LINKÖPINGS UNIVERSITET
Department of Computer and
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Division of Statistics and Machine learning/ANd

732A66 Decision theory
Fall semester 2024
Assignment 2

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Below are three tasks that you shall try to solve. All questions put should be answered. Prepare your solutions in a nice format that can be easily read. You are allowed to help each other but there must be individual submissions (that are not just copies of one submission).

You may use software to do the calculations and report accordingly, but you may not replace calculations by simulation.

Your solutions should be submitted at latest on **Friday 8 November 2024**.

1. One decision-making criterion for decision-making under ignorance (non-probabilistic criterion) involves the consideration of a weighted average of the highest and lowest payoffs for each action. The weights, which must sum to 1, can be thought of as an optimism-pessimism index. The action with the highest weighted average of the highest and lowest payoffs is the action chosen by this criterion.
Comment on this decision-making criterion and use it (i.e. find the optimal action) for payoff table (i) below with the highest payoff in each row receiving a weight of 0.4 and the lowest payoff receiving a weight of 0.6

Payoff table (i)

Action	State of the world				
	A	*B*	*C*	*D*	*E*
1	–50	80	20	100	0
2	30	40	70	20	50
3	10	30	–30	10	40
4	–10	–50	–70	–20	200

2. a) Find the optimal action for payoff table (ii) below using the decision-making criterion described in task 1, with the highest payoff in each row receiving a weight of 0.8 and the lowest payoff receiving a weight of 0.2.
   b) For payoff table (ii) the *ER* criterion (maximising the expected payoff) would also involve a weighted average of the two payoffs in each row. Assign the probability 0.8 to state I in payoff table (ii) and find the optimal action with the *ER*-criterion.
   c) Compare the outcomes with the two criteria and comment.

Payoff table (ii)

Action	State of the world	
	I	*II*
1	10	4
2	7	9

3. Consider payoff table (i) in task 1. Assume the utility function of a person is $U(R) = \log(R+71)$, where R is the payoff (and log is the natural logarithm with base e). Assign the probabilities $\boldsymbol{p} = (0.10, 0.20, 0.25, 0.10, 0.35)$ to the states vector (A, B, C, D, E) and find the optimal action according to the EU-criterion (maximising expected utility).

4. A bank official is concerned about the rate at which the bank's tellers provide service for their customers. He feels that all of the tellers work at about the same speed, which is either 30, 40 or 50 customers per hour. Furthermore, 40 customers per hour is twice as likely as each of the two other values, which are assumed to be equally likely. In order to obtain more information, the official observes all five tellers for a two-hour period, noting that 380 customers are served during that period.

   Show that the posterior probabilities for the three possible speeds are approximately 0.000045, 0.99996 and 0.00000012 respectively.

   (*Hint:* The total amount of 380 customers cannot be equalized with an average amount of customers per teller and hour. Using formal calculations instead of simulating data, it is easier to confirm the posterior probabilities.)

5. Assume you have decided to bet on a horse race, and that you have very little knowledge about the competing horses. You consider betting on Little Joe, and you see that the odds for this horse are 9 to 1 (i.e. odds against that the horse will win). You decide to look up some historical tracks on how Little Joe has performed recently and note that he has won in 2 of the last 10 races he competed in. You can assume that these races are fairly comparable with respect to the levels of his competitors.

   a) Using the above as your background information, what are your subjective odds for Little Joe?

   b) If you bet, you will obtain 9 times the money you have put in the bet. What is your subjective expected return from betting on Little Joe?