Static Analysis: Symbolic Execution and Inductive Verification Methods
TDDC90: Software Security

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Outline

Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning
Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning
We want to answer whether the program is **safe** or not (i.e., has some erroneous reachable configurations or not):
The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.
A sound analysis cannot give false negatives
A complete analysis cannot give false positives
Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- **Previous lecture:**
  - syntactic analysis: scalable but neither sound nor complete
  - abstract interpretation sound but not complete

- **This lecture:**
  - symbolic executions: complete but not sound
  - inductive methods: may require heavy human interaction in proving the program correct
Two Lectures on Static Analysis

These two lectures on static program analysis briefly introduce different types of analysis:

- Previous lecture:
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- These two lectures are only appetizers:
  - There will be a deeper course with more tools and applications in the spring.
  - Possibilities of exjobbs with applications to verification and security.
  - Contact me if interested :-}
First, What are SMT Solvers?

- Stands for *Satisfiability Modulo Theory*
- Intuitively, these are constraint solvers that extend *SAT solvers* to richer theories
- Many solvers exist (Yices, CVC, STP, OpenSMT, Princess, Z3, etc),
- You will be using Z3 [https://github.com/Z3Prover/z3](https://github.com/Z3Prover/z3) in the lab z3
- SAT solvers find a satisfying assignment to a formula where all variables are booleans or establishes its unsatisfiability
- SMT solvers find satisfying assignments to first order formulas where some variables may range over other values than just booleans
Introduction

Originates from automating proof-search for first order logic.

- Variables: \(x, y, z, \ldots\)
- Constants: \(a, b, c, \ldots\)
- N-ary functions: \(f, g, h, \ldots\)
- N-ary predicates: \(p, q, r, \ldots\)
- Atoms: \(\bot, \top, p(t_1, \ldots, t_n)\)
- Literals: atoms or their negation
- A FOL formula is a literal, boolean combinations of formulas, or quantified \((\exists, \forall)\) formulas.

Evaluation of formula \(\varphi\), with respect to interpretation \(I\) over non-empty (possibly infinite) domains for variables and constants gives true or false (resp. \(I \models \varphi\) or \(I \not\models \varphi\))
A formula \( \varphi \) is:

- satisfiable if \( I \models \varphi \) for some interpretation \( I \)
- valid if \( I \models \varphi \) for all interpretations \( I \)

Satisfiability of FOL is undecidable. Instead, target decidable or domain-specific fragments.
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

$$\varphi \triangleq g(a) = c \land (f(g(a)) \neq f(c) \lor g(a) = d) \land c \neq d$$

- EUF: Equality over Uninterpreted functions
- Satisfiable?
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq (x_1 \geq 0) \land (x_1 < 1) \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)) \]
Introduction

Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \equiv (x_1 \geq 0) \land (x_1 < 1) \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)) \]

- Linear Integer Arithmetic (LIA)
Introduction

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- Linear Integer Arithmetic (LIA)
- Equality over Uninterpreted functions (EUF)
- Arrays (A)
Given a quantifier free FOL formula and a combination of theories, is there an interpretation to the free variables that makes the formula true?

\[ \varphi \triangleq (x_1 \geq 0) \land (x_1 < 1) \]
\[ \land ((f(x_1) = f(0)) \Rightarrow (rd(wr(P, x_2, x_3), x_2 + x_1) = x_3 + 1)) \]

- **LIA**: \( x_1 = 0 \)
- **EUF**: \( f(x_1) = f(0) \)
- **A**: \( rd(wr(P, x_2, x_3), x_2) = x_3 \)
- **Bool**: \( rd(wr(P, x_2, x_3), x_2) = x_3 + 1 \)
- **LIA**: \( \perp \)
Sometimes more natural to express in logics other than propositional logic

SMT decide satisfiability of ground FO formulas wrt. background theories

Many applications: Model checking, predicate abstraction, symbolic execution, scheduling, test generation, ...
Outline

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Symbolic Execution

Hoare Triples and Deductive Reasoning
Most common form of software validation

Explores only one possible execution at a time

For each new value, run a new test.

On a 32 bit machine, \( \text{if}(i==2021) \ \text{bug}() \) would require \( 2^{32} \) different values to make sure there is no bug.

The idea in symbolic testing is to associate \textit{symbolic values} to the variables
Symbolic Testing

- Use symbolic values instead of concrete ones
- Along the path, maintain a *Path Constraint* (\(PC\)) and a symbolic state (\(\sigma\))
- \(PC\) collects constraints on variables' values along a path,
- \(\sigma\) associates variables to symbolic expressions,
- We get concrete values if \(PC\) is satisfiable
- The program can be run on these values
- Negate a condition in the path constraint to get another path
Symbolic Execution: a simple example

- Can we get to the ERROR? explore using SSA forms.
- Useful to check array out of bounds, assertion violations, etc.

```c
foo(int x,y,z){
    x = y - z;
    if(x==z){
        z = z - 3;
        if(4*z < x + y){
            if(25 > x + y) {
                ...}
            else{
                ERROR;
            }
        }
    }else{
        ERROR;
    }
}
```

\[ PC = (x_1 = y_0 - z_0 \land x_1 = z_0 \land z_1 = z_0 - 3 \land 4 \times z_1 < x_1 + y_0 \land \neg(25 > x_1 + y_0)) \]

Check satisfiability with a solver (e.g., Alt-Ergo, Boolector, CVC4, MathSAT5, OpenSMT2, STP, Yices2, Z3)
Symbolic execution today

- Leverages on the impressive advancements of SMT solvers
- Modern symbolic execution frameworks are not purely symbolic, and not necessarily purely static:
  - They can follow a concrete execution while collecting constraints along the way, or
  - They can treat some of the variables concretely, and some other symbolically
- This allows them to scale, to handle closed code or complex queries
Overview

Symbolic Execution

Hoare Triples and Deductive Reasoning
Contract between the caller and the implementation. **Total Correctness** requires that:

- if the pre-condition \((-100 \leq x && x \leq 100)\) holds
- then the implementation terminates,
- after termination, the following post-condition holds

\[(x \geq 0 && \text{result} == x \lor x < 0 && \text{result} == -x)\]

**Partial Correctness** does not require termination

```c
/*@ requires -100 <= x && x <= 100;
@ ensures x>=0 && \text{result} == x || x<0 && \text{result} == -x;
*/
int abs(int x){
    if(x < 0)
        return -x;
    return x;
}
```
a Hoare triple \( \{P\} \ \text{stmt} \ \{R\} \) consists in:
- a predicate pre-condition \( P \)
- an instruction \( \text{stmt} \)
- a predicate post-condition \( R \)

intuitively, \( \{P\} \ \text{stmt} \ \{R\} \) holds if whenever \( P \) holds and \( \text{stmt} \) is executed and terminates (partial correctness), then \( R \) holds after \( \text{stmt} \) terminates.

For example:
- \( \{true\} \ x = y \ \{(x = y)\} \)
- \( \{(x = 1) \land (y = 2)\} \ x = y \ \{(x = 2)\} \)
- \( \{(x \geq 1)\} \ y = 2 \ \{(x = 0) \lor (y \leq 10)\} \)
- \( \{(x \geq 1)\} \ (\text{if}(y == 2 \text{ then } x = 0) \ {(x \geq 0)} \)
- \( \{false\} \ x = 1 \ \{(x = 2)\} \)
Weakest Precondition

- If \( \{P\} \text{ stmt } \{R\} \) and \( P' \Rightarrow P \) for any \( P' \) s.t. \( \{P'\} \text{ stmt } \{R\} \), then \( P \) is the strongest precondition of \( R \) wrt. \text{ stmt }, written \( \text{ wp}(\text{ stmt }, R) \)

\[ \text{ wp}(x = x + 1, x \geq 1) = (x \geq 0). \]

\((x \geq 5), (x = 6), (x \geq 0 \land y = 8)\) are all valid preconditions, but they are not weaker than \( x \geq 0 \).

- Intuitively \( \text{ wp}(\text{ stmt }, R) \) is the weakest predicate \( P \) for which \( \{P\} \text{ stmt } \{R\} \) holds
Weakest Precondition of assignments

- $\text{wp}(x = E, R) = R[x/E]$, i.e., replace each occurrence of $x$ in $R$ by $E$.

- For instance:
  - $\text{wp}(x = 3, x == 5) = (x == 5)[x/3] = (3 == 5) = \text{false}$
  - $\text{wp}(x = 3, x >= 0) = (x >= 0)[x/3] = (3 >= 0) = \text{true}$
  - $\text{wp}(x = y + 5, x >= 0) = (x >= 0)[x/y + 5] = (y + 5 >= 0)$
  - $\text{wp}(x = 5 * y + 2 * z, x + y >= 0) = (x + y >= 0)[x/5 * y + 2 * z] = (6 * y + 2 * z >= 0)$
Weakest Precondition of sequences

- Assume a sequence of two instructions \( stmt; stmt' \), for example \( x = 2 \times y; y = x + 3 \times y \);
- the weakest precondition is given by:
  \[
  wp(stmt; stmt', R) = wp(stmt, wp(stmt', R)),
  \]
  
  \[
  wp(x = 2 \times y; y = x + 3 \times y, y > 10)
  = wp(x = 2 \times y, wp(y = x + 3 \times y, y > 10))
  = wp(x = 2 \times y, (y > 10)[y/x + 3 \times y])
  = wp(x = 2 \times y, x + 3 \times y > 10)
  = (x + 3 \times y > 10)[x/2 \times y]
  = (2 \times y + 3 \times y > 10)
  = y > 2
  \]
  
  \[
  = y > 2
  \]
Weakest Precondition of conditionals

- Assume a conditional (if($B$) then $stmt$ else $stmt'$), for example (if($x > y$) then $z = x$ else $z = y$)

- The weakest precondition is given by:
  \[
  \wp((\text{if}(B) \text{ then } stmt \text{ else } stmt'), R) = (B \Rightarrow \wp(stmt, R)) \land (\neg B \Rightarrow \wp(stmt', R))
  \]

- For example,
  \[
  \wp((\text{if}(x > y) \text{ then } z = x \text{ else } z = y), z \leq 10) = (x > y \Rightarrow \wp(z = x, z \leq 10)) \\
  \quad \land (x \leq y \Rightarrow \wp(z = y, z \leq 10)) \\
  = (x > y \Rightarrow x \leq 10) \land (x \leq y \Rightarrow y \leq 10)
  \]
In order to establish \( \{P\} \ (\text{while}(B)\text{do}\{stmt\}) \ \{R\} \), you will need to find an invariant \( Inv \) such that:

- \( P \Rightarrow Inv \)
- \( \{Inv\&\&B\} \ stmt \ \{Inv\} \)
- \( (Inv\&\&!B) \Rightarrow R \)

For example \( \{i == j == 0\} \ (\text{while}(i < 10)\text{do}\{i = i + 1; j = j + 1\}) \ \{j == 10\} \), we need to find \( Inv \) such that:

- \( (i == j == 0) \Rightarrow Inv \)
- \( \{Inv\&\&(i < 10)\} \ i = i + 1; j = j + 1 \ \{Inv\} \)
- \( (Inv\&\&i >= 10) \Rightarrow j == 10 \)
Hoare Triples for Loops, Total Correctness

\[ \{P\} \ (\text{while}(B)\text{do}\{\text{stmt}\}) \ \{R\} \]

- Partial correctness: if we start from \( P \) and \((\text{while}(B)\text{do}\{\text{stmt}\})\) terminates, then \( R \) terminates.
  - \( P \Rightarrow \text{Inv} \)
  - \( \{\text{Inv}&&B\} \ \text{stmt} \ \{\text{Inv}\} \)
  - \( (\text{Inv}&&!B) \Rightarrow R \)

- Total correctness: the loop does terminate: find a **variant function** \( v \) such that:
  - \( (\text{Inv}&&B) \Rightarrow (v > 0) \)
  - \( \{\text{Inv}&&B&&v = v_0\} \ \text{stmt} \ \{v < v_0\} \)

- For example \((\text{while}(i < 10)\text{do}\{i = i + 1; j = j + 1\})\) can be shown to terminate with \( v = (10 - i) \) and \( \text{Inv} = (i \leq 10) \)