Static Analysis: Overview, Syntactic Analysis and Abstract Interpretation
TDDC90: Software Security

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Outline

Overview

Syntactic Analysis

Abstract Interpretation
Outline

Overview

Syntactic Analysis

Abstract Interpretation
Static Program Analysis analyses computer programs *statically*, i.e., without executing them (as opposed to *dynamic analysis* that does execute the programs wrt. some specific input):

- No need to run programs, before deployment
- No need to restrict to a single input as for testing
- Useful in compiler optimization, program analysis, finding security vulnerabilities and verification
- Often performed on (models of) source code, sometimes on object code
- Usually highly automated though with the possibility of some user interaction
- From scalable bug hunting tools without guarantees to heavy weight verification frameworks for safety critical systems
We want to answer whether the program is **safe** or not (i.e., has some erroneous reachable configurations or not):
Static Program Analysis is a difficult problem

- Checking whether all possible behaviors are error-free is so hard that if we could write a program that could always do it for arbitrary computer programs then we would always be able to answer whether a Turing machine halts.

- This problem is proven to be undecidable, i.e., there is no algorithm that is guaranteed to terminate and to give an exact answer to the problem.
An analysis procedure takes as input a program to be checked against a property. The analysis procedure is an analysis algorithm if it is guaranteed to terminate in a finite number of steps.

An analysis algorithm is **sound** in the case where each time it reports the program is safe wrt. some errors, then the original program is indeed safe wrt. those errors (informally, pessimistic analysis)

An algorithm is **complete** in the case where each time it is given a program that is safe wrt. some errors, then it does report it to be safe wrt. those errors (informally, optimistic analysis)
The idea is then to come up with efficient approximations and algorithms to give correct answers in as many cases as possible.
Static Program Analysis and Approximations

- A sound analysis cannot give **false negatives**
- A complete analysis cannot give **false positives**

**False Positive**

**False Negative**
These Two Lectures

These two lectures on static program analysis will briefly introduce different types of analysis:

▶ This lecture:
  ▶ syntactic analysis: scalable but neither sound nor complete
  ▶ abstract interpretation sound but not complete

▶ Next lecture:
  ▶ symbolic executions: complete but not sound
  ▶ inductive methods: may require heavy human interaction in proving the program correct

These two lectures are only appetizers:
There will be a deeper course with more tools and applications in the spring.
Possibilities of exjobbs with applications to verification and security.
Contact me if interested :-)
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- **This lecture:**
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  - abstract interpretation sound but not complete

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  - Contact me if interested :-}
Administrative Aspects:

- Lab sessions might not be enough and you might have to work outside these sessions.
- You will need to write down your answers to each question on a draft.
- You will need to demonstrate (individually) your answers in a lab session on a computer to me or to Ulf.
- Once you get the green light, you can write your report in a pdf form and send it (in pairs) to the person you got the green light from.
- You will get questions in the final exam about these two lectures.
Outline

Overview

Syntactic Analysis

Abstract Interpretation
Automatic Unsound and Incomplete Analysis

- Tools such as the open source Splint or the commercial Clockworck and Coverity trade guarantees for scalability.
- Not all reported errors are actual errors (false positives) and even if the program reports no errors there might still be uncovered errors (false negatives).
- A user needs therefore to carefully check each reported error, and to be aware that there might be more uncovered errors.
Some tools are augmented versions of grep and look for occurrences of memcpy, pointer dereferences ... 

The open source Splint tool checks C code for security vulnerabilities and programming errors.

Splint does parse the source code and looks for certain patterns such as:

- unused method parameters
- loop tests that are not modified by the loop,
- variables used before definitions,
- null pointer dereference
- overwriting allocated structures
- and many more ...
Unsound and Incomplete analysis: Splint

return *s; // warning about dereference of possibly null pointer

if (s != NULL)
    return *s; // does not give warnings because s was checked

int dumbfunc ()
{
    int i;
    if (i = 0) return 1;
    int j = i;
    while (i > 0){
        j--;
    }
    return 0;
}
Still, the number of false positives remains very important, which may diminish the attention of the user since splint looks for “dangerous” patterns.

An important number of flags can be used to enable, inhibit or organize the kind of errors Splint should look for.

Splint gives the possibility to the user to annotate the source code in order to eliminate warnings.

Real errors can be made quite with annotations. In fact real errors will remain unnoticed with or without annotations.
Outline

Overview

Syntactic Analysis

Abstract Interpretation
Abstract Interpretation

- Suppose you have a program analysis that captures the program behavior but that is too inefficient to be feasible in practice (e.g. enumerating all possible values at each program location)

- You want an analysis that is efficient but that can also over-approximate all behaviors of the program (e.g. tracking only key properties of the values)
The sign example

Consider a language where you can multiply ($\times$), sum ($+$) and substract ($-$) integer variables.

If you are only interested in the signs of the variables values, then you can associate, at each position of the program, a subset of $\{+, 0, -\}$, instead of a subset of $\mathbb{Z}$, to each variable.

For an integer variable, the set of concrete values at a location is in $\mathcal{P}(\mathbb{Z})$. Concrete sets are ordered with the subset relation $\subseteq_c$ on $\mathcal{P}(\mathbb{Z})$. We can associate $\mathbb{Z}$ to each variable in each location, but that is not precise. We write $S_1 \subseteq_c S_2$ to mean that $S_1$ is more precise than $S_2$.

We approximate concrete values with an element in $\mathcal{P}(\{-, 0, +\})$. For instance, $\{0, +\}$ means the variable is larger or equal than zero. For $A_1, A_2$ in $\mathcal{P}(\{-, 0, +\})$, we write $A_1 \subseteq_a A_2$ to mean that $A_1$ is more precise than $A_2$. 
The sign example: concrete and abstract lattices

- A pair \((Q, \leq)\) is a lattice if each pair \(p, q\) in \(Q\) has
  - a greatest lower bound \(p \cap q\) wrt. \(\leq\) (aka meet), and
  - a least upper bound \(p \cup q\) wrt. \(\leq\) (aka join)

- \((\mathcal{P}(\mathbb{Z}), \subseteq_c)\) and \((\mathcal{P}(\{-, 0, +\}), \subseteq_a)\) are lattices

- For any \(S \in \mathcal{P}(\mathbb{Z})\), \(\{\}\ \subseteq_c S\)

- If \(A_1 = \{-, 0\}\) and \(A_2 = \{0, +\}\), then \(A_1 \cap_a A_2 = \{0\}\) and \(A_1 \cup_a A_2 = \{-, 0, +\}\)
The sign example: Galois connections

- $(\alpha, \gamma)$ is a Galois connection if, for all $S \in \mathcal{P}(\mathbb{Z})$ and $A \in \mathcal{P}(\{ -, 0, + \})$, $\alpha(S) \sqsubseteq_a A$ iff $S \sqsubseteq_c \gamma(A)$
- E.g. here, $\alpha(S) = \{ + \}$ if non-empty $S \subseteq \{ i | i > 0 \}$ and $\gamma(A) = \{ i | i \leq 0 \}$ if $A$ is $\{ -, 0 \}$
- Interestingly: $S \sqsubseteq_c \gamma \circ \alpha(S)$ and $\alpha \circ \gamma(A) \sqsubseteq_a A$ for any concrete and abstract elements $S, A$.

Concrete lattice  A Galois connection  Abstract lattice

You can play with more numerical domains at this web interface
http://pop-art.inrialpes.fr/interproc/interprocweb.cgi
Sound approximations: \( f(S) \subseteq \gamma \circ g \circ \alpha(S) \)

Let \( A, B \) be two abstract elements.

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\[
A \otimes B = \bigcup_{a \in A, b \in B} a \otimes b
\]

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\[
A \oplus B = \bigcup_{a \in A, b \in B} a \oplus b
\]

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<td>{-,0}</td>
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\[
A^{++} = \bigcup_{a \in A} a^{++}
\]

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<tbody>
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<td>\underline{\underline{--}}</td>
<td>{-}</td>
<td>{-}</td>
<td>{0,+}</td>
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\[
A^{--} = \bigcup_{a \in A} a^{--}
\]
Example 1

```plaintext
while (x > 0) {
    if (x > 0) {
      x --;
    } else {
      x ++;
    }
    assert (x >= 0);
}
```

---

```plaintext
while (x > 0) {
    if (x > 0) {
      x --;
      assert (x >= 0);
    } else { 
      x ++;
      assert (x >= 0);
    }
    assert (x >= 0);
}
```

---

```plaintext
while (x > 0) {
    if (x > 0) {
      x --;
      assert (x >= 0);
    } else {
      x ++;
      assert (x >= 0);
    }
    assert (x >= 0);
}
```
Example 2

while (x != 0)
    assert (x != 0);
    if (x > 0)
        x, y = x --, 1;
    else
        x, y = x ++, -1;
assert (y != 0);

// x: {...,0,+}; y: {...,0,+}
while (x != 0)
    // x: {...,0,+}; y: {...,0,+}
    assert (x != 0);
    if (x > 0)
        // x: {...,0,+}; y: {...,0,+}
        if (x > 0)
            // x: {+}; y: {...,0,+}
            x, y = x --, 1;
            // x: {}; y: {}
        // x: {0,+}; y: {+}
    else
        // x: {0,+}; y: {+}
        x, y = x ++, -1;
        // x: {}; y: {}
    // x: {0,+}; y: {+}
assert (y != 0);

// x: {0}; y: {...,0,+}

// x: {...,0,+}; y: {...,0,+}
while (x != 0)
    // x: {...,0,+}; y: {...,0,+}
    assert (x != 0);
    if (x > 0)
        // x: {...,0,+}; y: {...,0,+}
        if (x > 0)
            // x: {+}; y: {...,0,+}
            x, y = x --, 1;
            // x: {}; y: {}
        // x: {0,+}; y: {+}
    else
        // x: {0,+}; y: {+}
        x, y = x ++, -1;
        // x: {}; y: {}
    // x: {0,+}; y: {+}
assert (y != 0);

// x: {0}; y: {...,0,+}
Example 3: more precise abstract domain

while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x--, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x--, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: { - ,0 ,+}; y: { - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x--, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: { - ,+}; y: { - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x--, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+, - ,0 ,+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}

// x: {+}; y: {+, - ,0 ,+}
while (x != 0) {
    assert (x != 0);
    if (x > 0) {
        x, y = x++, 1;
    } else {
        x, y = x++, -1;
    }
    assert (y != 0);
}
Example 4: interval domain

\[
(-\infty, +\infty) \subseteq [c, d] \quad \text{iff} \quad c \leq a \text{ and } b \leq d
\]

\[
[a, b] \cup [c, d] \quad \text{is} \quad [\inf\{a, c\}, \sup\{b, d\}]
\]

\[
[a, b] \cap [c, d] \quad \text{is} \quad [\sup\{a, c\}, \inf\{b, d\}]
\]

\[
\begin{align*}
x, y &= 0, 0; \\
\text{while} \ (x \neq 100) \{} & \\
\quad x, y &= x++, y++; \\
\}\ \\
\text{assert} \ (x = 100); \\
\text{assert} \ (y = 100);
\end{align*}
\]

// x:[-oo,+oo], y:[-oo,+oo]
// x:[0,0], y:[0,0]
// x:[0,1], y:[0,1]
// x:[1,1], y:[1,1]
// x:[1,2], y:[1,2]
// x:[], y:[]

// x:[-oo,+oo], y:[-oo,+oo]
// x:[0,0], y:[0,0]
// x:[0,1], y:[0,1]
// x:[1,1], y:[1,1]
// x:[1,2], y:[1,2]
// x:[], y:[]
Example 4: interval domain, widening

\[ \{0, 0\}, \{0, 1\}, \{0, 2\}, \{0, 3\}, \ldots \]

would take 100 steps to converge.
Sometimes too many steps.
For this use some widening operator \( \nabla \).
Intuitively, an acceleration that ensures termination.

\[ x,y = 0,0; \]
\[ \text{while} (x != 100) \{ \]
\[ \quad x,y = x++, y++; \]
\[ \} \]
\[ \text{assert} (x == 100); \]
\[ \text{assert} (y == 100); \]

You can play with more numerical domains at this web interface:
http://pop-art.inrialpes.fr/interproc/interprocweb.cgi