LINKÖPINGS UNIVERSITET
Department of Computer and
Information Science
Division of Statistics and Machine learning/ANd

732A66 Decision theory
Fall semester 2021
Assignment 3

Assignment 3

Below are two tasks that you shall try to solve. All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You can help each other but there must be individual submissions (that are not just copies of one submission).

Your solutions should be submitted at latest on **Friday 10 December 2021**.

1. (This is to some parts Exercises 12,13 and 14 in Chapter 5 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed.")
   One non-probabilistic decision-making criterion involves the consideration of a weighted average of the highest and lowest payoffs for each action. The weights, which must sum to 1, can be thought of as an optimism-pessimism index. The action with the highest weighted average of the highest and lowest payoffs is the action chosen by this criterion.

   a) Comment on this decision-making criterion and use it for payoff table (i) below with the highest payoff in each row receiving a weight of 0.4 and the lowest payoff receiving a weight of 0.6

Payoff table (i)

Action	State of the world				
	A	*B*	*C*	*D*	*E*
1	–50	80	20	100	0
2	30	40	70	20	50
3	10	30	–30	10	40
4	–10	–50	–70	–20	200

   b) Consider payoff table (i) above. Assume the utility function of a person is $U(R) = \log(R+71)$, where R is the payoff (and log is the natural logarithm with base e). Moreover, assume that person's prior probabilities for the five states of nature are $P(\text{A}) = 0.10$; $P(\text{B}) = 0.20$; $P(\text{C}) = 0.25$; $P(\text{D}) = 0.10$ and $P(\text{E}) = 0.35$. Find the optimal action for this person according to the *EU* criterion.

2. (This task is constructed from Exercises 18 and 29 in Chapter 6 of "Winkler: An Introduction to Bayesian inference and decision, 2nd ed.")

   Consider a big box filled with an enormous amount of poker chips. You know that either 70% of the chips are red and the remainder blue, or 70% are blue and the remainder red. You must guess whether the big box has 70% red / 30% blue or 70% blue / 30% red. If you guess correctly, you win US$5. If you guess incorrectly, you lose US$3. Your prior probability that the big box contains 70% red / 30% blue is 0.40, and you are risk neutral in your decision making (i.e. your utility is linear in money).

a) If you could purchase sample information in the form of one draw of a chip from the big box, how much should you be willing to pay for it?

Assume now that the cost of sampling is US$0.25 (i.e. 25 US cents) per draw.

b) What is the ENGS for a sample of 10 chips using a single-stage sampling plan.