# Artificial Intelligence Planning 2: Abstraction Heuristics

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based on slides by Thomas Keller and Malte Helmert (University of Basel)

# Introduction

# **Planning Heuristics**

#### General Procedure for Obtaining a Heuristic

Solve a simplified version of the problem.

#### there are many ideas for domain-independent planning heuristics:

- $\blacksquare$  abstraction  $\rightsquigarrow$  now
- delete relaxation → tomorrow
- landmarks
- critical paths
- network flows
- potential heuristics

# **Planning Heuristics**

#### Abstraction: Idea

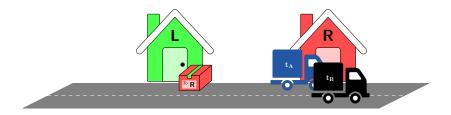
Estimate solution costs by considering a smaller planning task where states are merged to abstract states.

there are many ideas for domain-independent planning heuristics:

- $\blacksquare$  abstraction  $\rightsquigarrow$  now
- delete relaxation ~> tomorrow
- landmarks
- critical paths
- network flows
- potential heuristics

Pattern Databases

# Example: Logistics Task with One Package, Two Trucks



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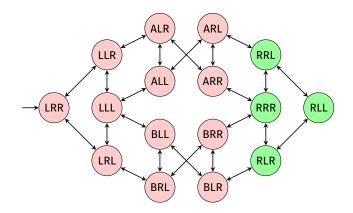


- $\bullet V = \{p, t_{\mathsf{A}}, t_{\mathsf{B}}\}$
- $dom(p) = \{L, R, A, B\}$  and  $dom(t_A) = dom(t_B) = \{L, R\}$

$$\blacksquare I = \{p \mapsto L, t_A \mapsto R, t_B \mapsto R\}$$

- $\blacksquare G = \{p \mapsto \mathbf{R}\}\$
- $A = \{load_{i,j} \mid i \in \{A, B\}, j \in \{L, R\}\}$ 
  - $\cup \{unload_{i,j} \mid i \in \{A, B\}, j \in \{L, R\}\}$
  - $\cup \{move_{i,j,j'} \mid i \in \{A, B\}, j, j' \in \{L, R\}, j \neq j'\} \text{ with:}$ 
    - *load*<sub>*i*,*j*</sub> has preconditions  $\{t_i \mapsto j, p \mapsto j\}$ , effects  $\{p \mapsto i\}$
    - *unload*<sub>*i*,*j*</sub> has preconditions  $\{t_i \mapsto j, p \mapsto i\}$ , effects  $\{p \mapsto j\}$
    - $move_{i,j,j'}$  has preconditions  $\{t_i \mapsto j\}$ , effects  $\{t_i \mapsto j'\}$
    - all actions have cost 1

#### State Space for Example Task



- state  $\{p \mapsto i, t_A \mapsto j, t_B \mapsto k\}$  denoted as *ijk*
- annotations of edges not shown for simplicity
- for example, edge from LLL to ALL has annotation load<sub>A,L</sub>

# Abstractions

#### Abstraction

abstractions drop distinctions between certain states, but preserve the state space behavior as well as possible.

- an abstraction of a state space S is defined by an abstraction function α that determines which states can be distinguished in the abstraction
- based on S and α, we compute the abstract state space S<sup>α</sup> which is "similar" to S but smaller

#### idea of the abstraction heuristic $h^{\alpha}$ :

use abstract solution costs (solution costs in  $S^{\alpha}$ ) as heuristic values for concrete solution costs (solution costs in S)

# Induced Abstraction

#### Definition (induced abstraction)

Let  $S = \langle S, A, cost, T, s_0, S_* \rangle$  be a state space, and let  $\alpha : S \to S'$  be a surjective function.

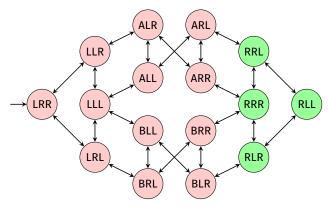
The abstraction of S induced by  $\alpha$ , denoted as  $S^{\alpha}$ , is the state space  $S^{\alpha} = \langle S', A, cost, T', s'_0, S'_{\star} \rangle$  with:

$$T' = \{ \langle \alpha(s), a, \alpha(t) \rangle \mid \langle s, a, t \rangle \in T \}$$

$$s_0' = \alpha(s_0)$$

$$S'_{\star} = \{ \alpha(s) \mid s \in S_{\star} \}$$

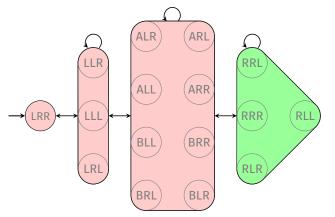
#### Abstraction: Example



concrete state space

## Abstraction: Example

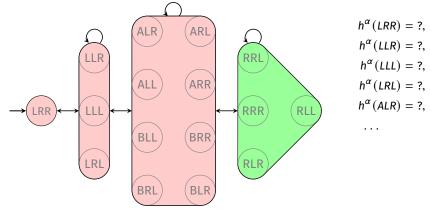
#### (an) abstract state space



remark: most edges correspond to several (parallel) transitions with different annotations

# Abstraction Heuristic: Example

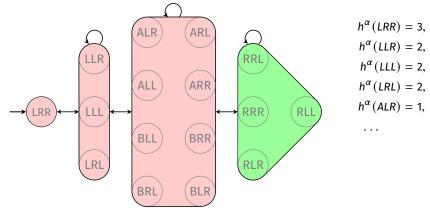
#### (an) abstract state space



remark: most edges correspond to several (parallel) transitions with different annotations

## Abstraction: Example

#### (an) abstract state space



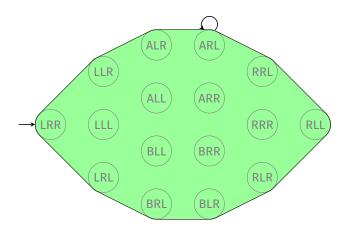
remark: most edges correspond to several (parallel) transitions with different annotations

# Abstraction Heuristics: Discussion

- every abstraction heuristic is admissible and consistent
- the choice of the abstraction function  $\alpha$  is very important
  - every  $\alpha$  yields an admissible and consistent heuristic
  - but most  $\alpha$  lead to poor heuristics
- a "good"  $\alpha$  must yield an informative heuristic ...
- …as well as being efficiently computable

#### How can we find a suitable $\alpha$ ?

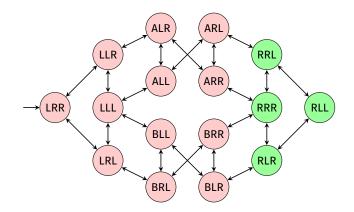
# Usually a Bad Idea: Single-State Abstraction



one state abstraction:  $\alpha(s) := const$ 

- + compactly representable and  $\alpha$  easy to compute
- very uninformed heuristic

#### Usually a Bad Idea: Identity Abstraction



identity abstraction:  $\alpha(s) := s$ 

- + perfect heuristic and  $\alpha$  easy to compute
- too many abstract states  $\rightsquigarrow$  computation of  $h^{\alpha}$  too hard

# Automatic Computation of Suitable Abstractions

Main Problem with Abstraction Heuristics

How to find a good abstraction?

several successful methods:

pattern databases (PDBs)

(Culberson & Schaeffer, 1996)

- merge-and-shrink abstractions (Dräger, Finkbeiner & Podelski, 2006)
- Cartesian abstractions (Seipp & Helmert, 2013)

# Pattern Databases

#### Pattern Databases: Background

- the most common abstraction heuristics are pattern database heuristics
- originally introduced for the 15-puzzle (Culberson & Schaeffer, 1996) and for Rubik's Cube (Korf, 1997)
- introduced for automated planning by Edelkamp (2001)
- for many search problems the best known heuristics
- many research papers studying
  - theoretical properties
  - efficient implementation and application
  - pattern selection

• • • •

#### Pattern Databases: Projections

a PDB heuristic is an abstraction heuristic where

- some aspects (= state variables) of the task are preserved with perfect precision while
- all other aspects are not preserved at all

formalized as projections on a pattern P, e.g.:

$$\mathbf{s} = \{\mathbf{v}_1 \mapsto d_1, \mathbf{v}_2 \mapsto d_2, \mathbf{v}_3 \mapsto d_3\}$$

- projection on  $P = \{v_1\}$  (= ignore  $v_2, v_3$ ):  $\alpha(s) = s|_P = \{v_1 \mapsto d_1\}$
- **projection on**  $P = \{v_1, v_3\}$  (= ignore  $v_2$ ):

$$\alpha(s) = s|_P = \{v_1 \mapsto d_1, v_3 \mapsto d_3\}$$

#### Pattern Databases: Definition

#### Definition (pattern database heuristic)

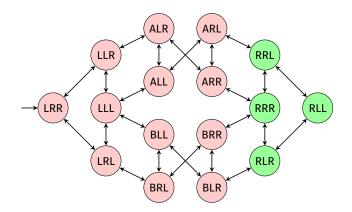
Let P be a subset of the variables of a planning task.

The abstraction heuristic induced by the projection  $\pi_P$  on P is called pattern database heuristic (PDB heuristic) with pattern P.

abbreviated notation:  $h^{P}$  for  $h^{\pi_{P}}$ 

remark: "pattern databases" in analogy to endgame databases (which have been successfully applied in 2-person-games)

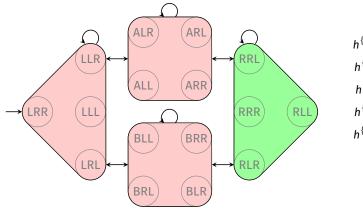
#### Example: Concrete State Space



- state variable for package, p: {L, R, A, B}
- state variable for truck A, t<sub>A</sub>: {L, R}
- state variable for truck B, t<sub>B</sub>: {L, R}

## Example: Projection (1)

#### abstraction induced by $\pi_{\{p\}}$ :

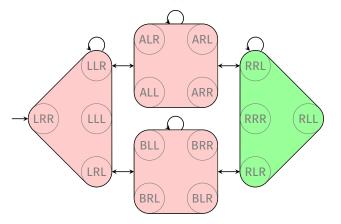


 $h^{\{p\}}(LRR) = 2,$   $h^{\{p\}}(LLR) = 2,$   $h^{\{p\}}(LLL) = 2,$   $h^{\{p\}}(LRL) = 2,$   $h^{\{p\}}(LRL) = 2,$  $h^{\{p\}}(ALR) = 1,$ 

. . .

# Example: Projection (1)

#### abstraction induced by $\pi_{\{p\}}$ :

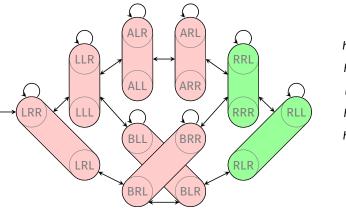




$$\{p \mapsto L\} = 2,$$
  
$$\{p \mapsto A\} = 1,$$
  
$$\{p \mapsto B\} = 1,$$
  
$$\{p \mapsto R\} = 0$$

# Example: Projection (2)

### abstraction induced by $\pi_{\{p,t_A\}}$ :

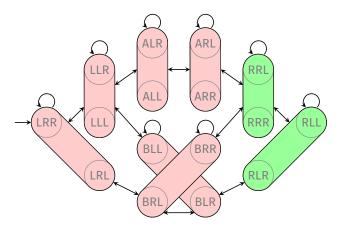


 $h^{\{p,t_{A}\}}(LRR) = 2,$   $h^{\{p,t_{A}\}}(LLR) = 2,$   $h^{\{p,t_{A}\}}(LLL) = 2,$   $h^{\{p,t_{A}\}}(LRL) = 2,$   $h^{\{p,t_{A}\}}(LRL) = 2,$  $h^{\{p,t_{A}\}}(ALR) = 2,$ 

. . .

# Example: Projection (2)

#### abstraction induced by $\pi_{\{p,t_A\}}$ :



 $\begin{array}{l} \mathsf{PDB} :\\ \{p \mapsto L, t_A \mapsto L\} = 2,\\ \{p \mapsto L, t_A \mapsto R\} = 2,\\ \{p \mapsto A, t_A \mapsto L\} = 2,\\ \{p \mapsto A, t_A \mapsto L\} = 2,\\ \{p \mapsto A, t_A \mapsto L\} = 1,\\ \{p \mapsto B, t_A \mapsto L\} = 1,\\ \{p \mapsto B, t_A \mapsto R\} = 1,\\ \{p \mapsto R, t_A \mapsto L\} = 0,\\ \{p \mapsto R, t_A \mapsto R\} = 0\end{array}$ 

#### Pattern Databases in Practice

practical aspects which we do not discuss in detail:

- How to automatically find good patterns?
- How to combine multiple PDB heuristics?
- How to implement PDB heuristics efficiently?
  - good implementations efficiently handle abstract state spaces with 10<sup>7</sup>, 10<sup>8</sup> or more abstract states
  - effort independent of the size of the concrete state space
  - usually all heuristic values are precomputed
    - $\rightsquigarrow$  space complexity = number of abstract states



# Kahoot!

#### Summary

#### planning formalisms:

- STRIPS: particularly simple, easy to handle for algorithms
  - binary state variables
  - preconditions, add and delete effects, goals: sets of variables
- SAS<sup>+</sup>: extension of STRIPS
  - state variables with arbitrary finite domains
- PDDL: input language used in practice
  - based on predicate logic (more compact than propositional logic)
  - only partly supported by most algorithms

#### abstraction heuristics:

- estimate solution cost by considering a smaller planning task
- Pattern database heuristics are abstraction heuristics based on projections onto state variable subsets (patterns): states are distinguishable iff they differ on the pattern.