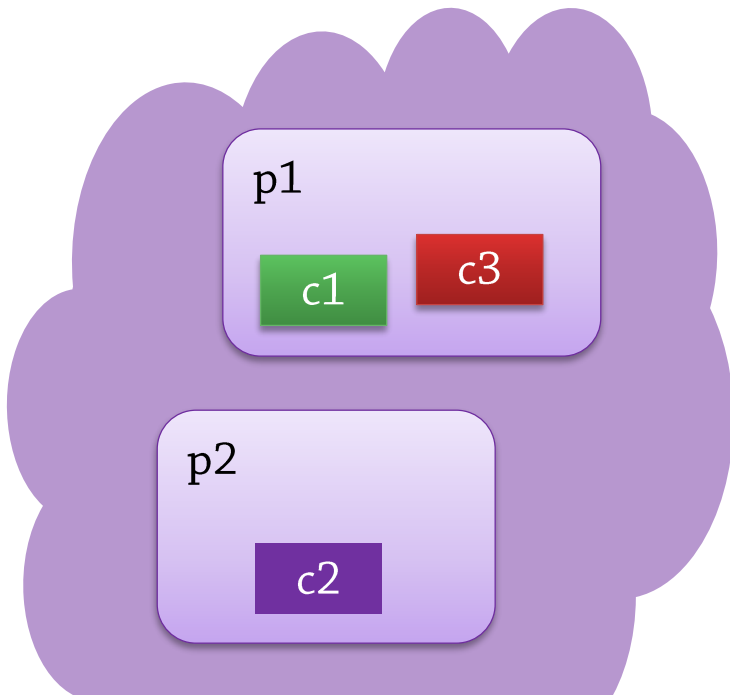
Solve optimally, compute cost
➔ admissible heuristic!



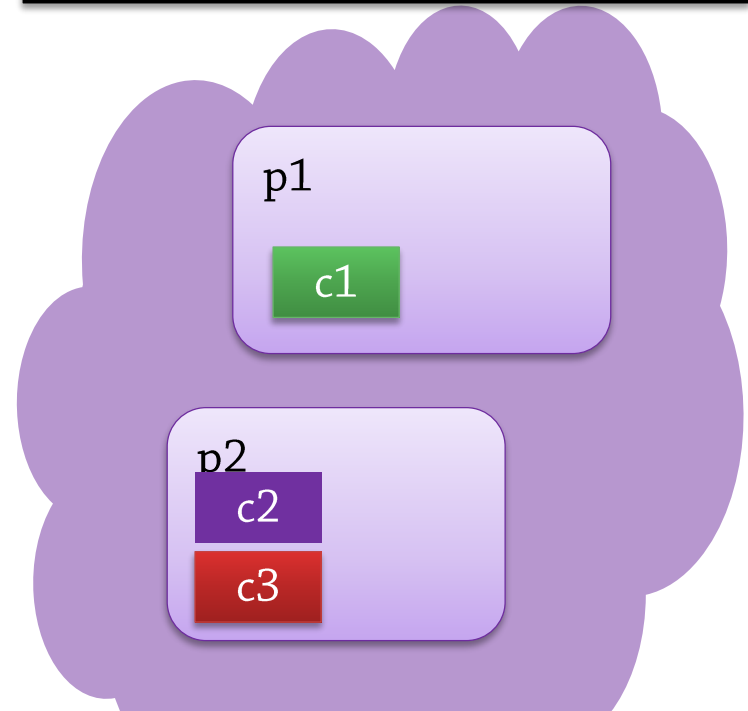
PDB 4: Computing a Heuristic Value

- **Solve $P'(s)$ optimally**, compute cost \rightarrow admissible heuristic $h(s)$!
 - Take **c2** with the crane (it's in the way)
 - Take **c3** with the crane [relaxation – not checking if the crane is busy]
 - Place **c3** at the bottom
 - Place **c2** on the top

Abstract current state s



Abstract goal



Let's formalize!

Pattern Database Heuristics: Intro

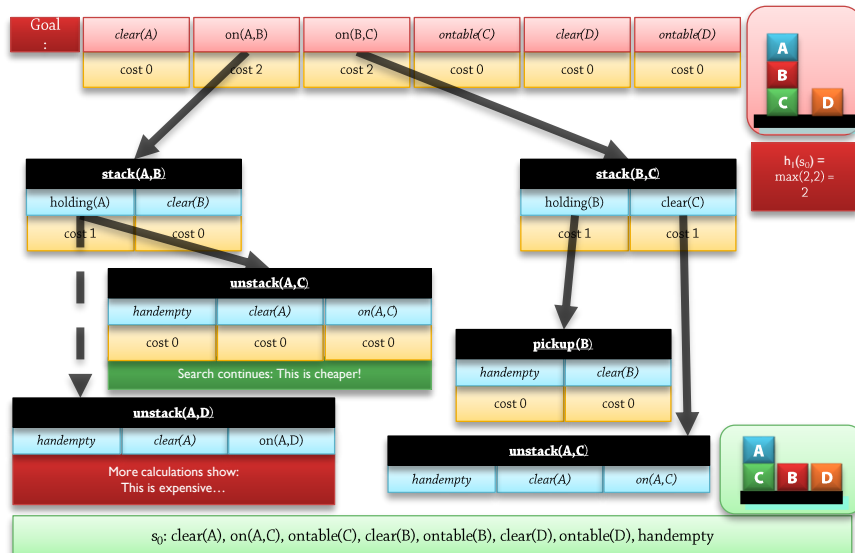
Many heuristics solve subproblems, combine their cost

In each subproblem for
the h_m heuristics:

Pick m goal literals at a time
Ignore the others
Solve a subproblem optimally

In each subproblem for
Pattern Database (PDB) Heuristics

Pick some ground atoms (facts)
Ignore the others
Solve a subproblem optimally





-

- All ground atoms (facts) in this problem instance:

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| (on A A) | (on A B) | (on A C) | (on A D) |
| (on B A) | (on B B) | (on B C) | (on B D) |
| (on C A) | (on C B) | (on C C) | (on C D) |
| (on D A) | (on D B) | (on D C) | (on D D) |

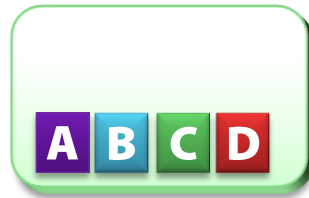
(ontable A) (ontable B) (ontable C) (ontable D)
(clear A) (clear B) (clear C) (clear D)
(holding A) (holding B) (holding C) (holding D)

(handempty)

BW4: Potential Subproblem

- Example: only consider 5 ground facts related to block A
 - "Pattern": $p = \{(\text{on A B}), (\text{on A C}), (\text{on A D}), (\text{clear A}), (\text{ontable A})\}$

- **Initial state:**



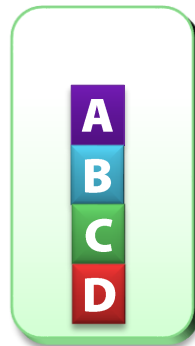
ontable(A)
ontable(B)
ontable(C)
ontable(D)
clear(A)
clear(B)
clear(C)
clear(D)
handempty



ontable(A)
ontable(B)
ontable(C)
ontable(D)
clear(A)
clear(B)
clear(C)
clear(D)
handempty

An "abstract state"

- **Goal:**



clear(A)
on(A,B)
on(B,C)
on(C,D)
ontable(D)
handempty



clear(A)
on(A,B)
on(B,C)
on(C,D)
ontable(D)
handempty

An "abstract goal"

BW4: Potential Subproblem (2)

- Pattern $p = \{(\text{on A B}), (\text{on A C}), (\text{on A D}), (\text{clear A}), (\text{ontable A})\}$

- **Example action:** (unstack A B)

- **Before transformation:**

- :precondition (and (handempty) (**clear A**) (**on A B**))

- :effect (and (not (handempty)) (holding A) (**not (clear A)**) (clear B) (**not (on A B)**))

Loses **some** preconditions and effects

- **After transformation:**

- :precondition (and (clear A) (on A B))

- :effect (and (not (clear A)) (not (on A B)))

Let's call this action $\text{transform}(a, p)$

- **Example action:** (unstack C D)

- **Before transformation:**

- :precondition (and (handempty) (clear C) (on C D))

- :effect (and (not (handempty)) (holding C) (not (clear C)) (clear D) (not (on C D)))

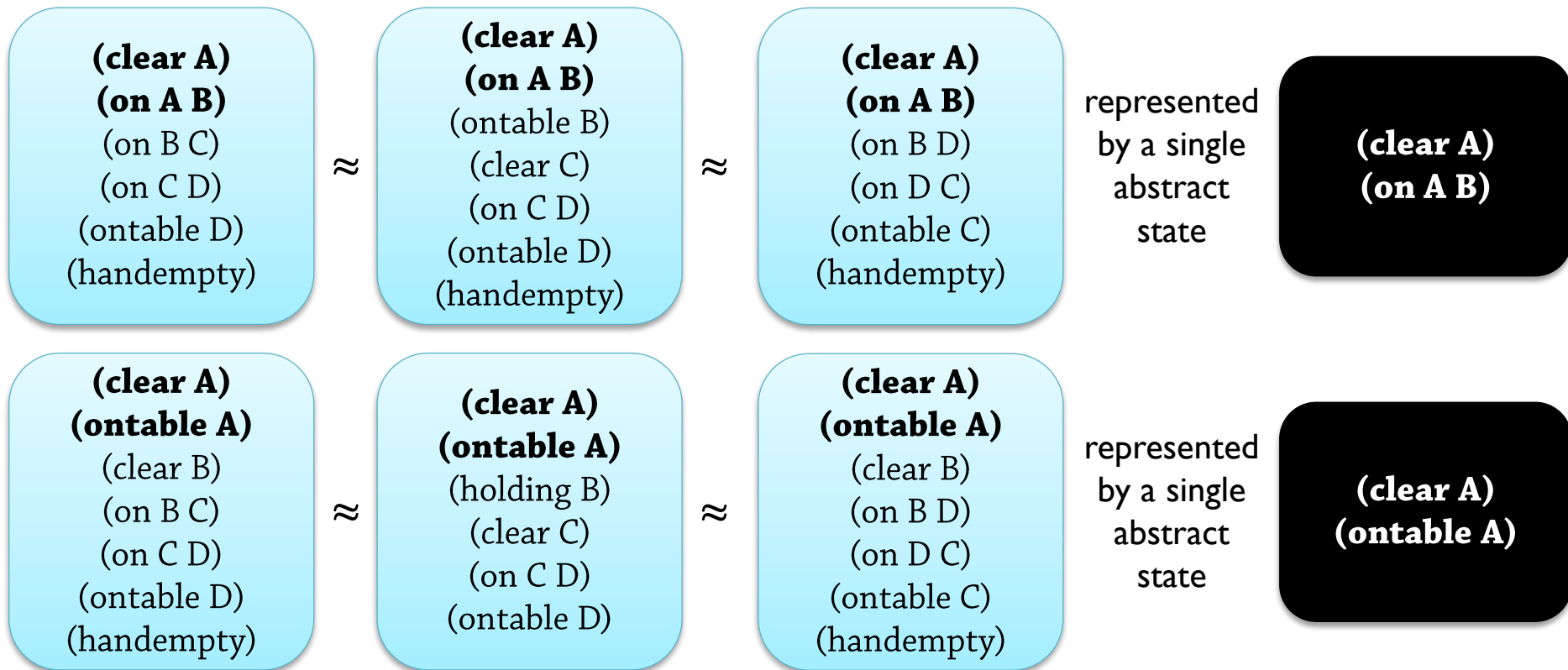
- **After transformation:**

- :precondition (and)

- :effect (and)

Loses **all** preconditions and effects → never used!

- The set of ground facts is called a pattern p
 - A state s is represented by the abstract state $s \cap p$
 - If $s \cap p = s' \cap p$, the two states are considered *equivalent*



A pattern generally contains few facts!

■ Is this a relaxation?

■ Yes

■ Facts disappear from states...

- $S' = \{s \cap p | s \in S\}$

■ But also from precond/goal requirements!

- If a_i could be executed in s ,
 $\text{transform}(a_i)$ can be executed in $s \cap p$

- If γ' is the state transition function given transformed actions, then
$$\gamma'(\text{transform}(a_i), s \cap p) = \gamma(a_i, s) \cap p$$

- ➔ executable action sequences are preserved

- If $g \subseteq s$, then $g \cap p \subseteq s \cap p$

- So: Solutions are preserved (but new solutions may arise)

ontable(A)
ontable(B)
ontable(C)
ontable(D)
clear(A)
clear(B)
clear(C)
clear(D)
handempty



ontable(A)
ontable(B)
ontable(C)
ontable(D)
clear(A)
clear(B)
clear(C)
clear(D)
handempty

BW4: State Transition Graph

■ New reachable state transition graph:

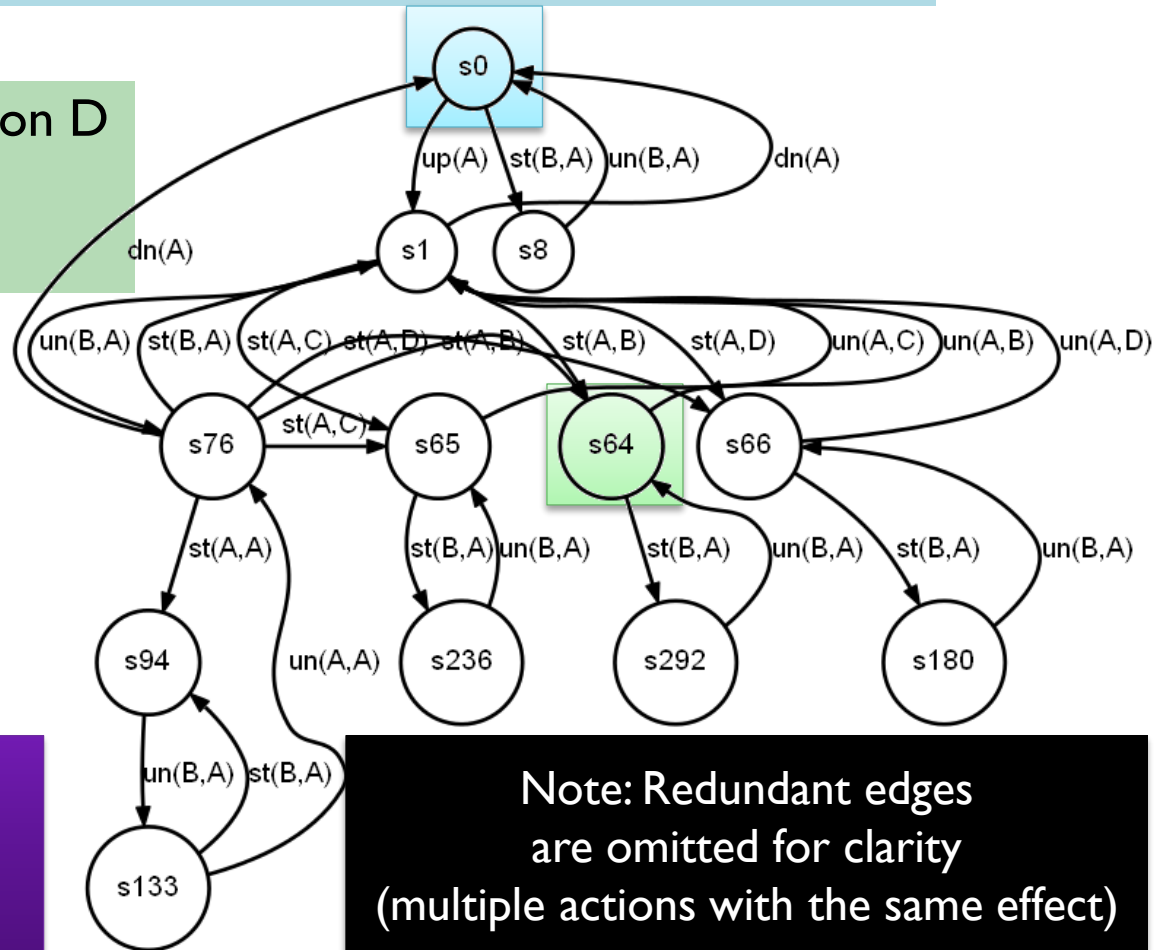
- Current state: Everything on the table, hand empty, all blocks clear

- Abstract state: $s_0 = \{ (\text{ontable } A), (\text{clear } A) \}$

- Goal state: A on B on C on D

- Abstract goal: $s_{64} = \{ (\text{on } A \ B), (\text{clear } A) \}$

- Sufficiently few states to **quickly** compute **optimal** costs
 - Cost is *at least* 2:
Shortest path $s_0 \rightarrow s_{64}$



Optimal cost of a relaxation
→
admissible heuristic

Note: Redundant edges
are omitted for clarity
(multiple actions with the same effect)

As in h_m , use multiple subproblems!

■ Subproblem 2: Some facts related to B

- **Current state:** Everything on the table, hand empty, all blocks clear

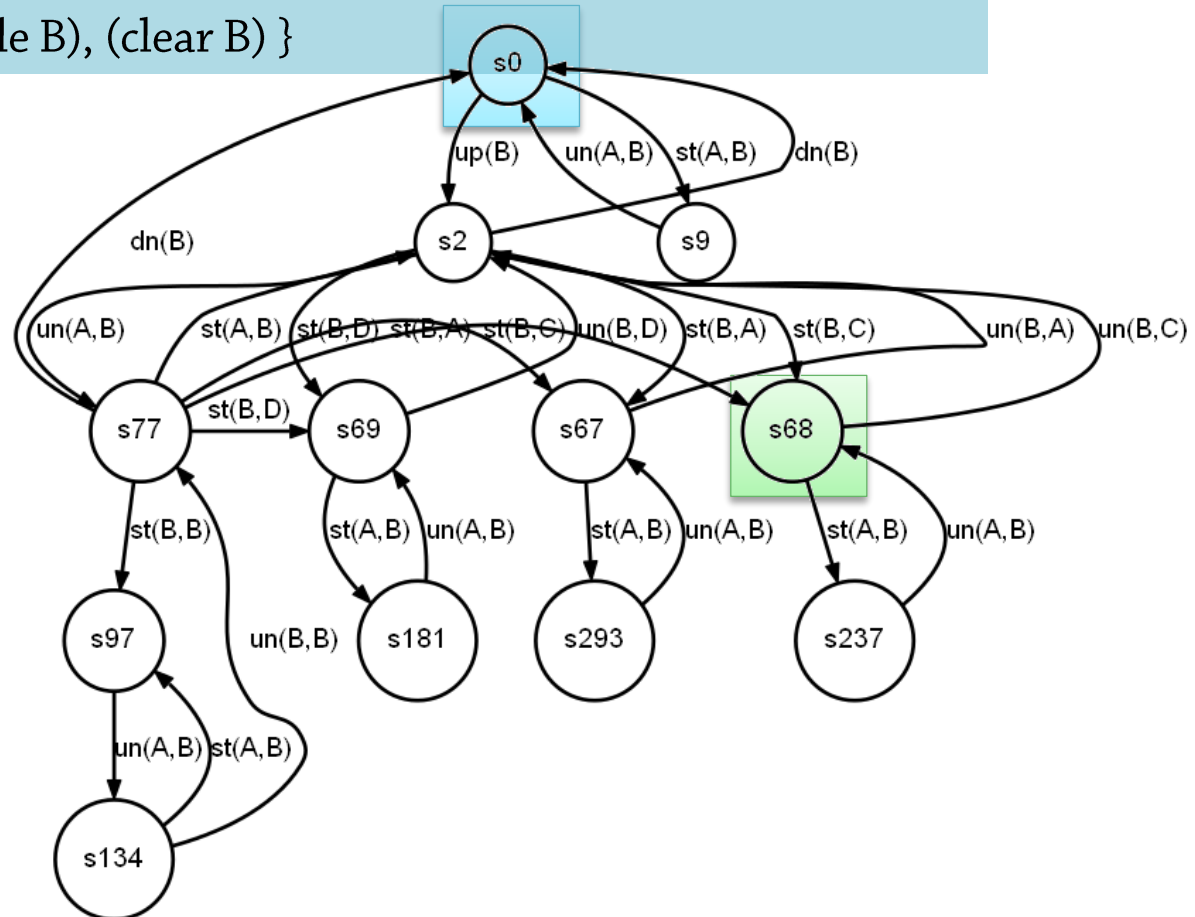
- Abstract state: $\{ (\text{ontable B}), (\text{clear B}) \}$

- **Goal state:**

A on B on C on D

- Abstract goal:
 $\{ (\text{on B C}) \}$

- Find a path,
compute its cost



- yields a cost
-
- The graph illustrates a state transition system for a 4-disk Tower of Hanoi problem. The nodes represent states, and the directed edges represent transitions between states, labeled with actions like 'up(A)', 'dn(B)', 'up(C)', and 'dn(D)'. The graph is highly interconnected, showing the complex nature of the problem space.

As in h_m , take the maximum of these costs \rightarrow admissible heuristic

Pattern Database Heuristics:

State Representation

- For PDB heuristics, a state variable representation is useful
 - Typically:
 - Reduces the number of facts
 - Provides more information about which states are actually reachable!
 - Model problems using the state variable representation, or let planners convert automatically from predicate representation

PDB Heuristics: State Variables (2)

- Example: Blocks world with 4 blocks
 - **536,870,912 states** (reachable and unreachable) in the standard predicate representation
 - But in **all states reachable** from "all-on-table" (all "normal" states):
 - Block A is:
 - Held in the gripper
 - Clear – at the top of a tower (possibly a tower of one block)
 - Below B
 - Below C, or
 - Below D
 - Equivalently: Exactly one of these facts is true *in every reachable state* (mutex!)
 - **(holding A), (clear A), (on B A), (on C A), (on D A)**
 - ➔ Remove those facts, introduce state variable **aboveA** $\in \{ \text{clear, B, C, D, gripper} \}$

■ Example, continued

- 536,870,912 states (reachable and unreachable) in predicate representation
- 20,000 states (reachable and unreachable) in state variable representation:
 - $\text{aboveA} \in \{\text{clear}, B, C, D, \text{gripper}\}$
 - $\text{aboveB} \in \{\text{clear}, A, C, D, \text{gripper}\}$
 - $\text{aboveC} \in \{\text{clear}, A, B, D, \text{gripper}\}$
 - $\text{aboveD} \in \{\text{clear}, A, B, C, \text{gripper}\}$
 - $\text{posA} \in \{\text{on-table}, \text{other}\}$
 - $\text{posB} \in \{\text{on-table}, \text{other}\}$
 - $\text{posC} \in \{\text{on-table}, \text{other}\}$
 - $\text{posD} \in \{\text{on-table}, \text{other}\}$
 - $\text{hand} \in \{\text{empty}, \text{full}\}$

The state variable *translation* is not part of the PDB heuristic!

Using state variables is *useful* because PDBs work better with fewer "irrelevant states" in the state space...

...so we can model using state variables, **or** let the planner rewrite the problem from PDDL predicates/atoms.

Provides more structure: Obvious that A can't be under B and under C

Useful when ignoring facts: Ignore where A is, care about where B is

PDB Heuristics: Rewriting the Problem

- **Rewriting** works as before

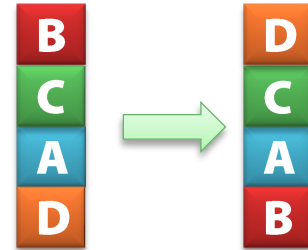
- Suppose the pattern is { **aboveB**, **aboveD**, **posB**, **posD** }

- **Rewrite** the goal

- Suppose that the original goal is expressed as

Original: { aboveB = A, aboveA = C, aboveC = D, aboveD = clear, hand = empty }

- Abstract: { aboveB = A, aboveD = clear }



- **Rewrite** actions, removing some preconds / effects

- (unstack A D) no longer requires aboveA = clear
- (unstack B C) still requires aboveB = clear

- ...

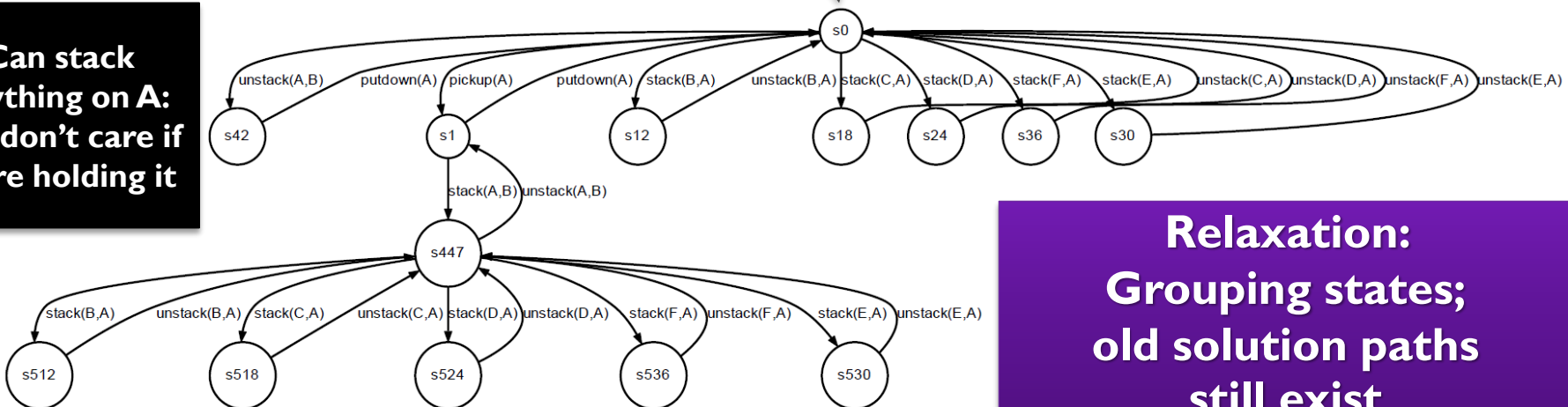
aboveA	∈ { clear, B, C, D, gripper }
aboveB	∈ { clear, A, C, D, gripper }
aboveC	∈ { clear, A, B, D, gripper }
aboveD	∈ { clear, A, B, C, gripper }
posA	∈ { on-table, other }
posB	∈ { on-table, other }
posC	∈ { on-table, other }
posD	∈ { on-table, other }
hand	∈ { empty, full }

PDB Heuristics: State Space Size

- **Abstract** states reachable from "all on table", **by pattern**...

Blocks	All variables	Pattern={aboveA}	{aboveA,aboveB}
4	125	10	96
5	866	12	140
6	7057	14	192
7	65990	16	252
8	695417	18	320
9	8145730	20	396

Can stack anything on A:
We don't care if we're holding it



Relaxation:
Grouping states;
old solution paths
still exist

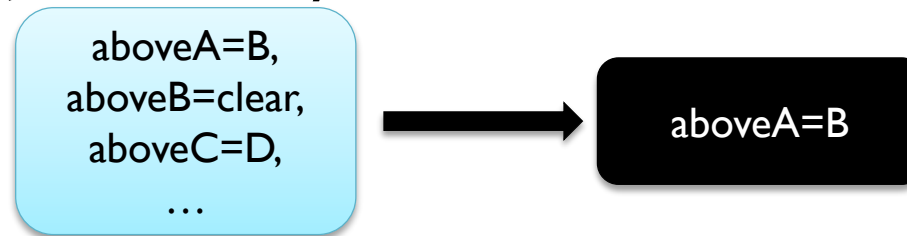
Pattern Database Heuristics:

Computation

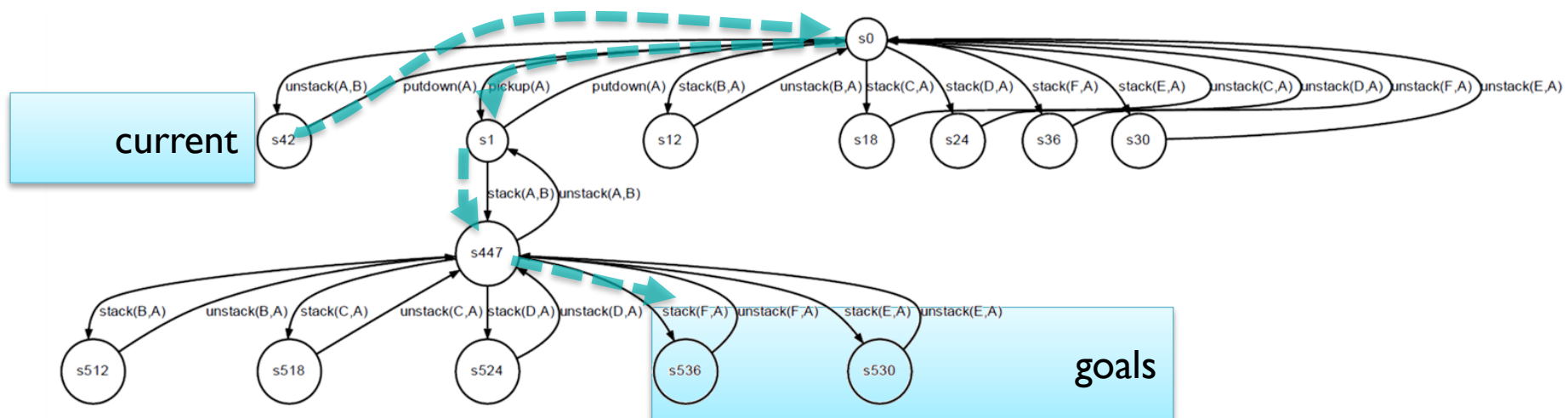
PDB Computation: Main Idea

- To calculate $h(s)$ for a newly encountered state s :

- Convert to abstract state

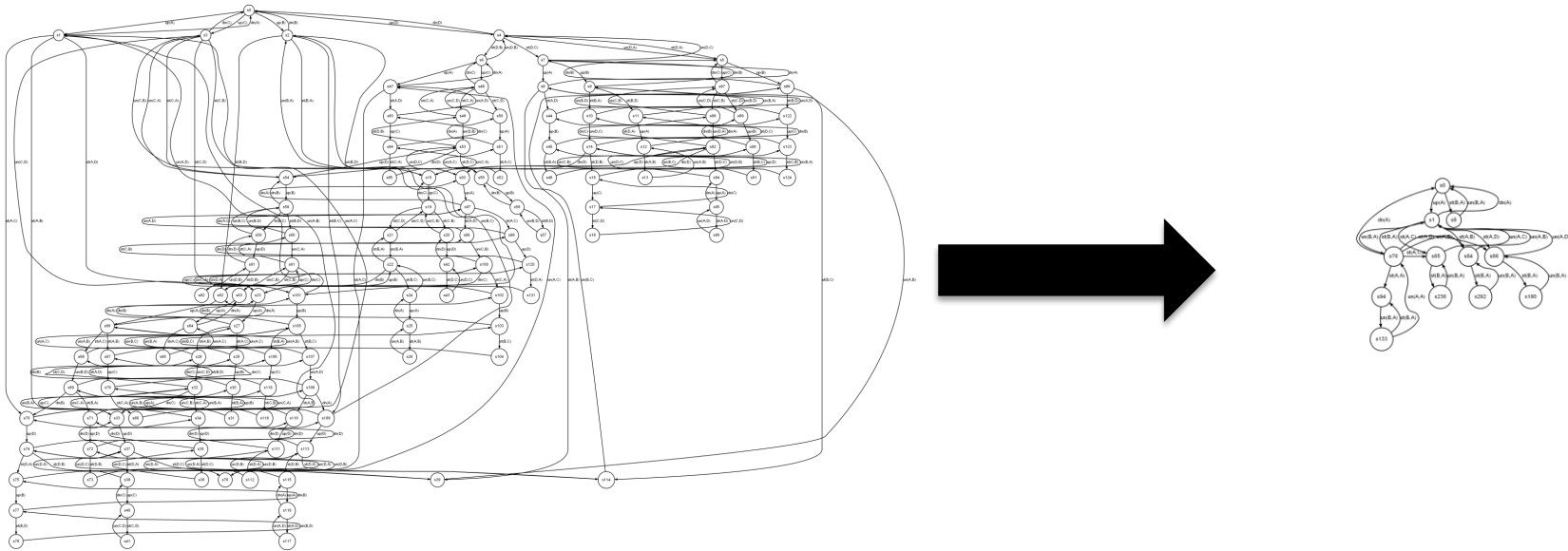


- Find **optimal** path to abstract goal state – in a much smaller search space!
 - Fast, using (for example) Dijkstra
 - Relaxation → **path cost** is an admissible heuristic



PDB Heuristics: Databases!

- Because we keep *few* state variables:
 - Many real states map to the *same* abstract state
 - → Every abstract state may be encountered many times during search
 - → **Cache** calculated costs



- Dijkstra efficiently finds optimal paths from *all* abstract states
 - → Precalculate **all** heuristic values for each pattern
 - Store in a look-up table – a **database**

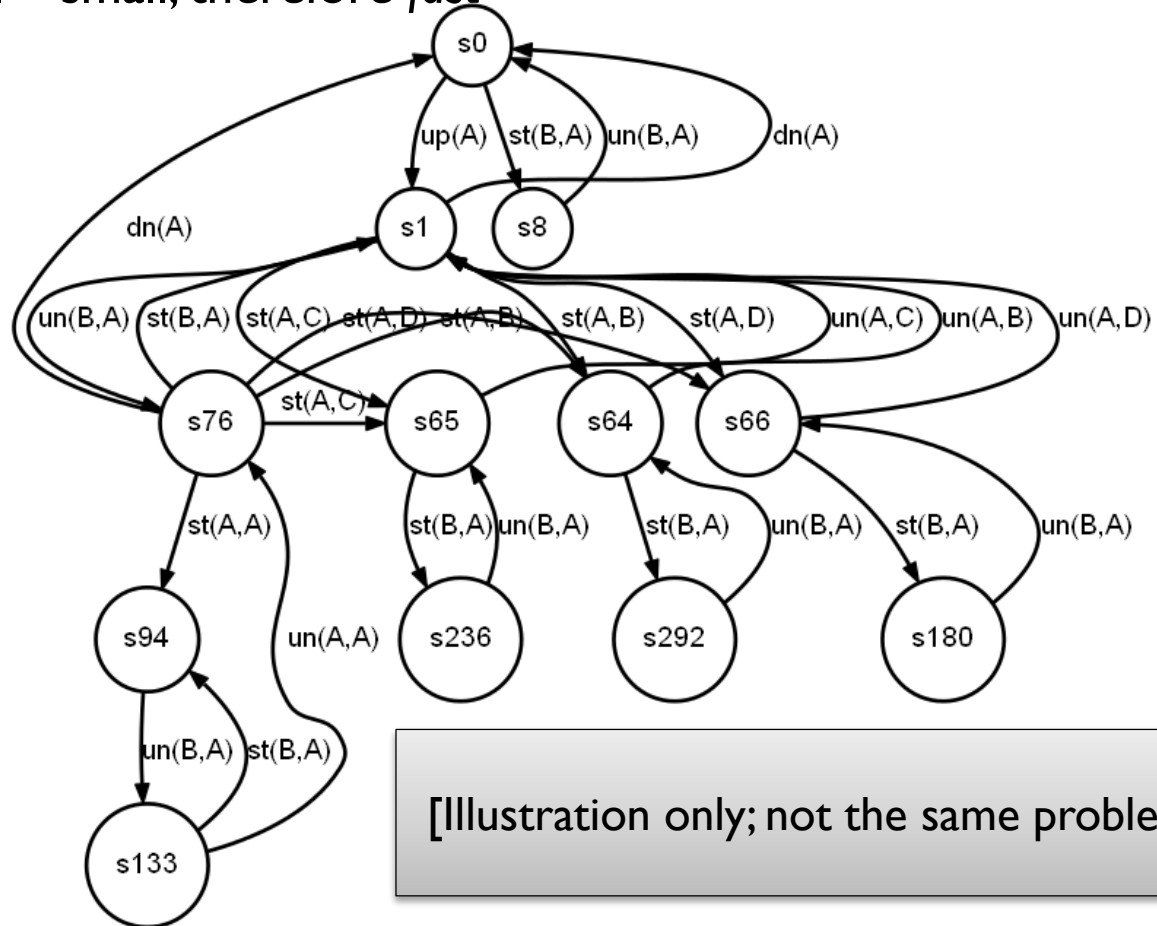
PDB Heuristics: Calculating (1)

- Preprocessing step I:

Find all abstract states *reachable* from the abstract initial state

- Exhaustive search – small, therefore *fast*

aboveA=clear,
aboveB=clear,
aboveC=clear,
aboveD=clear,
posA=on-table,
posB=on-table,
posC=on-table,
posD=on-table,
hand=empty

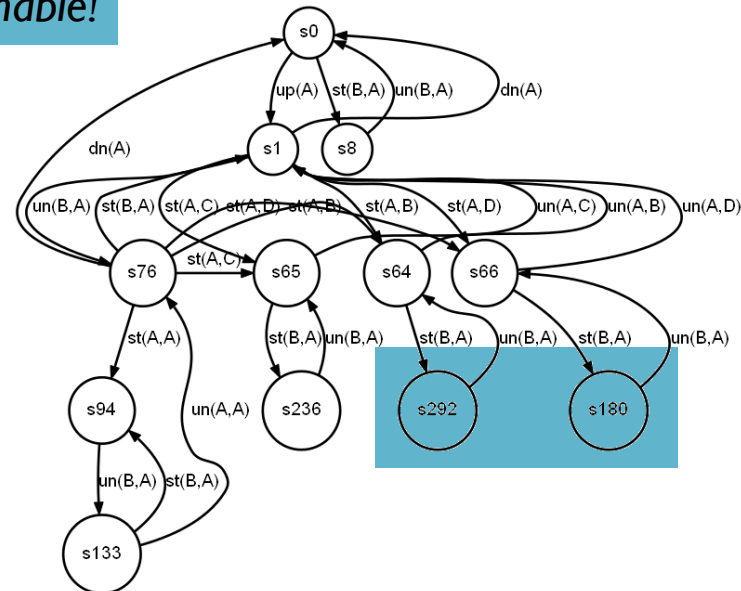


[Illustration only; not the same problem]

PDB Heuristics: Calculating (2)

■ Preprocessing step 2: Which states satisfy the *abstract goal*?

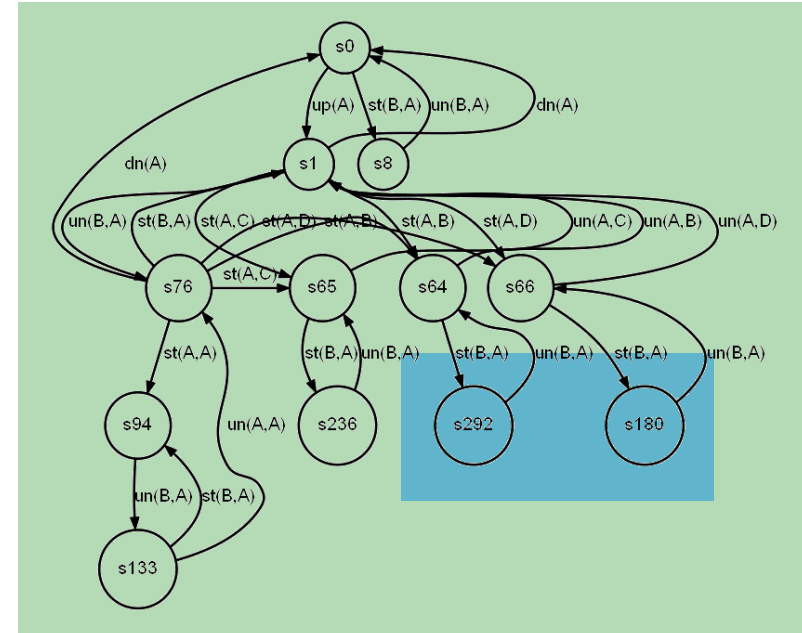
- Real goal = { aboveB = A, aboveA = C, aboveC = D, aboveD = clear, hand = empty }
- Abs. goal = { aboveB = A, aboveD = clear }
- Abs. goal states = { aboveB = A, aboveD = clear, posB = on-table, posD = on-table },
{ aboveB = A, aboveD = clear, posB = on-table, posD = other },
{ aboveB = A, aboveD = clear, posB = other, posD = on-table },
{ aboveB = A, aboveD = clear, posB = other, posD = other }
- Maybe only a subset of these are **reachable!**



PDB Heuristics: Calculating (3)

■ Preprocessing step 3: Compute the database

- For every abstract state reachable from the abstract initial state,
- find a cheapest path to any abstract goal state



- Can be done with backward search from the set of reachable abstract goal states, using Dijkstra's algorithm

PDB Heuristics: Calculating (4)

Reachable abstract goal states

aboveB = A,
aboveD = clear,
posB = on-table,
posD = on-table

aboveB = A,
aboveD = clear,
posB = on-table,
posD = other

aboveB = A,
aboveD = clear,
posB = other,
posD = on-table

aboveB = A,
aboveD = clear,
posB = other,
posD = other

stack(A,B)

putdown(D)

putdown(D)

aboveB = **clear**,
aboveD = clear,
posB = on-table,
posD = on-table

aboveB = A,
aboveD = **gripper**,
posB = on-table,
posD = on-table

aboveB = A,
aboveD = **gripper**,
posB = on-table,
posD = **other**

.....

PDB Heuristics: Databases

Abstract goal states

aboveB = A,
aboveD = clear,
posB = on-table,
posD = on-table

cost 0

aboveB = A,
aboveD = clear,
posB = on-table,
posD = other

cost 0

aboveB = A,
aboveD = clear,
posB = other,
posD = on-table

cost 0

aboveB = A,
aboveD = clear,
posB = other,
posD = other

cost 0

stack(A,B)

putdown(D)

putdown(D)

Assuming $\text{cost}(\text{stack/unstack})=2$,
 $\text{cost}(\text{pickup/putdown})=1$

aboveB = **clear**,
aboveD = clear,
posB = on-table,
posD = on-table

cost 2

aboveB = A,
aboveD = **gripper**,
posB = on-table,
posD = on-table

cost 1

aboveB = A,
aboveD = **gripper**,
posB = on-table,
posD = other

cost 1

...and so on
for all reachable
abstract states

This database represents an *admissible heuristic*!

Given a *real* state:

Find the unique abstract state that matches; **return** its precomputed cost

- Database:
 - Stores one cost for every abstract state s
 - Cost is optimal within the relaxed problem
 - Cost is admissible for the “real” problem
- For the database to be computable in polynomial time:
 - As problem instances grow, the pattern can (only) grow to include a *logarithmic* number of variables
 - Problem size n , maximum number of values for a state variable $d \rightarrow$
number of pattern variables: $O(\log n)$,
number of abstract states for the pattern: $O(d^{\log n}) = O(n^{\log d})$
 - Dijkstra is polynomial in the number of states

PDB Heuristics: Gripper Example

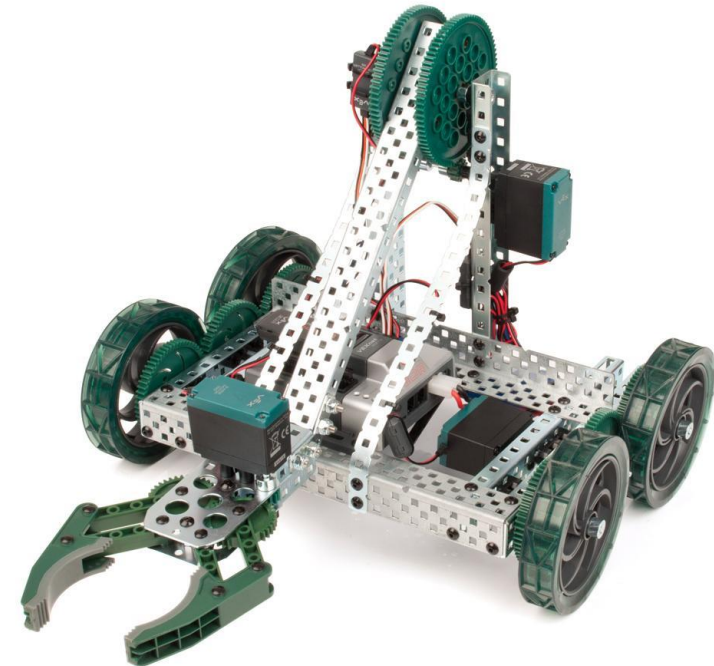
- A common *restricted* gripper domain:
 - **One** robot with **two** grippers
 - **Two** rooms
 - All n balls originally in the first room
 - Objective: All balls in the second room

Compact state variable representation:
 $\text{loc}(\text{ball}_k) \in \{ \text{room1}, \text{room2}, \text{gripper1}, \text{gripper2} \}$
 $\text{loc-robot} \in \{ \text{room1}, \text{room2} \}$

$2 * 4^n$ states, some unreachable – which ones?

Some possible patterns for $n \geq 1$ balls:

$\{ \text{loc}(\text{ball}_1) \}$	→ 4 abstract states
$\{ \text{loc}(\text{ball}_1), \text{loc-robot} \}$	→ 8 abstract states
$\{ \text{loc}(\text{ball}_k) \mid k \leq n \}$	→ 4^n abstract states
$\{ \text{loc}(\text{ball}_k) \mid k \leq \log(n) \}$	→ $4^{\log(n)}$ abstract states



**How are PDBs used
when solving the original planning problem?**

Step 1: Using a single pattern

PDB Heuristics in Forward Search (1)



- Step 1: **Automatically** generate a pattern
 - A selection of state variables to consider
 - Choosing a **good** pattern is a **difficult problem!**
 - Different approaches exist...

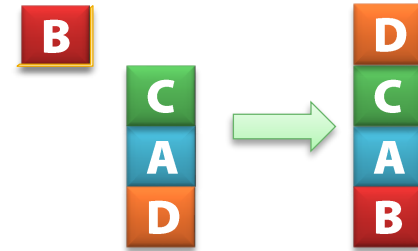
- Step 2: Calculate the pattern database
 - As already discussed

PDB Heuristics in Forward Search (2)

■ Step 3: Forward search in the original problem

- For each new successor state s_1 , calculate heuristic value $h_{pdb}(s_1)$

- Example: $s_1 = \{$ aboveD = A, aboveA = C, aboveC = clear, aboveB = gripper,
 posA = other, posB = other,
 posC = other, posD = on-table,
 hand = full $\}$



- Convert this to an *abstract* state
 - Example: $s'_1 = \{$ aboveB = gripper, aboveD = A, posB = other, posD = on-table $\}$
- Use the database to quickly look up $h_{pdb}(s_1) =$
the cost of reaching the nearest abstract goal from s'_1

aboveB = gripper, aboveD = A, posB = other, posD = on-table \rightarrow cost $n1$
aboveB = gripper, aboveD = A, posB = other, posD = other \rightarrow cost $n2$
...

**How can PDB heuristics
become more informative?**

- How close to $h^*(n)$ can an admissible PDB-based heuristic be?
 - Assuming we require polynomial computation:
 - Problem size n grows \rightarrow number of variables in a pattern can grow as $O(\log n)$
 - $h(n) \leq$ cost of reaching the **most expensive subgoal** of size $O(\log n)$

Significant differences compared to h_m heuristics!

Subgoal size is not constant but grows with problem size

On the other hand, does not consider *all* subgoals of a particular size

Decides state variables *in advance* – for h_m , facts are chosen *on each level*

- But still, $\log(n)$ grows much slower than n
 - \rightarrow For any given pattern, asymptotic accuracy is (often) 0
 - As before, *practical results* can be better!

- How to increase information?
 - Can't increase the size of a pattern beyond logarithmic growth...

- Can use multiple patterns!
 - For each pattern, compute a separate pattern database
 - Each such cost is an admissible heuristic
 - So the maximum over many different patterns is also an admissible heuristic

- What is the new level of accuracy?
 - Still 0... *asymptotically*
 - But this can still help *in practice*!

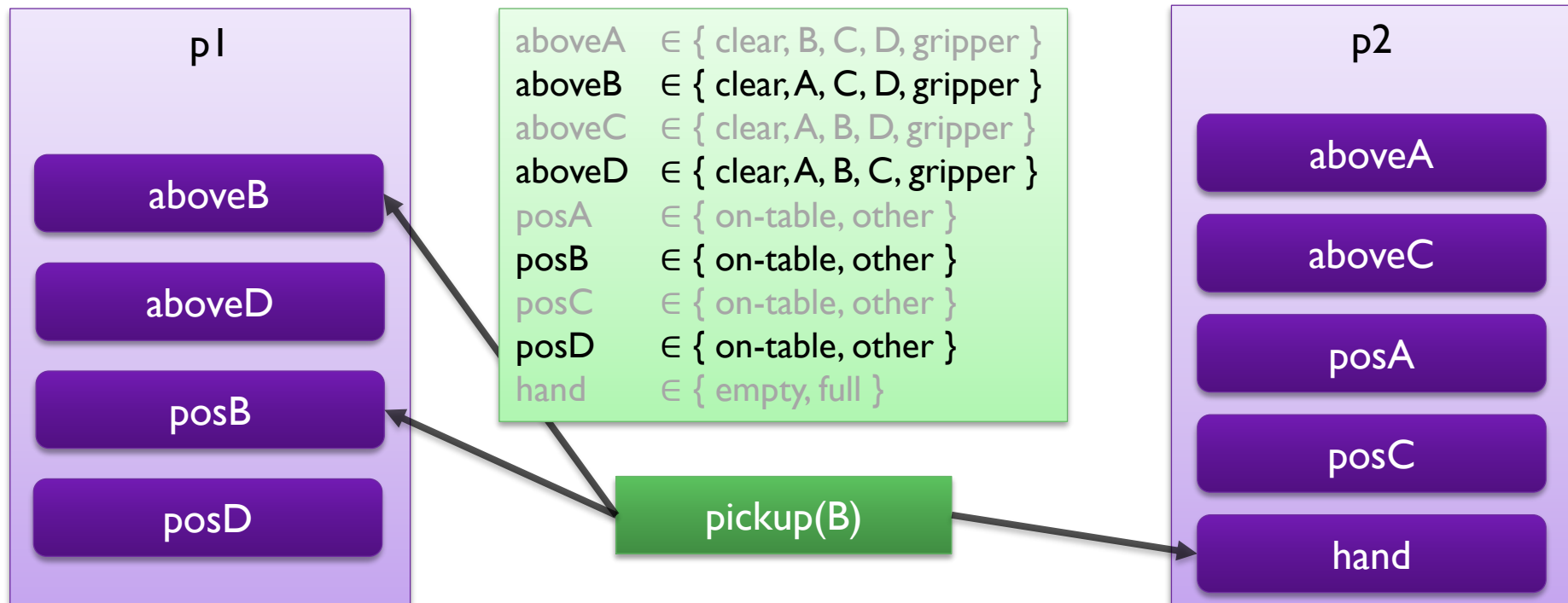
Additive PDB Heuristics (1)



- To improve further:
 - Define multiple patterns
 - Sum the heuristic values given by each pattern
- As in h_{add} , this could lead to overestimation problems
 - Some of the effort necessary to reach the goal is counted twice
- To avoid this and create an admissible heuristic:
 - Each fact should be in *at most* one pattern
 - Each action should affect facts in *at most* one pattern
 - → Additive pattern database heuristics

Additive PDB Heuristics (2)

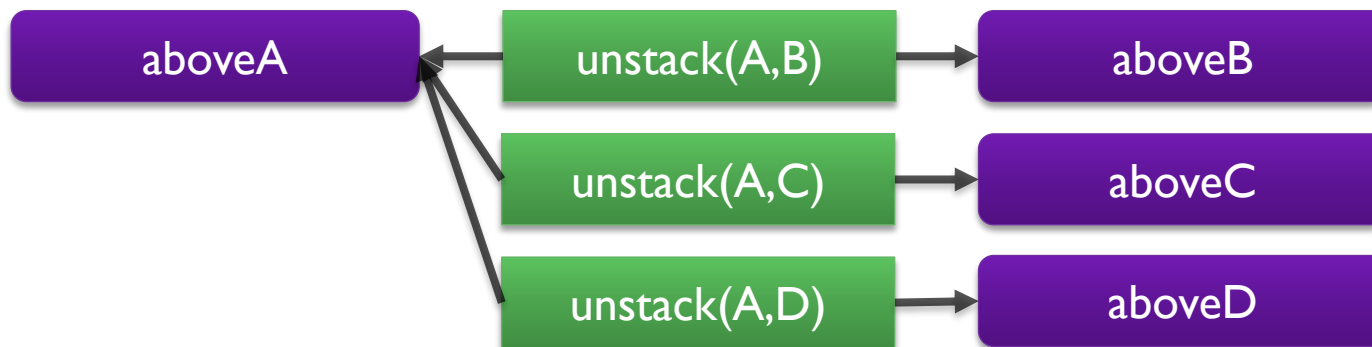
- BW: Is $p1 = \{\text{facts in even rows}\}$, $p2 = \{\text{facts in odd rows}\}$ *additive*?
 - No: pickup(B) affects {aboveB, posB} in $p1$, {hand} in $p2$



One potential problem:
Both patterns could use pickup(B) in their optimal solutions
→ sum counts this twice! This is what we're trying to avoid...

Additive PDB Heuristics (3)

- BW: Is $p1=\{\text{aboveA}\}$, $p2=\{\text{aboveB}\}$ additive?
 - No: $\text{unstack}(A,B)$ affects $\{\text{aboveB}\}$ in $p1$, $\{\text{aboveA}\}$ in $p2$
 - True for *all* combinations of aboveX

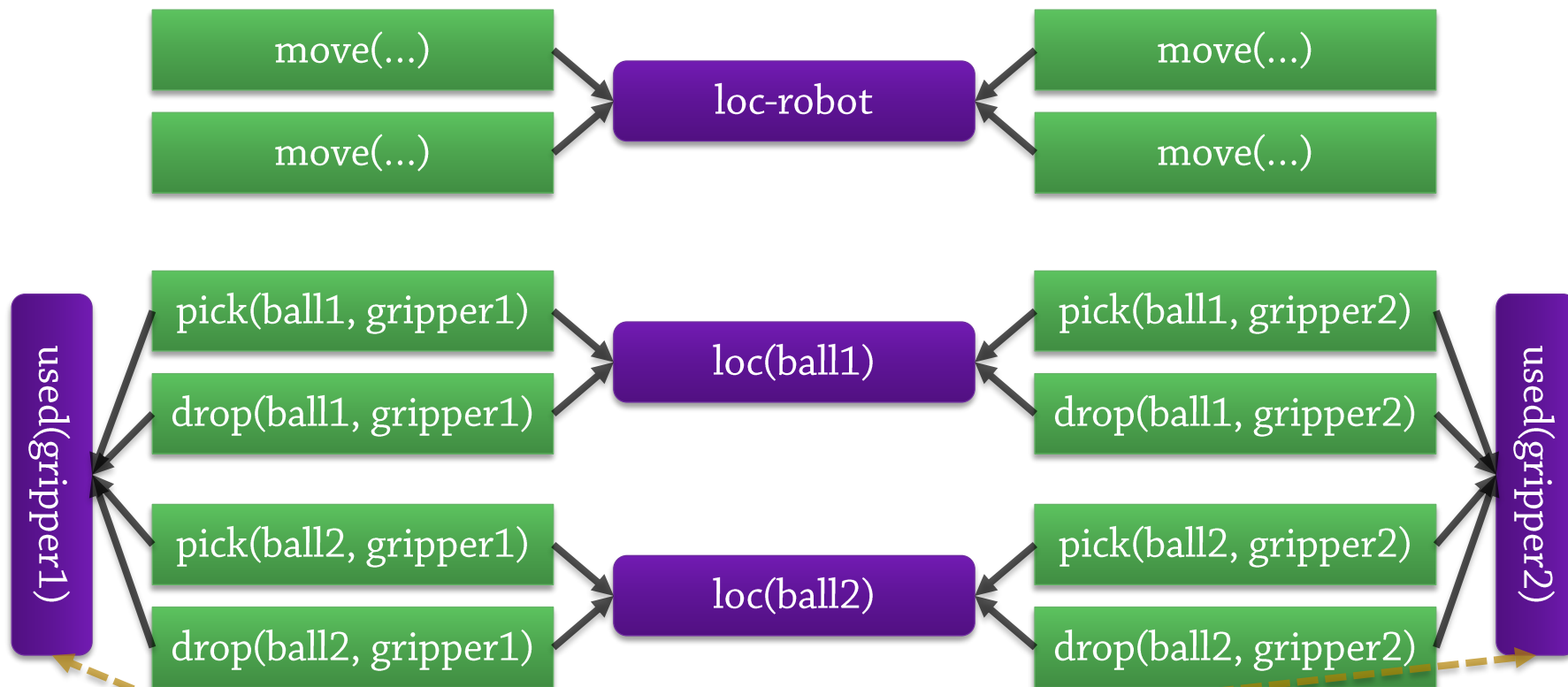


- An additive PDB heur. could use **one** of these:
 - $p1 = \{\text{aboveA}\}$
 - $p1 = \{\text{aboveA}, \text{aboveC}, \text{aboveD}\}$
 - ...
- Can't have **two** separate patterns $p1, p2$ both of which include an aboveX
 - Those aboveX will be directly connected by some unstack action

**This formulation of the
Blocks World is
"connected in the wrong way"
for this approach
to work well**

Additive PDB Heuristics (4)

- "Separating" patterns in the Gripper domain:

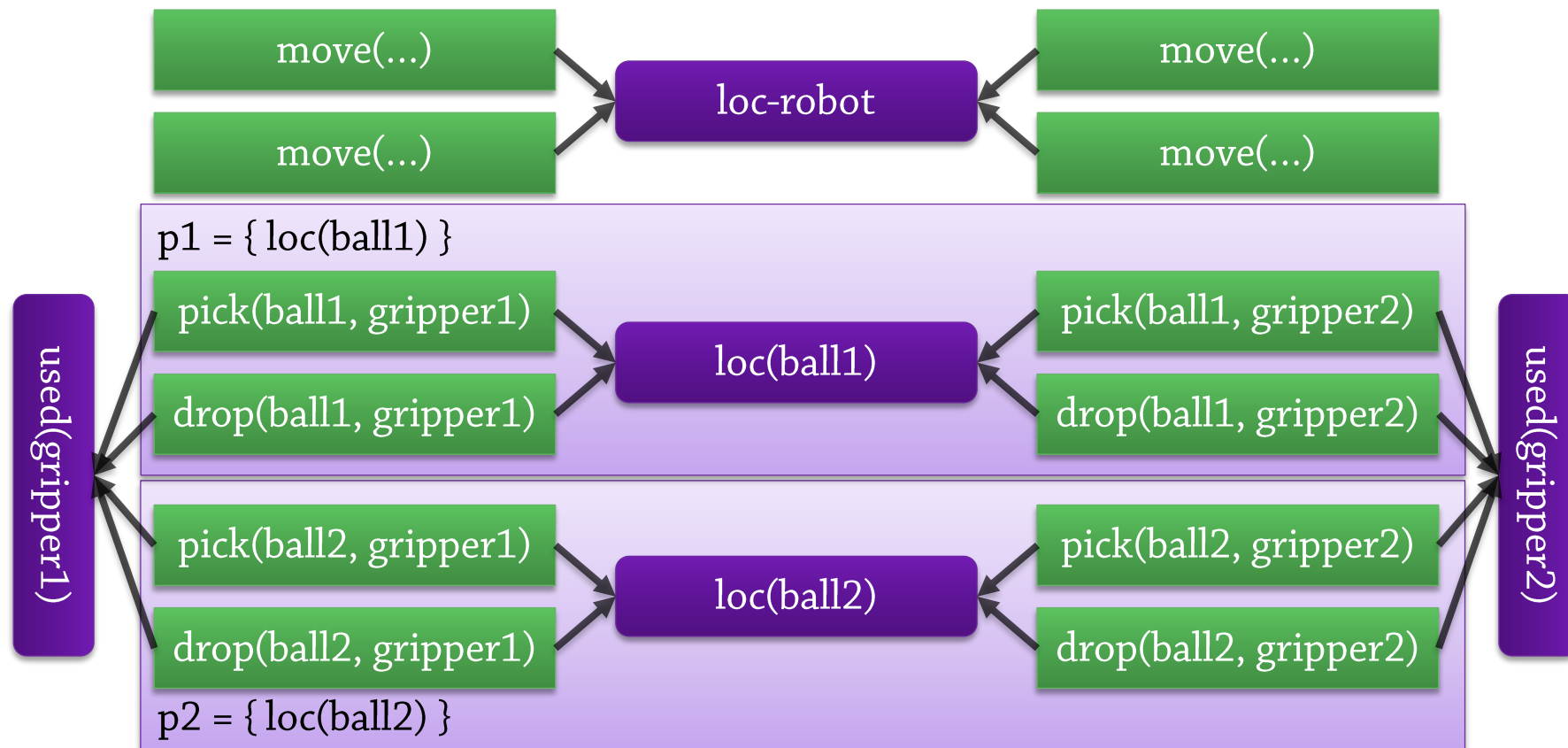


Are these a problem?

$loc(ball_k) \in \{ room1, room2, gripper1, gripper2 \}$
 $loc-robot \in \{ room1, room2 \}$
 $used(gripper_k) \in \{ true, false \}$

Additive PDB Heuristics (5)

- No problem: We don't have to use all variables in patterns!



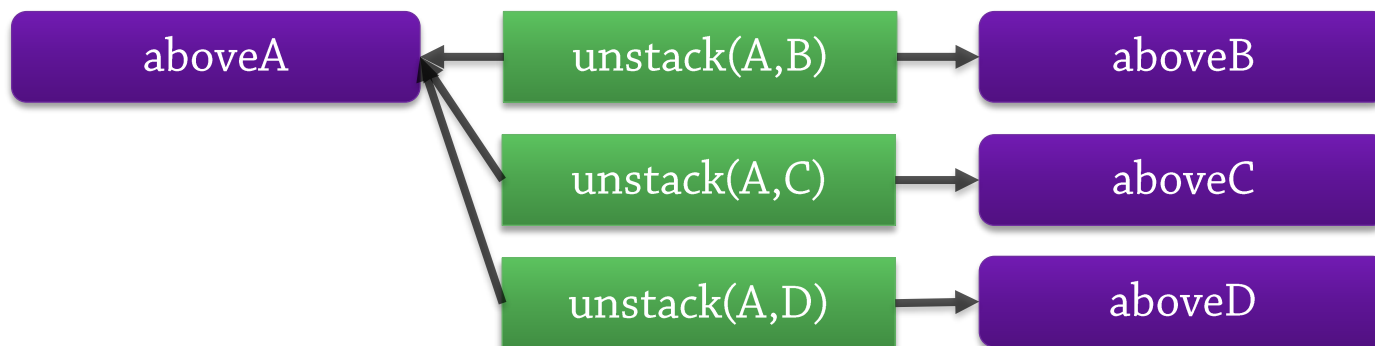
For each pattern we chose one variable

Then we have to include all actions affecting it

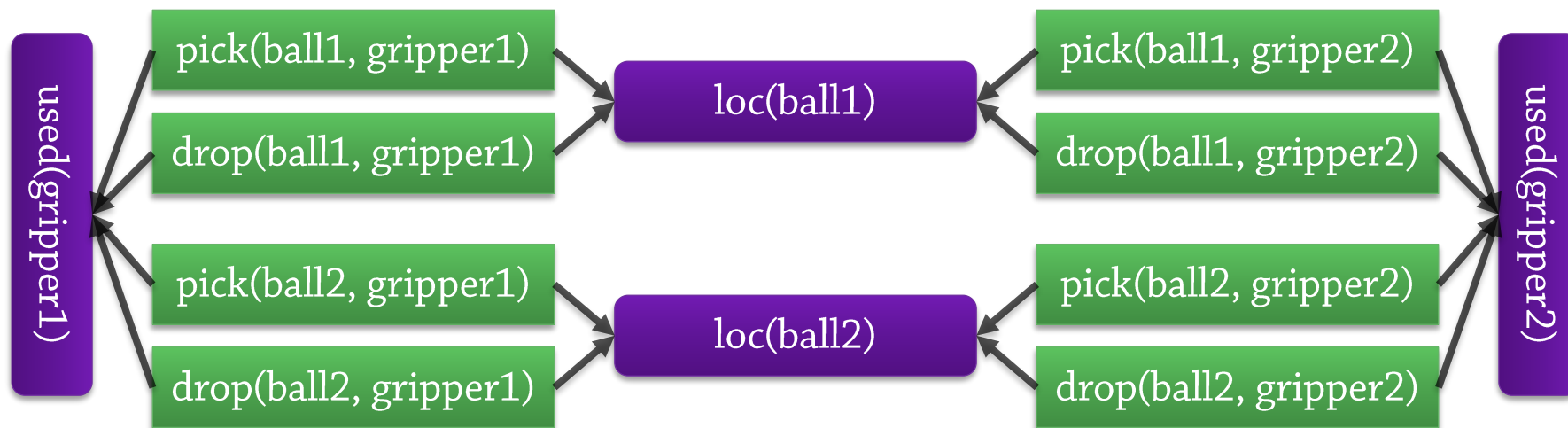
The other variables those actions affect [`used()`] don't have to be part of *any* pattern!

Additive PDB Heuristics (6)

- Notice the difference in structure!



BW: Every pair of aboveX facts has a *direct connection* through an action

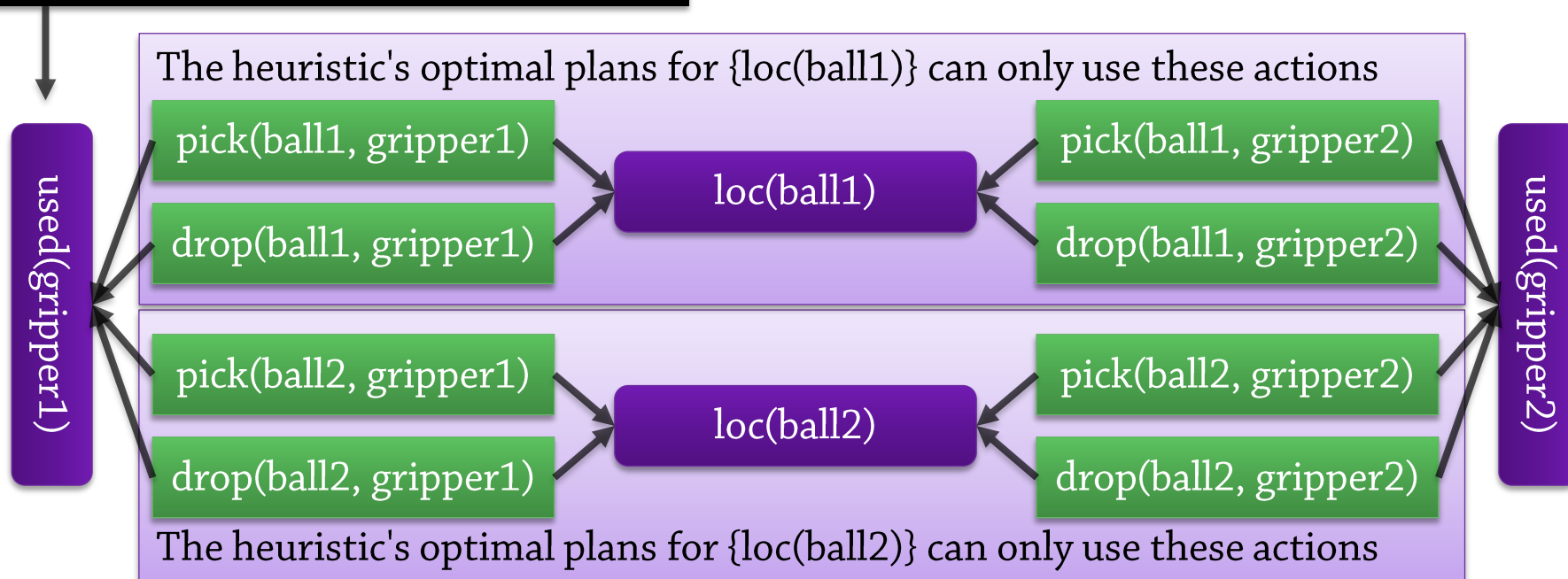


Gripper: No pair of loc() facts has a *direct connection* through an action

Additive PDB Heuristics (7)

- When every action affects facts in at most *one* pattern:
 - The subproblems we generated are completely *disjoint*
 - They achieve **different aspects** of the *goal*
 - Optimal solutions **must** use **different actions**

The heuristic never tries to generate optimal plans for `used(gripper1)` – we have not included it in any pattern



Additive PDB Heuristics (8)



- Avoids the overestimation problem we had with h_{add}

Problem earlier:

Goal: p and q

A1: effect p

A2: effect q

A3: effect p and q

To achieve p: Heuristic uses A1

To achieve q: Heuristic uses A2

Sum of costs is 2 – optimal cost is 1, using A3

This cannot happen

when every action affects facts in at most one pattern

- The costs are additive for multiple patterns
- Adding costs from multiple heuristics yields an *admissible* heuristic!

Additive PDB Heuristics (9)

- Can be taken one step further...
 - Suppose we have several sets of additive patterns:
 - Can calculate an admissible heuristic from **each** additive set, then take the **maximum** of the results as a **stronger** admissible heuristic

$$\text{Max} \rightarrow \text{admissible heuristic } h_{pdb}^3(s) = \max(h_{pdb}^1(s), h_{pdb}^2(s))$$

$$\text{Sum} \rightarrow \text{admissible heuristic } h_{pdb}^1(s)$$

p1

p2

p3

p4

4 patterns satisfying
additive constraints

$$\text{Sum} \rightarrow \text{admissible heuristic } h_{pdb}^2(s)$$

p5

p6

p7

p8

p9

5 patterns satisfying
additive constraints

- **How close** to $h^*(n)$ can an **additive** PDB-based heuristic be?
 - For additive PDB heuristics with a single sum, **asymptotic accuracy** as problem size approaches infinity...
- **In Gripper:**
 - In state s_n there are n balls in room 1, and no balls are carried
 - Additive PDB heuristic $h_{add}^{PDB}(s_n)$:
 - One singleton pattern for each ball location variable $loc(ball_k)$
 - For each pattern, the optimal cost is 2
 - $pick(ball, room1, gripper1): loc(ball)=room1 \rightarrow loc(ball)=gripper1$
 - $drop(ball, room2, gripper1): loc(ball)=gripper1 \rightarrow loc(ball)=room2$
 - $h_{add}^{PDB}(s_n) = \text{sum for } n \text{ balls} = 2n$
 - Real cost:
 - Use both grippers: $pick, pick, move(room1, room2), drop, drop, move(room2, room1)$
 - Repeat $n/2$ times, total cost $\approx 6n/2 = 3n$
 - \rightarrow Asymptotic accuracy $2n/3n = 2/3$

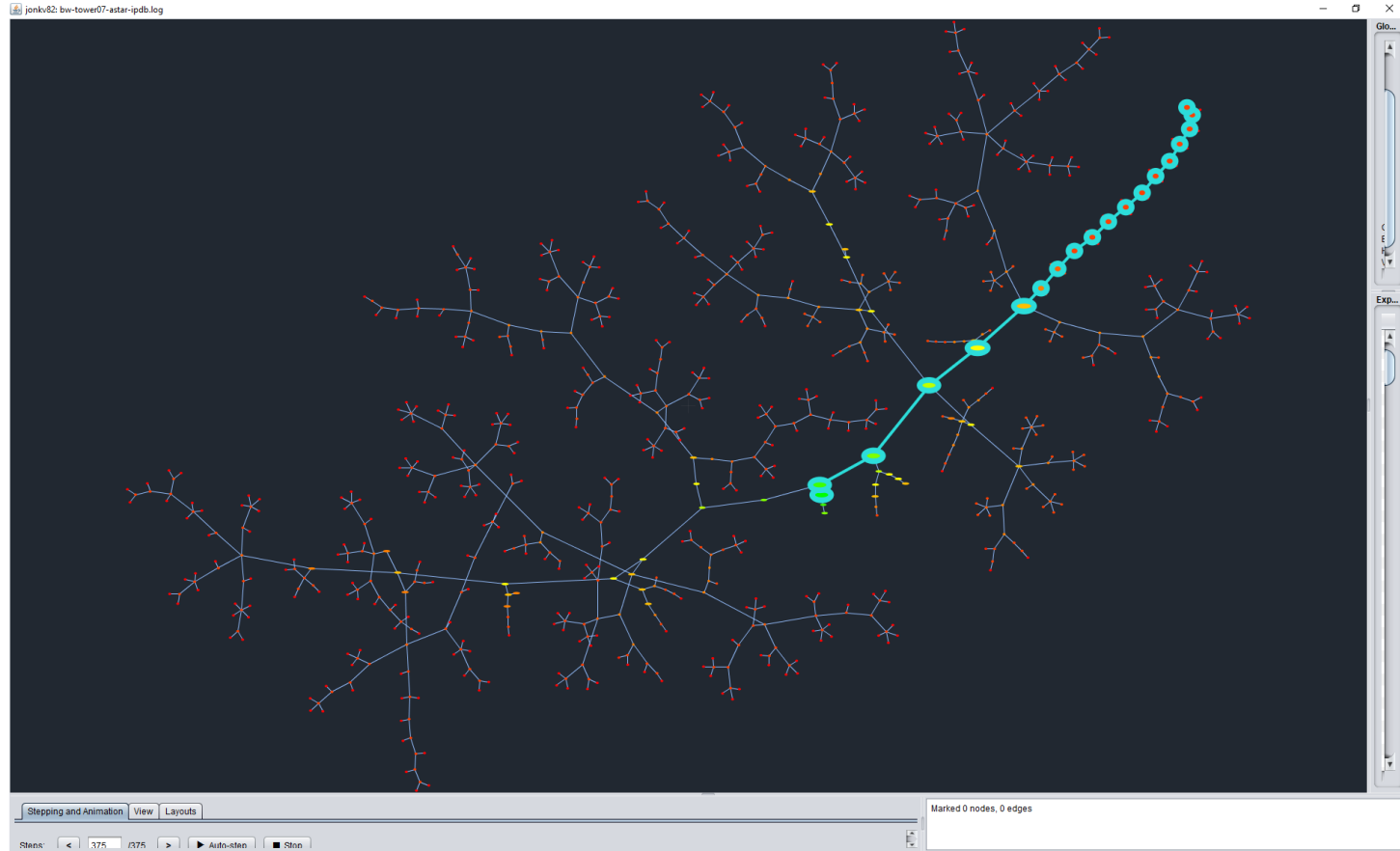
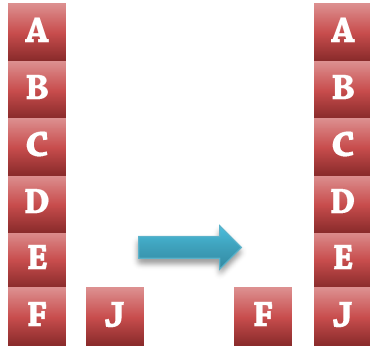
Additive PDB Heuristics (11)

- **How close** to $h^*(n)$ can an **additive** PDB-based heuristic be?
 - For additive PDB heuristics with a single sum,
asymptotic accuracy as problem size approaches infinity:

	h^+ (too slow!)	h_2	Additive PDB
Gripper	2/3	0	2/3
Logistics	3/4	0	1/2
Blocks world	1/4	0	0
Miconic-STRIPS	6/7	0	1/2
Miconic-Simple-ADL	3/4	0	0
Schedule	1/4	0	1/2
Satellite	1/2	0	1/6

- **Only guaranteed** if the planner **finds** the best combination of patterns!
 - This is a very difficult problem in itself!
- But as usual, this is a worst-case analysis...

bw-tower07-astar-ipdb: Only 7 blocks, A* search, based on PDB variation



- Blind A*: 43150 states calculated, 33436 visited
- A* + goal count: 6463 states calculated, 3222 visited
- A* + iPDB: 1321 states calculated, 375 visited

No heuristic is perfect – visiting some additional states is fine!

Heuristics part IV

An Overview of Landmark Heuristics

Landmark Heuristics (1)

Landmark:

"a **geographic feature** used by explorers and others to **find their way** back or through an area"



Landmark Heuristics (2)

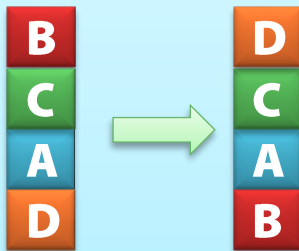
Landmarks in planning:

Something you must *pass by/through* in every solution to a specific planning problem

Assume we are currently in state s ...

Fact Landmark for s :

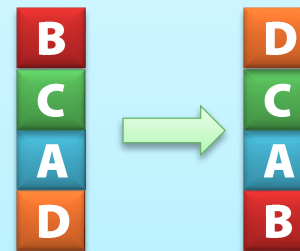
A **fact** that is **not true** in s ,
but must be true at some point
in every solution starting in s



clear(A)
holding(C)
...

Formula Landmark for s :

A **formula** that is **not true** in s ,
but must be true at some point
in every solution starting in s



clear(A) \wedge handempty
...

Landmark Heuristics (3)

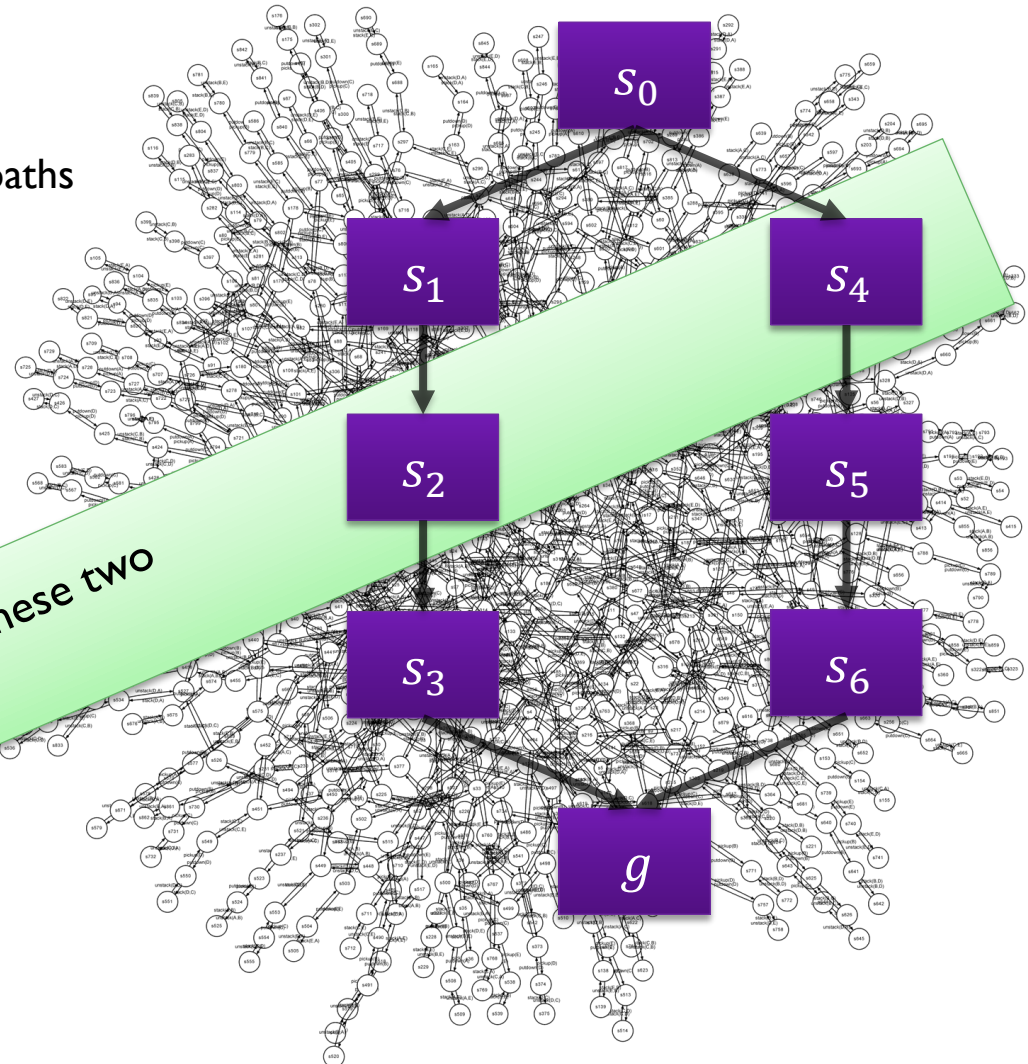
Facts and formulas, not states! Why?

- Usually **many** paths lead from s to a goal state
 - Few **states** are shared among **all** paths
 - Many **facts** occur along **all** paths

Not "we must reach **the** landmark state"!

Instead "we must reach **some state** that satisfies the fact/formula landmark"

Many facts shared among these two
→ occur along all paths



Landmark Heuristics (4)

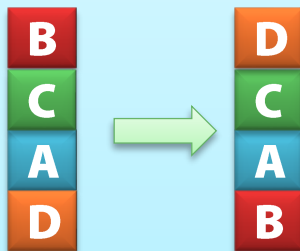
Landmarks in planning:

Something you must *pass by/through* in every solution to a specific planning problem

Assume we are currently in state $s...$

Fact Landmark for s :

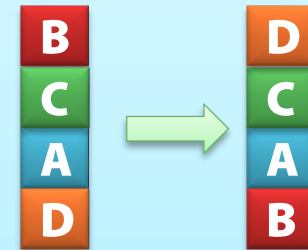
A **fact** that is not true in s , but must be true at some point in every solution starting in s



clear(A)
holding(C)
...

Action Landmark for s :

An **action** that must be used in every solution starting in s



unstack(B,C)
putdown(B)
stack(D,C)

...but *not* putdown(C)! (Why?)

...so the effects of action landmarks are *fact landmarks*, and so are their *preconds*

(except those facts that are already true in s)

Landmark Heuristics (5)



- Generalization:
 - **Disjunctive** action landmark $\{a_1, a_2, a_3\}$ for state s
 - Every solution starting in state s and reaching a goal must use *at least one* of these actions

Finding Landmarks: A (Too) General Technique

Finding Landmarks: General Technique

- One general technique for discovering landmarks:

Current planning problem, P

Initial state does not include atom A



Modified planning problem, P'

*Removed all actions
that add atom A*



...then **every** solution to P
must use one of the removed actions

→ Action set is a disj. act. landmark

→ Atom A is a fact landmark



If this problem (P') is **unsolvable**...

Test:

Delete relaxation of P' is
unsolvable,

or $h_m(s_0) = \infty$, or ...

→ P' is unsolvable

Unsolvable when removing a set of actions

→ some action in the set must be used → **disjunctive action landmark!**

Finding Landmarks: General Technique (2)



- This technique is very general
 - Applicable to *any* planning problem, *any* atom
- General techniques tend to be widely applicable but slow...

Verifying Landmarks (1)



- How difficult is it to verify that an action is an action landmark, in the general case?
 - Suppose we can verify this
 - Then given any STRIPS problem P , we can determine if it has a solution:
 - Add a new action:
 - cheat
 - :precond true
 - :effects goal-formula
 - If cheat is an action landmark, then it is *needed* in order to solve the problem
 - ➔ the original problem was *unsolvable*
 - ➔ As difficult as solving the planning problem (PSPACE-complete)

Verifying Landmarks (2)

- How difficult is it to verify that a fact is a fact landmark, in the general case?
 - Suppose we can verify this
 - Then given any STRIPS problem P , we can determine if it has a solution:
 - Add a new fact:
 - cheated (false in the initial state)
 - Add new action:
 - cheat
:precond true :effects
(and cheated goal-formula)
 - If cheated is a fact landmark,
then cheat was necessary → the original problem was unsolvable
 - → Again , as difficult as solving the planning problem

But of course there are special cases...

Finding Landmarks: Efficiently

Means-Ends Analysis

- Discover landmarks using means-ends analysis

Unachieved goals are (obviously)
fact landmarks:

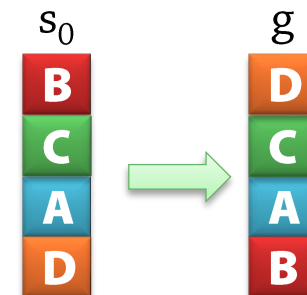
clear(D), on(D,C), on(A,B), ontable(B)

on(D,C) is a landmark,
on(D,C) is not true in the current state (s)
→ we must *cause* **on(D,C)** with an action
→ compute *achievers* = { **stack(D,C)** }

All achievers require candidates =
{ **holding(D), handempty, clear(C), ...** }

handempty is already true, but
 $new = \{ \mathbf{holding(D)}, \mathbf{clear(C)} \}$ are *not*

Maybe we can find more landmarks
related to achieving *those*!



$\text{fact-landmarks} \leftarrow g - s$

```
do {  
  for each p in fact-landmarks {  
    // Create disjunctive action landmark  
    achievers  $\leftarrow \{a \in A \mid p \in \text{eff}(a)\}$ 
```

$\text{candidates} \leftarrow \bigcap_{a \in \text{achievers}} \text{pre}(a)$

```
new  $\leftarrow$  candidates – s  
fact-landmarks  $\leftarrow$  fact-landmarks  $\cup$  new
```

```
}  
} until no more fact-landmarks found
```

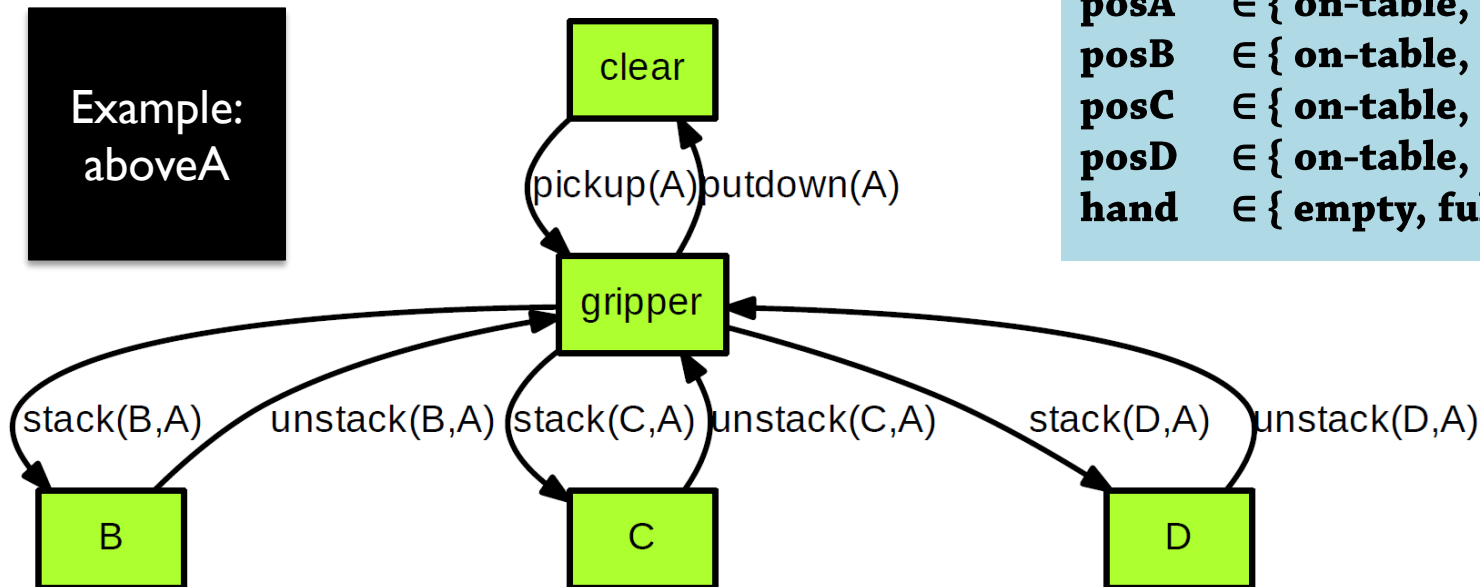
- Extensions to backwards means-ends analysis:
 - **Effects of disjunctive action landmarks:**
 - All shared **effects** must also take place regardless of the "chosen" action, similarly to shared *preconditions* on the previous page
 - Given a disjunctive action landmark, every fact in $(\bigcap \{\text{eff}(a) \mid a \in \text{landmark}\} - s)$ is a fact landmark for s

Domain Transition Graphs (1)

■ General concept: domain transition graphs

- Assume a state variable representation
 - Each variable has a domain, a set of possible values
- For each state variable:
 - Add a node for each value
 - Add an edge for each action changing the value

$\text{aboveA} \in \{ \text{clear}, B, C, D, \text{gripper} \}$
 $\text{aboveB} \in \{ \text{clear}, A, C, D, \text{gripper} \}$
 $\text{aboveC} \in \{ \text{clear}, A, B, D, \text{gripper} \}$
 $\text{aboveD} \in \{ \text{clear}, A, B, C, \text{gripper} \}$
 $\text{posA} \in \{ \text{on-table}, \text{other} \}$
 $\text{posB} \in \{ \text{on-table}, \text{other} \}$
 $\text{posC} \in \{ \text{on-table}, \text{other} \}$
 $\text{posD} \in \{ \text{on-table}, \text{other} \}$
 $\text{hand} \in \{ \text{empty}, \text{full} \}$



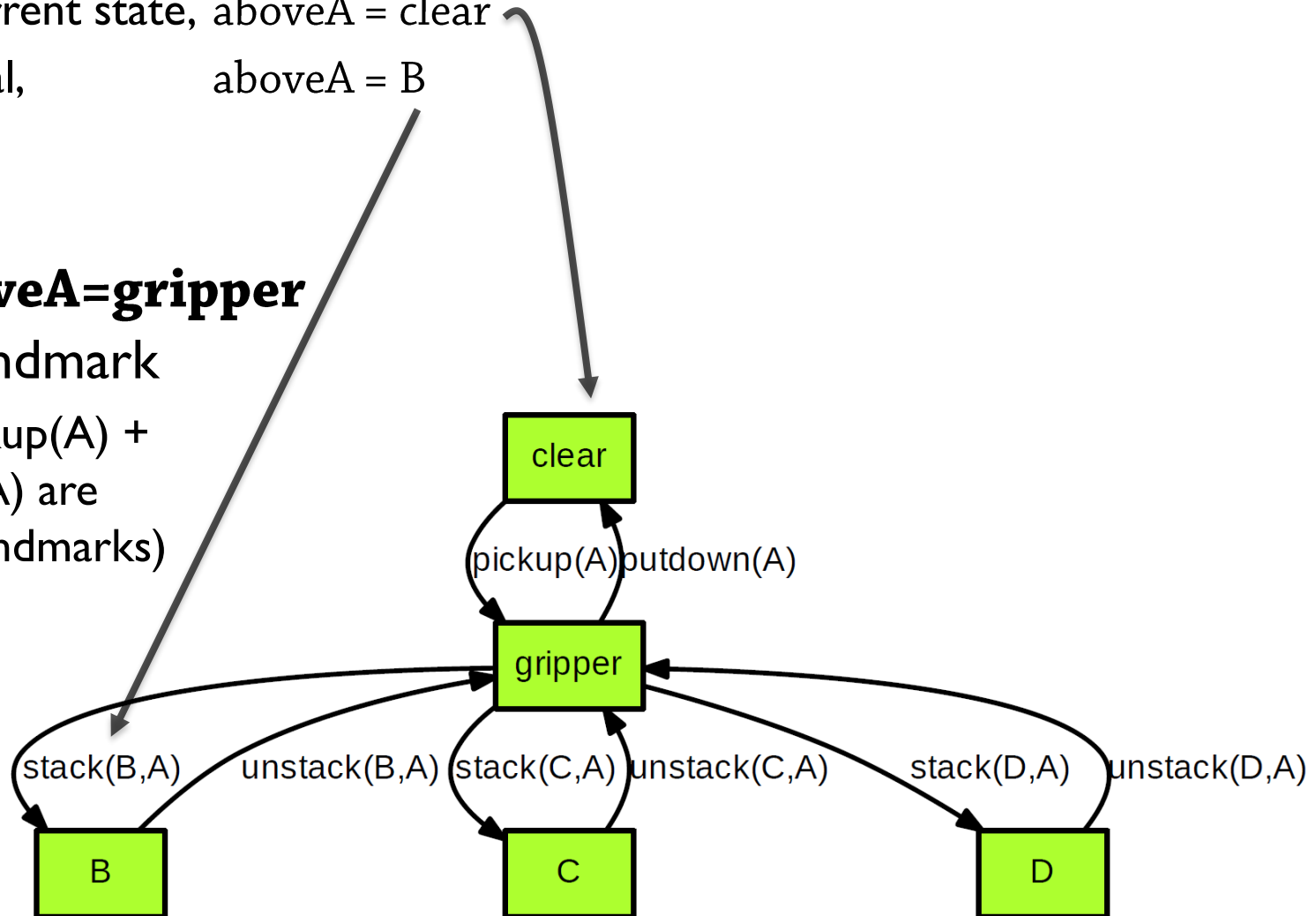
Landmarks from DTGs

- Suppose:

- In the current state, $\text{aboveA} = \text{clear}$
- In the goal, $\text{aboveA} = \text{B}$

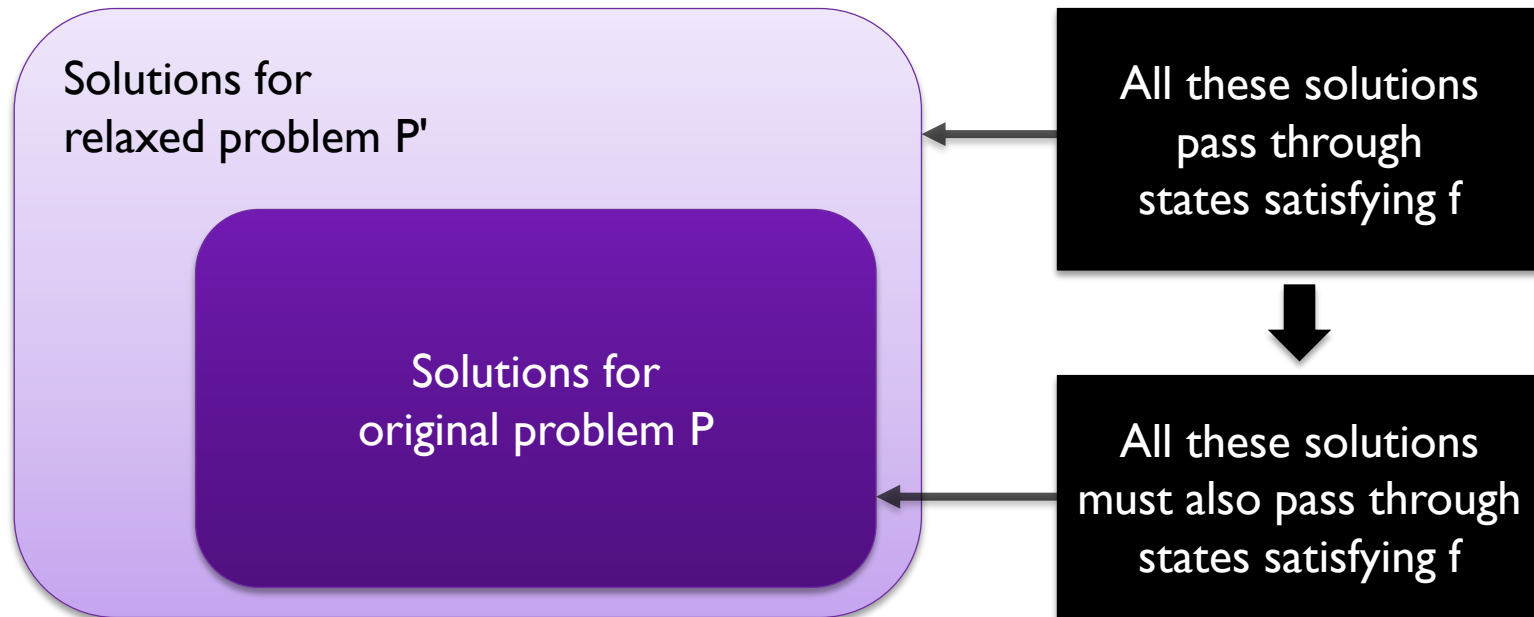
- Then **$\text{aboveA} = \text{gripper}$** is a fact landmark

- (And pickup(A) + stack(B,A) are action landmarks)



Landmarks and Relaxation

- Assume a problem P , and a relaxed problem P'
 - Suppose f is a fact landmark for P'



- Then f is a fact landmark for the original problem as well!
- Similarly for action landmarks, etc.

- Many other techniques exist...
 - Beyond the scope of the course

Landmark Ordering

Landmark Ordering (1)

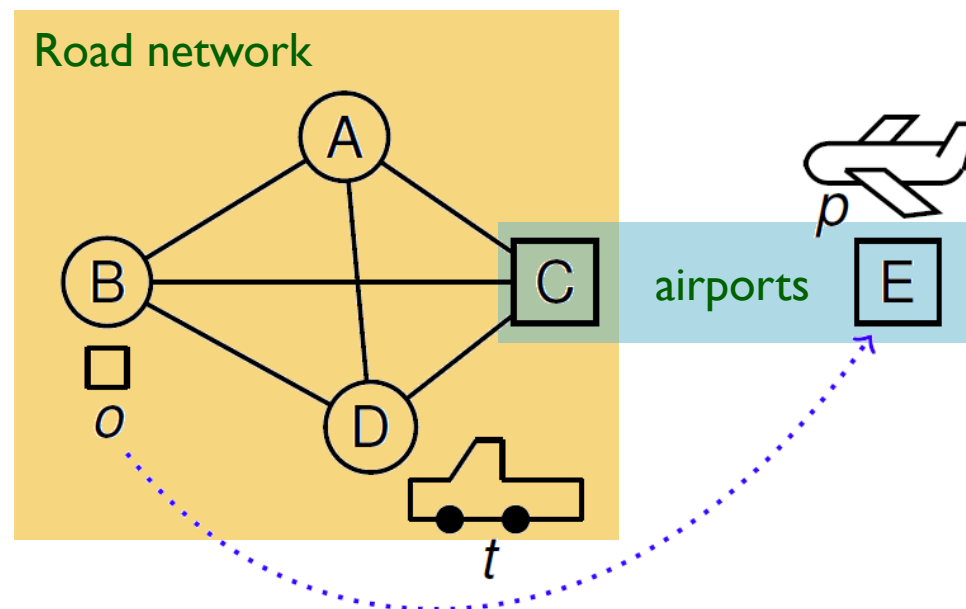


- Sometimes we can find or approximate necessary orderings
 - We must achieve $\text{holding}(A)$, *then* $\text{holding}(B)$

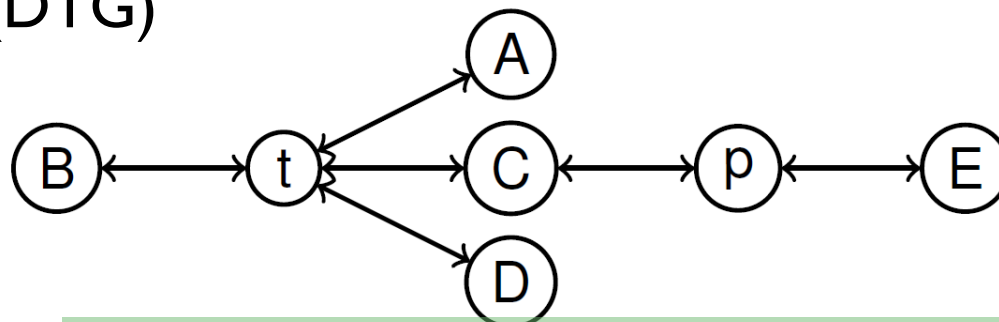
Landmark Ordering (2): Example Problem

■ Example Problem:

- Truck t transports object o within **road network** A/B/C/D
- Airplane p transports object between **airports** C/E
- Goal: Object at E

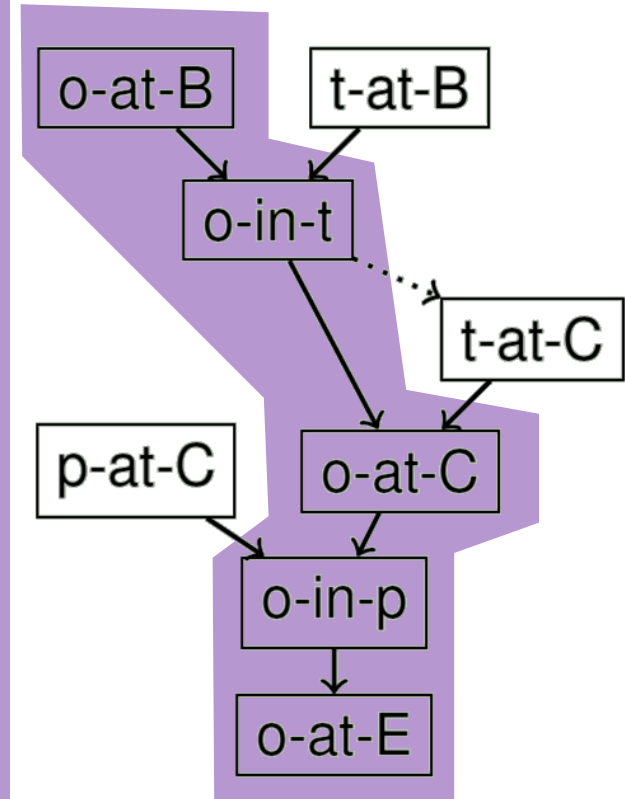
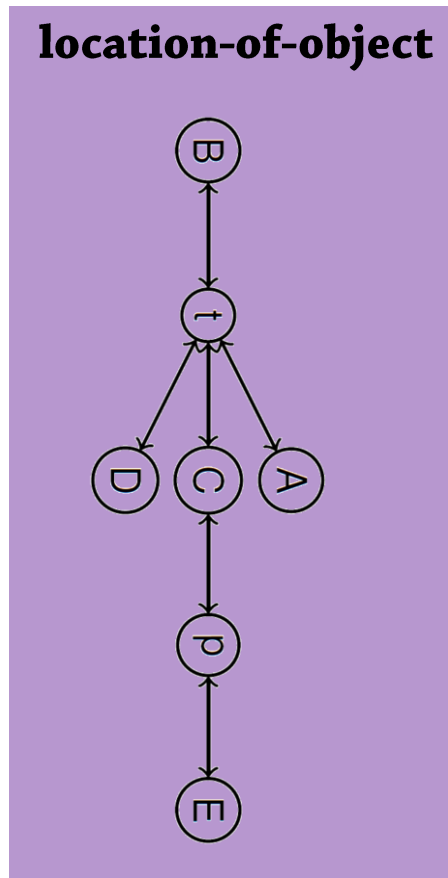
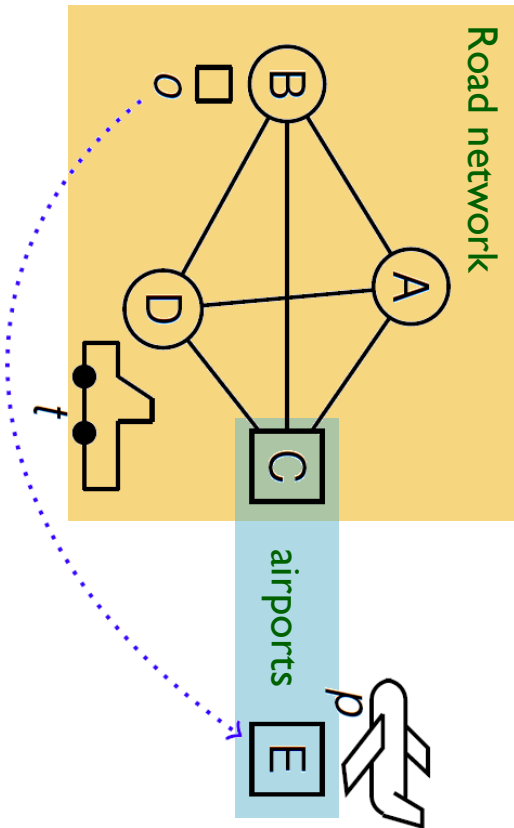


■ Domain transition graph (DTG) for **location-of-object**:



Note: Every **edge** in the road network corresponds to a **path** through t in the DTG!

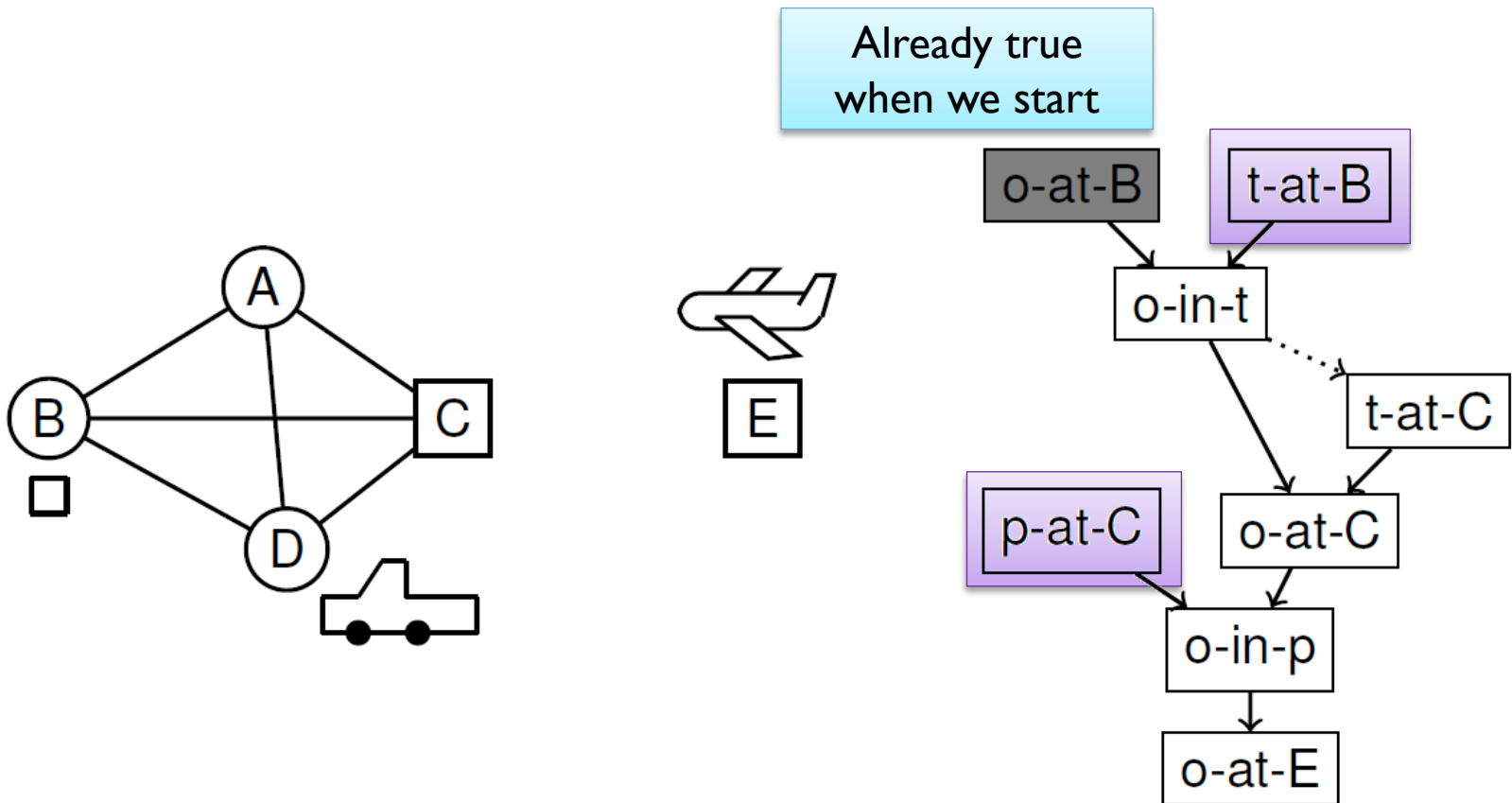
- **Directly from the DTG!**



Using Ordered Landmarks as Subgoals

Landmarks as Subgoals (1)

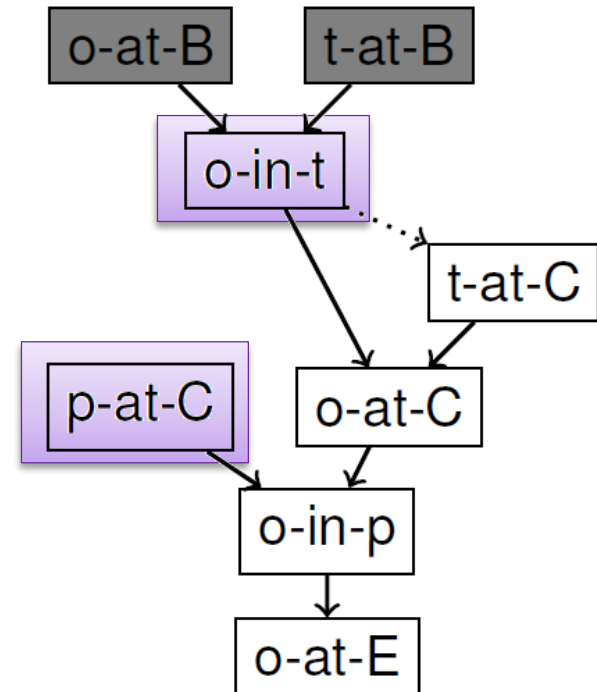
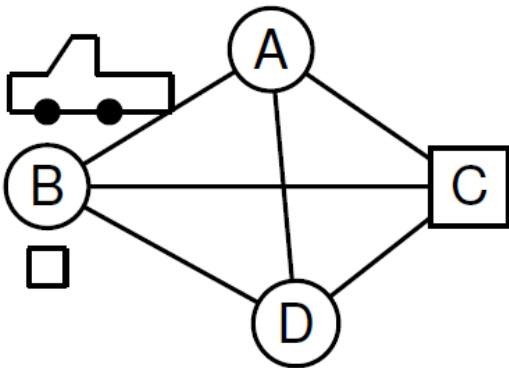
- One use of **ordered** landmarks:
 - As **subgoals**: Try to plan for each landmark **separately** in the inferred **order**



Two landmarks could be "first" (all predecessors achieved)
Current goal: $t\text{-at-B} \vee p\text{-at-C}$ (disjunctive!)

Landmarks as Subgoals (2)

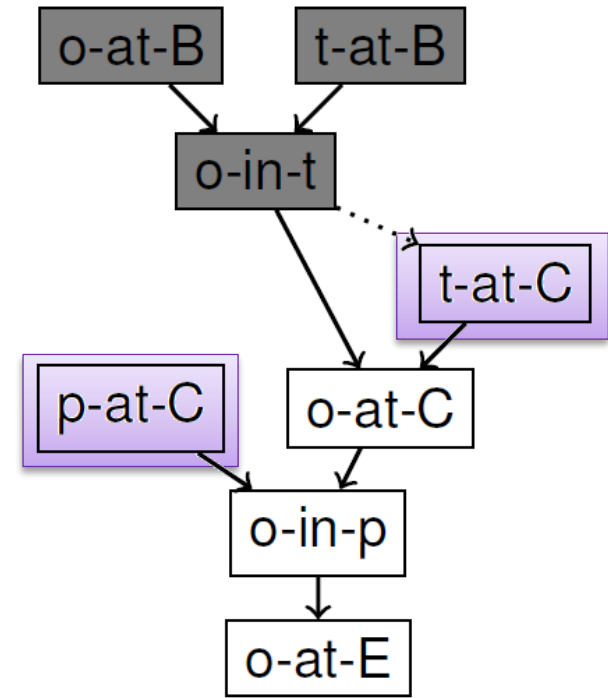
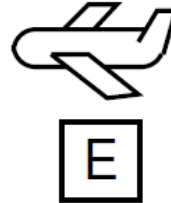
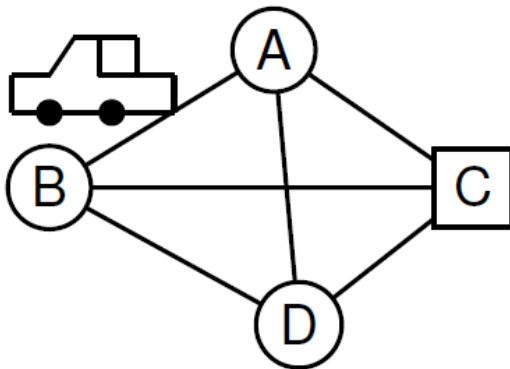
Suppose we begin by achieving t-at-B:
Simple planning problem,
results in a single action -- drive(t, B)



Current goal: o-in-T or p-at-C

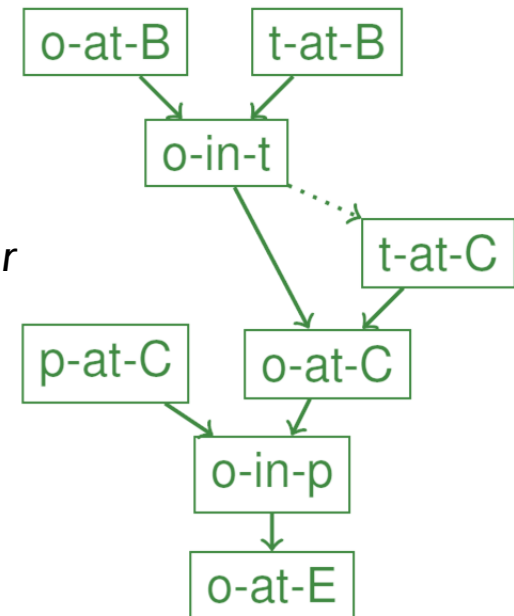
Landmarks as Subgoals (3)

Suppose we continue by achieving o-in-T:
Simple planning problem,
results in a single action -- load-truck(o,t,B)



Landmarks as Subgoals (4)

- Sometimes very helpful, but:
 - There are still **choices** to be made – backtrack points!
 - Optimizing for one **part** of the overall goal at a time:
 - Can't see the whole picture
 - Can miss opportunities:
Cheapest solution *here* → **more expensive** solution *later*
 - Can be incomplete:
Cheapest solution *here* → **impossible** to solve *later*

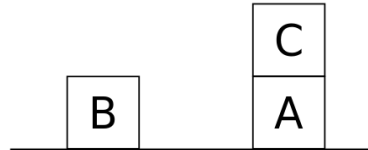


Sussman Anomaly

- The Sussman Anomaly (Gerald Sussman)

- Goal is $\text{on}(A,B), \text{on}(B,C)$

- Now:

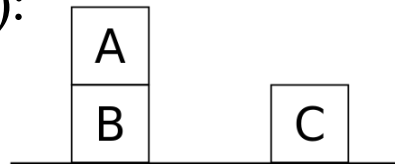


- Separate into subgoals:

- First achieve $\text{on}(A,B)$
- Then achieve $\text{on}(B,C)$

- Achieve first subgoal, $\text{on}(A,B)$:

- $\text{unstack}(C,A); \text{putdown}(C);$
 $\text{pickup}(A); \text{stack}(A,B)$



- Achieve second subgoal, $\text{on}(B,C)$:

- $\text{unstack}(A,B); \text{putdown}(A);$
 $\text{pickup}(B); \text{stack}(B,C) \rightarrow \text{original goal destroyed!}$

Landmark Counts and Costs

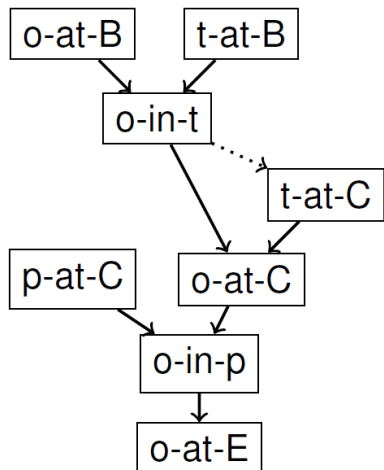
- Use of landmarks:
 - As a basis for non-admissible heuristic estimates in standard forward state space search
 - Pioneered by LAMA
 - The winner of the **sequential satisficing** track of the 2008/2011 competitions
 - If LAMA-2011 had participated in **IPC-2014** (the latest competition):
 - Would have been 12th of 21 planners
 - But LAMA is **part** of the following planners from the 2014 competition:
 - **IBaCoP2**, 1st place in the sequential satisficing track
 - **IBaCoP**, 2nd place in the sequential satisficing track
 - **ArvandHerd**, 1st place in the sequential multi-core track
 - **IBaCoP**, 2nd place in the sequential multi-core track

Results from 2018
will be
presented in June,
analyzed in July!

Landmark Counts and Costs (1)

- LAMA counts landmarks:
 - Identifies a set of landmarks that still need to be achieved after reaching state s through path (action sequence) π

■ $L(s, \pi) =$



$(L \setminus \text{Accepted}(s, \pi))$

All discovered landmarks,
minus those that are
accepted as achieved
(has become true *after*
predecessors are achieved!)

\cup

$\text{ReqAgain}(s, \pi)$

Plus those we can show will
have to be re-achieved

Not admissible: One action may achieve multiple landmarks!

Landmark Counts and Costs (2)



- The **LAMA heuristic** combines:
 - The **number** of landmarks still to be achieved in a state
 - FF heuristics (relaxed planning graph)
 - Searches for **low-cost plans**
 - But we also want to find plans quickly!
 - **Search strategy:**
 - First, **greedy best-first** (create a solution as quickly as possible)
 - Only care about $h(n)$
 - Ignore $g(n)$ = cost of reaching n
 - Then, **repeated weighted A*** search with decreasing weights
 - A* with $f(n) = g(n) + \text{weight} * h(n)$, where $\text{weight} > 1$
 - Iteratively improve the plan – **anytime planning!**

Landmark Counts and Costs (3)

- Other uses of landmarks:

- As a basis for admissible heuristic estimates

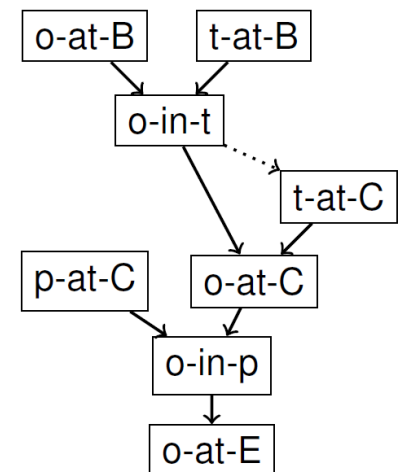
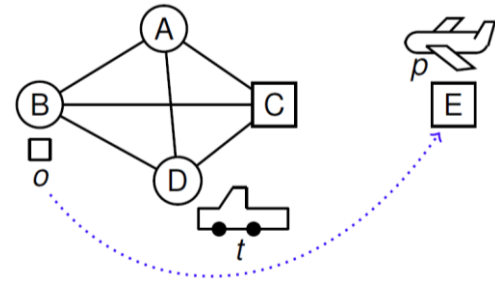
- Idea: The cost of each action is *divided* across the landmarks it achieves

- Simplified example:**

- Suppose there is a goto-and-pickup action of cost 10, that achieves both t-at-B and o-in-t
- Suppose *no other action* can achieve these landmarks
- One can then let (for example)
 $\text{cost}(\text{t-at-B})=3$ and $\text{cost}(\text{o-in-t})=7$

- The sum of the cost of remaining landmarks is then an admissible heuristic

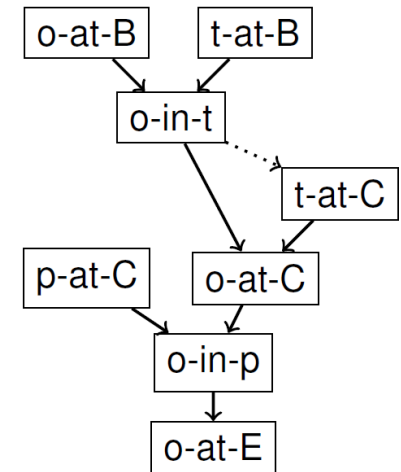
- Must decide how to split costs across landmarks
- Optimal split *can* be computed polynomially, but is still expensive



Landmarks: Modified Problem

- Landmarks as a basis for a modified planning problem

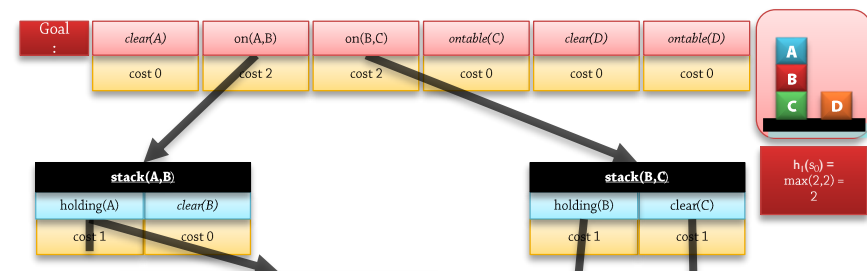
- Add new facts "achieved-landmark-*n*"
 - Concretely: *object-has-been-in-plane*
- An action achieving a landmark makes the corresponding facts true
 - (load object plane) \rightarrow object-has-been-in-plane := true



- The goal requires all such facts to be true
 - (**goal** object-has-been-in-plane ...)

- \rightarrow Any *other* heuristic can be applied to the modified problem!

- $h_1(s)$: What is the cost of achieving object-has-been-in-plane?



Search Techniques

Dual Queue Techniques

Helpful Actions and Completeness

- Recall FF's helpful actions

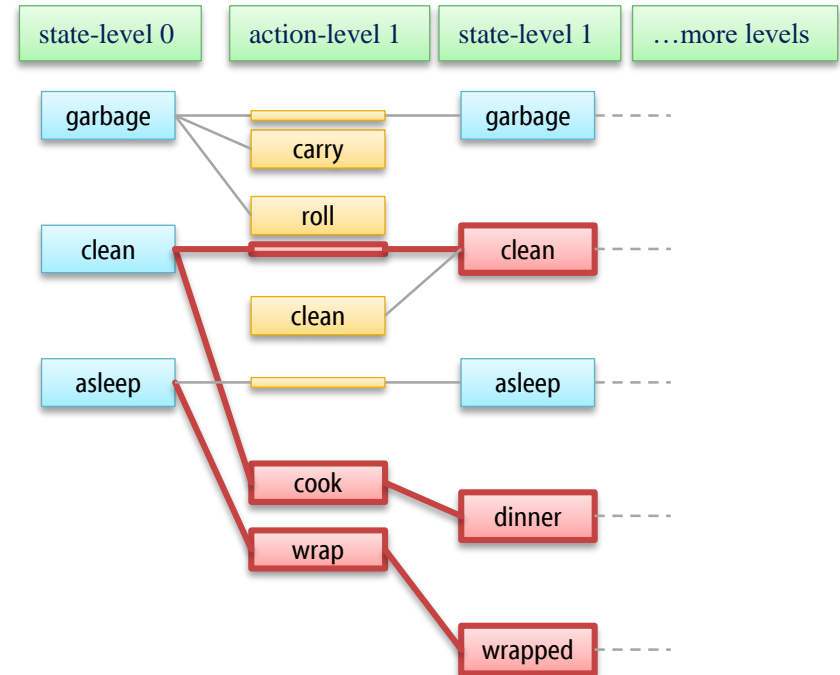
- \approx Actions chosen in the first level of the relaxed planning graph when computing the heuristic

- FF uses these to prune the tree in Enforced Hill Climbing

- Leads to incompleteness
- May search for a long time, exhaust the search space, then start over using complete search

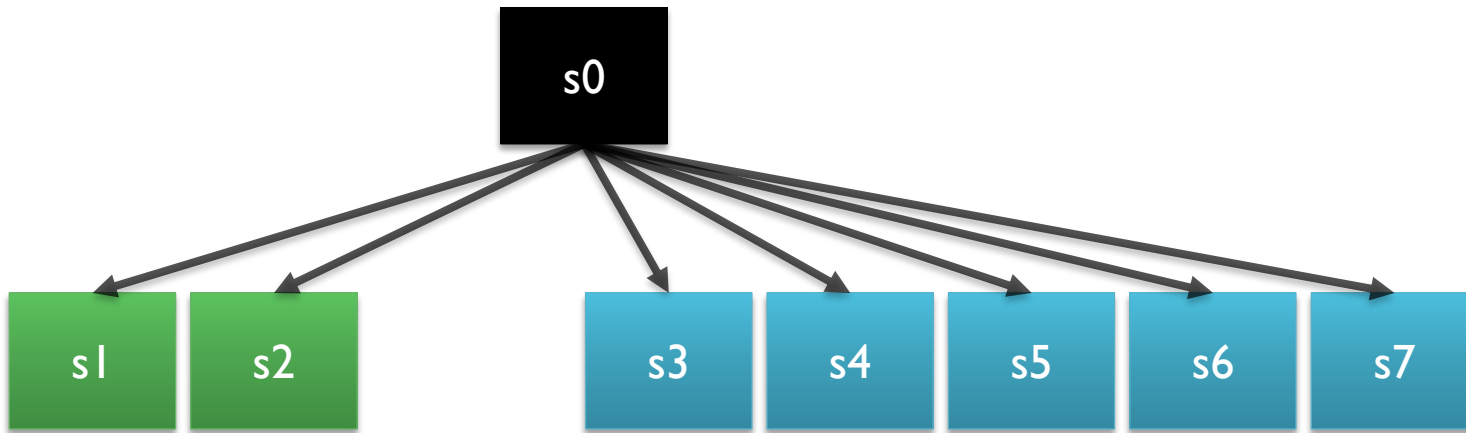
- "Helpful actions" are more likely to be helpful

- But skipping the other actions completely is too strict!
- Fast Downward: Prioritize helpful actions ("preferred successors")



Dual Queues (1)

- When we expand a state:
 - Successors created by helpful actions are preferred successors
 - Successors created by non-helpful actions are ordinary successors



Generally
much fewer!

Dual Queues (2)

- Fast Downward introduced **dual queues** (two "open lists")
 - One for states generated as **preferred** successors
 - One for the **ordinary** states

Preferred

s299

s95

s42

s102

s150

"Ordinary"

s522

s293

s7

s222

s856

Priority queues!

Dual Queues (3)

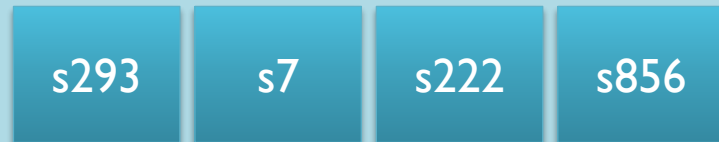
- To expand a state:
 - Pick the **best** state from the **preferred** queue, and expand it
 - Pick the **best** state from the **ordinary** queue, and expand it



Preferred

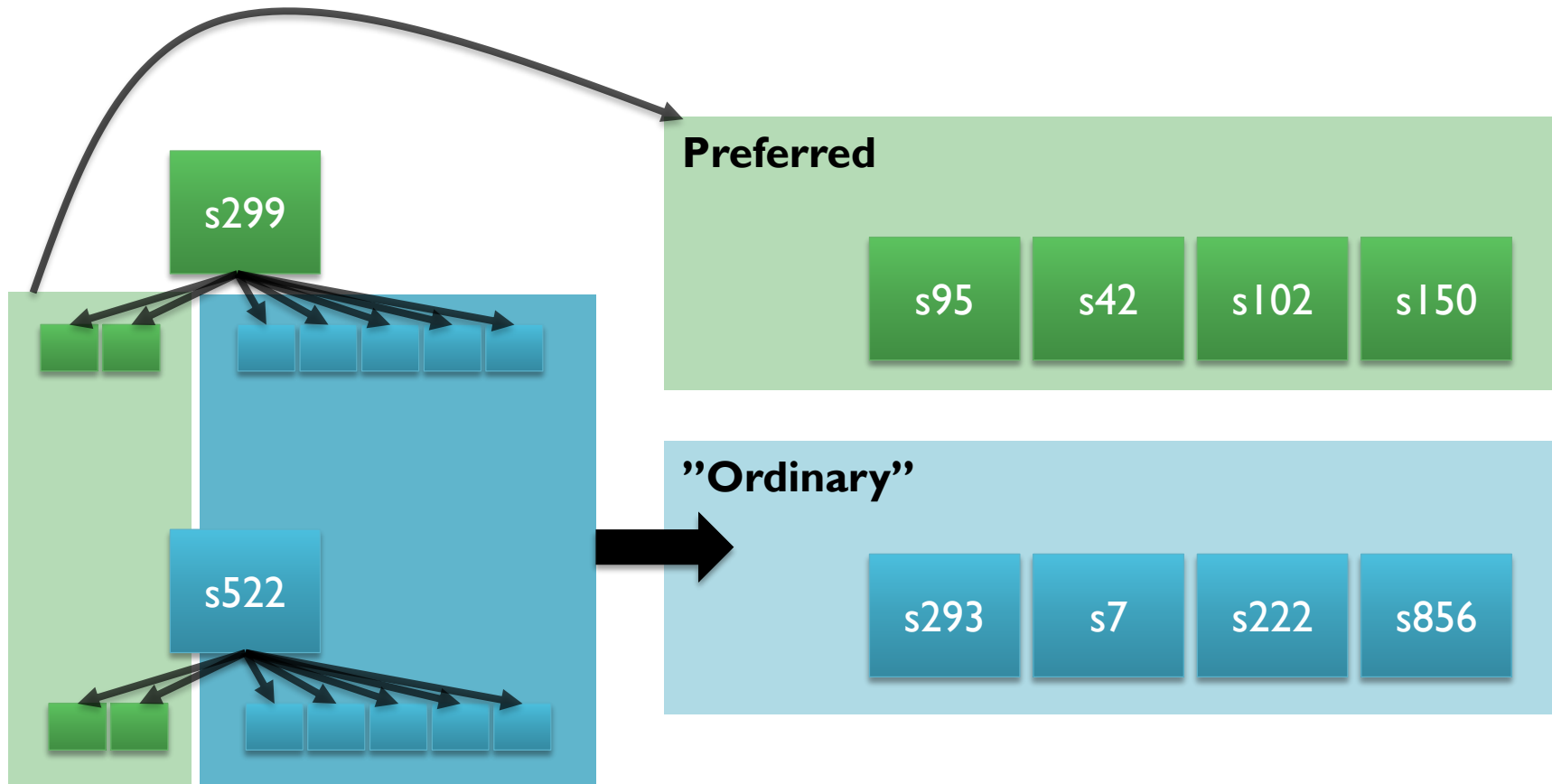


"Ordinary"



Dual Queues (4)

- After expansion:
 - Place all new states where they belong



Dual Queues (5)

- **Fewer** states are preferred
 - Reached more quickly in the queue
- If we "**misclassified**" an action as non-helpful:
 - Don't have to exhaust the "preferred part" of the search space before we can "recover"
 - Search is *complete*

Preferred

s95

s42

s102

s150

"Ordinary"

s293

s7

s222

s856

■ Boosted Dual Queues:

- Used in later versions of Fast Downward and LAMA
- Whenever progress is made (better h -value reached):
 - Expand **1000** preferred states

Preferred

s95

s42

s102

s150

"Ordinary"

s293

s7

s222

s856

- If progress is made again within these 1000 successors:
 - Add another 1000, accumulating
 - (Progress made after 300 → keep expanding 1700 more)

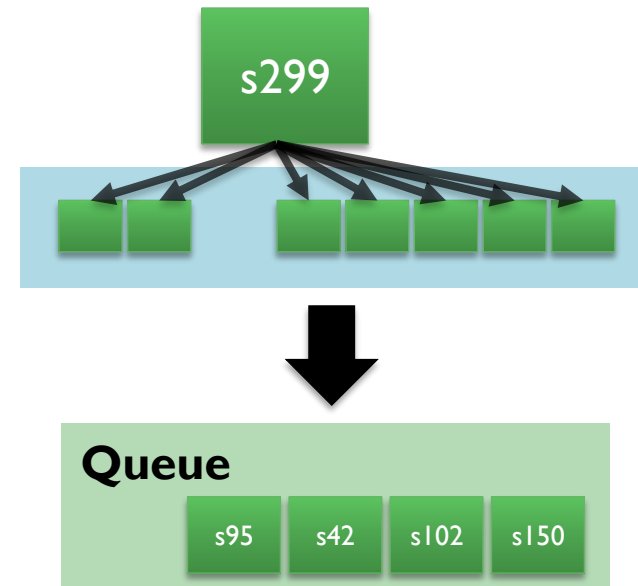
- **Boosted** Dual Queues:
 - After reaching the preferred successor limit:
 - Expand a **single** node from the non-preferred queue
 - Still complete
 - More aggressive than ordinary dual queues
 - Less aggressive than pure pruning

Deferred Evaluation / Lazy Search

- Standard **best-first** search:

- Remove the "best" (most promising) state from the priority queue
- Check whether it satisfies the goal
- Generate all successors
- Calculate their heuristic values
- Place in priority queue(s)

Typically takes most of the time



Deferred Evaluation (2)

- Potentially faster: **Deferred Evaluation** (Fast Downward, ...)
 - Remove the "best" state from the priority queue
 - Check whether it satisfies the goal
 - Calculate **its** heuristic value (**only one!**)
 - Generate all successors
 - Place in priority queue using the **parent's** heuristic value

Takes less time, but less accurate heuristic – "one step behind"
Often **faster** but **lower-quality** plans

Parameter Optimization and Portfolio Planners

A general technique – not limited to state-space search!

Parameter Optimization (1)



- Some planners have many parameters to tweak
 - In early planning competitions, domains were known in advance
 - Participants could manually adapt their "domain-independent" planners...
 - Somewhat exaggerated quote from IPC-2008 results:
 - if domain name begins with "PS" and part after first letter is "SR":
use algorithm 100
 - else if there are 5 actions, all with 3 args, and 12 non-ground facts:
use algorithm -1000
 - else if all facts ground and 10th/11th domain name letters "PA":
use algorithm -1004
 - else if there are 11 actions and action name lengths range from 5 to 28:
use algorithm 107
 - From 2008, this was no longer allowed
 - Planners were handed in
 - Then the organizers ran the planners

Parameter Optimization (2)

- How about *automatically* learning parameters?
 - One specific form of learning in planning – others exist
 - Experimental application to **Fast Downward**
 - Optimization for speed: 45 params, $2.99 * 10^{13}$ possible configurations
 - Optimization for quality: 77 params, $1.94 * 10^{26}$ possible configurations
 - Example parameters:
 - **Heuristics used:**
 $h_{\max} = h_0, h_m, h_{\text{add}}, h_{\text{FF}}, h_{\text{LM}}$ (landmarks), h_{LA} (admissible landmarks), goal count, ...
 - Method used to **combine heuristics**: Max, sum, selective max (learns which heuristic to use per state), tie-breaking, Pareto-optimal, alternation
 - **Preferred operators** used or not, for each heuristic
 - Like FF's helpful actions, but used for *prioritization*, not pruning
 - **Search strategy** combinations: Eager best-first, lazy best-first, EHC
 - ...
 - Parameter learning framework **ParamILS** used

Parameter Optimization (3): Results

- **Under** the diagonal = **faster** than default configuration

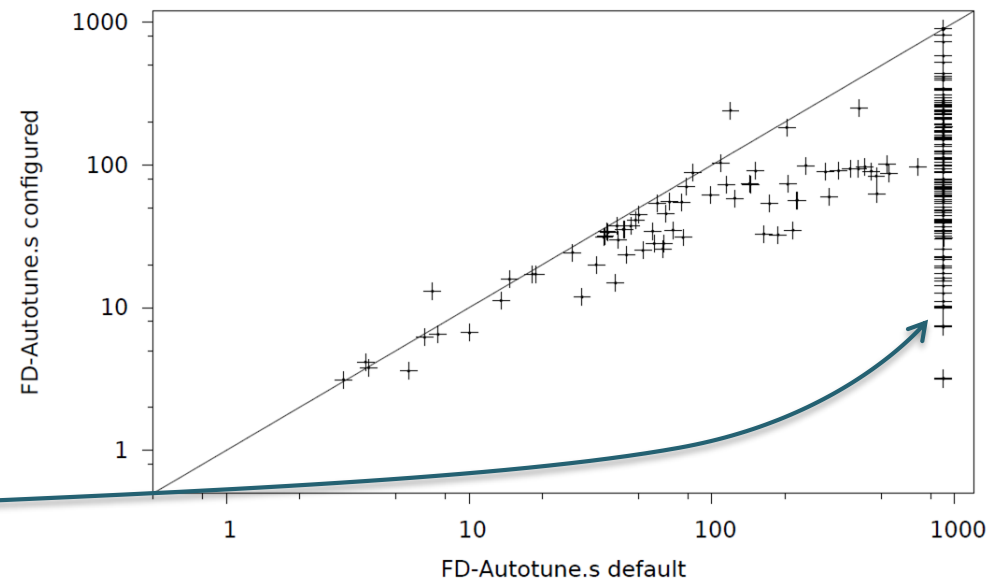
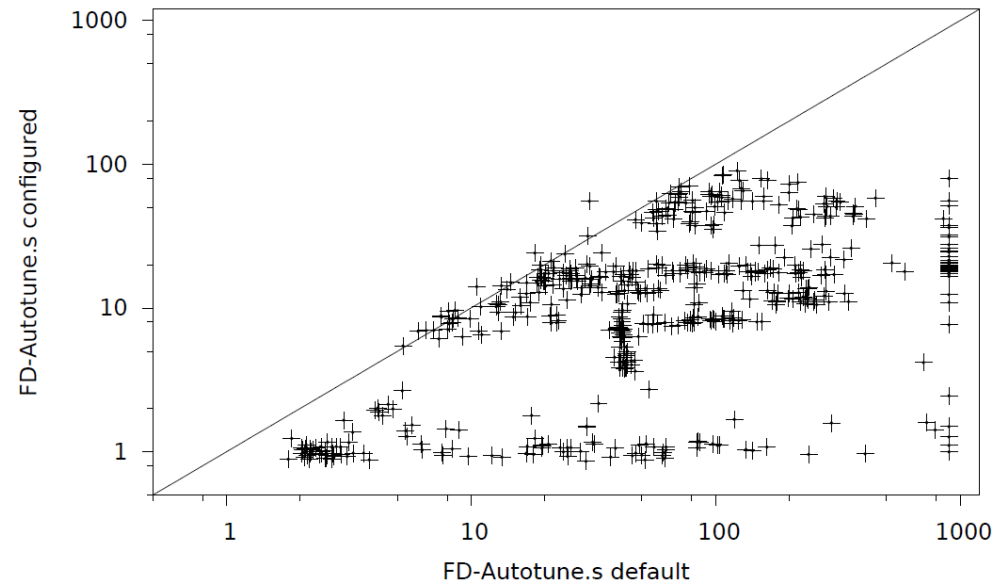
- For 540 small **training instances**:

- Very good results
- To be expected – parameters tuned for these specific instances!

- For 270 larger **test instances**:

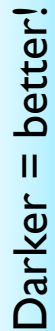
- From the same domains
- Performance still improves

Unsolvable in 900 seconds
by the default configuration





- ## Sequential Satisficing track: Results



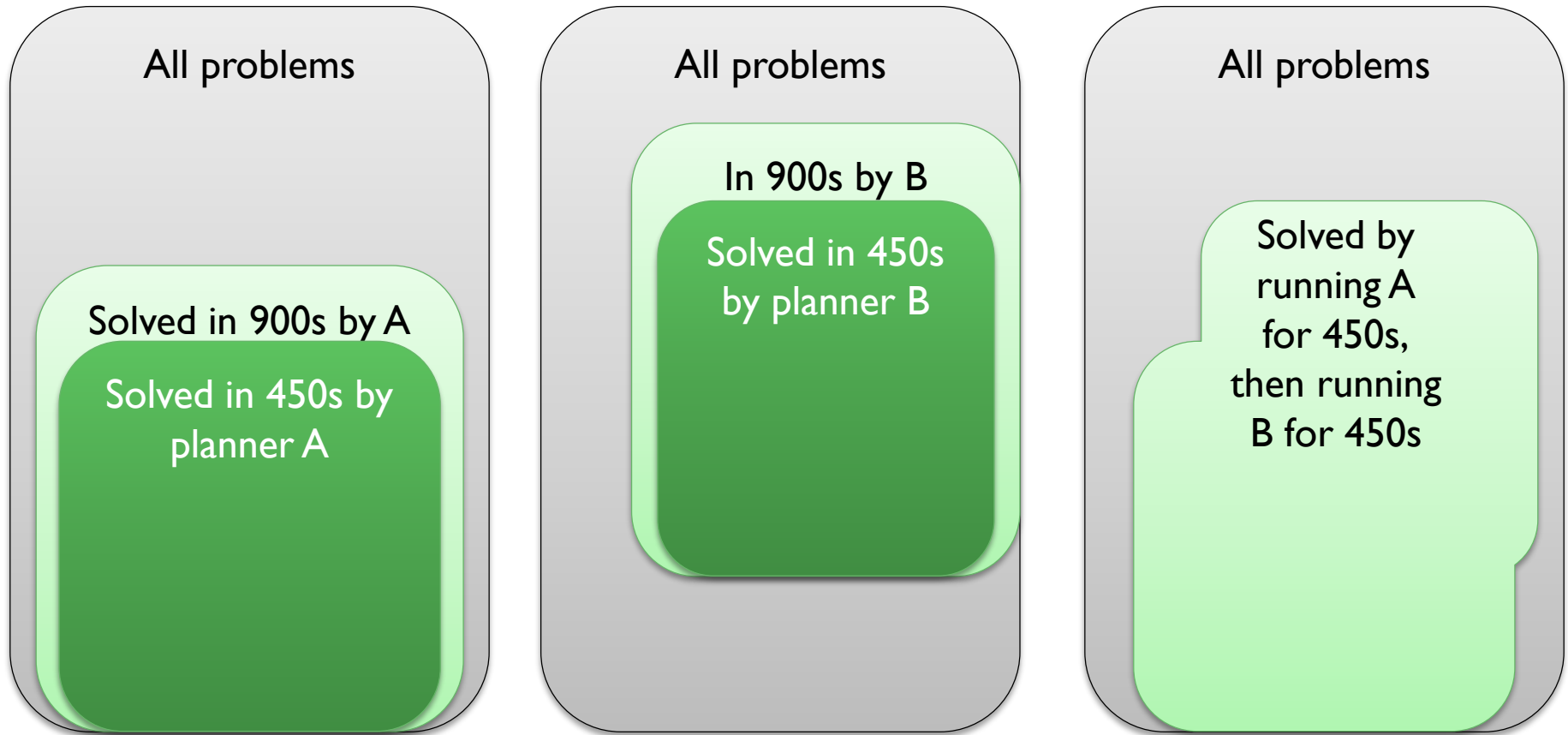


- # Darker = better!

[illegible]

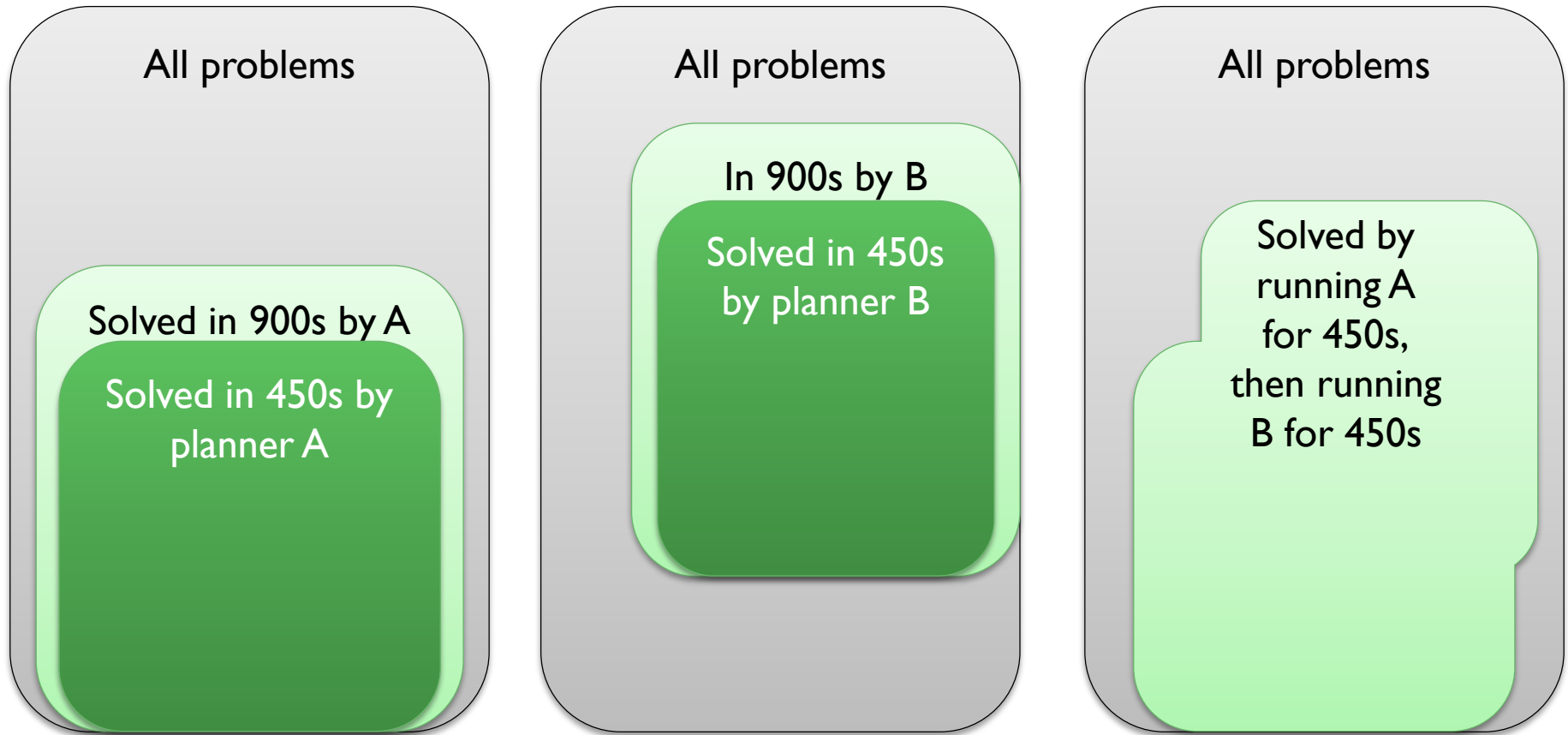
Portfolio Planning (2)

- Further analysis would show:
 - Even if two planners solve equally many problems in one domain, they may solve **different** problems
 - Also, planners often return plans **quickly** or **not at all**



Portfolio Planning (3)

- The competition has a fixed time limit
 - Can benefit from splitting this across **multiple algorithms!**
 - ➔ **Portfolio** planning



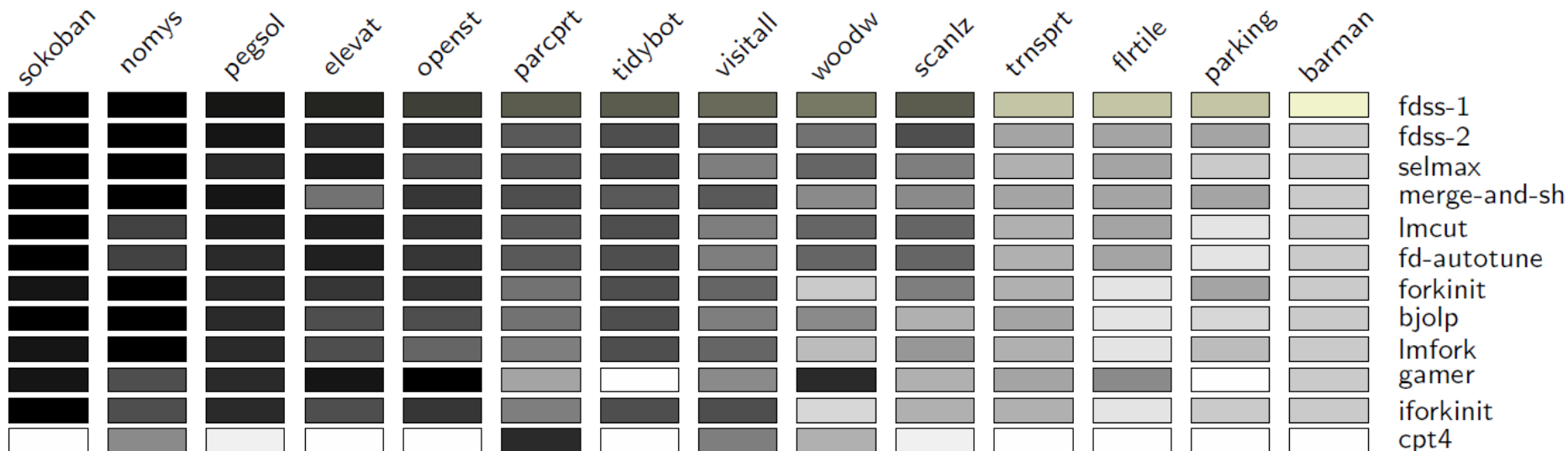
■ Fast Downward Stone Soup: Learning

- Which configurations to use
- How much time to assign to each one
- Given test examples from older domains

Algorithm	Score	Time	Marginal
BJOLP	605	455	46
RHW landmarks	597	0	—
LM-cut	593	569	26
h^1 landmarks	588	0	—
M&S-bisim 1	447	175	8
h^{\max}	427	0	—
M&S-bisim 2	426	432	20
blind	393	0	—
M&S-LFPA 10000	316	0	—
M&S-LFPA 50000	299	0	—
M&S-LFPA 100000	286	0	—
Portfolio	654	1631	
“Holy Grail”	673		

Configurations
learned for
sequential optimal
planning

- ## Sequential Optimization track: Results



Portfolio Planning (6)



- Results from IPC-2014:
 - Sequential Satisficing Track
 - #1: **IBaCoP** -- portfolio planner
 - #2: **IBaCoP2** -- portfolio planner
 - (Instance-Based Configured Portfolios)