Examination
Formal Languages and Automata Theory
TDDD14/TDDD85

(Formella Språk och Automatateori)

2025–06–03

1. You may answer in Swedish or English.
2. Allowed help materials
   - A sheet of handwritten notes - 2 sided A5 or 1 sided A4.
     The contents is up to you. Return the notes together with the exam. The notes should be signed in the same way as the exam sheets and returned together with the exam.
   - English dictionary
3. The maximum number of points is 28. The grades are as follows:

Grade	TDDD14	TDDD85
3	14–19	12–17
4	20–23	18–22
5	24–28	23–28

Make sure that you justify your answers! Unexplained answers will be granted 0 points. (For instance, if you are writing a grammar for a given language then you should also explain that the grammar indeed generates the language. If you apply some known method then you should explain each step. And so on.)

GOOD LUCK !

1. *(4p)* Which of the following claims are true?

   (a) If A, B are regular languages then $A \cap B$ is regular.

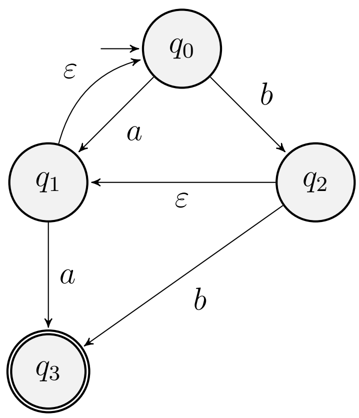
   (b) The regular expression $a(a^*b)^*$ generates the same language as a^+b^* (where $a^+ = \{a\}^+ = \{a\}^* \cup \{a\}$, i.e., one or more as).

   (c) If Σ and Δ are two alphabets then there exists a homomorphism from Σ^* to Δ^*.

   (d) There exists a finite language which is not LR(0).

   Answer each claim by first stating whether the claim is true or false, and then motivate your claim. A substantial motivation is necessary in order to get points.

2. *(4p)* Consider the following NFA.



   Using the subset construction method, construct an equivalent DFA. **Explain each step of the construction. A table/figure without any explanation will be given 0 points.**

3. *(4p)*

   (a) Consider the DFA M given by the table

		a	b
$\rightarrow$	1	3	2
	2	4	2
	$3F$	6	2
	$4F$	6	1
	5	2	4
	6	1	3

The states are $\{1, 2, 3, 4, 5, 6\}$, 1 is the start state, 3 and 4 the final states, and the the input alphabet is $\Sigma = \{a, b\}$. Construct the minimal DFA equivalent to this DFA by using the marking algorithm (or a similar algorithm).

Include all relevant calculations. In particular, if you in an iteration detect that two states are not equivalent, then you have to include the corresponding calculation.

(b) Let $A = L(M)$ be the language of the machine above. Consider the equivalence relation R_A on Σ^* defined by: xR_Ay if and only if for all $z \in \Sigma^*$, $xz \in A$ iff $yz \in A$. How many equivalence classes does R_L have?

Attempt to relate the equivalence classes to the minimal DFA produced before.

4. *(6p)* Consider the language $L = \{a^n b^m \mid n, m \geq 0, m = n^2\}$.

   (a) Give examples of strings in L and outside L. (*1p*)

   (b) Show that L is not context-free by using the (inverted) pumping lemma for context-free languages. (*5p*)

   Optionally: show instead that L is not regular, by using similar techniques for regular languages. (*3p*)

   You can choose at most one subtask in (b), i.e., either prove that the language is not context-free, or prove that it is not regular, but not both.

5. *(2p)* Consider the language $L = \{v0w \mid v, w \in \{a, b\}^*, |v| = |w|\}$.

   Construct a PDA accepting strings in L. Give the PDA in tuple form, except for the transition relation which you may give in table form (with current-state and top-of-stack along the vertical dimension and symbol-read along the horizontal dimension).

   Explain the purpose of each state in the PDA, and the purpose of each symbol in the stack alphabet.

6. *(4p)* Given the follwing grammar, G_1:

   $E \to E + E \mid (E)$

   $E \to a \mid b \mid c \mid d$

   (a) Consider the string $w = a + b + (c + d)$. Give a leftmost derivation of w (show each step) and its corresponding derivation tree. (*1p*)

(b) Give another leftmost derivation of w, and show that its derivation tree is different from the one in (a). (*1p*)

(c) Construct a new grammar G_2 such that $L(G_2) = L(G_1)$. Your new grammar has to be unambiguous, that is, for a given string, there is only one possible (leftmost) derivation tree. Show the derivation tree for w in this new grammar G_2. (*2p*)

7. *(4p)* Let $A, B \subseteq \Sigma^*$ be two languages. Answer the following statements. Proofs or counter examples are needed.

   (a) If A and B are decidable then $A \cup B$ is decidable.

   (b) If A or B is undecidable then $A \cup B$ is undecidable.