Static Analysis: Symbolic Execution and Inductive Verification Methods
TDDC90: Software Security

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Outline

Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning
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Symbolic Execution

Hoare Triples and Deductive Reasoning
We want to answer whether the program is **safe** or not (i.e., has some erroneous reachable configurations or not):
The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.
A sound analysis cannot give **false negatives**

A complete analysis cannot give **false positives**
Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- **Previous lecture:**
  - syntactic analysis: scalable but neither sound nor complete
  - abstract interpretation sound but not complete

- **This lecture:**
  - symbolic executions: complete but not sound
  - inductive methods: may require heavy human interaction in proving the program correct

These two lectures are only appetizers: More concepts and ideas are discussed in TDDE34 under VT2
Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- Previous lecture:
  - syntactic analysis: scalable but neither sound nor complete
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First, What are SMT Solvers?

- Stands for *Satisfiability Modulo Theory*
- Intuitively, these are constraint solvers that extend *SAT solvers* to richer theories
- Many solvers exist (Yices, CVC, STP, OpenSMT, Princess, Z3, etc),
- You will be using Z3 https://github.com/Z3Prover/z3 in the lab z3
- SAT solvers find a satisfying assignment to a formula where all variables are booleans or establishes its unsatisfiability
- SMT solvers find satisfying assignments to first order formulas where some variables may range over other values than just booleans
Introduction

Originates from automating proof-search for first order logic.

- **Variables:** $x, y, z, ...$
- **Constants:** $a, b, c, ...$
- **N-ary functions:** $f, g, h, ...$
- **N-ary predicates:** $p, q, r, ...$
- **Atoms:** $\bot, \top, p(t_1, \ldots, t_n)$
- **Literals:** atoms or their negation
- **A FOL formula is a literal, boolean combinations of formulas, or quantified ($\exists, \forall$) formulas.**

Evaluation of formula $\varphi$, with respect to interpretation $I$ over non-empty (possibly infinite) domains for variables and constants gives true or false (resp. $I \models \varphi$ or $I \not\models \varphi$)
A formula $\varphi$ is:

- satisfiable if $I \models \varphi$ for some interpretation $I$
- valid if $I \models \varphi$ for all interpretations $I$

Satisfiability of FOL is undecidable. Instead, target decidable or domain-specific fragments.
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq g(a) = c \land (f(g(a)) \neq f(c) \lor g(a) = d) \land c \neq d \]

- EUF: Equality over Uninterpreted functions
- Satisfiable?
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq (x_1 \geq 0) \land (x_1 < 1) \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)) \]
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq (x_1 \geq 0) \land (x_1 < 1) \]
\[ \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)) \]

▶ Linear Integer Arithmetic (LIA)
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \equiv (x_1 \geq 0) \land (x_1 < 1) \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1) \]

- Linear Integer Arithmetic (LIA)
- Equality over Uninterpreted functions (EUF)
- Arrays (A)
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq (x_1 \geq 0) \land (x_1 < 1) \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1) \]

- **LIA:** \( x_1 = 0 \)
- **EUF:** \( f(x_1) = f(0) \)
- **A:** \( rd(wr(P, x_2, x_3), x_2) = x_3 \)
- **Bool:** \( rd(wr(P, x_2, x_3), x_2) = x_3 + 1 \)
- **LIA:** \( \perp \)
Introduction

- Sometimes more natural to express in logics other than propositional logic
- SMT decide satisfiability of ground FO formulas wrt. background theories
- Many applications: Model checking, predicate abstraction, symbolic execution, scheduling, test generation, ...
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Testing

- Most common form of software validation
- Explores only one possible execution at a time
- For each new value, run a new test.
- On a 32 bit machine, if(i==2021) bug() would require $2^{32}$ different values to make sure there is no bug.
- The idea in symbolic testing is to associate **symbolic values** to the variables
Symbolic Testing

- Use symbolic values instead of concrete ones
- Along the path, maintain a Path Constraint \((PC)\) and a symbolic state \((\sigma)\)
- \((PC)\) collects constraints on variables’ values along a path,
- \((\sigma)\) associates variables to symbolic expressions,
- We get concrete values if \((PC)\) is satisfiable
- The program can be run on these values
- Negate a condition in the path constraint to get another path
Symbolic Execution: a simple example

- Can we get to the ERROR? explore using SSA forms.
- Useful to check array out of bounds, assertion violations, etc.

```c
foo(int x, y, z){
    x = y - z;
    if(x==z){
        z = z - 3;
        if(4*z < x + y){
            if(25 > x + y) {
                ...}
            else{
                ERROR;
                }
        }
    }
    }
}
```

- $PC_1 = true$
- $PC_2 = PC_1$
- $PC_3 = PC_2 \land x_1 = y_0 - z_0$
- $PC_4 = PC_3 \land x_1 = z_0$
- $PC_5 = PC_4 \land z_1 = z_0 - 3$
- $PC_6 = PC_5 \land 4 \times z_1 < x_1 + y_0$
- $PC_{10} = PC_6 \land \neg(25 > x_1 + y_0)$

$PC = (x_1 = y_0 - z_0 \land x_1 = z_0 \land z_1 = z_0 - 3 \land 4 \times z_1 < x_1 + y_0 \land \neg(25 > x_1 + y_0))$

Check satisfiability with a solver (e.g., Alt-Ergo, Boolector, CVC4, MathSAT5, OpenSMT2, STP, Yices2, Z3)
Symbolic execution today

- Leverages on the impressive advancements of SMT solvers
- Modern symbolic execution frameworks are not purely symbolic, and not necessarily purely static:
  - They can follow a concrete execution while collecting constraints along the way, or
  - They can treat some of the variables concretely, and some other symbolically
- This allows them to scale, to handle closed code or complex queries
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Function Specifications and Correctness

- **Contract between the caller and the implementation.** Total Correctness requires that:
  1. if the pre-condition \((-100 \leq x \&\& x \leq 100)\) holds
  2. then the implementation terminates,
  3. after termination, the following post-condition holds
      \((x\geq 0 \&\& \text{result} == x \mid\mid x<0 \&\& \text{result} == -x)\)

- **Partial Correctness** does not require termination

```c
/*@ requires -100 <= x && x <= 100;
@ ensures x>=0 && \result == x || x<0 && \result == -x;
*/
int abs(int x){
    if(x < 0)
        return -x;
    return x;
}
```
Hoare Triples and Partial Correctness

- a Hoare triple \( \{ P \} \ stmt \ { R \} \) consists in:
  - a predicate pre-condition \( P \)
  - an instruction \( stmt \),
  - a predicate post-condition \( R \)
- intuitively, \( \{ P \} \ stmt \ { R \} \) holds if whenever \( P \) holds and \( stmt \) is executed and terminates (partial correctness), then \( R \) holds after \( stmt \) terminates.
- For example:
  - \( \{ true \} \ x := y \ \{(x = y)\} \)
  - \( \{(x = 1) \land (y = 2)\} \ x := y \ \{(x = 2)\} \)
  - \( \{(x \geq 1)\} \ y := 2 \ \{(x = 0) \lor (y \leq 10)\} \)
  - \( \{(x \geq 1)\} \ (if(y == 2) then x := 0) \ \{(x \geq 0)\} \)
  - \( \{false\} \ x := 1 \ \{(x = 2)\} \)
Weakest Precondition

- If \( \{ P \} \) stmt \( \{ R \} \) and \( P' \Rightarrow P \) for any \( P' \) s.t. \( \{ P' \} \) stmt \( \{ R \} \), then \( P \) is the **weakest precondition** of \( R \) wrt. stmt, written \( wp(stmt, R) \)

- \( wp(x := x + 1, x \geq 1) = (x \geq 0) \).
  
  \( (x \geq 5), (x = 6), (x \geq 0 \land y = 8) \) are all valid preconditions, but they are not weaker than \( x \geq 0 \).

- Intuitively \( wp(stmt, R) \) is the weakest predicate \( P \) for which \( \{ P \} \) stmt \( \{ R \} \) holds
Weakest Precondition of assignments

- $wp(x = E, R) = R[x/E]$, i.e., replace each occurrence of $x$ in $R$ by $E$.

For instance:

- $wp(x := 3, x == 5) = (x == 5)[x/3] = (3 == 5) = false$
- $wp(x := 3, x >= 0) = (x >= 0)[x/3] = (3 >= 0) = true$
- $wp(x := y + 5, x >= 0) = (x >= 0)[x/y + 5] = (y + 5 >= 0)$
- $wp(x := 5 * y + 2 * z, x + y >= 0) = (x + y >= 0)[x/5 * y + 2 * z] = (6 * y + 2 * z >= 0)$
Assume a sequence of two instructions $stmt; stmt'$, for example $x := 2 * y; y := x + 3 * y$.

the the weakest precondition is given by:

$$wp(stmt; stmt', R) = wp(stmt, wp(stmt', R)),$$

$$\begin{align*}
wp(x := 2 \cdot y; y := x + 3 \cdot y, y > 10) \\
= & \quad wp(x := 2 \cdot y, wp(y := x + 3 \cdot y, y > 10)) \\
= & \quad wp(x := 2 \cdot y, (y > 10)[y/x + 3 \cdot y]) \\
= & \quad wp(x := 2 \cdot y, x + 3 \cdot y > 10) \\
= & \quad (x + 3 \cdot y > 10)[x/2 \cdot y] \\
= & \quad (2 \cdot y + 3 \cdot y > 10) \\
= & \quad y > 2
\end{align*}$$
Assume a conditional \((\text{if}(B) \text{ then } stmt \text{ else } stmt')\), for example \((\text{if}(x > y) \text{ then } z := x \text{ else } z := y)\)

The weakest precondition is given by:

\[
(\wp((\text{if}(B) \text{ then } stmt \text{ else } stmt'), R)) = (B \Rightarrow \wp(stmt, R)) \land (\neg B \Rightarrow \wp(stmt', R))
\]

For example, \(\wp((\text{if}(x > y) \text{ then } z := x \text{ else } z := y), z <= 10)\)

\[
= (x > y \Rightarrow \wp(z := x, z <= 10)) \land (x <= y \Rightarrow \wp(z := y, z <= 10))
\]

\[
= (x > y \Rightarrow x <= 10) \land (x <= y \Rightarrow y <= 10)
\]
In order to establish \( \{ P \} \ (\textbf{while}(B)\textbf{do}\{stmt\}) \ \{ R \} \), you will need to find an invariant \( Inv \) such that:

- \( P \Rightarrow Inv \)
- \( \{Inv\&\&B\} \ stmt \ \{Inv\} \)
- \( (Inv\&\&!B) \Rightarrow R \)

For example \( \{i == j == 0\} \ (\textbf{while}(i < 10)\textbf{do}\{i := i + 1; j := j + 1\}) \ \{j == 10\} \), we need to find \( Inv \) such that:

- \( (i == j == 0) \Rightarrow Inv \)
- \( \{Inv\&\&(i < 10)\} \ i = i + 1; j = j + 1 \ \{Inv\} \)
- \( (Inv\&\&i >= 10) \Rightarrow j == 10 \)
Hoare Triples for Loops, Total Correctness

- \( \{P\} \ (\text{while}(B)\text{do}\{\text{stmt}\}) \ \{R\} \)
- Partial correctness: if we start from \( P \) and \( (\text{while}(B)\text{do}\{\text{stmt}\}) \) terminates, then \( R \) terminates.
  - \( P \Rightarrow Inv \)
  - \( \{Inv\&\&B\} \text{ stmt } \{Inv\} \)
  - \( (Inv\&\&!B) \Rightarrow R \)
- Total correctness: the loop does terminate: find a **variant function** \( v \) such that:
  - \( (Inv\&\&B) \Rightarrow (v > 0) \)
  - \( \{Inv\&\&B\&\&v = v_0\} \text{ stmt } \{v < v_0\} \)
- For example, \( (\text{while}(i < 10)\text{do}\{i := i + 1; j := j + 1\}) \) can be shown to terminate with \( v = (10 - i) \) and \( Inv = (i \leq 10) \)