Examination
Formal Languages and Automata Theory
TDDD14/TDDD85

(Formella Språk och Automatateori)

2021–06–02

1. You may answer in Swedish or English.
2. The maximum number of points is 28. The grades are as follows:

Grade	TDDD14	TDDD85
3	14–19	12–17
4	20–23	18–22
5	24–28	23–28

Make sure that you justify your answers! Unexplained answers will be granted 0 points. (For instance, if you are writing a grammar for a given language then you should also explain that the grammar indeed generates the language. If you apply some known method then you should explain each step. And so on.)

GOOD LUCK !

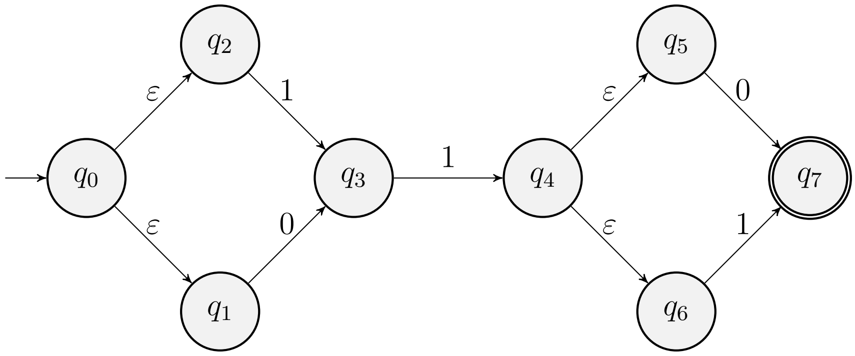


Figure 1: The NFA in Exercise 2.

1. *(2p)* For each pair of regular expressions R_1 and R_2 below, answer whether they generate the same language ($L(R_1) = L(R_2)$). If no, give a string which belongs to one of the languages and does not belong to the other. If yes, show that they are equivalent, e.g., by (1) computing $L(R_1)$ and $L(R_2)$ as far as you can and (2) verifying that the two resulting sets are equal. For the last step, an informal explanation is sufficient.

   (a) $a\emptyset^*$ and $a\varepsilon^*$.

   (b) $a\emptyset$ and a.

   (c) $(a + \varepsilon + \emptyset)b^*$ and $ab^* + b^*$.

   (d) a^* and $(aa)^* + a(aa)^*$.

2. *(4p)* Consider the NFA M in Figure 1.

   (a) Describe $L(M)$, as precise as possible, using the set builder notation.

   (b) Using the subset construction method, construct an equivalent DFA.

   Explain each step of the construction. A table/figure without any explanation will be given 0 points.

3. *(4p)* Which of the following claims are true?

   (a) Every finite language is regular.

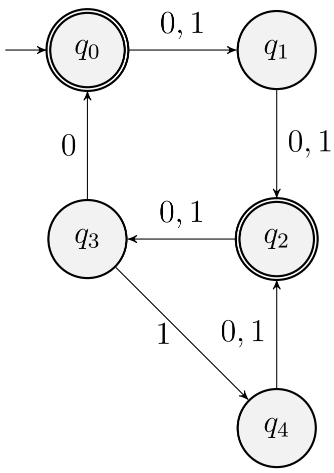
   (b) There exists a subset of a regular language which is not Turing-recognisable.

(c) A regular expression without any occurrence of the star operator (R^*) or the $+$ operator as a superscript (R^+) must describe a finite language.

(d) Every Σ^* has a homomorphism to $\emptyset^*$ (where Σ is an arbitrary alphabet).

Answer each claim by first stating whether the claim is true or false, and then motivate your claim. A substantial motivation is necessary in order to get points.

4. *(4p)* Consider the following DFA.



(a) Construct the minimal DFA equivalent to this DFA by using the marking algorithm (or a similar algorithm).

Include all relevant calculations. In particular, if you in an iteration detect that two states are not equivalent, then you have to include the corresponding calculation.

(b) Recall that $\hat{\delta}(q_i, x)$ for a string $x \in \{0,1\}^*$ is the state resulting from "simulating" the DFA on the input string x, starting from the state q_i. Define the binary relation R as $\{(q_i, q_j) \mid q_i, q_j \in \{q_0, q_1, q_2, q_3, q_4\}, \hat{\delta}(q_i, x) \in \{q_0, q_2\} \Leftrightarrow \delta(q_j, x) \in \{q_0, q_2\}, x \in \{0,1\}^*\}$. Describe the relation R by listing all its tuples.

5. *(6p)* For a string $x \in \{a, b\}^*$, let x^R denote the string x reversed. Consider the language $L = \{\, ww^R \mid w \in \{a, b\}^* \}$.

(a) Prove that L is not regular by using the pumping lemma for regular languages.

(b) Prove that L is not regular by using the Myhill-Nerode theorem.

6. *(4p)* Prove that the language $\{ a^i b^j c^{ij} \mid i, j > 0 \}$ is not context-free by using the pumping lemma for context-free languages.

7. *(4p)* Assume that A and B are non-empty languages over the Boolean alphabet $\{0, 1\}$ where we know that $A \leq_m B$ (I.e., there is a mapping reduction from A to B).

   (a) Prove or disprove that $A \cup X \leq_m B$ always holds when $X \subseteq \{0, 1\}^*$ is a *finite* set where $A \cap X = \emptyset$.

   (b) Prove or disprove that $A \cup X \leq_m B$ always holds when $X \subseteq \{0, 1\}^*$ is an *infinite* set where $A \cap X = \emptyset$.