LINKÖPINGS UNIVERSITET
Department of Computer and
Information Science
Division of Statistics and Machine learning/ANd

732A66 Decision theory
Fall semester 2021
Assignment 4

Assignment 4

Below are two tasks that you shall try to solve using **analytical methods** (i.e. not by simulation). All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You can help each other but there must be individual submissions (that are not just copies of one submission).

Your solutions should be submitted at latest on **Friday 14 January 2022**.

1. Assume some budget calculations depend on whether a certain cost will be at least SEK 120 000 or lower than this amount. A reasonable model for this cost is a normal distribution with standard deviation SEK 12 000 (independent of the mean) and a mean that can be modelled as normally distributed with mean 115 000 (SEK) and standard deviation 9 000 (SEK). No trend is anticipated for this cost and for the 6 previous periods the average cost was SEK 121 000.
   Note that the hypotheses are about the actual cost, not the expected cost.

   a) Show that the prior odds for the hypothesis that the cost will exceed SEK 120 000 (against the alternative that it will not) is about 0.59. [*hint:* write the observed variable $\tilde{x}$ as a sum of two independent random variables $\tilde{x} = \tilde{\mu} + \tilde{\varepsilon}$]
   b) Show that the Bayes factor (considering the average cost for the previous 6 periods) for the hypothesis that the cost will exceed SEK 120 000 (against the alternative that it will not) is about 1.63.
   c) If the loss of accepting the hypothesis that the cost will be lower than SEK 120 000 while the opposite will be true is SEK 4 000, and the loss of accepting the hypothesis that the cost will be at least SEK 120 000 while the opposite will be true is SEK 6 000, which decision should be made for the budget (according to the rule of minimizing the expected loss)?

2. *Curiosity:* Quite recently (2019) the international prototype kilogram (IPK) in Paris was replaced by a definition in terms of the Planck constant.

   Assume we have a scale with no systematic error and that we have weighed the IPK 3 times and obtained the values 1.0076, 1.0015 and 0.9971 (kg).

   a) Assume the measurements are normally distributed with variance $\widetilde{\sigma^2}$, where a prior distribution for $\widetilde{\sigma^2}$ is assumed to be Inverse Gamma, i.e. with probability density function

   $$f'(\sigma^2|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}(\sigma^2)^{-(\alpha+1)}e^{-\beta/\sigma^2} \ \ , \ \ \sigma^2 > 0$$

   where the hyperparameters α and β have been assigned to 2 and 10^{-5} respectively. What is the point estimate $\widehat{\sigma^2}$ of the true measurement variance σ^2 using a decision-theoretic approach with loss function

   $$L\left(\widehat{\sigma^2},\sigma^2\right) = \begin{cases} 0 & \left|\widehat{\sigma^2} - \sigma^2\right| \le 2\cdot 10^{-5} \\ 1 & \left|\widehat{\sigma^2} - \sigma^2\right| > 2\cdot 10^{-5} \end{cases}$$

   b) What is the point estimate of the true measurement variance σ^2 using a decision-theoretic approach with the same loss function as in a), but with prior probability density function

   $$f'(\sigma^2) = 0.164/\sigma^2 \ \ , \ \ \sigma^2 > 0$$