Problem Set for Tutorial 1 — TDDD08[*]

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1. Which of the following expressions are well-formed formulas in first-order logic?

   (a) $\forall x(p(x) \to q(x))$ where p and q are predicate symbols.

   (b) $p(x) \to q(x)$ where p and q are predicate symbols.

   (c) $\exists x(p(f(x)) \wedge q(x))$ where p and q are predicate symbols and f a function symbol.

   (d) $\exists x(p(f(x)) \wedge \exists x q(x))$ where p and q are predicate symbols and f a function symbol.

   (e) $\forall x(f(x) \wedge p(x))$ where f is a function symbol and p a predicate symbol.

   (f) $\forall x p(f(x))$ where p is a predicate symbol and f a function symbol.

   (g) $\forall x f(p(x))$ where p is a predicate symbol and f a function symbol.

2. Translate the following sentences into first-order logic:

   (a) "All employees have income."

   (b) "Some employees are on holidays."

   (c) "No employees are unemployed."

   (d) "Some employees are not satisfied by their salary policy."

3. Translate the following formulas into natural language:

   (a) $\forall x((\text{strongEngine}(x) \wedge \text{car}(x) \wedge \text{wheels}(x, 4)) \to \text{fast}(x))$.

   (b) $\forall x \forall y((\text{parent}(x, y) \wedge \text{ancestor}(y)) \to \text{ancestor}(x))$.

   (c) $\forall x \forall y((\text{car}(x) \wedge \text{onRoad}(x, y) \wedge \text{highway}(y) \wedge \text{normalConditions}(y)) \to \text{fastSpeedAllowed}(x))$.

4. Consider the formula
$$\forall x(\text{shaves}(barber, x) \leftrightarrow \neg\text{shaves}(x, x)),$$
which in natural language can be read as "The barber is a man in a town who shaves all those, and only those, men in town who do not shave themselves.". Does it have a model, i.e., is it satisfiable?

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[*]Some exercises taken from TDDD72.
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5. Let a and b be constant symbols, f a binary function symbol, and l a unary predicate symbol. Consider the first-order sentence $l(a) \wedge \forall x(l(x) \to l(f(b,x)))$. Provide two interpretations: one which is a model and one which is not.

6. Consider the formula $\mathrm{nat}(zero) \wedge \forall x\,(\mathrm{nat}(x) \to \mathrm{nat}(f(x)))$. Provide two interpretations: one which is a model and one which is not.