TDDD14/TDDD85
Formal Languages and Automata Theory
2018-05-31

Materials allowed (Tillåtna hjälpmedel):

- A sheet of notes - 2-sided A5 or 1-sided A4. These notes must be handed in together with the answers and signed in the same way as the exam papers. (Ett blad med anteckningar - 2-sidigt A5 eller 1-sidigt A4. Detta blad ska lämnas in med svaren och signeras på samma sätt som övriga papper.)
- An english dictionary. (Engelsk ordbok).

Instructions:

- You may answer in english or swedish.
- Make sure your text and figures are big and clear enough to read easily.
- All answers must be motivated. A correct answer without reasonable motivation may result in zero points!

Grading: The maximum number of points is 34. The grades are as follows:

grade	TDDD14	TDDD85
3:	18–24 p.	15–21 p.
4:	25–29 p.	22–27 p.
5:	30–34 p.	28–34 p.

Problems

1. For each of the following languages, give a regular expression for it and explain the expression. The expressions should be as simple as possible. Clearly state any assumptions you make. (4 p)

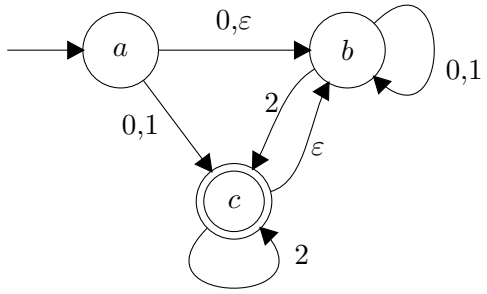
   (a) L_a consists of all strings over the alphabet $\{0, 1\}$ that are even binary numbers of odd length.

   (b) L_b consists of all strings over the alphabet $\{0, 1\}$ where every 1 is immediately preceeded by at least two 0's.

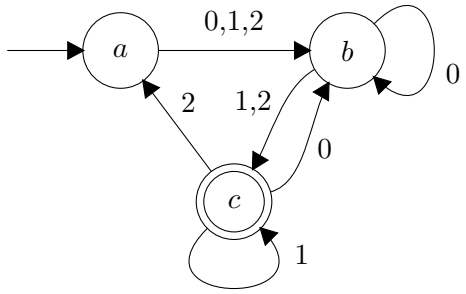
   (c) L_c consists of all strings over the alphabet $\{0, 1\}$ where every 1 is immediately preceeded by another 1, unless it is the first symbol in the string.

   (d) L_d consists of all strings over the alphabet $\{0, 1, 2, 3\}$ where the elements are sorted in numerical order, i.e. a string $s = x_1 x_2 \dots x_n$ must satisfy that $x_i \leq x_{i+1}$ for all i $(1 \leq i < n)$.

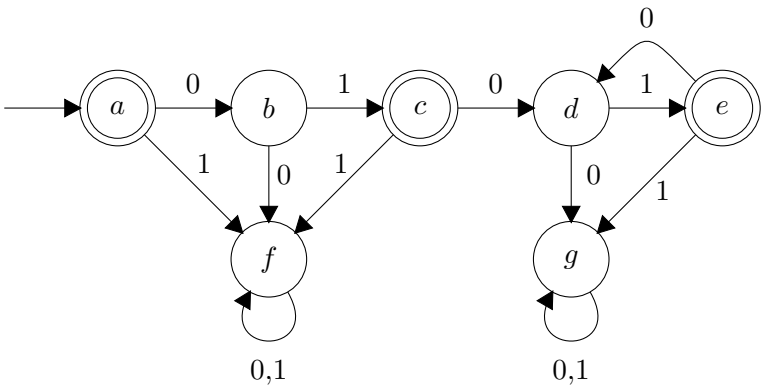
2. Convert the following NFA to an equivalent DFA, using the subset construction method. You must give the transition table for the construction and draw the state diagram of the resulting DFA. (4 p)



3. Convert the following DFA to an equivalent regular expression using the GNFA method (or some other standard method from the course). (4 p)



4. Show that the following DFA has a minimal number of states or construct an equivalent DFA with a minimal number of states. Use the minimization algorithm from the course. (4 p)



5. (6 p)

(a) Prove that the language $L_1 = \{0^k 1^m 2^n \mid m < k + n\}$ is not regular, by using the pumping lemma for regular languages.

(b) Prove that the language $L_2 = \{0^k 1^m 2^n \mid k < m \text{ and } n < m\}$ is not context-free by using the pumping lemma for context-free languages.

6. Consider the following two context-free grammars: (6 p)

$$
\begin{array}{lrcl}
G_1: & S & \to & A \,|\, aB \\
 & A & \to & aA \,|\, B \\
 & B & \to & A \,|\, b
\end{array}
$$

$$
\begin{array}{lrcl}
G_2: & S & \to & A \\
 & A & \to & aB \,|\, b \\
 & B & \to & A
\end{array}
$$

(a) What is the language $L(G_1)$?

(b) Are G_1 and G_2 equivalent? Explain why.

(c) Prove or disprove that G_1 is an LR(0) grammar.

(d) Prove or disprove that G_2 is an LR(0) grammar.

7. Let A, B and C be languages of which we know that $A \leq_m B$ and $A \leq_m C$ (where $L \leq_m L'$ denotes that there exists a mapping reduction from L to L'). (6 p)

(a) Prove or disprove that $A \leq_m B \cup C$ always holds.

(b) Prove or disprove that $A \leq_m B \cap C$ always holds.

(c) Is it possible that $C \leq_m A$?