TDDD08 – Tutorial 3

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1. Determine which of the following pairs of terms that are unifiable, and provide the most general unifier (mgu) in case there is one:

1. `p(f(X),X,f(Y))` and `p(Y,f(Z),Z)`
2. `p(f(X),f(Y),X)` and `p(Z,Z,W)`
3. `p(X1,X2,X3)` and `p(f(X2,X2),f(X3,X3),a)`
4. `[X,f(X)|X]` and `[Z,Y,Z]`

2. Draw the SLD-tree for the program below and the query `member(X,[a,b]).`

```
member(X, [X|_]).
member(X, [_|L]) :- member(X,L).
```

Draw the SLD-tree for query `member(X,[a,b]),member(X,[b,c])`, under a selection rule chosen by you.

3. Define the following relations on lists by means of Prolog programs. The predicates `member/2`, `append/3` and `dif/2`, may be used, if required. (`dif(X,Y)` is a "right" implementation of inequality in Prolog.)

- `longer(Xs,Ys)` — the list `Xs` has more elements than the list `Ys`. Do this without computing the length of a list. (It is simpler.)
- `append(A, B, C, ABC)` — the list `ABC` is the result of appending `A`, `B` and `C`.
- Relations named `replace1`, `replace`, `replaceall`, describing replacing an element E by EE in a list: replacing exactly one occurrence of E, any number of occurrences, and all occurrences.

4. Consider the following definite program P:

```
p(X) :- r(X),p(X).
q(f(X),X).
r(a).
r(Y) :- q(Y,X),r(X).
```

What are the constants, function symbols and predicate symbols of the program? What is the Herbrand universe $\mathbf{U}_P$, and the Herbrand base? Which of the following Herbrand interpretations are models of P?

- $I_1 = \{q(f(x), x) \mid x \in U_P\}$
- $I_2 = I_1 \cup \{r(f^{2n}(a)) \mid n \geq 0\}$
- $I_3 = I_1 \cup \{r(f^n(a)) \mid n \geq 0\}$
- $I_4 = I_1 \cup \{p(a)\} \cup \{r(f^n(a)) \mid n \geq 0\}$

Is any of them the least Herbrand model $\mathbf{M}_P$ of P?

5. Consider the immediate consequence operator T_P. Prove that I is a Herbrand model of P if and only if $T_P(I) \subseteq I$.

6. Formulate the completeness theorem of SLD-resolution and give an example of a definite program P and a definite query A such that $P \models A\theta$, but there is no computed answer substitution σ such that $A\sigma = A\theta$.