Static Analysis: Symbolic Execution and Inductive Verification Methods
TDDC90: Software Security

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Outline

Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning
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Static Program Analysis and Approximations

We want to answer whether the program is **safe** or not (i.e., has some erroneous reachable configurations or not):

![Diagram showing safe and unsafe programs with reachable configurations and errors](image-url)
The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.
A sound analysis cannot give **false negatives**

A complete analysis cannot give **false positives**
These two lectures on static program analysis briefly introduce different types of analysis:

- **Previous lecture:**
  - syntactic analysis: scalable but neither sound nor complete
  - abstract interpretation sound but not complete

- **This lecture:**
  - symbolic executions: complete but not sound
  - inductive methods: may require heavy human interaction in proving the program correct

These two lectures are only appetizers: More concepts and ideas are discussed in TDDE34 under VT2.
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First, What are SMT Solvers?

- Stands for *Satisfiability Modulo Theory*
- Intuitively, these are constraint solvers that extend *SAT solvers* to richer theories
- Many solvers exist (Yices, CVC, STP, OpenSMT, Princess, Z3, etc),
- You will be using Z3 [https://github.com/Z3Prover/z3](https://github.com/Z3Prover/z3) in the lab z3
- SAT solvers find a satisfying assignment to a formula where all variables are booleans or establishes its unsatisfiability
- SMT solvers find satisfying assignments to first order formulas where some variables may range over other values than just booleans
Originates from automating proof-search for first order logic.

- **Variables**: $x, y, z, ...$
- **Constants**: $a, b, c, ...$
- **N-ary functions**: $f, g, h, ...$
- **N-ary predicates**: $p, q, r, ...$
- **Atoms**: $\bot, \top, p(t_1, \ldots, t_n)$
- **Literals**: atoms or their negation
- **A FOL formula** is a literal, boolean combinations of formulas, or quantified ($\exists, \forall$) formulas.

Evaluation of formula $\varphi$, with respect to interpretation $I$ over non-empty (possibly infinite) domains for variables and constants gives true or false (resp. $I \models \varphi$ or $I \not\models \varphi$)
A formula $\varphi$ is:

- satisfiable if $I \models \varphi$ for some interpretation $I$
- valid if $I \models \varphi$ for all interpretations $I$

Satisfiability of FOL is undecidable. Instead, target decidable or domain-specific fragments.
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq g(a) = c \land (f(g(a)) \neq f(c) \lor g(a) = d) \land c \neq d \]

- EUF: Equality over Uninterpreted functions
- Satisfiable?
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[
\varphi \triangleq (x_1 \geq 0) \land (x_1 < 1) \\
\land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1))
\]
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

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- Linear Integer Arithmetic (LIA)
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

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- Linear Integer Arithmetic (LIA)
- Equality over Uninterpreted functions (EUF)
- Arrays (A)
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \overset{\Delta}{=} (x_1 \geq 0) \land (x_1 < 1) \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)) \]

- **LIA:** \[ x_1 = 0 \]
- **EUF:** \[ f(x_1) = f(0) \]
- **A:** \[ rd(wr(P, x_2, x_3), x_2) = x_3 \]
- **Bool:** \[ rd(wr(P, x_2, x_3), x_2) = x_3 + 1 \]
- **LIA:** \[ \bot \]
Sometimes more natural to express in logics other than propositional logic

SMT decide satisfiability of ground FO formulas wrt. background theories

Many applications: Model checking, predicate abstraction, symbolic execution, scheduling, test generation, ...
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Testing

- Most common form of software validation
- Explores only one possible execution at a time
- For each new value, run a new test.
- On a 32 bit machine, \( \text{if}(i==2021) \) \( \text{bug}() \) would require \( 2^{32} \) different values to make sure there is no bug.
- The idea in symbolic testing is to associate **symbolic values** to the variables
Symbolic Testing

- Use symbolic values instead of concrete ones
- Along the path, maintain a *Path Constraint* \((PC)\) and a symbolic state \((\sigma)\)
- \(PC\) collects constraints on variables’ values along a path,
- \(\sigma\) associates variables to symbolic expressions,
- We get concrete values if \(PC\) is satisfiable
- The program can be run on these values
- Negate a condition in the path constraint to get another path
Symbolic Execution: a simple example

- Can we get to the ERROR? explore using SSA forms.
- Useful to check array out of bounds, assertion violations, etc.

```plaintext
foo(int x,y,z){
  x = y - z;
  if(x==z){
    z = z - 3;
    if(4*z < x + y){
      if(25 > x + y) {
        ...
      }
      else{
        ERROR;
      }
    }
  }
  }
  else{
    ERROR;
  }
}
```

\[
PC_1 = true
\]
\[
PC_2 = PC_1
\]
\[
PC_3 = PC_2 \land x_1 = y_0 - z_0
\]
\[
PC_4 = PC_3 \land x_1 = z_0
\]
\[
PC_5 = PC_4 \land z_1 = z_0 - 3
\]
\[
PC_6 = PC_5 \land 4 \times z_1 < x_1 + y_0
\]

\[
PC_{10} = PC_6 \land \neg(25 > x_1 + y_0)
\]

\[
PC = (x_1 = y_0 - z_0 \land x_1 = z_0 \land z_1 = z_0 - 3 \land 4 \times z_1 < x_1 + y_0 \land \neg(25 > x_1 + y_0))
\]

Check satisfiability with a solver (e.g., Alt-Ergo, Boolector, CVC4, MathSAT5, OpenSMT2, STP, Yices2, Z3)
Symbolic execution today

- Leverages on the impressive advancements of SMT solvers
- Modern symbolic execution frameworks are not purely symbolic, and not necessarily purely static:
  - They can follow a concrete execution while collecting constraints along the way, or
  - They can treat some of the variables concretely, and some other symbolically
- This allows them to scale, to handle closed code or complex queries
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Two Lectures on Static Analysis

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Function Specifications and Correctness

- **Contract** between the caller and the implementation. **Total Correctness** requires that:
  - if the pre-condition \((-100 \leq x \&\& x \leq 100)\) holds
  - then the implementation terminates,
  - after termination, the following post-condition holds
    \((x \geq 0 \&\& result == x || x < 0 \&\& result == -x)\)

- **Partial Correctness** does not require termination

```c
/*@ requires -100 <= x && x <= 100;@
@ ensures x>=0 && result == x || x<0 && result == -x; */
int abs(int x){
    if(x < 0){
        return -x;
    }
    return x;
}
```
a Hoare triple \{P\} stmt \{R\} consists in:
- a predicate pre-condition \(P\)
- an instruction \(stmt\)
- a predicate post-condition \(R\)

intuitively, \{P\} stmt \{R\} holds if whenever \(P\) holds and \(stmt\) is executed and terminates (partial correctness), then \(R\) holds after \(stmt\) terminates.

For example:

\[
\{\text{true}\} \ x := y \ \{(x = y)\}
\]
\[
\{(x = 1) \land (y = 2)\} \ x := y \ \{(x = 2)\}
\]
\[
\{(x \geq 1)\} \ y := 2 \ \{(x = 0) \lor (y \leq 10)\}
\]
\[
\{(x \geq 1)\} \ (\text{if}(y == 2) \text{ then } x := 0) \ \{(x \geq 0)\}
\]
\[
\{\text{false}\} \ x := 1 \ \{(x = 2)\}
\]
Weakest Precondition

- if \{P\} stmt \{R\} and \(P' \Rightarrow P\) for any \(P'\) s.t. \{P'\} stmt \{R\},
  then \(P\) is the **weakest precondition** of \(R\) wrt. \(stmt\), written
    \(wp(\text{stmt}, R)\)

- \(wp(x := x + 1, x \geq 1) = (x \geq 0)\).
  \((x \geq 5), (x = 6), (x \geq 0 \land y = 8)\) are all valid preconditions,
  but they are not weaker than \(x \geq 0\).

- Intuitively \(wp(\text{stmt}, R)\) is the weakest predicate \(P\) for which
  \{\(P\}\} \(\text{stmt} \{R\}\) holds
Weakest Precondition of assignments

1. \( wp(x = E, R) = R[x/E] \), i.e., replace each occurrence of \( x \) in \( R \) by \( E \).

2. For instance:
   - \( wp(x := 3, x == 5) = (x == 5)[x/3] = (3 == 5) = false \)
   - \( wp(x := 3, x >= 0) = (x >= 0)[x/3] = (3 >= 0) = true \)
   - \( wp(x := y + 5, x >= 0) = (x >= 0)[x/y + 5] = (y + 5 >= 0) \)
   - \( wp(x := 5 * y + 2 * z, x + y >= 0) = (x + y >= 0)[x/5 * y + 2 * z] = (6 * y + 2 * z >= 0) \)
Weakest Precondition of sequences

Assume a sequence of two instructions $stmt; stmt'$, for example $x := 2 \times y; y := x + 3 \times y$;

the the weakest precondition is given by:

$wp(stmt; stmt', R) = wp(stmt, wp(stmt', R))$,

$wp(x := 2 \times y; y := x + 3 \times y, y > 10)$

$= wp(x := 2 \times y, wp(y := x + 3 \times y, y > 10))$

$= wp(x := 2 \times y, (y > 10)[y/x + 3 \times y])$

$= wp(x := 2 \times y, x + 3 \times y > 10)$

$= (x + 3 \times y > 10)[x/2 \times y]$

$= (2 \times y + 3 \times y > 10)$

$= y > 2$
Weakest Precondition of conditionals

- Assume a conditional (if($B$) then $stmt$ else $stmt'$), for example (if($x > y$) then $z := x$ else $z := y$)

- The weakest precondition is given by:

  \[
  wp((\text{if}(B) \text{ then } stmt \text{ else } stmt'), R) = (B \Rightarrow wp(stmt, R)) && (!B \Rightarrow wp(stmt', R))
  \]

- For example,

  \[
  wp((\text{if}(x > y) \text{ then } z := x \text{ else } z := y), z <= 10) = (x > y \Rightarrow wp(z := x, z <= 10)) && (x <= y \Rightarrow wp(z := y, z <= 10)) = (x > y \Rightarrow x <= 10) && (x <= y \Rightarrow y <= 10)
  \]
In order to establish \( \{P\} \textbf{while}(B)\textbf{do}\{stmt\}\} \{R\}, you will need to find an invariant \( Inv\) such that:

1. \( P \Rightarrow Inv\)
2. \( \{Inv\&\&B\} \textbf{stmt} \{Inv\}\)
3. \( (Inv\&\&\neg B) \Rightarrow R\)

For example \( \{i == j == 0\} \textbf{while}(i < 10)\textbf{do}\{i := i + 1; j := j + 1\}\} \{j == 10\}, we need to find \( Inv\) such that:

1. \( (i == j == 0) \Rightarrow Inv\)
2. \( \{Inv\&\&(i < 10)\} \textbf{stmt} \{Inv\}\)
3. \( (Inv\&\&i >= 10) \Rightarrow j == 10\)
{P} (while(B)do{stmt}) {R}

Partial correctness: if we start from \( P \) and (while(B)do{stmt}) terminates, then \( R \) terminates.

- \( P \Rightarrow Inv \)
- \{Inv\&\&B\} stmt \{Inv\}
- (Inv\&\&!B)\Rightarrow R

Total correctness: the loop does terminate: find a **variant function** \( v \) such that:

- (Inv\&\&B) \Rightarrow (v > 0)
- \{Inv\&\&B\&\&v = v_0\} stmt \{v < v_0\}

For example (while(\( i < 10 \))do{\( i := i + 1; j := j + 1 \)}) can be shown to terminate with \( v = (10 - i) \) and \( Inv = (i <= 10) \)