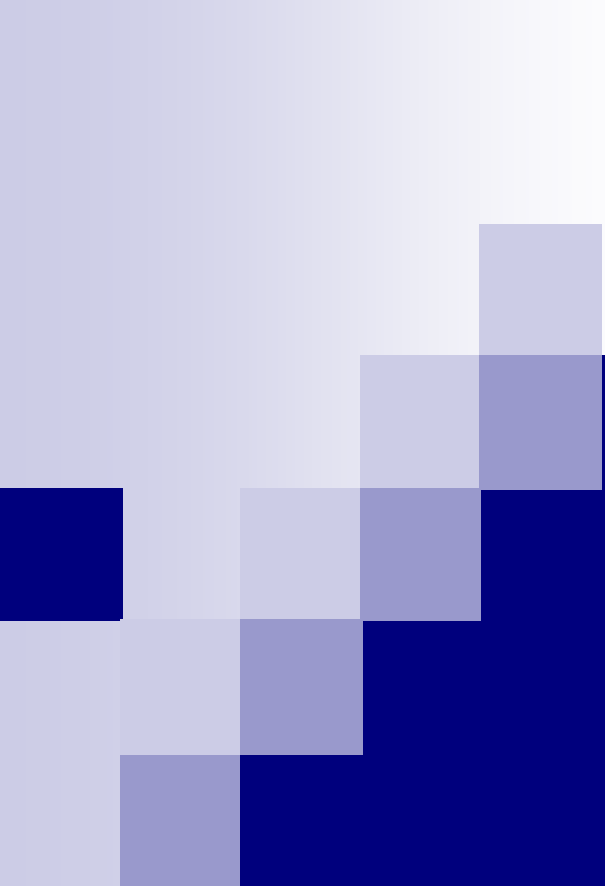




Description logics

Description Logics

- ❑ A family of KR formalisms, based on FOPL decidable, supported by automatic reasoning systems
- ❑ Used for modelling of application domains
- ❑ Classification of concepts and individuals
concepts (unary predicates), subconcept (subsumption), roles (binary predicates), individuals (constants), constructors for building concepts, equality ...

[Baader et al. 2002]



Applications

- software management
 - configuration management
 - natural language processing
 - clinical information systems
 - information retrieval
 - ...
-
- Ontologies and the Web



Ontologies, Description Logics and OWL terminology

Ontologies

DL

OWL

concept

concept

class

relation

role (binary)

property

axiom

axiom

axiom

instance

individual

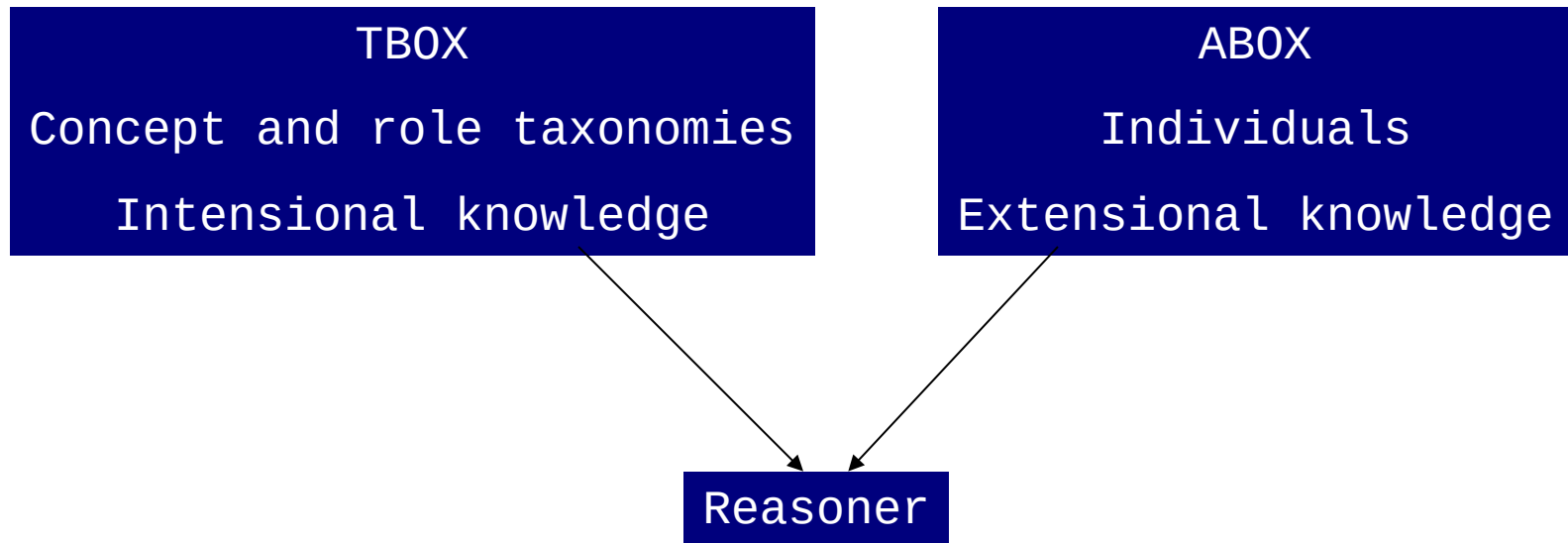
individual



Outline

- DL languages
 - syntax and semantics
- DL reasoning services
 - algorithms, complexity
- DL systems
- DLs for the web

Tbox and Abox



Syntax - $\mathcal{AL}$

R atomic role, A atomic concept

$C, D \rightarrow A$ | (atomic concept)

T | (universal concept, top)

$\perp$ | (bottom concept)

$\neg A$ | (atomic negation)

$C \cap D$ | (conjunction)

$\forall R.C$ | (value restriction)

$\exists R.T$ (limited existential quantification)

$\mathcal{AL}[\mathcal{X}]$

$\mathcal{C}$ $\neg \mathcal{C}$ (concept negation)

$\mathcal{U}$ $\mathcal{C} \cup \mathcal{D}$ (disjunction)

$\mathcal{E}$ $\exists R.C$ (existential quantification)

$\mathcal{N}$ $\geq n R, \leq n R$ (number restriction)

$\mathcal{Q}$ $\geq n R.C, \leq n R.C$ (qualified number restriction)

Example

Team

Team $\cap \geq 10$ hasMember

Team $\cap \geq 11$ hasMember
 $\cap \forall$ hasMember.Soccer-player

$\mathcal{AL}[\mathcal{X}]$

$\mathcal{R}$ $R \cap S$ (role conjunction)

$\mathcal{I}$ R^- (inverse roles)

$\mathcal{H}$ (role hierarchies)

$\mathcal{F}$ $u_1 = u_2, u_1 \neq u_2$ (feature (dis)agreements)

$S[\mathcal{X}]$

S $\mathcal{ALC}$ + transitive roles

$S\mathcal{HIQ}$ $\mathcal{ALC}$ + transitive roles
 + role hierarchies
 + inverse roles
 + number restrictions

Tbox

- Terminological axioms:

- $C = D$ ($R = S$)

- owl:equivalentClass / owl:equivalentProperty

- $C \sqsubseteq D$ ($R \sqsubseteq S$)

- rdfs:subClassOf / rdfs:subPropertyOf

- (disjoint $C D$)

- owl:disjointWith



Tbox

- An equality whose left-hand side is an atomic concept is a definition.
- A finite set of definitions T is a Tbox (or terminology) if no symbolic name is defined more than once.

Example Tbox

Soccer-player $\subseteq$ T

Team $\subseteq \geq 2$ hasMember

Large-Team = Team $\cap \geq 10$ hasMember

S-Team = Team $\cap \geq 11$ hasMember
 $\cap \forall$ hasMember.Soccer-player

DL as sublanguage of FOPL

Team(this)

$\wedge$

$(\exists x_1, \dots, x_{11}:$

$\text{hasMember}(\text{this}, x_1) \wedge \dots \wedge \text{hasMember}(\text{this}, x_{11})$

$\wedge x_1 \neq x_2 \wedge \dots \wedge x_{10} \neq x_{11})$

$\wedge$

$(\forall x: \text{hasMember}(\text{this}, x) \rightarrow \text{Soccer-player}(x))$

Abox

- Assertions about individuals:

- $C(a)$

- $R(a,b)$



Example

Ida-member(Sture)

Individuals in the description language

- $O \{i_1, \dots, i_k\}$ (one-of)
- $R:a$ (fills)

Example

$(\text{S-Team} \cap \text{hasMember:Sture})(\text{IDA-FF})$



Knowledge base

A knowledge base is a tuple $\langle T, A \rangle$
where T is a Tbox and A is an Abox.

Example KB

Soccer-player $\subseteq$ T

Team $\subseteq \geq 2$ hasMember

Large-Team = Team $\cap \geq 10$ hasMember

S-Team = Team $\cap \geq 11$ hasMember
 $\cap \forall$ hasMember.Soccer-player

Ida-member(Sture)

(S-Team $\cap$ hasMember:Sture)(IDA-FF)

Example - OWL

<Declaration> <ObjectProperty IRI="#hasmember"/> </Declaration>

<Declaration> <Class IRI="#soccer-player"/> </Declaration>

<Declaration> <Class IRI="#ida-member"/> </Declaration>

<Declaration> <Class IRI="#team"/> </Declaration>

<Declaration> <Class IRI="#large-team"/> </Declaration>

<Declaration> <Class IRI="#s-team"/> </Declaration>

<Declaration> <NamedIndividual IRI="#IDA-FF"/> </Declaration>

<Declaration> <NamedIndividual IRI="#Sture"/> </Declaration>

Example - OWL

```
<EquivalentClasses>
  <Class IRI="#large-team"/>
  <ObjectIntersectionOf>
    <Class IRI="#team"/>
    <ObjectMinCardinality cardinality="10">
      <ObjectProperty IRI="#hasmember"/>
    </ObjectMinCardinality>
  </ObjectIntersectionOf>
</EquivalentClasses>
```

Example - OWL

```
<EquivalentClasses>
  <Class IRI="#s-team"/>
  <ObjectIntersectionOf>
    <Class IRI="#team"/>
    <ObjectAllValuesFrom>
      <ObjectProperty IRI="#hasmember"/>
      <Class IRI="#soccer-player"/>
    </ObjectAllValuesFrom>
    <ObjectMinCardinality cardinality="11">
      <ObjectProperty IRI="#hasmember"/>
    </ObjectMinCardinality>
  </ObjectIntersectionOf>
</EquivalentClasses>
```


Example - OWL

```
<ClassAssertion>
  <ObjectIntersectionOf>
    <Class IRI="#s-team"/>
    <ObjectHasValue>
      <ObjectProperty IRI="#hasmember"/>
      <NamedIndividual IRI="#Sture"/>
    </ObjectHasValue>
  </ObjectIntersectionOf>
  <NamedIndividual IRI="#IDA-FF"/>
</ClassAssertion>
```

```
<ClassAssertion>
  <Class IRI="#ida-member"/>
  <NamedIndividual IRI="#Sture"/>
</ClassAssertion>
```

$\mathcal{AL}$ (Semantics)

An interpretation $\mathcal{I}$ consists of a non-empty set $\Delta^{\mathcal{I}}$ (the domain of the interpretation) and an interpretation function $\cdot^{\mathcal{I}}$ which assigns to every atomic concept A a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and to every atomic role R a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

The interpretation function is extended to concept definitions using inductive definitions.

$\mathcal{AL}$ (Semantics)

$C, D \rightarrow A$ | (atomic concept)

$\top$ | (universal concept)

$\perp$ | (bottom concept)

$\neg A$ | (atomic negation)

$C \cap D$ | (conjunction)

$\forall R.C$ | (value restriction)

$\exists R.T$ | (limited existential
quantification)

$$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$$

$$\perp^{\mathcal{I}} = \emptyset$$

$$(\neg A)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$$

$$(C \cap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$$

$$(\forall R.C)^{\mathcal{I}} =$$

$$\{a \in \Delta^{\mathcal{I}} \mid \forall b.(a,b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$$

$$(\exists R.T)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a,b) \in R^{\mathcal{I}} \wedge b \in T^{\mathcal{I}}\}$$

$\mathcal{ALC}$ (Semantics)

$$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$$

$$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$

$$(\geq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \# \{b \in \Delta^{\mathcal{I}} \mid (a,b) \in R^{\mathcal{I}}\} \geq n\}$$

$$(\leq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \# \{b \in \Delta^{\mathcal{I}} \mid (a,b) \in R^{\mathcal{I}}\} \leq n\}$$

$$(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b \in \Delta^{\mathcal{I}} : (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$$

Semantics

Individual i

$$i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$$

Unique Name Assumption:

$$\text{if } i_1 \neq i_2 \text{ then } i_1^{\mathcal{I}} \neq i_2^{\mathcal{I}}$$

Semantics

An interpretation $\mathcal{I}$ is a model for a terminology T iff

$$C^{\mathcal{I}} = D^{\mathcal{I}} \text{ for all } C = D \text{ in } T$$

$$C^{\mathcal{I}} \subseteq D^{\mathcal{I}} \text{ for all } C \subseteq D \text{ in } T$$

$$C^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset \text{ for all (disjoint } C \text{ } D) \text{ in } T$$

Semantics

An interpretation $\mathcal{I}$ is a model for a knowledge base $\langle T, A \rangle$ iff

$\mathcal{I}$ is a model for T

$a^{\mathcal{I}} \in C^{\mathcal{I}}$ for all $C(a)$ in A

$\langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$ for all $R(a,b)$ in A

Semantics - acyclic Tbox

$\text{Bird} = \text{Animal} \cap \forall \text{Skin.Feather}$

$\Delta^{\mathcal{I}} = \{\text{tweety}, \text{goofy}, \text{fea1}, \text{fur1}\}$

$\text{Animal}^{\mathcal{I}} = \{\text{tweety}, \text{goofy}\}$

$\text{Feather}^{\mathcal{I}} = \{\text{fea1}\}$

$\text{Skin}^{\mathcal{I}} = \{\langle \text{tweety}, \text{fea1} \rangle, \langle \text{goofy}, \text{fur1} \rangle\}$

$\rightarrow \text{Bird}^{\mathcal{I}} = \{\text{tweety}\}$

Semantics - cyclic Tbox

$\text{QuietPerson} = \text{Person} \cap \forall \text{Friend. QuietPerson}$
($A = F(A)$)

$\Delta^{\mathcal{I}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

$\text{Person}^{\mathcal{I}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

$\text{Friend}^{\mathcal{I}} = \{<\text{john}, \text{sue}>, <\text{andrea}, \text{bill}>, <\text{bill}, \text{bill}>\}$

$\rightarrow \text{QuietPerson}^{\mathcal{I}} = \{\text{john}, \text{sue}\}$

$\rightarrow \text{QuietPerson}^{\mathcal{I}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

Semantics - cyclic Tbox

Descriptive semantics: $A = F(A)$ is a constraint stating that A has to be some solution for the equation.

- Not appropriate for defining concepts
- Necessary and sufficient conditions for concepts

Human = Mammal $\cap \exists$ Parent

$\cap \forall$ Parent.Human

Semantics - cyclic Tbox

Least fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the smallest solution (if it exists) for the equation.

- Appropriate for inductively defining concepts

$DG = \text{EmptyDG} \cup \text{Non-Empty-DG}$

$\text{Non-Empty-DG} = \text{Node} \cap \forall \text{ Arc. Non-Empty-DG}$

$\text{Human} = \text{Mammal} \cap \exists \text{ Parent} \cap \forall \text{ Parent. Human}$
 $\rightarrow \text{Human} = \perp$

Semantics - cyclic Tbox

Greatest fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the greatest solution (if it exists) for the equation.

- Appropriate for defining concepts whose individuals have circularly repeating structure

$\text{FoB} = \text{Blond} \cap \exists \text{Child.FoB}$

$\text{Human} = \text{Mammal} \cap \exists \text{Parent} \cap \forall \text{Parent.Human}$

$\text{Horse} = \text{Mammal} \cap \exists \text{Parent} \cap \forall \text{Parent.Horse}$

$\rightarrow \text{Human} = \text{Horse}$



Open world vs closed world semantics

Databases: closed world reasoning

database instance represents one interpretation

→ absence of information interpreted as negative information

“complete information”

query evaluation is finite model checking

DL: open world reasoning

Abox represents many interpretations (its models)

→ absence of information is lack of information

“incomplete information”

query evaluation is logical reasoning

Open world vs closed world semantics

hasChild(Jocasta, Oedipus)

hasChild(Jocasta, Polyneikes)

hasChild(Oedipus, Polyneikes)

hasChild(Polyneikes, Thersandros)

patricide(Oedipus)

¬ patricide(Thersandros) *(not represented in DB)*

Does it follow from the Abox that

$\exists \text{hasChild.}(\text{patricide} \sqcap \exists \text{hasChild.} \neg \text{patricide})(\text{Jocasta})$?



Reasoning services

- Satisfiability of concept
- Subsumption between concepts
- Equivalence between concepts
- Disjointness of concepts

- Classification

- Instance checking
- Realization
- Retrieval
- Knowledge base consistency

Reasoning services

- Satisfiability of concept
 - C is satisfiable w.r.t. $\mathcal{T}$ if there is a model I of $\mathcal{T}$ such that C^I is not empty.
- Subsumption between concepts
 - C is subsumed by D w.r.t. $\mathcal{T}$ if $C^I \subseteq D^I$ for every model I of $\mathcal{T}$.
- Equivalence between concepts
 - C is equivalent to D w.r.t. $\mathcal{T}$ if $C^I = D^I$ for every model I of $\mathcal{T}$.
- Disjointness of concepts
 - C and D are disjoint w.r.t. $\mathcal{T}$ if $C^I \cap D^I = \emptyset$ for every model I of $\mathcal{T}$.

Reasoning services

- Reduction to subsumption
 - C is unsatisfiable iff C is subsumed by $\perp$
 - C and D are equivalent iff C is subsumed by D and D is subsumed by C
 - C and D are disjoint iff $C \cap D$ is subsumed by $\perp$
- The statements also hold w.r.t. a Tbox.

Reasoning services

- Reduction to unsatisfiability
 - C is subsumed by D iff $C \cap \neg D$ is unsatisfiable
 - C and D are equivalent iff
 - both $(C \cap \neg D)$ and $(D \cap \neg C)$ are unsatisfiable
 - C and D are disjoint iff $C \cap D$ is unsatisfiable
- The statements also hold w.r.t. a Tbox.

Tableau algorithms

- To prove that C subsumes D:
 - If C subsumes D, then it is impossible for an individual to belong to D but not to C.
 - Idea: Create an individual that belongs to D and not to C and see if it causes a contradiction.
 - If **always** a contradiction (clash) then subsumption is proven. Otherwise, we have found a model that contradicts the subsumption.

Tableau algorithms

- Based on constraint systems.
 - $S = \{ x: \neg C \cap D \}$
 - Add constraints according to a set of propagation rules
 - Until clash or no constraint is applicable

Tableau algorithms – de Morgan rules

$$\neg \neg C \rightarrow C$$

$$\neg (A \cap B) \rightarrow \neg A \cup \neg B$$

$$\neg (A \cup B) \rightarrow \neg A \cap \neg B$$

$$\neg (\forall R.C) \rightarrow \exists R.(\neg C)$$

$$\neg (\exists R.C) \rightarrow \forall R.(\neg C)$$

Tableau algorithms – constraint propagation rules

- $S \rightarrow_{\cap} \{x:C_1, x:C_2\} \cup S$

if $x: C_1 \cap C_2$ in S

and either $x:C_1$ or $x:C_2$ is not in S

- $S \rightarrow_{\cup} \{x:D\} \cup S$

if $x: C_1 \cup C_2$ in S and neither $x:C_1$ or $x:C_2$ is in S , and $D = C_1$ or $D = C_2$

Tableau algorithms – constraint propagation rules

- $S \rightarrow_{\forall} \{y:C\} \cup S$

if $x: \forall R.C$ in S and xRy in S and $y:C$ is not in S

- $S \rightarrow_{\exists} \{xRy, y:C\} \cup S$

if $x: \exists R.C$ in S and y is a new variable and there is no z such that both xRz and $z:C$ are in S

Example

- ST: Tournament

$\cap \exists \text{ hasParticipant.Swedish}$

- SBT: Tournament

$\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$

Example 1

- $SBT \Rightarrow ST?$
- $S = \{ x:$
 - $\neg(\text{Tournament} \cap \exists \text{hasParticipant.Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
 - $\}$

Example 1

- $S = \{ x:$
 $(\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.} (\text{Swedish} \cap \text{Belgian}))$
 $\}$

Example 1

$\cap$ -rule:

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{hasParticipant}.\neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian}),$
 - $x: \neg \text{Tournament}$**
 - $\cup \forall \text{hasParticipant}.\neg \text{Swedish},$**
 - $x: \text{Tournament},$**
 - $x: \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian})$**
 - $\}$

Example 1

$\exists$ -rule:

- $S = \{$
 $x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant}.\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant} . (\text{Swedish} \cap \text{Belgian})) ,$
 $x: \neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant} . \neg \text{Swedish} ,$
 $x: \text{Tournament} ,$
 $x: \exists \text{ hasParticipant} . (\text{Swedish} \cap \text{Belgian}) ,$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian})$
 $\}$

Example 1

$\cap$ -rule:

- $S = \{x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})),$
 $x: \neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish},$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}),$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 $y: \text{Swedish}, y: \text{Belgian} \quad \}$

Example 1

U-rule, choice 1

- $S = \{ x: (\neg \text{Tournament} \cup \forall \text{hasParticipant}.\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian}),$
 $x: \neg \text{Tournament} \cup \forall \text{hasParticipant}.\neg \text{Swedish},$
 $x: \text{Tournament},$
 $x: \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian},$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 $y: \text{Swedish}, y: \text{Belgian},$
 $x: \neg \text{Tournament}$
 $\}$

→ clash

Example 1

U-rule, choice 2

- $S = \{x: (\neg \text{Tournament} \cup \forall \text{hasParticipant}.\neg \text{Swedish})$
 $\cap (\text{Tournament}$
 $\cap \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian}),$
 $x: \neg \text{Tournament} \cup \forall \text{hasParticipant}.\neg \text{Swedish},$
 $x: \text{Tournament},$
 $x: \exists \text{hasParticipant}.\text{Swedish} \cap \text{Belgian},$
 $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 $y: \text{Swedish}, y: \text{Belgian},$
 $x: \forall \text{hasParticipant}.\neg \text{Swedish}$
}

Example 1

choice 2 – continued

$\forall$ -rule

■ $S = \{$
x: ($\neg$ Tournament $\cup \forall$ hasParticipant. $\neg$ Swedish)
 $\cap$ (Tournament $\cap \exists$ hasParticipant.(Swedish $\cap$ Belgian)),
x: $\neg$ Tournament $\cup \forall$ hasParticipant. $\neg$ Swedish,
x: Tournament,
x: $\exists$ hasParticipant.(Swedish $\cap$ Belgian),
x hasParticipant y, y: (Swedish $\cap$ Belgian),
y: Swedish, y: Belgian,
x: $\forall$ hasParticipant. $\neg$ Swedish,
y: $\neg$ Swedish
}

$\rightarrow$ clash

Example 2

- $ST \Rightarrow SBT?$
- $S = \{ x:$
 - $\neg (\text{Tournament}$
 - $\cap \exists \text{hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish})$
 - $\}$

Example 2

- $S = \{ x:$
 $(\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant.}(\neg \text{Swedish } \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish})$
 $\}$

Example 2

$\cap$ -rule

- $S = \{$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$,
 $x: \text{Tournament},$
 $x: \exists \text{hasParticipant.Swedish}$
 $\}$

Example 2

$\exists$ -rule

- $S = \{$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 $x: \text{Tournament},$
 $x: \exists \text{ hasParticipant.Swedish},$
 $x \text{ hasParticipant } y, y: \text{Swedish}$
 $\}$

Example 2

U –rule, choice 1

- $S = \{$
 - $x: (\neg \text{Tournament}$
 $\quad U \forall \text{hasParticipant}.(\neg \text{Swedish } U \neg \text{Belgian}))$
 $\quad \cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 $\quad U \forall \text{hasParticipant}.(\neg \text{Swedish } U \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \neg \text{Tournament}$**

→ clash

Example 2

U –rule, choice 2

- $S = \{$
 - $x: (\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant}.(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 $\cup \forall \text{hasParticipant}.(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{hasParticipant}.(\neg \text{Swedish} \cup \neg \text{Belgian})$**
- $\}$

Example 2

choice 2 continued

$\forall$ -rule

- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 - $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}),$
 - $y: (\neg \text{Swedish} \cup \neg \text{Belgian})$**
- $\}$

Example 2

choice 2 continued

U-rule, choice 2.1

- $S = \{$
 - $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}),$
 - $y: (\neg \text{Swedish} \cup \neg \text{Belgian}),$
 - $y: \neg \text{Swedish}$**
- $\} \rightarrow \text{clash}$

Example 2

choice 2 continued

U-rule, choice 2.2

- $S = \{$
 - $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}))$
 $\cap (\text{Tournament} \cap \exists \text{ hasParticipant.Swedish}),$
 - $x: (\neg \text{Tournament}$
 $\cup \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian})),$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.Swedish},$
 - $x \text{ hasParticipant } y, y: \text{Swedish},$
 - $x: \forall \text{ hasParticipant.}(\neg \text{Swedish} \cup \neg \text{Belgian}),$
 - $y: (\neg \text{Swedish} \cup \neg \text{Belgian}),$
 - $y: \neg \text{Belgian}$**
 - $\} \rightarrow \text{ok, model}$

Complexity - languages

- Overview available via the DL home page at <http://dl.kr.org>

Example tractable language:

$A, T, \perp, \neg A, C \cap D, \forall R.C, \geq n R, \leq n R$

Reasons for intractability:

choices, e.g. $C \cup D$

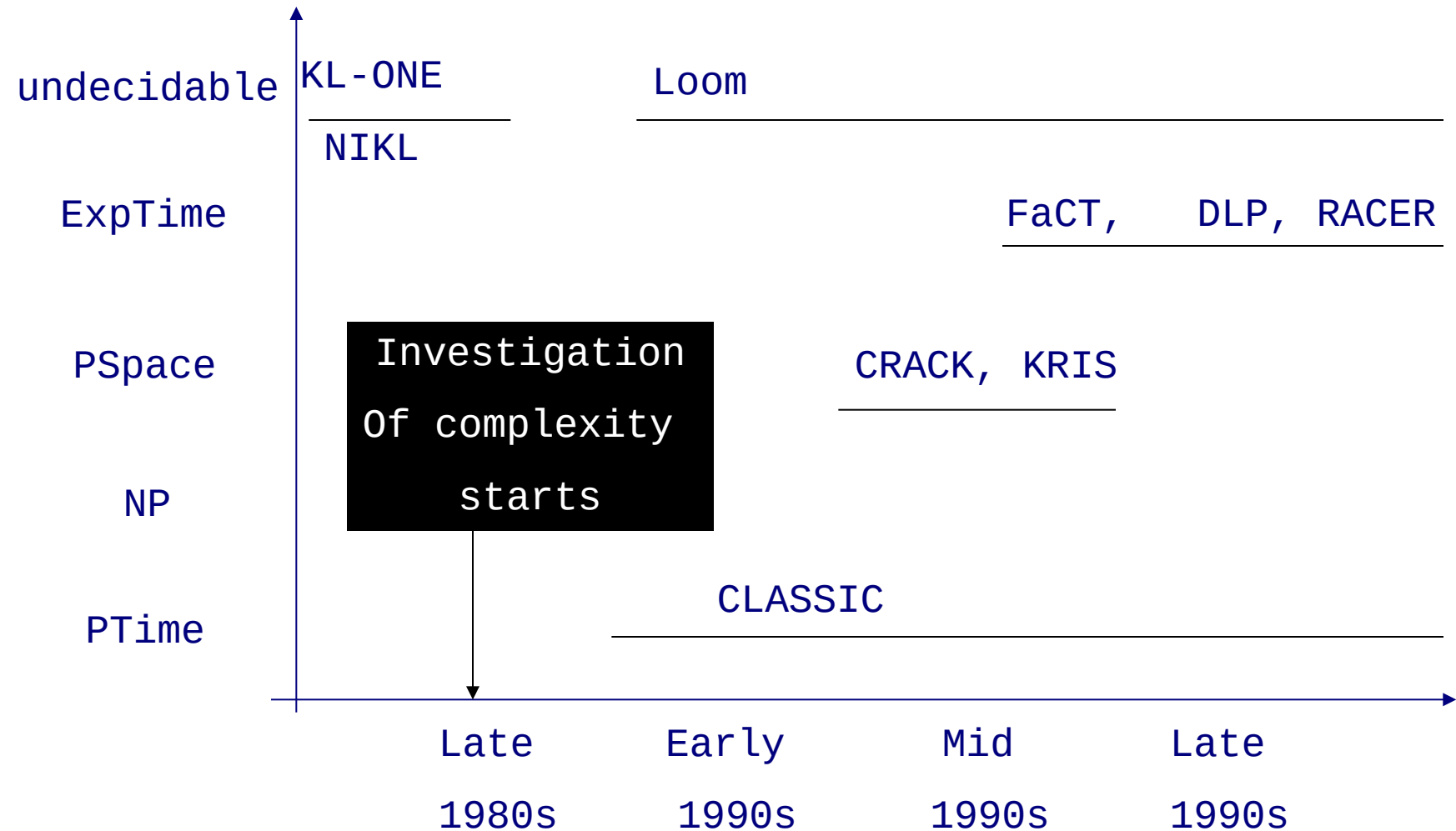
exponential size models,

e.g. interplay universal and existential quantification

Reasons for undecidability:

e.g. role-value maps $R=S$

Systems



Systems

- Overview available via the DL home page at <http://dl.kr.org>
- Current systems include: CEL, Cerebra Enginer, FaCT++, fuzzyDL, HermiT, KAON2, MSPASS, Pellet, QuOnto, RacerPro, SHER



Extensions

- Time
- Defaults
- Part-of
- Knowledge and belief
- Uncertainty (fuzzy, probabilistic)

DAML+OIL Class Constructors

Constructor	DL Syntax	Example
intersectionOf	$C_1 \sqcap \dots \sqcap C_n$	Human $\sqcap$ Male
unionOf	$C_1 \sqcup \dots \sqcup C_n$	Doctor $\sqcup$ Lawyer
complementOf	$\neg C$	$\neg$ Male
oneOf	$\{x_1 \dots x_n\}$	{john, mary}
toClass	$\forall P.C$	$\forall$ hasChild.Doctor
hasClass	$\exists P.C$	$\exists$ hasChild.Lawyer
hasValue	$\exists P.\{x\}$	$\exists$ citizenOf.{USA}
minCardinalityQ	$\geq n P.C$	≥ 2 hasChild.Lawyer
maxCardinalityQ	$\leq n P.C$	≤ 1 hasChild.Male
cardinalityQ	$= n P.C$	$= 1$ hasParent.Female

- XMLS **datatypes** as well as classes
- Arbitrarily complex **nesting** of constructors
 - E.g., Person $\sqcap \forall$ hasChild.(Doctor $\sqcup \exists$ hasChild.Doctor)

DAML+OIL Axioms

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
sameClassAs	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
samePropertyAs	$P_1 \equiv P_2$	cost $\equiv$ price
sameIndividualAs	$\{x_1\} \equiv \{x_2\}$	{President_Bush} $\equiv$ {G_W_Bush}
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$ Female
differentIndividualFrom	$\{x_1\} \sqsubseteq \neg\{x_2\}$	{john} $\sqsubseteq \neg$ {peter}
inverseOf	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent ⁻
transitiveProperty	$P^+ \sqsubseteq P$	ancestor ⁺ $\sqsubseteq$ ancestor
uniqueProperty	$\top \sqsubseteq \leq 1P$	$\top \sqsubseteq \leq 1$ hasMother
unambiguousProperty	$\top \sqsubseteq \leq 1P^-$	$\top \sqsubseteq \leq 1$ isMotherOf ⁻

 Axioms (mostly) **reducible to subClass/PropertyOf**

OWL

- OWL-Lite, OWL-DL, OWL-Full: increasing expressivity
- A legal OWL-Lite ontology is a legal OWL-DL ontology is a legal OWL-Full ontology
- OWL-DL: expressive description logic, decidable
- XML-based
- RDF-based (OWL-Full is extension of RDF, OWL-Lite and OWL-DL are extensions of a restriction of RDF)

OWL-Lite

- **Class**, subClassOf, equivalentClass
- intersectionOf (only named classes and restrictions)
- **Property**, subPropertyOf, equivalentProperty
- domain, range (global restrictions)
- inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- allValuesFrom, someValuesFrom (local restrictions)
- minCardinality, maxCardinality (only 0/1)
- **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

OWL-DL

- **Type separation** (class cannot also be individual or property, property cannot be also class or individual), Separation between DatatypeProperties and ObjectProperties
- **Class –complex classes**, subClassOf, equivalentClass, *disjointWith*
- intersectionOf, *unionOf*, *complementOf*
- **Property**, subPropertyOf, equivalentProperty
- domain, range (global restrictions)
- inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- allValuesFrom, someValuesFrom (local restrictions), *oneOf*, *hasValue*
- minCardinality, maxCardinality
- **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

OWL2

- OWL2 Full and OWL2 DL
- OWL2 DL compatible with SROIQ
- Punning
 - IRI may denote both class and individual
 - For reasoning they are considered separate entities



OWL2 profiles

- OWL2 EL (based on EL++)
 - Essentially intersection and existential quantification
 - SNOMED CT, NCI Thesaurus
- OWL2 QL (“query language”)
 - Can be realized using relational database technology
 - RDFS + small extensions
- OWL2 RL (“rule language”)

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