723A75 Advanced Data Mining
TDDD41 Data Mining - Clustering and Association Analysis

Lecture 7: FP Grow Algorithm

Johan Alenlöv
IDA, Linköping University, Sweden

Outline

$\arg\min_{x \in D}$

- Content
  - Recap
  - Frequent Pattern (FP) Grow Algorithm
  - Exercise
  - Summary
- Litterature
  - Course Book. 2nd ed.: 5.2.4. 3rd ed.: 6.2.4
  - Han, J., Pei, J., and Yin, Y. **Mining Frequent Patterns without Candidate Generation**. In Proc. of the 2000 ACM SIGMOD Int. Conf. on Management of Data, 2000.

Recap

- Given a database of transactions we want to find association rules,

$$\text{Item}_1, \dots, \text{Item}_m \rightarrow \text{Item}_{m+1}, \dots, \text{Item}_n$$
$$(X \rightarrow Y)$$

  with a **user-specified** minimum **support** and **confidence**.
- **support**: Fraction of transactions that contains th full rule $\text{Item}_1, \dots, \text{Item}_n$. $(p(X, Y))$
- **confidence**: Fraction of transactions that contain $\text{Item}_1, \dots, \text{Item}_m$ which also contain $\text{Item}_{m+1} \rightarrow \text{Item}_n$. $(p(Y \mid X))$
- We find the rules in two steps:
  1. Find all **frequent itemsets**
  2. Find all rules with minimum confidence from these sets.

Recap: Apriori Algorithm

- Using the following **apriori propeerty**:
  - Every subset of a frequent itemset is frequent.
  - Alternatively, every superset of an infrequent itemset is infrequent.
- The Apriori Algorithm works as follows:
  1. Find all 1-itemsets.
  2. Use the previous found frequent itemsets and the apriori property to generate candidates for the next frequent itemsets.
  3. Go through the candidates to find the itemsets.

  Step 2 and 3 are repeated until no new frequent itemsets are found.
- We proved by induction that the algorithm is correct.

Recap: Generate rules

- Given a large itemset L we wish to generate rules

$$X \rightarrow L \setminus X,$$

  where $X \subset L$.
- These rules should have a **minimum confidence**.
- The algorithm uses the following **apriori property**:
  - If X does not result in a rule with minimum confidence for L, then neither does **any subset** $X' \subset X$,

$$\mathrm{confidence}(X \rightarrow L \setminus X) = \frac{\mathrm{support}(L)}{\mathrm{supportX}} \geq \frac{\mathrm{support}(L)}{\mathrm{support}(X')} = \mathrm{confidence}(\mathrm{X}' \rightarrow \mathrm{L} \setminus \mathrm{X}')$$

for all large itemsets l_k with $k \geq 2$ do
    call genrules(l_k, l_k, *minconf*)

Algorithm: genrules(l_k, a_m, *minconf*)
Input: A large itemset l_k, a set $a_m \subseteq l_k$, the minimum confidence *minconf*.
Output: All the rules of the form $a \rightarrow l_k \setminus a$ with $a \subseteq a_m$ and confidence equal or above *minconf*.

$\mathbb{A} = \{(m-1)$-itemsets $a_{m-1} | a_{m-1} \subseteq a_m\}$
for all $a_{m-1} \in \mathbb{A}$ do
    conf $=$ support(l_k) / support(a_{m-1})   // Confidence of the rule $a_{m-1} \rightarrow l_k \setminus a_{m-1}$
    if conf $\geq$ *minconf* then
        output the rule $a_{m-1} \rightarrow l_k \setminus a_{m-1}$ with confidence=conf and support=support(l_k)
        if $m-1 > 1$ then call genrules(l_k, a_{m-1}, *minconf*)

Recap: Rule Generation Algorithm Proof

for all large itemsets l_k with $k \geq 2$ do
    call genrules(l_k, l_k, *minconf*)

Algorithm: genrules(l_k, a_m, *minconf*)
Input: A large itemset l_k, a set $a_m \subseteq l_k$, the minimum confidence *minconf*.
Output: All the rules of the form $a \rightarrow l_k \setminus a$ with $a \subseteq a_m$ and confidence equal or above *minconf*.

$\mathbb{A} = \{(m-1)\text{-itemsets } a_{m-1} | a_{m-1} \subseteq a_m\}$
for all $a_{m-1} \in \mathbb{A}$ do
    conf = support(l_k) / support(a_{m-1}) // Confidence of the rule $a_{m-1} \rightarrow l_k \setminus a_{m-1}$
    if conf $\geq$ *minconf* then
        output the rule $a_{m-1} \rightarrow l_k \setminus a_{m-1}$ with confidence=conf and support=support(l_k)
        if $m-1 > 1$ then call genrules(l_k, a_{m-1}, *minconf*)

- We prove by contradiction that the rule generation algorithm is correct.
- Assume that the algorithm missed a rule. Let $a_{m-1} \rightarrow l_k \setminus a_{m-1}$ denote one of the missing rules with the largest antecedent. Then,
  - Note that l_k has minimum support and, thus, it is outputted by the apriori algorithm since this is correct.
  - Then, the rule generation algorithm cannot have missed the rule if $m = k$.
  - Moreover if $m < k$, then

$$\begin{aligned}\text{confidence}(a_m \rightarrow l_k \setminus a_m) = \text{support}(l_k)/\text{support}(a_m) &\geq \text{support}(l_k)/\text{support}(a_{m-1}) \\ &= \text{confidence}(a_{m-1} \rightarrow l_k \setminus a_{m-1}) \geq minconf.\end{aligned}$$

  - Note that the algorithm didn't miss the rule $a_m \rightarrow l_k \setminus a_m$.
  - Then the algorithm couldn't have missed the rule $a_{m-1} \rightarrow l_k \setminus a_{m-1}$.
  - This contradicts our assumption and, thus, the algorithm is correct.

FP Grow Algorithm

- As previous, assume that we have access to some transactional data,

Tid	Items
1	F, A, C, D, G, I, M, P
2	A, B, C, F, L, M, O
3	B, F, H, J, O, W
4	B, C, K, S, P
5	A, F, C, E, L, P, M, N

- The FP grow algorithm returns all frequent itemsets **without candidate generation** and may save time and space.
- First, it finds frequent 1-itemsets and **sorts the frequent items within each transaction in support decending order**, e.g. with $minsup = 3$

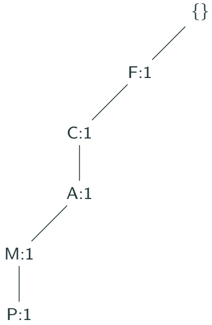
Tid	Items
1	F, C, A, M, P
2	F, C, A, B, M
3	F, B
4	C, B, P
5	F, C, A, M, P

- Then it outputs the frequent 1-itemsets, F, C, A, B, M, and P.

FP Grow Algorithm

- Given the new sorted set it constructs a so-called FP tree.

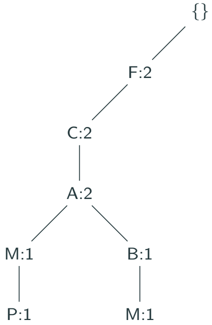
Tid	Items
1	**F, C, A, M, P**
2	F, C, A, B, M
3	F, B
4	C, B, P
5	F, C, A, M, P



FP Grow Algorithm

- Given the new sorted set it constructs a so-called FP tree.

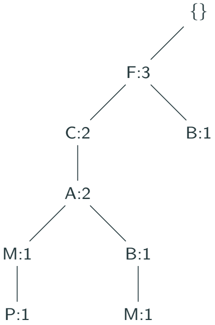
Tid	Items
1	F, C, A, M, P
2	**F, C, A, B, M**
3	F, B
4	C, B, P
5	F, C, A, M, P



FP Grow Algorithm

- Given the new sorted set it constructs a so-called FP tree.

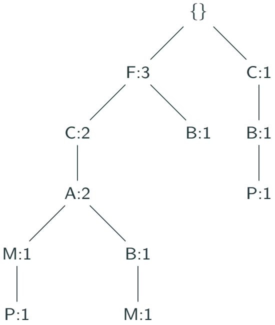
Tid	Items
1	F, C, A, M, P
2	F, C, A, B, M
3	**F, B**
4	C, B, P
5	F, C, A, M, P



FP Grow Algorithm

- Given the new sorted set it constructs a so-called FP tree.

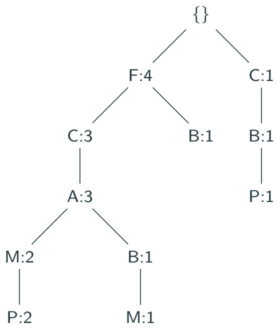
Tid	Items
1	F, C, A, M, P
2	F, C, A, B, M
3	F, B
4	**C, B, P**
5	F, C, A, M, P



FP Grow Algorithm

- Given the new sorted set it constructs a so-called FP tree.

Tid	Items
1	F, C, A, M, P
2	F, C, A, B, M
3	F, B
4	C, B, P
5	**F, C, A, M, P**



- Finally, it mines the FP tree for frequent itemsets instead of the original database.

FP Grow Algorithm

Algorithm: FP-tree(D, $minsup$)
Input: A transactional database D, and the minimum support $minsup$.
Output: The FP tree for D and $minsup$.

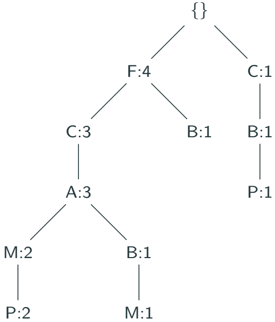
Count support for each item in D
Remove the infrequent items from the transactions in D
Sort the items in each transaction in D in support descending order
Create a FP tree with a single node T with $T.name = NULL$
for each transaction $I \in D$ do
    insert-tree(I, T)

Algorithm: insert-tree($I_1, \ldots I_m$, T)
Input: An itemset $I_1, \ldots, I_m$, and a node T in the FP tree.
Output: Modified FP tree.

if T has a child N such that $N.name = I_1.name$ then
    $N.count++$
else
    create a new child N of T with $N.name = I_1.name$ and $N.count = 1$
if $m > 1$ then
    insert-tree ($I_2, \ldots, I_m, N$)

FP Grow Algorithm

- The X-conditional database consists of all the prefix paths leading to X in the FP tree.



Item	Conditional database
F	-
C	F:3
A	FC:3
B	FCA:1, F:1, C:1
M	FCA:2, FCAB:1
P	FCAM:2, CB:1

- The support of each prefix path in the conditional database is equal to the count of X for that prefix path.
- The X-conditional database contains all the itemsets in *D* that end with X.
- It is enough to mine the X-conditional database to find all the frequent itemsets in *D* that end with X.
- Re-start the algorithm for the X-conditional database, i.e. call the FP grow algorithm recursively.

FP Grow Algorithm

- If we look at the M-conditional database, $(\{\text{FCA} : 2, \text{FCAB} : 1\})$

Tip	Items
1	F, C, A
2	F, C, A
3	F, C, A, B

- After finding the frequent 1-itemsets and sorting the transactions we have

Tid	Items
1	F, C, A
2	F, C, A
3	F, C, A

- Output the frequent 1-itemsets, adding M as suffix (FM, CM, AM)
- Build the FP tree and the conditional databases.

Item	Conditional database
F	-
C	F:3
A	FC:3

- Restart the algorithm for the FM, CM, and AM condtional databases.

FP Grow Algorithm

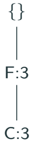
- For the AM-conditional database ({FC : 3}), or

Tid	Items
1	F,C
2	F,C
3	F,C

- After finding the 1-itemsets and sorting the transactions we have

Tid	Items
1	F,C
2	F,C
3	F,C

- Output th 1-itemsets, adding AM as a suffix. (FAM and CAM).
- Build the FP tree and the econditional databases.



Item	Conditional database
F	-
C	F:3

- Restart the algorithm for the FAM and CAM conditional databases.

FP Grow Algorithm

- For the CAM-conditional database ({F : 3}), or

Tid	Items
1	F
2	F
3	F

- After finding the 1-itemsets and sorting the transactions we have

Tid	Items
1	F
2	F
3	F

- Output th 1-itemsets, adding CAM as a suffix. (FCAM).
- Build the FP tree and the econditional databases.

Item	Conditional database
F	-

- Conditional database is empty.
  Backtrack.

FP Grow Algorithm

- To mine the FP tree *Tree*, call FP-grow(*Tree*, NULL, *minsup*).

Algorithm: FP-grow(*Tree*, α, *minsup*)
Input: A FP tree *Tree*, an itemset α, and the minimum support *minsup*.
Output: All the itemsets in *Tree* that end with α and have *minsup*.

for each item X in *Tree* do
    output the itemset $\beta = X \cup \alpha$ with support=$X.count$
    build the β conditional database and the corresponding FP tree $Tree_\beta$
    if $Tree_\beta$ is not empty then call FP-grow($Tree_\beta$, β, *minsup*)

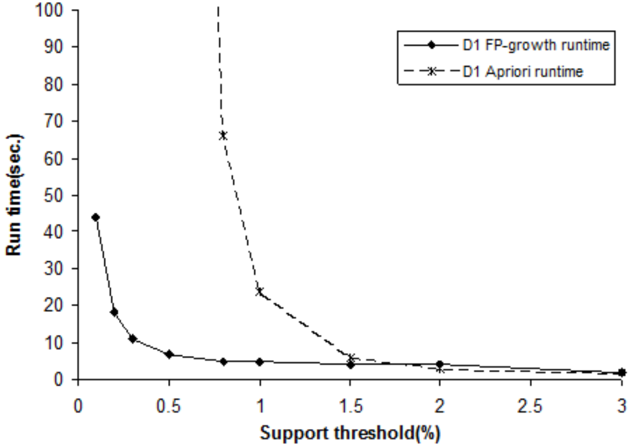
- The algorithm above can be made more efficient by adding the lines below.

if *Tree* has a single branch then
    for each combination β of the nodes in the branch do
        output the itemset $\beta \cup \alpha$ with support $= \min_{X \in \beta} X.count$
else

- The FP grow algorithm is correct.

FP Grow Algorithm

- With small values for *minsup*, there are many and long candidates, which implies long runtime due to expensive operations such as pattern matching, subset checking, storing, etc.



Exercise

- Run the FP grow algorithm on the database below with *minsup* 2.

Tid	Items
1	A, B, E
2	B, D
3	B, C
4	A, B, D
5	A, C
6	B, C
7	A, C
8	A, B, C, E
9	A, B, C

- Show the execution details (i.e. FP tree construction, conditional databases, recursive calls), not just the frequent itemsets found.
- Solution : $\{A, B, C, D, E, AB, AC, AE, BC, BD, BE, ABC, ABE\}$

Summary

- Mining transactions to find rules of the form

$$\text{Item}_1, \ldots, \text{Item}_m \rightarrow \text{Item}_{m+1}, \ldots, \text{Item}_n$$

  with **user-defined minimum support** and **confidence**.
- Two-step solution:
  1. Find all the large itemsets.
  2. Generate all the rules with minimum confidence.
- We have seen two solutions for step 1. Apriori and FP grow algorithm.
- The runtime can differ a lot for small values of *minsup*.