Problem Set for Tutorial 5 — TDDD08

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1. Consider the following definite program P:

$$o(s(z)). \tag{1}$$
$$o(s(s(N))) \leftarrow o(N). \tag{2}$$
$$e(z). \tag{3}$$
$$e(s(s(N))) \leftarrow e(N). \tag{4}$$
$$n(N) \leftarrow o(N). \tag{5}$$
$$n(N) \leftarrow e(N). \tag{6}$$

   (a) Assume that the vocabulary $\mathcal{A}$ contains only the constant z and the unary function symbol s. What is the Herbrand universe $\mathbf{U}_\mathcal{A}$?

   (b) What is the Herbrand base?

   (c) Find the least Herbrand model of the program.

   (d) Give an example of a model of the program which is *not* an Herbrand model.

2. Consider the following definite program P:

$$p(\, X, Y\,) \leftarrow r(\, g(X), X\,). \tag{1}$$
$$r(\, g(Z), f(Z)\,). \tag{2}$$
$$r(\, g(X), Y\,) \leftarrow r(\, X, f(Y)\,). \tag{3}$$

   (a) Assume that the vocabulary $\mathcal{A}$ contains one constant a and two one-argument function symbols f, g. What is the Herbrand universe $\mathbf{U}_\mathcal{A}$ corresponding to $\mathcal{A}$?

   (b) Which of the following Herbrand interpretations are models of the program?

$$I_0 = \emptyset$$
$$I_1 = \{\, r(g(t), f(t)) \mid t \in \mathbf{U}_\mathcal{A}\,\}$$
$$I_2 = I_1 \cup \{\, r(g^i(f^j(t)), t) \mid i, j \geq 0,\ t \in \mathbf{U}_\mathcal{A}\,\}$$
$$I_3 = I_2 \cup \{\, p(t, u) \mid t, u \in \mathbf{U}_\mathcal{A}\,\}$$

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(c) Find the least Herbrand model of the program.

(d) Give an example of a ground atom which is a logical consequence of P, but is not an instance of $r(g(Z), f(Z))$.

(e) Give an example of a ground atom which is not a logical consequence of P, but it is an instance of $r(\, g(X), Y\,)$.

(f) Give an example of a non-ground atom which is a logical consequence of P, but is not an instance of $r(g(Z), f(Z))$.

3. Write a DCG which recognises whether a string is a *palindrome*, i.e., whether the string reads the same forwards and backwards. Translate your DCG to Prolog using the translation described in Section 10.5 of the course book.

4. Consider the following fragment of the syntax of a programming language:

```
<exp> ::= begin <exp> end | skip | if <b_exp> exp | <id> := <num>
<b_exp> ::= <id> < <id> | <id> = <id>
```

First, write a DCG which recognises the above language, under reasonable assumptions with respect to `<id>` and `<num>` (e.g., you may assume that `<id> ::= x | <id> ::= y`, to avoid specifying exactly what strings constitute variables). Second, write a DCG `exp(T)` which recognises the language and where `T` is a term corresponding to the accepted string. For example, to parse strings of the form `if <b_exp> exp` your program should include a DCG rule along the lines of:

```
exp(if(B_term, Exp_term)) --> [if], b_exp(B_term), exp(Exp_term).
```

5. Say that a string of left and right parenthesis (i.e., a string consisting of '(', and ')')) is *balanced* if each left parenthesis has a matching right parenthesis. For example, (())() is balanced but (()() is not. Write a DCG which recognises the language of all balanced strings of parenthesises, i.e., $\{\alpha \in \{(,)\}^* \mid \alpha \text{ is balanced}\}$. Would your solution work as expected in Prolog (under the standard translation)?

6. Write a DCG which recognises the language $\{a^n b^n c^n \mid n \geq 1\}$.