Confidence-regions, Simultaneous confidence-intervals and Confidence-interval with simultaneous confidence level

In this short review I will try to make clear for you the difference between these three types of confidence-intervals given in the headline. I present the intervals only for the case when we have two populations. The assumptions we have to make here are that all of the variables are normally distributed.

Confidence-regions
To get some sort of confidence-intervals of the difference between mean vectors \( \mu_1 - \mu_2 \) we have to define a region. This region has a shape of an ellipsoid.

The region \( T^2 = \frac{n_1n_2}{n_1+n_2} \left( \bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2) \right)'C_p^{-1} \left( \bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2) \right) \) is an ellipsoid.

So a \( 100(1-\alpha)\% \) confidence-region for \( \mu_1 - \mu_2 \) is defined by

\[
T^2 < \frac{(n_1+n_2-2)p}{n_1+n_2-p-1} F_{a}(p, n_1+n_2-p-1)
\]

Try different values of \( \mu_1 - \mu_2 \) to see if the inequality holds or not.

Simultaneous confidence-intervals
If we put a box that encloses the ellipsoid as tight as possible we get simultaneous confidence-intervals. These intervals are to large and the erroneous region grows positively with the dependence of the \( p \) variables.

So a \( 100(1-\alpha)\% \) confidence-interval for \( a'(\mu_1 - \mu_2) \sigma \) given by

\[
a'(\bar{X}_1 - \bar{X}_2) \pm K_{\alpha/2} s_a \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \text{ where } K_{\alpha/2} = \left( \frac{n_1+n_2-2)p}{n_1+n_2-p-1} F_{a}(p, n_1+n_2-p-1) \right) \]

and \( s_a^2 = a'C_p a \)

As an example, the vector \( a' = (1 \ 0 \ ... \ 0) \) if you want an interval for the first par of mean-values.

Confidence-interval with simultaneous confidence level
We base these intervals on Bonferroni’s methodology.

Construct \( p \) ordinary confidence-intervals based on the t-distribution. If you want a simultaneous confidence-level of at least \( 1-\alpha \), then each interval should have confidence-level \( 1-\frac{\alpha}{p} \)

These intervals are parallel with the simultaneous intervals but a bit smaller. The problem is then that a part of the region in the ellipsoid is not contained in these intervals.