



LL Parsing Issues Beyond Recursive Descent

LL(k)
LL items
Finite pushdown automaton
FIRST and FOLLOW
Table-driven Predictive Parser

Peter Fritzzon, Christoph Kessler,
IDA, Linköpings universitet, 2008.

LL(k)



- Given:
 - Context-free grammar $G = (N, \Sigma, P, S)$
 - Integer $k > 0$
- G is (in) **LL(k)** if:
 - for any two leftmost derivations
 - $S \Rightarrow_{lm}^* uY\alpha \Rightarrow u\beta\alpha \Rightarrow^* ux$ and
 - $S \Rightarrow_{lm}^* uY\alpha \Rightarrow u\gamma\alpha \Rightarrow^* uy$
 with $x[1:k] = y[1:k]$
 - the k first tokens of x and y are equal
 it holds $\beta = \gamma$.
- That is, for fixed left context u , the choice for the "right" production to apply to Y is uniquely determined by the next k input tokens.

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Example



- The following grammar is LL(1)
(terminals are bold-face):

```

S -> if ident then S else S fi
    | while ident do S od
    | begin S end
    | ident := ident
    
```

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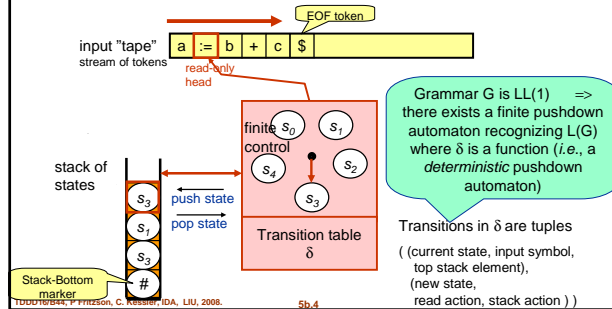
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Automaton Model for Parsing Context-Free Languages



Finite pushdown automaton (FPA)

- a finite automaton with a stack of states



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Context-Free Items



Given CFG G , construct states of the finite pushdown automaton:

- Add new start symbol S' with $S' \rightarrow S \$$
- For each production $A \rightarrow \alpha_1 \dots \alpha_k$ e.g. $A \rightarrow aBc$ create $k+1$ **context-free items** (= states)
 - e.g., $[A \rightarrow aBc]$, $[A \rightarrow a.Bc]$, $[A \rightarrow aB.c]$, $[A \rightarrow aBc.]$
- Construct a **predictive parser** as finite pushdown automaton:
 - start in state $[S' \rightarrow S \$]$ with empty stack ($\#$)
 - halt and accept in state $[S' \rightarrow S \$.]$ with empty stack ($\#$)
 - at $[A \rightarrow \alpha.b\gamma]$: read input symbol, i.e., $[A \rightarrow \alpha.b\gamma] \rightarrow [A \rightarrow \alpha b.\gamma]$
 - at $[A \rightarrow \alpha.B\gamma]$: push $[A \rightarrow \alpha.B.\gamma]$, determine new production $B \rightarrow \beta$ and start from $[B \rightarrow \beta]$ Prediction!
 - at $[B \rightarrow \beta.]$: pop state $[A \rightarrow \alpha.B.\gamma]$ to restore context (if $\#$, error)

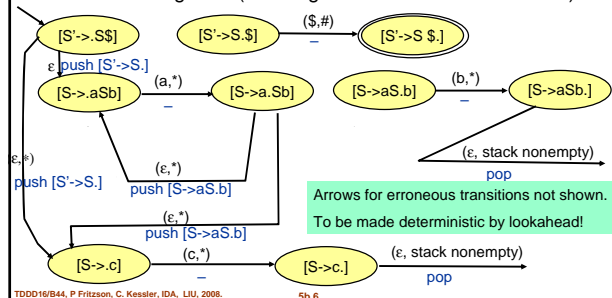
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Example



- Grammar with productions $\{ S \rightarrow aSb \mid c \}$
- Add new start symbol S' : $\{ S' \rightarrow S; S \rightarrow aSb; S \rightarrow c \}$
- Transition diagram (showing **stack actions** below arrows):

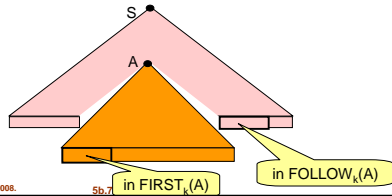


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FIRST and FOLLOW

- For a sentential form α in $(N \cup \Sigma)^+$, $FIRST(\alpha)$ denotes the set of all terminals with which a string derived from α may begin.
- For a nonterminal A in N , $FOLLOW(A)$ denotes the set of all terminals that could appear in a sentential form immediately after A , i.e., there exists $S \Rightarrow^* \alpha A \beta$ for arbitrary α, β



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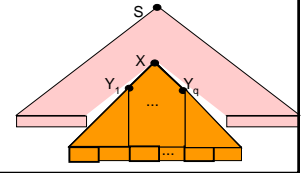
Computing $FIRST = FIRST_1$

For all grammar symbols X :

- If X is a terminal, then $FIRST(X) = \{X\}$.
- If $X \rightarrow \epsilon$ is a production, then add ϵ to $FIRST(X)$.
- If X is a nonterminal and $X \rightarrow Y_1 Y_2 \dots Y_q$ is a production,
 - then place all those a of Σ in $FIRST(X)$ where for some i , a is in $FIRST(Y_i)$ and ϵ is in all of $FIRST(Y_1), \dots, FIRST(Y_{i-1})$ (that is, Y_1, \dots, Y_{i-1} all may derive ϵ).
 - If ϵ is in $FIRST(Y_j)$ for all $j=1,2,\dots,q$ then add ϵ to $FIRST(X)$.

Apply these rules until no more terminals or ϵ can be added to any $FIRST$ set.

For the example grammar
 $S' \rightarrow S; S \rightarrow aSb; S \rightarrow c$
 $FIRST(a) = \{a\}, FIRST(b) = \{b\},$
 $FIRST(c) = \{c\}$
 $FIRST(S') = FIRST(S)$
 $FIRST(S) = \{a, c\}$



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Computing FIRST (cont.)

For any string $X_1 X_2 \dots X_n$ of grammar symbols:

- Add to $FIRST(X_1 X_2 \dots X_n)$ all non- ϵ symbols of $FIRST(X_1)$.
- If ϵ in $FIRST(X_1)$, add also all non- ϵ symbols of $FIRST(X_2)$, otherwise done.
- If ϵ also in $FIRST(X_2)$, add also all non- ϵ symbols of $FIRST(X_3)$, otherwise done.
- ...
- If ϵ also in $FIRST(X_n)$, add ϵ to $FIRST(X_1 X_2 \dots X_n)$

For the example grammar
 $S' \rightarrow S; S \rightarrow aSb; S \rightarrow c$
 $FIRST(abc) = \{a\}$
 $FIRST(Sb) = FIRST(S) = \{a, c\}$

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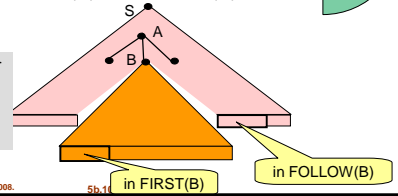
Computing FOLLOW

Compute $FOLLOW(B)$ for each nonterminal B :

- Add $\$$ to $FOLLOW(S)$
- If there is a production $A \rightarrow^* \alpha B \beta$ for arbitrary α, β then add all of $FIRST(\beta)$ except ϵ to $FOLLOW(B)$
- If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$ where ϵ in $FIRST(\beta)$, i.e. $\beta \Rightarrow^* \epsilon$, then add all of $FOLLOW(A)$ to $FOLLOW(B)$.

Apply these rules until no more terminals or ϵ can be added to any $FOLLOW$ set.

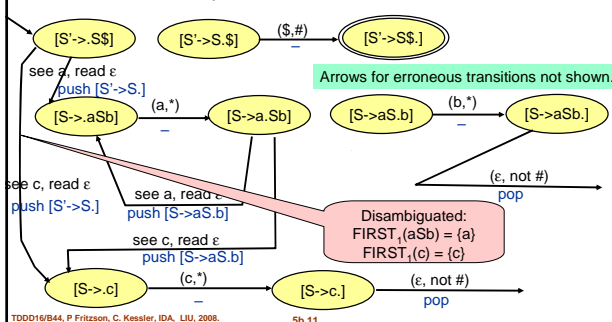
For the example grammar
 $S \rightarrow aSb; S \rightarrow c$
 $FOLLOW(S) = \{\$, b\}$



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Example Cont.: Finite Pushdown Automaton (FPA) Made Deterministic

- Grammar with productions $\{ S \rightarrow aSb \mid c \}$
- Added new start symbol S' : $\{ S' \rightarrow S\$; S \rightarrow aSb; S \rightarrow c \}$



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Example (cont.): Transition table ($k=1$)

state	final ?	lookahead a	lookahead b	lookahead c	lookahead \$
$[S' \rightarrow S \$]$	no	push $[S' \rightarrow S \$]$; $[S \rightarrow aSb]$	[Error]	push $[S' \rightarrow S \$]$; $[S \rightarrow c]$	[Error]
$[S' \rightarrow S \$]$	no	[Error]	[Error]	[Error]	read \$; $[S' \rightarrow S \$]$
$[S' \rightarrow S \$]$	yes				
$[S \rightarrow aSb]$	no	read a; $[S \rightarrow a.Sb]$	[Error]	[Error]	[Error]
$[S \rightarrow a.Sb]$	no	push $[S \rightarrow aS.b]$; $[S \rightarrow aSb.]$	[Error]	push $[S \rightarrow aS.b]$; $[S \rightarrow c]$	[Error]
$[S \rightarrow aS.b]$	no	[Error]	read b; $[S \rightarrow aSb.]$	[Error]	[Error]
$[S \rightarrow aSb.]$	no	[Error]	pop state	[Error]	pop state
$[S \rightarrow c]$	no	[Error]	[Error]	read c; $[S \rightarrow c.]$	[Error]
$[S \rightarrow c.]$	no	[Error]	pop state	[Error]	pop state

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General Approach: Predictive Parsing



At any production $A \rightarrow \alpha$

- If ϵ is not in $\text{FIRST}(\alpha)$:
 - Parser expands by production $A \rightarrow \alpha$ if current lookahead input symbol is in $\text{FIRST}(\alpha)$.
- otherwise (i.e., ϵ in $\text{FIRST}(\alpha)$):
 - Expand by production $A \rightarrow \alpha$ if current lookahead symbol is in $\text{FOLLOW}(A)$ or if it is $\$$ and $\$$ is in $\text{FOLLOW}(A)$.

Use these rules to fill the transition table.
(pseudocode: see [ASU86] p. 190, [ALSU06] p. 224)

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Summary: Parsing $\text{LL}(k)$ Languages



- **Predictive LL parser**
 - iterative, based on finite pushdown automaton
 - transition-table-driven
 - can be generated automatically
- **Recursive-descent parser**
 - recursive
 - manually coded
 - easier to fix intermediate code generation, error handling
- **Both require lookahead** (or backtracking) to predict the next production to apply
 - Removes nondeterminism
 - Necessary checks derived from FIRST and FOLLOW sets
 - FIRST and FOLLOW are also useful for syntax error recovery

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Homework



- Now, read again the part on recursive descent parsers and find the equivalent of
 - Context-free items (Pushdown automaton (PDA) states)
 - The stack of states
 - Pushing a state to stack
 - Popping a state from stack
 - Start state, final statein a recursive descent parser.

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