Examination
Formal Languages and Automata Theory
TDDD85

(Formella Språk och Automatateori)

2020–06–04, 08.00 – 13.00

1. You may answer in Swedish or English.
2. Email your solution as a single PDF file to `victor.lagerkvist@liu.se` with the subject line "TDDD85: exam LiU-ID" where LiU-ID is your LiU-ID.
3. The maximum number of points is 30. The grades are as follows:

Grade	TDDD85
3	13–18
4	19–23
5	24–30

4. *Jour* (contact person): Victor Lagerkvist (`victor.lagerkvist@liu.se`, tel. +46730817584)

GOOD LUCK !

Make sure that you justify your answers! Unexplained answers will be granted 0 points. (For instance, if you are writing a grammar for a given language then you should also explain that the grammar indeed generates the language. If you apply some known method then you should explain each step. And so on.)

1. *(2p)* For each pair of regular expressions R_1 and R_2 below, answer whether they generate the same language ($L(R_1) = L(R_2)$). If no, give a string which belongs to one of the languages and does not belong to the other. If yes, prove that they are equivalent by (1) computing $L(R_1)$ and $L(R_2)$ as far as you can and (2) verifying that the two resulting sets are equal.

(a)	$(a^*bc)^*$	$(bc + a)^*$
(b)	$(ab + a)^*a$	$a(ba + a)^*$

2. *(2p)* Let the languages L_1 and L_2 be defined as follows:

   - L_1 is defined by the regular expression $(a + b)^*bba(a + b)^*$.
   - L_2 is the language of strings over $\{a, b\}^*$ containing the string ab.

   Give a regular expression R such that $L(R) = L_1 - L_2$, *i.e,* $L(R) = \{w \mid w \in L_1 \wedge w \notin L_2\}$. Explain your reasoning and why your solution is correct.

   Hint: while it is possible to solve the problem by a systematic approach by constructing a DFA for $L_1 \cap \overline{L_2}$ and converting this DFA to a regular expression, it is *much* easier to construct the regular expression directly.

3. *(4p)* The NFA $N = (Q, \Sigma, \Delta, S, F)$ is defined as follows:

$$Q = \{0, 1, 2, 3\} \qquad \Sigma = \{a, b\} \qquad S = \{0\} \qquad F = \{2, 3\}$$

   with the transition function Δ given by

	ϵ	a	b
$\rightarrow 0$	$\emptyset$	$\emptyset$	$\{1, 3\}$
1	$\emptyset$	$\{0\}$	$\{2\}$
2 F	$\{1\}$	$\emptyset$	$\{3\}$
3 F	$\emptyset$	$\{2\}$	$\emptyset$

   Using the subset construction method, construct an equivalent DFA M. **Explain each step of the construction. A table/figure without any explanation will be given 0 points.**

4. *(4p)* Using the GNFA method, construct a regular expression defining the same language as the DFA whose transition function δ is given by

	a	b
$\rightarrow A$ F	B	A
B	B	C
C F	A	B

Explain each step of the construction. Hence, when removing a state and updating transitions, motivate the new transitions by the $R_1 R_2^* R_3 + R_4$ rule. Explain any simplifications that you make.

5. *(4p)* Construct the minimal DFA equivalent to the DFA given by the table below by using the marking algorithm (or a similar algorithm). Its set of states is $Q = \{A, B, C, D, E, F, G, H\}$, the input alphabet is $\Sigma = \{0, 1\}$, the start state is A, and $\{A, E\}$ is the set of final states.

	0	1
$\rightarrow A$ F	B	F
B	C	C
C	D	C
D	D	E
E F	F	F
F	G	C
G	H	G
H	D	A

Include all relevant calculations. In particular, if you in an iteration detect that two states are not equivalent, then you have to include the corresponding calculation.

6. *(6p)* Consider the language L generated by the context-free grammar

$$S \rightarrow \epsilon \mid aaSA \qquad\qquad A \rightarrow bb \mid bbb$$

where S is the start symbol, A and S are nonterminals, and a and b are terminals.

(a) Prove that L is not regular by using the pumping lemma for regular languages.

(b) Prove that L is not regular by using the Myhill-Nerode theorem.

7. *(4p)* Consider the set of strings over $\{0, 1, \dots, 9\}$ containing an even, respectively odd, number of digits. Let E and O denote these languages. For example, $\varepsilon, 00, 09, 99 \in E$, and $5, 000, 083 \in O$.

   (a) Prove that E and O are context-free by constructing context-free grammars G_1 and G_2 such that $L(G_1) = E$ and $L(G_2) = O$.

   (b) Are G_1 and G_2 LR(0)? Properly motivate your answer (a yes/no answer will not give any points).

8. *(4p)* Let L be a decidable language, L' a Turing-recognisable language, and X a finite, non-empty language.

   (a) Prove or disprove that $L \leq_m X$ always holds.

   (b) Prove or disprove that $L' \leq_m X$ always holds.

   Recall that we write $L_1 \leq_m L_2$ if there exists a mapping reduction from L_1 to L_2.