TDDD14/TDDD85
Formal Languages and Automata Theory
2017-08-26

Materials allowed (Tillåtna hjälpmedel):

- A sheet of notes - 2-sided A5 or 1-sided A4. These notes must be handed in together with the answers and signed in the same way as the exam papers. (Ett blad med anteckningar - 2-sidigt A5 eller 1-sidigt A4. Detta blad ska lämnas in med svaren och signeras på samma sätt som övriga papper.)
- An english dictionary. (Engelsk ordbok).

Instructions:

- You may answer in english or swedish.
- Make sure your text and figures are big and clear enough to read easily.
- All answers must be motivated. A correct answer without reasonable motivation may result in zero points!

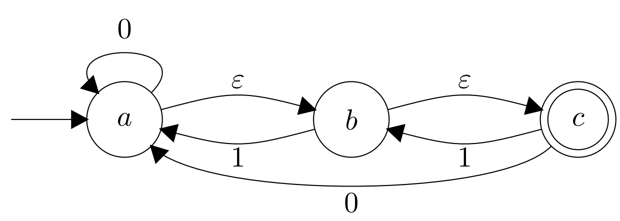
Grading: The maximum number of points is 34. The grades are as follows:

grade	TDDD14	TDDD85
3:	18–24 p.	15–21 p.
4:	25–29 p.	22–27 p.
5:	30–34 p.	28–34 p.

Problems

1. Assume the alphabet $\Sigma = \{0, 1\}$. Draw the state transition diagram for a DFA that accepts exactly those non-empty strings over Σ^* that end with an even number of 1's. For example, the DFA should accept 0110011 and 111101111 but not 1000111 or 0101. (4 p)

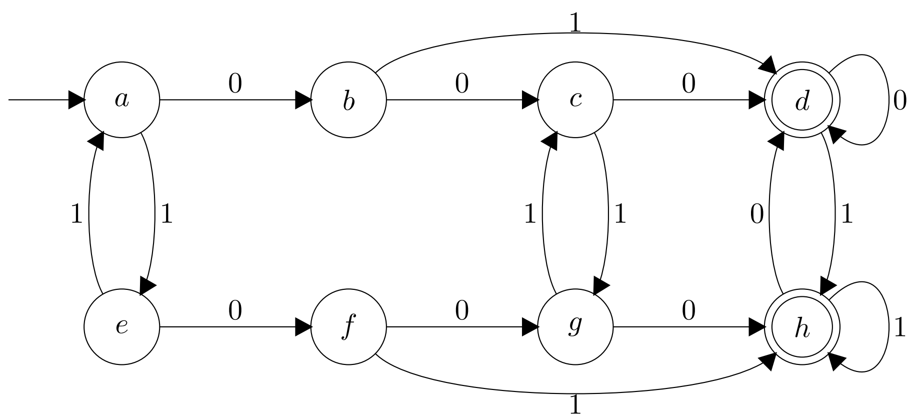
2. Consider the following NFA. (4 p)



(a) Convert the NFA to an equivalent DFA, using the standard method. Also draw the state transition diagram of the resulting DFA.

(b) Give a regular expression for the language accepted by this NFA. The expression should be reasonably simple.

3. Show that the following DFA has a minimal number of states or construct an equivalent DFA with a minimal number of states. Use the algorithm from the lectures and specify clearly what is marked in each stage of the algorithm. (4 p)



4. Let L_1 and L_2 be languages over some alphabet Σ. Define the difference language $L = L_1 \setminus L_2$, i.e. L contains all strings in L_1 that are not also in L_2. Show that if L is not regular, then L_1 and L_2 cannot both be regular (i.e. at least one of L_1 and L_2 is not regular). (4 p)

5. Consider the following context-free grammar, where $\triangle$ and $\square$ are terminals (i.e. non-variables). (6 p)

$$S \rightarrow A \mid B \mid \triangle\triangle \mid \square\square$$
$$A \rightarrow \triangle A \mid \square A \mid \square$$
$$B \rightarrow \square B \mid \triangle B \mid \triangle$$

(a) What is the language generated by this grammar?

(b) Show that this grammar is ambiguous.

(c) Give an equivalent grammar that is unambigious and that is as simple as possible.

6. (6 p)

(a) Prove that the language $L_1 = \{0^m 1^k 2^n \mid m + k = n \text{ or } m = n + k\}$ is not regular, by using the pumping lemma for regular languages.

(b) Prove that the language $L_2 = \{0^m 1^k 2^n \mid n = km\}$ is not context-free by using the pumping lemma for context-free languages.

7. (6 p)

(a) Suppose L is a language that is not context free. Does the claim that $L \leq_m \{0, 1\}$ hold always, never or only under certain conditions? Prove your answer and specify the conditions if the third case applies.

(b) Suppose L_1, L_2 and L_3 are languages of which we know that $L_1 \leq_m L_3$ and $L_2 \leq_m L_3$. Does the claim that $L_1 \leq_m L_2$ hold always, never or sometimes. Prove your answer.

(c) Suppose L_1, L_2 and L_3 are languages of which we know that $L_1 \leq_m L_2$ and $L_1 \leq_m L_3$. Does the claim that $L_2 \leq_m L_3$ hold always, never or sometimes. Prove your answer.