

Description logics

Description Logics

- A family of KR formalisms, based on FOPL decidable, supported by automatic reasoning systems
- Used for modelling of application domains
- Classification of concepts and individuals
concepts (unary predicates), subconcept (subsumption), roles (binary predicates), individuals (constants), constructors for building concepts, equality ...

[Baader et al. 2002]

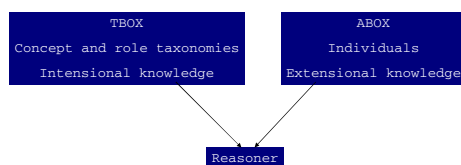
Applications

- software management
- configuration management
- natural language processing
- clinical information systems
- information retrieval
- ...
- Ontologies and the Web

Outline

- DL languages
 - syntax and semantics
- DL reasoning services
 - algorithms, complexity
- DL systems
- DLs for the web

Tbox and Abox



Syntax - \mathcal{AL}

R atomic role, A atomic concept

$C, D \rightarrow A$ | (atomic concept)

\top | (universal concept, top)

\perp | (bottom concept)

$\neg A$ | (atomic negation)

$C \sqcap D$ | (conjunction)

$\forall R.C$ | (value restriction)

$\exists R.T$ | (limited existential quantification)

$\mathcal{AL}[X]$

$C \quad \neg C$ (concept negation)

$U \quad C \sqcup D$ (disjunction)

$E \quad \exists R.C$ (existential quantification)

$\mathcal{N} \quad \geq n R, \leq n R$ (number restriction)

$\mathcal{Q} \quad \geq n R.C, \leq n R.C$ (qualified number restriction)

Example

Team

Team $\sqcap \geq 10$ hasMember

Team $\sqcap \geq 11$ hasMember
 $\sqcap \forall$ hasMember.Soccer-player

$\mathcal{AL}[X]$

$R \quad R \sqcap S$ (role conjunction)

$I \quad R^-$ (inverse roles)

\mathcal{H} (role hierarchies)

$\mathcal{F} \quad u_1 = u_2, u_1 \neq u_2$ (feature (dis)agreements)

$S[X]$

$S \quad \mathcal{ALC} + \text{transitive roles}$

$\mathcal{SHIQ} \quad \mathcal{ALC} + \text{transitive roles}$
 $+ \text{role hierarchies}$
 $+ \text{inverse roles}$
 $+ \text{number restrictions}$

Tbox

■ Terminological axioms:

- $C = D$ ($R = S$)
- $C \sqsubseteq D$ ($R \sqsubseteq S$)
- (disjoint $C \sqcap D$)

- An equality whose left-hand side is an atomic concept is a definition.
- A finite set of definitions T is a Tbox (or terminology) if no symbolic name is defined more than once.

Example Tbox

Soccer-player $\sqsubseteq T$

Team $\sqsubseteq \geq 2$ hasMember

Large-Team = Team $\sqcap \geq 10$ hasMember

S-Team = Team $\sqcap \geq 11$ hasMember
 $\sqcap \forall$ hasMember.Soccer-player

DL as sublanguage of FOPL

Team(this)
^
($\exists x_1, \dots, x_{11}$:
hasMember(this, x1) ^ ... ^ hasMember(this, x11)
^ $x_1 \neq x_2$ ^ ... ^ $x_{10} \neq x_{11}$)
^
($\forall x$: hasMember(this, x) \rightarrow Soccer-player(x))

Abox

- Assertions about individuals:
 - C(a)
 - R(a, b)

Example

Ida-member(Sture)

Individuals in the description language

- $\bigcirc \{i_1, \dots, i_k\}$ (one-of)
- R:a (fills)

Example

(S-Team \cap hasMember:Sture)(IDA-FF)

Knowledge base

A knowledge base is a tuple $\langle T, A \rangle$
where T is a Tbox and A is an Abox.

Example KB

Soccer-player \subseteq T
 Team $\subseteq \geq 2$ hasMember
 Large-Team = Team $\cap \geq 10$ hasMember
 S-Team = Team $\cap \geq 11$ hasMember
 $\cap \forall$ hasMember.Soccer-player

Ida-member(Sture)

(S-Team \cap hasMember:Sture)(IDA-FF)

\mathcal{AL} (Semantics)

An interpretation \mathcal{I} consists of a non-empty set $\Delta^{\mathcal{I}}$ (the domain of the interpretation) and an interpretation function $\cdot^{\mathcal{I}}$ which assigns to every atomic concept A a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and to every atomic role R a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

The interpretation function is extended to concept definitions using inductive definitions.

\mathcal{AL} (Semantics)

$C, D \rightarrow A$ | (atomic concept)
 T | (universal concept) $T^{\mathcal{I}} = \Delta^{\mathcal{I}}$
 \perp | (bottom concept) $\perp^{\mathcal{I}} = \emptyset$
 $\neg A$ | (atomic negation) $(\neg A)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$
 $C \cap D$ | (conjunction) $(C \cap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
 $\forall R.C$ | (value restriction) $(\forall R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \forall b. (a,b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$
 $\exists R.T$ | (limited existential quantification) $(\exists R.T)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b. (a,b) \in R^{\mathcal{I}}\}$

\mathcal{ALC} (Semantics)

$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
 $(C \cup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
 $(\geq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \# \{b \in \Delta^{\mathcal{I}} \mid (a,b) \in R^{\mathcal{I}}\} \geq n\}$
 $(\leq n R)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \# \{b \in \Delta^{\mathcal{I}} \mid (a,b) \in R^{\mathcal{I}}\} \leq n\}$
 $(\exists R.C)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \exists b \in \Delta^{\mathcal{I}} : (a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$

Semantics

Individual i

$i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

Unique Name Assumption:

if $i_1 \neq i_2$ then $i_1^{\mathcal{I}} \neq i_2^{\mathcal{I}}$

Semantics

An interpretation $\cdot^{\mathcal{I}}$ is a model for a terminology T iff

$C^{\mathcal{I}} = D^{\mathcal{I}}$ for all $C = D$ in T

$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all $C \subseteq D$ in T

$C^{\mathcal{I}} \cap D^{\mathcal{I}} = \emptyset$ for all (disjoint $C D$) in T

Semantics

An interpretation \mathcal{I} is a model for a knowledge base $\langle T, A \rangle$ iff

\mathcal{I} is a model for T

$a^{\mathcal{I}} \in C^{\mathcal{I}}$ for all $C(a)$ in A

$\langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$ for all $R(a,b)$ in A

Semantics - acyclic Tbox

$\text{Bird} = \text{Animal} \cap \forall \text{Skin.Feather}$

$\Delta^{\mathcal{I}} = \{\text{tweety}, \text{goofy}, \text{fea1}, \text{fur1}\}$

$\text{Animal}^{\mathcal{I}} = \{\text{tweety}, \text{goofy}\}$

$\text{Feather}^{\mathcal{I}} = \{\text{fea1}\}$

$\text{Skin}^{\mathcal{I}} = \{\langle \text{tweety}, \text{fea1} \rangle, \langle \text{goofy}, \text{fur1} \rangle\}$

$\rightarrow \text{Bird}^{\mathcal{I}} = \{\text{tweety}\}$

Semantics - cyclic Tbox

$\text{QuietPerson} = \text{Person} \cap \forall \text{Friend.QuietPerson}$
($A = F(A)$)

$\Delta^{\mathcal{I}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

$\text{Person}^{\mathcal{I}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

$\text{Friend}^{\mathcal{I}} = \{\langle \text{john}, \text{sue} \rangle, \langle \text{andrea}, \text{bill} \rangle, \langle \text{bill}, \text{bill} \rangle\}$

$\rightarrow \text{QuietPerson}^{\mathcal{I}} = \{\text{john}, \text{sue}\}$

$\rightarrow \text{QuietPerson}^{\mathcal{I}} = \{\text{john}, \text{sue}, \text{andrea}, \text{bill}\}$

Semantics - cyclic Tbox

Descriptive semantics: $A = F(A)$ is a constraint stating that A has to be some solution for the equation.

- Not appropriate for defining concepts
- Necessary and sufficient conditions for concepts

$\text{Human} = \text{Mammal} \cap \exists \text{Parent}$

$\cap \forall \text{Parent.Human}$

Semantics - cyclic Tbox

Least fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the smallest solution (if it exists) for the equation.

- Appropriate for inductively defining concepts

$\text{DAG} = \text{EmptyDAG} \cup \text{Non-Empty-DAG}$

$\text{Non-Empty-DAG} = \text{Node} \cap \forall \text{Arc.Non-Empty-DAG}$

$\text{Human} = \text{Mammal} \cap \exists \text{Parent} \cap \forall \text{Parent.Human}$

$\rightarrow \text{Human} = \perp$

Semantics - cyclic Tbox

Greatest fixpoint semantics: $A = F(A)$ specifies that A is to be interpreted as the greatest solution (if it exists) for the equation.

- Appropriate for defining concepts whose individuals have circularly repeating structure

$\text{FoB} = \text{Blond} \cap \exists \text{Child.FoB}$

$\text{Human} = \text{Mammal} \cap \exists \text{Parent} \cap \forall \text{Parent.Human}$

$\text{Horse} = \text{Mammal} \cap \exists \text{Parent} \cap \forall \text{Parent.Horse}$

$\rightarrow \text{Human} = \text{Horse}$

Open world vs closed world semantics

Databases: closed world reasoning
 database instance represents one interpretation
 → absence of information interpreted as negative information
 “complete information”
 query evaluation is finite model checking

DL: open world reasoning
 Abox represents many interpretations (its models)
 → absence of information is lack of information
 “incomplete information”
 query evaluation is logical reasoning

Open world vs closed world semantics

hasChild(Jocasta, Oedipus)
 hasChild(Jocasta, Polyneikes)
 hasChild(Oedipus, Polyneikes)
 hasChild(Polyneikes, Thersandros)
 patricide(Oedipus)
 ¬ patricide(Thersandros)

Does it follow from the Abox that
 $\exists \text{hasChild}(\text{patricide} \cap \exists \text{hasChild} \neg \text{patricide})(\text{Jocasta})$?

Reasoning services

- Satisfiability of concept
- Subsumption between concepts
- Equivalence between concepts
- Disjointness of concepts
- Classification
- Instance checking
- Realization
- Retrieval
- Knowledge base consistency

Reasoning services

- Satisfiability of concept
 - C is satisfiable w.r.t. \mathcal{T} if there is a model I of \mathcal{T} such that C^I is not empty.
- Subsumption between concepts
 - C is subsumed by D w.r.t. \mathcal{T} if $C^I \subseteq D^I$ for every model I of \mathcal{T} .
- Equivalence between concepts
 - C is equivalent to D w.r.t. \mathcal{T} if $C^I = D^I$ for every model I of \mathcal{T} .
- Disjointness of concepts
 - C and D are disjoint w.r.t. \mathcal{T} if $C^I \cap D^I = \emptyset$ for every model I of \mathcal{T} .

Reasoning services

- Reduction to subsumption
 - C is unsatisfiable iff C is subsumed by \perp
 - C and D are equivalent iff C is subsumed by D and D is subsumed by C
 - C and D are disjoint iff $C \cap D$ is subsumed by \perp
- The statements also hold w.r.t. a Tbox.

Reasoning services

- Reduction to unsatisfiability
 - C is subsumed by D iff $C \cap \neg D$ is unsatisfiable
 - C and D are equivalent iff both $(C \cap \neg D)$ and $(D \cap \neg C)$ are unsatisfiable
 - C and D are disjoint iff $C \cap D$ is unsatisfiable
- The statements also hold w.r.t. a Tbox.

Tableau algorithms

- To prove that C subsumes D:
 - If C subsumes D, then it is impossible for an individual to belong to D but not to C.
 - Idea: Create an individual that belongs to D and not to C and see if it causes a contradiction.
 - If **always** a contradiction (clash) then subsumption is proven. Otherwise, we have found a model that contradicts the subsumption.

Tableau algorithms

- Based on constraint systems.
 - $S = \{ x: \neg C \cap D \}$
 - Add constraints according to a set of propagation rules
 - Until clash or no constraint is applicable

Tableau algorithms – de Morgan rules

- $\neg \neg C \rightarrow C$
- $\neg (A \cap B) \rightarrow \neg A \cup \neg B$
- $\neg (A \cup B) \rightarrow \neg A \cap \neg B$
- $\neg (\forall R.C) \rightarrow \exists R.(\neg C)$
- $\neg (\exists R.C) \rightarrow \forall R.(\neg C)$

Tableau algorithms – constraint propagation rules

- $S \rightarrow_{\cap} \{x:C_1, x:C_2\} \cup S$
 - if $x: C_1 \cap C_2$ in S
 - and either $x:C_1$ or $x:C_2$ is not in S
- $S \rightarrow_{\cup} \{x:D\} \cup S$
 - if $x: C_1 \cup C_2$ in S and neither $x:C_1$ or $x:C_2$ is in S, and $D = C_1$ or $D = C_2$

Tableau algorithms – constraint propagation rules

- $S \rightarrow_{\forall} \{y:C\} \cup S$
 - if $x: \forall R.C$ in S and xRy in S and $y:C$ is not in S
- $S \rightarrow_{\exists} \{xRy, y:C\} \cup S$
 - if $x: \exists R.C$ in S and y is a new variable and there is no z such that both xRz and $z:C$ are in S

Example

- ST: Tournament
 - $\cap \exists \text{hasParticipant.Swedish}$
- SBT: Tournament
 - $\cap \exists \text{hasParticipant.}(\text{Swedish} \cap \text{Belgian})$

Example 1

- SBT \Rightarrow ST?
- $S = \{ x:$
 - $\neg(\text{Tournament} \cap \exists \text{ hasParticipant.Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
 - $\}$

Example 1

- $S = \{ x:$
 - $(\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}))$
 - $\}$

Example 1

- \cap -rule:
- $S = \{$
 - $x: (\neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})),$
 - $x: \neg \text{Tournament}$**
 - $\cup \forall \text{ hasParticipant.} \neg \text{Swedish},$**
 - $x: \text{Tournament},$**
 - $x: \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})$**
 - $\}$

Example 1

- \exists -rule:
- $S = \{$
 - $x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})),$
 - $x: \neg \text{Tournament}$
 - $\cup \forall \text{ hasParticipant.} \neg \text{Swedish},$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}),$
 - $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian})$**
 - $\}$

Example 1

- \cap -rule:
- $S = \{ x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})),$
 - $x: \neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish},$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}),$
 - $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 - $y: \text{Swedish}, y: \text{Belgian} \}$**

Example 1

- U-rule, choice 1
- $S = \{ x: (\neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish})$
 - $\cap (\text{Tournament}$
 - $\cap \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian})),$
 - $x: \neg \text{Tournament} \cup \forall \text{ hasParticipant.} \neg \text{Swedish},$
 - $x: \text{Tournament},$
 - $x: \exists \text{ hasParticipant.}(\text{Swedish} \cap \text{Belgian}),$
 - $x \text{ hasParticipant } y, y: (\text{Swedish} \cap \text{Belgian}),$
 - $y: \text{Swedish}, y: \text{Belgian},$
 - $x: \neg \text{Tournament}$**
 - $\}$
- \rightarrow clash

Example 1

U-rule, choice 2

```

■ S = {x: (¬Tournament U ∀ hasParticipant.¬ Swedish)
  ∩ (Tournament
  ∩ ∃ hasParticipant.(Swedish ∩ Belgian)),
x: ¬Tournament U ∀ hasParticipant.¬ Swedish,
x: Tournament,
x: ∃ hasParticipant.(Swedish ∩ Belgian),
x hasParticipant y, y: (Swedish ∩ Belgian),
y: Swedish, y: Belgian,
x: ∀ hasParticipant.¬ Swedish
}

```

Example 1

choice 2 – continued

∀-rule

```

■ S = {
x: (¬Tournament U ∀ hasParticipant.¬ Swedish)
  ∩ (Tournament ∩ ∃ hasParticipant.(Swedish ∩ Belgian)),
x: ¬Tournament U ∀ hasParticipant.¬ Swedish,
x: Tournament,
x: ∃ hasParticipant.(Swedish ∩ Belgian),
x hasParticipant y, y: (Swedish ∩ Belgian),
y: Swedish, y: Belgian,
x: ∀ hasParticipant.¬ Swedish,
y: ¬ Swedish
}

```

→ clash

Example 2

■ ST => SBT?

```

■ S = { x:
  ¬ (Tournament
  ∩ ∃ hasParticipant.(Swedish ∩ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish)
}

```

Example 2

```

■ S = { x:
  (¬Tournament
  U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish)
}

```

Example 2

∩-rule

```

■ S = {
x: (¬Tournament
  U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
x: (¬Tournament
U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
x: Tournament,
x: ∃ hasParticipant.Swedish
}

```

Example 2

∃-rule

```

■ S = {
x: (¬Tournament
  U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
  ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
x: (¬Tournament
  U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
x: Tournament,
x: ∃ hasParticipant.Swedish,
x hasParticipant y, y: Swedish
}

```

Example 2

U –rule, choice 1

```

■ S = {
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
    ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
  x: Tournament,
  x: ∃ hasParticipant.Swedish,
  x hasParticipant y, y: Swedish,
  x: ¬Tournament
}
→ clash

```

Example 2

U –rule, choice 2

```

■ S = {
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
    ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
  x: Tournament,
  x: ∃ hasParticipant.Swedish,
  x hasParticipant y, y: Swedish,
  x: ∀ hasParticipant.(¬ Swedish U ¬ Belgian)
}

```

Example 2

choice 2 continued

∀ –rule

```

■ S = {
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
    ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
  x: Tournament,
  x: ∃ hasParticipant.Swedish,
  x hasParticipant y, y: Swedish,
  x: ∀ hasParticipant.(¬ Swedish U ¬ Belgian),
  y: (¬ Swedish U ¬ Belgian)
}

```

Example 2

choice 2 continued

U –rule, choice 2.1

```

■ S = {
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
    ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
  x: Tournament,
  x: ∃ hasParticipant.Swedish,
  x hasParticipant y, y: Swedish,
  x: ∀ hasParticipant.(¬ Swedish U ¬ Belgian),
  y: (¬ Swedish U ¬ Belgian),
  y: ¬ Swedish
}
→ clash

```

Example 2

choice 2 continued

U –rule, choice 2.2

```

■ S = {
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian))
    ∩ (Tournament ∩ ∃ hasParticipant.Swedish),
  x: (¬Tournament
    U ∀ hasParticipant.(¬ Swedish U ¬ Belgian)),
  x: Tournament,
  x: ∃ hasParticipant.Swedish,
  x hasParticipant y, y: Swedish,
  x: ∀ hasParticipant.(¬ Swedish U ¬ Belgian),
  y: (¬ Swedish U ¬ Belgian),
  y: ¬ Belgian
}
→ ok, model

```

Complexity - languages

- Overview available via the DL home page at <http://dl.kr.org>

Example tractable language:

$A, T, \perp, \neg A, C \cap D, \forall R.C, \exists n R, \leq n R$

Reasons for intractability:

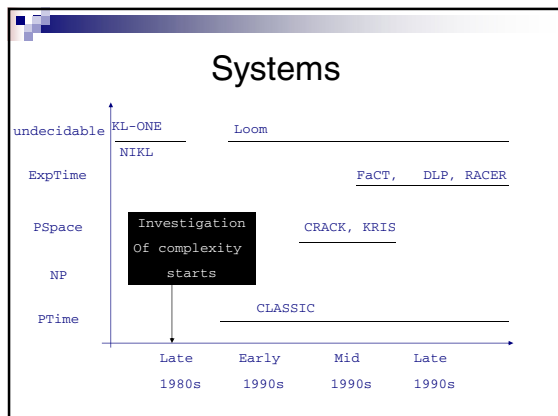
choices, e.g. $C \cup D$

exponential size models,

e.g. interplay universal and existential quantification

Reasons for undecidability:

e.g. role-value maps $R=S$



- ## Systems
- Overview available via the DL home page at <http://dl.kr.org>
 - Current systems include: CEL, Cerebra Engine, FaCT++, fuzzyDL, Hermit, KAON2, MSPASS, Pellet, QuOnto, RacerPro, SHER

- ## Extensions
- Time
 - Defaults
 - Part-of
 - Knowledge and belief
 - Uncertainty (fuzzy, probabilistic)

DAML+OIL Class Constructors

Constructor	DL Syntax	Example
intersectionOf	$C_1 \sqcap \dots \sqcap C_n$	Human \sqcap Male
unionOf	$C_1 \sqcup \dots \sqcup C_n$	Doctor \sqcup Lawyer
complementOf	$\neg C$	\neg Male
oneOf	$\{x_1, \dots, x_n\}$	{john, mary}
toClass	$\forall P.C$	\forall hasChild.Doctor
hasClass	$\exists P.C$	\exists hasChild.Lawyer
hasValue	$\exists P.\{x\}$	\exists citizenOf.{USA}
minCardinalityQ	$\geq n.P.C$	≥ 2 hasChild.Lawyer
maxCardinalityQ	$\leq n.P.C$	≤ 1 hasChild.Male
cardinalityQ	$= n.P.C$	$= 1$ hasParent.Female

✎ XMLS **datatypes** as well as classes
 ✎ Arbitrarily complex **nesting** of constructors
 • E.g., $\text{Person} \sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild.Doctor})$

EDBT 2002, DAML+OIL, p.15/32

DAML+OIL Axioms

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human \sqsubseteq Animal \sqcap Biped
sameClassAs	$C_1 \equiv C_2$	Man \equiv Human \sqcap Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter \sqsubseteq hasChild
samePropertyAs	$P_1 \equiv P_2$	cost \equiv price
sameIndividualAs	$\{x_1\} \equiv \{x_2\}$	{President_Bush} \equiv {G_W_Bush}
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$ Female
differentIndividualFrom	$\{x_1\} \sqsubseteq \neg \{x_2\}$	{john} $\sqsubseteq \neg$ {peter}
inverseOf	$P_1 \equiv P_2^-$	hasChild \equiv hasParent ⁻
transitiveProperty	$P^+ \sqsubseteq P$	ancestor ⁺ \sqsubseteq ancestor
uniqueProperty	$T \sqsubseteq \leq 1P$	$T \sqsubseteq \leq 1$ hasMother
unambiguousProperty	$T \sqsubseteq \leq 1P^-$	$T \sqsubseteq \leq 1$ isMotherOf ⁻

✎ Axioms (mostly) **reducible to subClass/PropertyOf**

EDBT 2002, DAML+OIL, p.14/32

- ## OWL
- OWL-Lite, OWL-DL, OWL-Full: increasing expressivity
 - A legal OWL-Lite ontology is a legal OWL-DL ontology is a legal OWL-Full ontology
 - OWL-DL: expressive description logic, decidable
 - XML-based
 - RDF-based (OWL-Full is extension of RDF, OWL-Lite and OWL-DL are extensions of a restriction of RDF)

OWL-Lite

- **Class**, subClassOf, equivalentClass
- intersectionOf (only named classes and restrictions)
- **Property**, subPropertyOf, equivalentProperty
- domain, range (global restrictions)
- inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- allValuesFrom, someValuesFrom (local restrictions)
- minCardinality, maxCardinality (only 0/1)
- **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

OWL-DL

- **Type separation** (class cannot also be individual or property, property cannot be also class or individual), Separation between DatatypeProperties and ObjectProperties
- **Class –complex classes**, subClassOf, equivalentClass, *disjointWith*
- *intersectionOf*, *unionOf*, *complementOf*
- **Property**, subPropertyOf, equivalentProperty
- domain, range (global restrictions)
- inverseOf, TransitiveProperty (*), SymmetricProperty, FunctionalProperty, InverseFunctionalProperty
- allValuesFrom, someValuesFrom (local restrictions), *oneOf*, *hasValue*
- *minCardinality*, *maxCardinality*
- **Individual**, sameAs, differentFrom, AllDifferent

(*) restricted

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