# Problem Set for Tutorial 8 — TDDD14/TDDD85

## 1 LR-Parsing

**Exercise 1.** Consider the following CFG G (S is the start symbol):

 $\begin{array}{l} S \rightarrow A\$ \\ A \rightarrow AB \mid B \\ B \rightarrow (A) \mid () \end{array}$ 

- 1. Construct an NFA which shows the valid LR(0) items for each viable prefix. (You may choose to skip this step and go directly to b).
- 2. Construct an equivalent DFA (exclude the error state and all transitions to it).
- 3. Is G an LR(0) grammar?

**Exercise 2.** Is the following CFG an LR(0) grammar? (S' is the start symbol.)

 $\begin{array}{l} S' \rightarrow S \\ S \rightarrow bA \mid aB \\ A \rightarrow a \mid aS \mid bAA \\ B \rightarrow b \mid bS \mid aBB \end{array}$ 

**Exercise 3.** Show how the following strings are parsed by the LR(0) parser whose finite control is given by the DFA in Exercise 1(2). For each step show the stack, the remaining input and whether the operation is "shift" or "reduce". For reduce operations show which productions are involved.

- 1. (()())\$
- 2. ()((()))()

**Exercise 4.** Is the following CFG an LR(0) grammar? (S is the start symbol.)

 $\begin{array}{l} S \rightarrow A \\ A \rightarrow aAa \mid bAb \mid c \end{array}$ 

**Exercise 5.** Consider the following CFG G (S is the start symbol):

 $\begin{array}{l} S \rightarrow A \\ A \rightarrow AB \mid \epsilon \\ B \rightarrow b \mid aB \end{array}$ 

- 1. Construct an NFA showing the valid LR(1) items for each viable prefix. (You may choose to skip this step and construct a DFA instead).
- 2. Is G an LR(1) grammar?

**Exercise 6.** Consider the following CFG G (S is the start symbol):

 $\begin{array}{l} S \rightarrow E \\ E \rightarrow E + T \mid T \\ T \rightarrow a \mid (E) \end{array}$ 

- 1. Construct a DFA showing the valid LR(1) items for a viable prefix.
- 2. Show how the string a + (a + a) is parsed by the LR(1) parser corresponding to the DFA in a) (\$ is used to denote 'end-of-input').

For each step show the stack, the remaining input, the kind of action ("shift" or "reduce") and the production used in "reduce".

## Solutions

#### Solution to Exercise 1.

1. The NFA is shown in Figure 1.

- 2. The DFA is shown in Figure 2.
- 3. Yes, the grammar is LR(0).

**Solution to Exercise 2.** If we construct a DFA which determines the set of valid LR(0) items for each viable prefix (see Figure 3), then we will find that both  $A \to a \bullet$  and  $S \to \bullet bA$  are valid for the viable prefix ab, for instance. Since  $A \to a \bullet$  is a complete item, and another item is valid for the same viable prefix, the grammar is *not* LR(0).

### Solution to Exercise 3.

Stack	Remaining input	Comment
0	(()())\$	Start
0(5	()())\$	Shift
0(5(5	)())\$	Shift
0(5(5)7)	())\$	Shift
0(5B4)	())\$	Reduce by $B \to ()$
0(5A6)	())\$	Reduce by $A \to B$
0(5A6(5	))\$	Shift
0(5A6(5)7	)\$	Shift
0(5A6B3	)\$	Reduce by $B \to ()$
0(5A6)	)\$	Reduce by $A \to AB$
0(5A6)8	\$	Shift
0B4	\$	Reduce by $B \to (A)$
0A1	\$	Reduce by $A \to B$
0A1\$2	-	Shift
0S	-	Reduce by $S \to A$ and accept

Solution to Exercise 4. If a DFA is constructed as in Exercise 1 (see Figure 4), we will find that the grammar is LR(0).

### Solution to Exercise 5.

1. See the NFA in Figure 5 (\$ denotes 'end-of-input').

2. If a DFA equivalent to the NFA in the first subtask is constructed (see Figure 6), we will find that the grammar is LR(1).

### Solution to Exercise 6.

1. See Figure 7.

2.	Stack	Remaining	Comment
		$\mathbf{input}$	
	A	a + (a + a)\$	Start
	AaE	+(a+a)\$	Shift
	ATF	+(a+a)\$	Reduce by $T \to a$
	AEB	+(a+a)\$	Reduce by $E \to T$
	AEB+C	(a+a)\$	Shift
	AEB+C(G	(a+a)	Shift
	AEB+C(GaK	+a)\$	Shift
	AEB+C(GTJ	+a)\$	Reduce by $T \to a$
	AEB+C(GEH	+a)\$	Reduce by $E \to T$
	AEB+C(GEH+M	a)\$	Shift
	AEB+C(GEH+MaK)	)\$	Shift
	AEB+C(GEH+MTN	)\$	Reduce by $T \to a$
	AEB+C(GEH	)\$	Reduce by
			$E \rightarrow E + T$
	AEB+C(GEH)I	\$	Shift
	AEB+CTD	\$	Reduce by
			$T \to (E)$
	AEB	\$	Reduce by
			$E \rightarrow E + T$
	AS	\$	Reduce by $S \to E$
			and accept

An LR(1) parser can be described by a decision table. For an input symbol and a DFA state from the stack, the table gives the parser's action. The decision table corresponding to the DFA above is shown here.

Stack	Next input symbol					
$\operatorname{top}$	a	+	(	)	\$	
А	Shift		Shift			
В		Shift			Reduce by $S \rightarrow E$ and accept	
С	Shift		Shift			
D		$\begin{array}{c} \text{Reduce by } E \to \\ E+T \end{array}$			Reduce by $E \rightarrow E + T$	
E		Reduce by $T \to a$			Reduce by $T \to a$	
F		$\begin{array}{c} \text{Reduce by } E \rightarrow \\ T \end{array}$			Reduce by $E \rightarrow T$	
G	Shift		Shift			
Н		Shift		Shift		
I		$\begin{array}{c} \text{Reduce by } T \to \\ (E) \end{array}$			Reduce by $T \to (E)$	
J		$\begin{array}{c} \text{Reduce by } E \rightarrow \\ T \end{array}$		Reduce by $E \rightarrow T$		
K		Reduce by $T \to a$		Reduce by $T \to a$		
L	Shift		Shift			
М	Shift		Shift			
Ν		$\begin{array}{c c} \text{Reduce by } E \to \\ E+T \end{array}$		Reduce by $E \rightarrow E + T$		
0		Shift		Shift		
Р		$\begin{array}{c} \text{Reduce by } T \rightarrow \\ (E) \end{array}$		Reduce by $T \rightarrow (E)$		

### Figures

This section contains the figures for the solutions to Exercise 1 – Exercise 6



Figure 1: The NFA for Exercise 1.



Figure 2: The DFA for Exercise 1.



Figure 3: The DFA for Exercise 2.



Figure 4: The DFA for Exercise 4.



Figure 5: The NFA for Exercise 5.



Figure 6: The DFA for Exercise 5.



Figure 7: The DFA for Exercise 6.

## 2 Advanced and Exam Like Exercises

**Exercise 7.** Consider the set of O of strings over  $\{0, 1, \ldots, 9\}$  containing an odd number of digits. For example,  $5,000,083 \in O$ .

Prove that O is context-free by constructing a context-free grammar for O. Your grammar should LR(0) or LR(1). To simplify the last task you may restrict yourself to a sufficiently rich subset of the grammar (e.g., instead of using 9 distinct digits, you could use a single symbol).

### Solutions

Solution to Exercise 7. A context-free grammar for the language can be given by:  $D \to 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$   $O \to D$   $O \to DDO$ 

This grammar can be proven to not be LR(0) by constructing the DFA with LR(0) items. However, it can be proven to be LR(1).