Examination
Formal Languages and Automata Theory
TDDD14 & TDDD85

(Formella Språk och Automatateori)

2019–10–31, 14.00 – 18.00

1. Allowed help materials

   - A sheet of notes - 2 sided A5 or 1 sided A4.
     The contents is up to you. Return the notes together with the exam.
   - English dictionary

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   Tillåtna hjälpmedel:

   - Ett papper med valfria anteckningar - 2 sidor A5 eller 1 sida A4. Anteckningarna ska bifogas tentamen vid inlämnandet.
   - Engelsk ordbok
)

2. You may answer in Swedish or English.

3. The maximum number of points is 34. The grades are as follows:

Grade	TDDD14	TDDD85
3	18–24	15–21
4	25–29	22–27
5	30–34	28–34

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GOOD LUCK !

Make sure that you justify your answers! Unexplained answers will be granted 0 points. (For instance, if you are writing a grammar for a given language then you should also explain that the grammar indeed generates the language. If you apply some known method then you should explain each step. And so on.)

1. *(4p)* Give regular expressions for each of the following subsets of $\{0,1\}^*$. The expressions should be as simple as possible. Clearly state any assumptions you make.

   (a) $\{x \mid x$ contains an even number of 0's$\}$,

   (b) $\{x \mid x$ contains an odd number of 1's$\}$,

   (c) $\{x \mid x$ contains an even number of 0's or an odd number of 1's$\}$,

   (d) $\{x \mid x$ contains two consecutive 0's but not three consecutive 0's$\}$.

2. *(4p)* The NFA with ϵ transitions N is defined via the transition function

	ϵ	a	b
$\rightarrow 1$	$\{2\}$	$\{3\}$	$\{3\}$
2	$\emptyset$	$\{1,4\}$	$\{3\}$
3 F	$\{4\}$	$\emptyset$	$\emptyset$
4	$\emptyset$	$\emptyset$	$\{2\}$

   Recall that $\rightarrow$ indicates the start state, and that F indicates an accept/final state.

   (a) Draw the transition diagram for N.

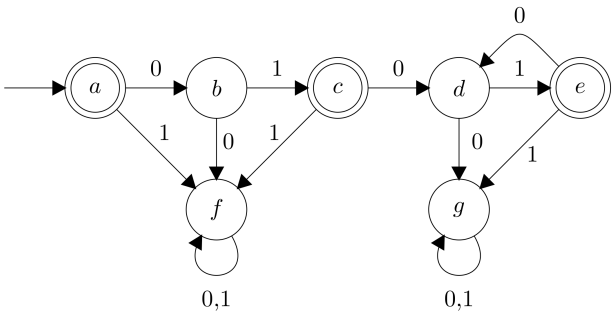
   (b) Using a standard method, construct an equivalent DFA M.

3. *(4p)* Using a standard method, construct a regular expression defining the same language as the DFA whose transition function δ is given by

	a	b
$\rightarrow A$	A	C
B F	A	B
C F	B	A

   Recall that $\rightarrow$ indicates the start state, and that F indicates an accept/final state.

4. *(4p)* Show that the following DFA has a minimal number of states or construct an equivalent DFA with a minimal number of states. Use the minimization algorithm from the course.



5. *(6p)* Consider the language P consisting of all properly balanced parenthesis. That is, strings over (and) where each left parenthesis (has a matching right parenthesis). For example, the strings (((())())()) and ()() are in P but the string)(() is not.

   (a) Prove that P is context-free by providing a context-free grammar for P. Is your grammar unambiguous?

   (b) Prove that P is not regular by using the pumping lemma for regular languages.

6. *(8p)* Which of the following statements are true, which are false? Why?

   (a) The union of a regular language and a context-free language is always context-free.

   (b) The intersection of a regular language and a context-free language is always regular.

   (c) There exists a recursive language whose complement is not recursive.

   (d) There exists an algorithm which finds out whether the languages defined by two NFA's are equal.

7. *(4p)* Let A, B and C be languages of which we know that $A \leq_m B$ and $A \leq_m C$ (where $L \leq_m L'$ denotes that there exists a mapping reduction from L to L').

   (a) Prove or disprove that $A \leq_m B \cup C$ always holds.

   (b) Prove or disprove that $A \leq_m B \cap C$ always holds.